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Production Inventory Model for Deteriorating Items with Shortages and Salvage Value Under Reverse Logistics

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Abstract. In this paper, a production inventory model is developed for the business enterprise which consists of three wings. The first wing is for manufacturing new items, the second wing is for collecting the returned items, while third wing is for remanufacturing the returned item. In this model we consider the fact that the storage item is deteriorated during storage periods and salvage value is incorporated to the deteriorated items. The demand, deterioration, production, remanufacturing and return rates are time dependent. The shortages are allowed and fully backlogged. The model is solved analytically by minimizing the total inventory cost. The model can be applied for optimizing the total inventory cost of deteriorating items inventory under reverse logistic for a business enterprise where demand and deterioration both is function of time.

Keywords: Inventory, Deteriorating items, Shortages, Time dependent Deterioration, Salvage value, Weibull distribution, Time varying holding cost.

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1. Introduction

Inventory of items is a very important part of the logistic system common to all economic sectors such as agriculture, industry, trade and business. Generally inventory is a balance between demand and supply. One of the most unrealistic assumptions in traditional inventory model was that items preserved their physical characteristics while they were kept stored in inventory. However, the deteriorating items are subject to a continuous loss in their masses or utility throughout their life time due to decay, damage, dryness, spoilage and penalty of other reasons. Owing to this fact controlling and maintaining inventory of deteriorating items becomes a challenging problem for decision makers.

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Maintenance of inventories of deteriorating items under reverse logistic is a problem of major concern in the supply chain of almost any business organization.

Collection of used products, as paper, bottle, and battery, is a known idea in modern economies. Reuse, remanufacturing and recycling of cars and electronic appliances, and disposal of hazardous waste are very recent research field. The listed activities include a very broad area, and it seems to have different management problems. Reverse logistics is an extension of logistics, which deals with handling and reuse of reusable used products withdrawn from production and consumption process. Such a reuse is e.g. recycling or repair of spare parts. An environmental conscious materials management and/or logistics can be achieved with reuse. It has an advantage from economic point of view, as reduction of environmental load through return of used items in the manufacturing process, but the exploitation of natural resources can be decreased with this reuse that saves the resources from extreme consumption for the future generation.

The first reverse logistic (repair/reuse/recycling) model was first investigated by Schrady (1967) in an EOQ context. The paper has examined the cost savings of repair of high cost items at the U.S. navy aviation supply office in opposite to procurement. The condition of the basic model is that there are only procurement and several repair batches. Richter, K. (1996) gave the EOQ repair and waste disposal model with variable setup numbers. In this model the author extended to the case of variable setup numbers n and m for production and repair within some collection time interval.

Teunter, R. H. (2001) developed an economic ordering quantities model for remanufacturable items inventory systems. Balkhi, Z. T. (2001) gave an optimal solution on a finite horizon production lot size inventory model for deteriorating items. Brito and Dekker (2002) have given a smooth to the theory of reverse logistics totally, from such three aspects as why people needs reverse logistics, for what products the reverse logistics should be used, and how to carry out reverse logistics. Dobos, I., and Richter, K. (2003, 2004) described a production/recycling model with stationary demand and return rates and an extended production/recycling model with stationary demand and return rates. Tang, O., and Grubbstron, R .W. (2005) Considering stochastic lead times in a manufacturing/remanufacturing system with deterministic demands and returns . Grubbstrm, R. W., and Tang, O. (2006) explain an optimal production opportunities in a remanufacturing system . Jaber, M. Y., Nuwayhid, R. Y., and Rosen, M. A .(2006) gave a thermodynamic approach to modelling the economic order quantity in which they consider some hidden costs that not accounted for when modelling inventory systems.

Konstantaras, I., and Papachristos, S. (2006) gave an inventory model of lot-sizing for a singleproduct recovery system with backordering. In this article, a single-product recovery system is studied. Used products are collected from customers and kept at the recoverable inventory warehouse for future recovery. King, A. M., Burgess, S. C., Ijomah, W., and McMahon, C. A. (2006) describes and compares the four alternative strategies to reducing end-of-life waste within the context of extended producer responsibility: namely repairing, reconditioning, remanufacturing or recycling. It also introduces a more robust definition of remanufacturing, validated by earlier research, which differentiates it from repair and reconditioning.

Jaber, M. Y., and Rosen, M. A. (2008) developed an economic order quantity repair and waste disposal model with entropy cost. In this model they suggest that improvements to production systems may be achievable by applying the first and second laws of thermodynamics to reduce system entropy (or disorder). El Saadany, A. M. A., and Jaber, M. Y. (2008) developed an EOQ repair and waste disposal model with switching costs. Jaber, M. Y., and El Saadany, A. M. A. (2009) gave the production, remanufacture and waste disposal model with lost sales. In this model they considered that the demand for manufactured items is different from that for remanufactured (repaired) ones. Omar, M., and Yeo, I. (2009) describe a model for a production repair system under a time varying demand process. In this study, they consider a production system that satisfies a continuous time-varying demand for a finished product over a known and finite planning horizon by supplying either new products or repaired used products. Liu, N., Kim, Y., and Hwang, H. (2009) gave an optimal operating policy for the production system with rework. In this paper they studied a production inventory system with rework where a stationary demand is satisfied either by production setup with new raw materials or by rework setup with defective items coming from production process. Behret, H., and Korugan, A. (2009) gave performance analysis of a hybrid system under quality impact of returns. In this paper they analyze a hybrid system that meets the demand with remanufactured or new products.

Konstantaras, I., and Skouri, K. (2010) studied about lot sizing for a single product recovery system with variable setup numbers. In this paper a production-remanufacturing inventory system is considered, where the demand can be satisfied by production and remanufacturing. Ahmed, M. A., Saadany, E., and Jaber, M. Y. (2010) gave a production/remanufacturing inventory model with price and quality dependant return rate. Adel A. Alamri (2011) developed theory and methodology on the global optimal solution to a general reverse logistics inventory model for deteriorating items. In this model he presents a unified general inventory model for integrated production of new items and remanufacturing of returned items for an infinite planning horizon.

The motivations for this work came from some reality issues. As we know shortages may occur during any manufacturing and remanufacturing life cycle of the product and a product life cycle, the demand rate at growth and/or ending stage of the product life cycle can be well approximated by a linear demand function, the assumptions that all returned items that are collected in the returned stock facility can be remanufactured, and that newly produced and/or remanufactured items are perfect are nearly unattainable. In fact, the variation of demand and/or product deterioration with time is a quite natural phenomenon. For instance, seasonal variations (e.g., summer, winter), occasions (e.g., new years, festivals) may cause an increase or a decrease in the demand of a certain commodity. Also, the increase of time storage as well as the changes in the environments of storage may also result in an increase or a decrease in the deterioration rate of certain items. Therefore, it is necessary to consider the variation of production, remanufacturing, demand, return, and product deterioration with time and also need to consider the fact that the shortages is to be occurred during production and remanufacturing time period, which may enhance this line of research. In this paper, we made the paper of A. A. Alamri (2011) more realistic by considering the fact that the shortages may occur during manufacturing and remanufacturing cycle and salvage value incorporate to the deteriorated items. The aim of this model is to find an optimal order quantity which minimizes the total inventory cost.

2. Assumptions and Notations

The mathematical model is based on the following assumptions and notations.

2.1 *Assumption*

Demand, deterioration, production, remanufacturing and return rates are time dependent and arbitrary function of time.
Shortages are allowed and completely backlogged.
The salvage value is associated to deteriorated units during the cycle time.
The deteriorated units cannot be repaired or replaced during the period under review.

2.2 *Notations*

$Q_m(t)$, $Q_r(t)$, $Q_R(t)$ are inventory level at time t in the manufacturing stock, remanufacturing stock and returned stock respectively.
$D(t)$, $\theta(t)$, $P_m(t)$, $P_r(t)$, $P_R(t)$, $\beta(t)$ are demand, deterioration, production, remanufacturing return and backlogging rates respectively.
K_m , K_r, are set up cost per cycle of production and remanufacturing stock respectively.
K_R is ordering cost per cycle of return stock.
h_m, h_r, h_R are holding cost per unit per unit time of manufacturing stock, remanufacturing stock and returned stock respectively.
s_m , s_r are per unit manufacturing and remanufacturing cost, which includes the cost components like labor, energy and machinery.
C_m is the per unit material cost.
γ_m, γ_r are shortages cost during manufacturing and remanufacturing cycle.
c_R is the per unit purchase cost of returned item.
$Q_{b,p}$, Qb, r are backordered inventory during production and remanufacturing life cycle.

3. Mathematical Formulation

If $Q(t)$ is the inventory level at any instant of time t then the states of inventory level without shortages and with shortages are governed by the following differential equations and changes in the inventory level is depicted in Figure 1.

$$\frac{dQ_m(t)}{dt} + \theta_m(t)Q_m(t) = P_m(t) - D(t) \qquad (T_0 \leqslant t < T_1),\ Q_m(T_0) = 0 \qquad (1)$$

$$\frac{dQ_m(t)}{dt} + \theta_m(t)Q_m(t) = -D(t) \qquad (T_1 \leqslant t \leqslant T_2),\ Q_m(T_1) = 0 \qquad (2)$$

$$\frac{dQ_m(t)}{dt} = -D(t)\beta(T_3 - t) \qquad (T_2 \leqslant t \leqslant T_3),\ Q_m(T_2) = 0 \qquad (3)$$

$$\frac{dQ_r(t)}{dt} + \theta_r(t)Q_r(t) = P_r(t) - D(t) \qquad (T_3 \leqslant t \leqslant T_4),\ Q_r(T_3) = 0 \qquad (4)$$

$$\frac{dQ_r(t)}{dt} + \theta_r(t)Q_r(t) = -D(t) \qquad (T_4 \leqslant t \leqslant T_5),\ Q_r(T_5) = 0 \qquad (5)$$

$$\frac{dQ_r(t)}{dt} - D(t)\beta(T_6 - t) \qquad (T_5 \leqslant t \leqslant T_6),\ Q_r(T_5) = 0 \qquad (6)$$

$$\frac{dQ_R(t)}{dt} + \theta_R(t)Q_R(t) = R(t) \qquad (T_0 \leqslant t \leqslant T_4),\ Q_R(T_4) = 0 \tag{7}$$

$$\frac{dQ_R(t)}{dt} + \theta_R(t)Q_R(t) = -P_r(t) + R(t) \qquad (T_4 \leqslant t \leqslant T_6),\ Q_R(T_4) = 0 \tag{8}$$

The realization of the inventory level for this inventory model is depicted in Figure 1

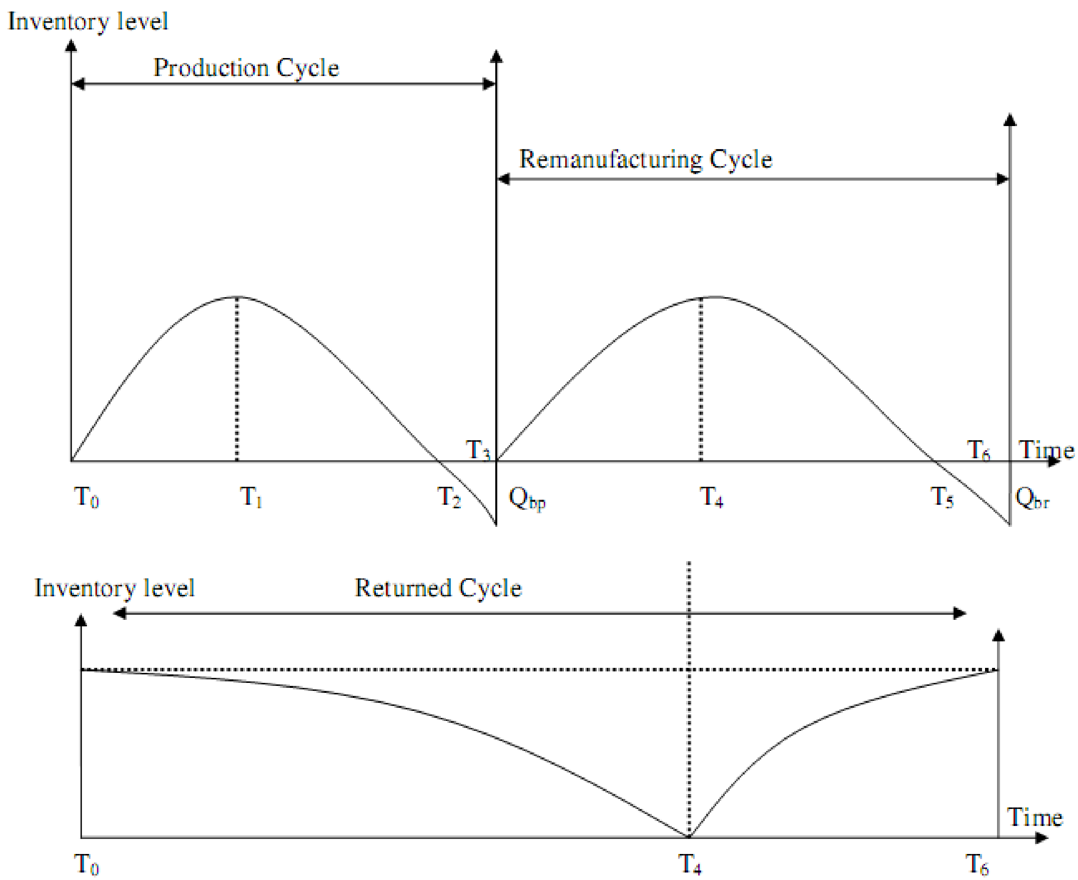

Figure 1. Inventory level for the model.

The solutions of the above differential equations are:

$$Q_m(t) = e^{-g_m(t)} \int_{T_0}^{t} [P_m(u) - D(u)] e^{g_m(u)} du \qquad (T_0 \leqslant t < T_1) \tag{9}$$

$$Q_m(t) = e^{-g_m(t)} \int_{t}^{T_2} D(u) e^{g_m(u)} du \qquad (T_1 \leqslant t \leqslant T_2) \tag{10}$$

$$Q_m(t) = \int_{T_2}^{t} -D(u)\beta(T_3 - u) du \qquad (T_2 \leqslant t \leqslant T_3) \tag{11}$$

$$Q_r(t) = e^{-g_r(t)} \int_{T_3}^{t} [P_r(u) - D(u)] e^{g_r(u)} du \qquad (T_3 \leqslant t \leqslant T_4) \tag{12}$$

$$Q_r(t) = e^{-g_r(t)} \int_{t}^{T_5} D(u) e^{g_r(u)} du \qquad (T_4 \leqslant t \leqslant T_5) \tag{13}$$

$$Q_r(t) = \int_{T_5}^{t} -D(u)\beta(T_6 - u) du \qquad (T_5 \leqslant t \leqslant T_6) \tag{14}$$

$$Q_R(t) = e^{-g_R(t)} \int_{T_0}^{t} R(u) e^{g_R(u)} du \qquad (T_0 \leqslant t < T_4) \tag{15}$$

$$Q_R(t) = e^{-g_R(t)} \int_{t}^{T_6} [P_r(u) - R(u)] e^{g_R(u)} du \qquad (T_4 \leqslant t \leqslant T_6) \tag{16}$$

where $g_x(t) = \int \theta_x(t) dt.$ (17)

The cumulative inventory in the respective class interval is as follows:

$$Q_m(T_0,T_1) = \int_{T_0}^{T_1} e^{-g_m(t)} (\int_{T_0}^{t} [P_m(u) - D(u)] e^{g_m(u)} du) dt \tag{18}$$

$$Q_m(T_1,T_2) = \int_{T_1}^{T_2} e^{-g_m(t)} (\int_{t}^{T_2} D(u) e^{g_m(u)du}) dt \tag{19}$$

$$Q_m(T_2,T_3) = \int_{T_2}^{T_3} (\int_{T_2}^{t} -D(u)\beta(T_3 - u) du) dt \tag{20}$$

$$Q_r(T_3,T_4) = \int_{T_3}^{T_4} e^{-g_r(t)} (\int_{T_3}^{t} [P_r(u) - D(u)] e^{g_r(u)} du) dt \tag{21}$$

$$Q_r(T_4,T_5) = \int_{T_4}^{T_5} e^{-g_r(t)} (\int_{t}^{T_5} D(u) e^{g_r(u)du}) dt \tag{22}$$

$$Q_r(T_5,T_6) = \int_{T_5}^{T_6} \int_{T_5}^{t} -D(u)\beta(T_6 - u) du dt \tag{23}$$

$$Q_R(T_0,T_4) = \int_{T_0}^{T_4} e^{-g_R(t)} (\int_{T_0}^{t} R(u) e^{g_R(u)du}) dt \tag{24}$$

$$Q_R(T_4,T_6) = \int_{T_4}^{T_6} e^{-g_R(t)} (\int_{t}^{T_6} [P_r(u) - R(u) e^{g_R(u)}] du) dt \tag{25}$$

Now using integration by parts, Eqsuations (18)-(25) reduce to

$$Q_m(T_0, T_1) = \int_{T_0}^{T_1} [G_m(T_1) - G_m(u)][P_m(u) - D(u)e^{g_m(u)}]du \tag{26}$$

$$Q_m(T_1, T_2) = \int_{T_1}^{T_2} [G_m(u) - G_m(T_1)][D(u)e^{g_m(u)}]du \tag{27}$$

$$Q_m(T_2, T_3) = \int_{T_2}^{T_3} [T_3 - u][-D(u)\beta(T_3 - u)]du \tag{28}$$

$$Q_r(T_3, T_4) = \int_{T_3}^{T_4} [G_r(T_4) - G_r(u)][P_r(u) - D(u)e^{g_r(u)}]du \tag{29}$$

$$Q_r(T_4, T_5) = \int_{T_4}^{T_5} [G_r(u) - G_r(T_1)][D(u)e^{g_r(u)}]du \tag{30}$$

$$Q_r(T_5, T_6) = \int_{T_5}^{T_6} [T_6 - u][-D(u)\beta(T_6 - u)]du \tag{31}$$

$$Q_R(T_0, T_4) = \int_{T_0}^{T_4} [G_R(T_4) - G_R(u)][R(u)e^{g_R(u)}]du \tag{32}$$

$$Q_R(T_4, T_6) = \int_{T_4}^{T_6} [G_R(u) - G_R(T_4)][P_r(u) - R(u)e^{g_R(u)}]du \tag{33}$$

$$\text{where } G_x(t) = \int e^{-g_x(t)}dt. \tag{34}$$

4. Analytical Solution of the Model

The total cost is the sum of item cost, production cost, deterioration cost, holding cost, shortages cost, remanufacturing cost and salvage values and is given by TC= Holding cost + Production cost + Remanufacturing cost + Shortages cost + Item cost (Salvage value and deterioration cost included in the item cost) + Set up cost. where

$$\text{Holding cost} = h_m(Q_m(T_0,T_1)+Q_m(T_1,T_2)) + h_r(Q_r(T_3,T_4) + Q_r(T_4,T_5)) + h_R(Q_R(T_0,T_4)+Q_R(T_4,T_6))$$

$$\text{Production cost} = s_m \int_{T_0}^{T_1} P - m(u)du$$

$$\text{Remanufacturing cost} = s_r \int_{T_3}^{T_4} P_r(u)du$$

$$\text{Shortages cost} = -\gamma_m Q_m(T_2,T_3) - \gamma_r Q_r(T_5,T_6)$$

$$\text{Item cost} = c_m \int_{T_0}^{T_1} P_m(u)du + c_R \int_{T_0}^{T_6} R(u)du$$

$$\text{Set up cost} = K_r + K_m + K_R$$

Now if we set $T_0 = 0$, then the total cost per unit of time of this inventory system during the cycle $[0,T_6]$, as a function of T_1, T_2, T_3, T_4, T_5, and T_6 say $TC(T_1,T_2,T_3,T_4,T_5,T_6)$ is given by

$$TC(T_1,T_2,T_3,T_4,T_5,T_6) = \frac{1}{T_6} \qquad (35)$$

$$\begin{bmatrix} K_r + K_m + K_R + h_m(\int_0^{T_1}[G_m(T_1)-G_m(u)][P_m(u)-D(u)e^{g_m(u)}]du + \int_{T_1}^{T_2} \\ [G_m(u)-G_m(T_1)][D(u)e^{g_m(u)}]du) + h_r(\int_{T_3}^{T_4}[G_r(T_4)-G_r(u)] \\ [P_r(u)-D(u)e^{g_r(u)}]du + \int_{T_4}^{T_5}[G_r(u)-G_r(T_4)][D(u)e^{g_r(u)}]du) \\ +h_R(\int_0^{T_4}[G_R(T_4)-G_R(u)][R(u)e^{g_R(u)}]du \\ + \int_{T_4}^{T_6}[G_R(u)-G_R(T_4)][P_r(u)-R(u)e^{g_R(u)}]du) + (S_m+C_m)\int_0^{T_1} P_m(u)du \\ +S_r\int_{T_3}^{T_4} P_r(u)du + C_R\int_0^{T_6} R(u)du \\ -\gamma_m\int_{T_2}^{T_3}[T_3-u][-D(u)\beta(T_3-u)]du - \gamma_r\int_{T_5}^{T_6}[T_6-u][-D(u)\beta(T_6-u)]du \end{bmatrix}$$

Where $g_x(u)$ and $G_x(u)$ are given by the equation (17) and (34) respectively.
Our objective is to find the value of T_1, T_2, T_3, T_4, T_5, and T_6 that Minimize $TC(T_1,T_2,T_3,T_4,T_5,T_6)$ given by equation (35). Subject to the following constraints

$$0 = T_0 < T_1 < T_2 < T_3 < T_4 < T_5 < T_6 \qquad (36)$$

$$Q_m(T_0,T_1) = Q_m(T_1,T_2) \text{ at } t = T_1 \qquad (37)$$

$$Q_r(T_3,T_4) = Q_r(T_4,T_5) \text{ at } t = T_4 \qquad (38)$$

$$(Q_R(T_0,T_4) \text{ at } t = 0) = (Q_R(T_4,T_6) \text{ at } t = T_6) \qquad (39)$$

Thus our goal is to solve the above optimization problem which is restated as follows, which we shall call problem p_1

$$P_1 = \begin{bmatrix} \text{Minimize} \quad TC(T_1, T_2, T_3, T_4, T_5, T_6) \\ \text{subject to } (36), q_1 = 0, q_2 = 0, q_3 = 0 \end{bmatrix} \tag{40}$$

where

$$q_1 = \int_0^{T_1} [P_m(u) - D(u)] e^{g_m(u)} du - \int_t^{T_2} D(u) e^{g_m(u)} du \tag{41}$$

$$q_2 = \int_{T_3}^{T_4} [P_r(u) - D(u)] e^{g_r(u)} du - \int_{T_4}^{T_5} D(u) e^{g_r(u)} du \tag{42}$$

$$q_3 = e^{-g_R(u)} \int_{T_0}^{T_4} R(u) e^{g_R(u)} du - e^{-g_R(T_6)} \int_{T_4}^{T_6} [P_r(u) - R(u)] e^{g_R(u)} du \tag{43}$$

If we ignore the monotony constraint of p_1 and state the resulting problem as p_2, then as a result from Kuhn-Tucker necessary condition relation (36) do satisfy a solution of p_2. Hence we conclude that the problem p_1 and p_2 are equivalent.
Suppose the total returned quantity of used item in the interval $[0, T_6]$ is

$$Q = \int_0^{T_6} R(u) du \tag{44}$$

Also suppose that, q be the sum of amount produced and remanufactured in the interval $[0, T_6]$ .i.e.

$$q = q_m + q_r$$

where

$$q_m = \int_0^{T_1} P_m(u) du \tag{45}$$

$$q_r = \int_{T_3}^{T_4} P_r(u) du \tag{46}$$

From equation (44) we note that the T6 can be expressed as function of Q i.e.

$$T_6 = f_6(Q) \tag{47}$$

From (47) and (39), T_4 can also be as function of Q. i.e.

$$T_4 = f_4(Q) \tag{48}$$

Similarly with the help of (36) to (37) and (47) to (48), we obtain T_1, T_2, T_3, and T_5 in term of Q. i.e.

$$T_1 = f_1(Q) \tag{49}$$

$$T_2 = f_2(Q) \tag{50}$$

$$T_3 = f_3(Q) \tag{51}$$

$$T_5 = f_5(Q) \tag{52}$$

Thus, if we substitute the value of T_1, T_2, T_3, T_4, T_5, and T_6 in problem $p1$ then problem p1 will be converted into the following unconstrained problem with the variable Q.

$$W(Q) = \frac{1}{f_6}\left[\begin{array}{c} K_r + K_m + K_R + h_m(\int_0^{f_1}[-G_m(u)][P_m(u) - D(u)e^{g_m(u)}]du + \int_{f_1}^{f_2} \\ [G_m(u)][D(u)e^{g_m(u)}]du) + h_r(\int_{f_3}^{f_4}[-G_r(u)] \\ [P_r(u) - D(u)e^{g_r(u)}]du + \int_{f_4}^{f_5}[G_r(u)][D(u)e^{g_r(u)}]du) \\ +h_R(\int_0^{f_4}[-G_R(u)][R(u)e^{g_R(u)}]du \\ + \int_{f_4}^{f_6}[G_R(u)][P_r(u) - R(u)e^{g_R(u)}]du) + (S_m + C_m)\int_0^{f_1} P_m(u)du \\ +S_r\int_{f_3}^{f_4} P_r(u)du + C_R\int_0^{f_6} R(u)du \\ -\gamma_m\int_{f_2}^{f_3}[f_3 - u][-D(u)\beta(f_3 - u)]du - \gamma_r\int_{f_5}^{f_6}[f_6 - u][-D(u)\beta(f_6 - u)]du \end{array}\right] \tag{53}$$

We will call the above problem as problem ($p3$). Now necessary condition for having a minimum for problem ($p3$) is

$$\frac{dW}{dQ} = 0 \tag{54}$$

To find the solution of (54), suppose $W = \frac{w_Q}{f_6}$ then

$$\frac{dW}{dQ} = \frac{w_Q f_6 - f_6 w_Q}{f_6^2} \tag{55}$$

from (54)

$$w_Q = \frac{w_Q f_6}{f_{6,Q}};\ \text{where } w_Q \text{is obtained from(53)} \tag{56}$$

and

$$W = \frac{w}{f_6} = \frac{w_Q}{f_6} \tag{57}$$

Now from equation (56) , we find the optimal value of Q after that we can obtain the optimal values of T_1, T_2, T_3, T_4, T_5, and T_6 from (47)-(52) and minimum total cost can be determined from (57).

5. Conclusion

In this paper a Production inventory model for deteriorating items with shortages and salvage value under reverse logistics is presented and give analytical solution of the model that minimize the total inventory cost. The model is very practical for the industries where the manufacturing as well as remanufacturing unit is working for new and returned items separately. The proposed model can further be extended by considering multiple production and remanufacturing batches.

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