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Cost and Allocative Efficiency in Buyer-Supplier

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Abstract. The aim of this paper is the evaluation overall performance of buyer-supplier relationships. In some situations the cost of inputs in a buyer-supplier chain are available, on the other hand, achieving a low cost position is necessary for most businesses. And, in a buyer-supplier chain wants to know how assign inputs with the least cost. This paper introduces cost efficiency with same and different costs of inputs. Moreover, cost efficiency not only does it improve profitability, but businesses with a low cost position are better able to drive growth through higher investment in sales, marketing, improve customer service and new product development, as well as more competitive pricing in buyer-supplier. Moreover, in this paper we introduce allocative and technical efficiency of buyer-supplier that based on the Data envelopment analysis (DEA). And show how far the buyer-supplier is from the point of maximum profitability given the existing market prices for inputs and products, and also is achieved when the value consumers place on a good or service equals the cost of the resources used up in production. Finally, we illustrate the proposed models by an example.

Keywords: Buyer-supplier, Data envelopment analysis, Cost and Allocative efficiency.

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1. Introduction

Supply chain management (SCM), which appeared in the early 1990s and is the management of a network of interconnected businesses involved in the ultimate provision of product and service packages required by end consumers [5], offers a way to improve the industrial environment, business administration and become more competitive [8]. Market pressures, organizations (suppliers, manufacturers, distributors and retailers) are forming strategies under various industry initiatives to gain competitive advantage and better service, lower cost and prices. One of the major problems of supply chain management is ignore costs associated with performance evaluation of buyer-supplier, so that, comprehensive supply chain management in todays market requires a command of social and environmental topics and also

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consider the cost in performance evaluation of buyer-supplier. The major reason for absence of performance measurement tools for buyer-supplier lies on the fact that attentions are paid to the tradeoff or harmony among members, rather than the technical efficiency and cost efficiency of the overall buyer-supplier. For example, in a buyer-supplier chain, the outputs for the buyer are usually the inputs of the supplier. The buyer wants to maximize its performance by increasing the outputs and the prices, which usually results increasing the costs and decreasing of the supplier performance, therefore, if buyer and supplier fail to work together to find an optimal balance, then cost, reliability, customer satisfaction, profitability, and continued technology advancements cannot exist in harmony. Recent years have seen a great variety of applications of DEA (Data Envelopment Analysis) for use performance evaluation of supply chain. For example, [3], [6], [10]. But this approach is not always valid in actual business because of variability in the prices and costs that might need to be considered, ([4] and [9]). Therefore, in this paper we will propose another supply chain DEA approach of the "cost, technical and allocative efficiency" which can appear for treatment when information on prices costs is known exactly. Finally, we illustrate the model using a numerical example.

2. Data Envelopment Analysis

Data envelopment analysis (DEA) has been utilized worldwide for measuring efficiencies of banks, companies, electric utilities, transportation systems and so forth, [1]. The Production Possibility Set (PPS) is defined as the set of all inputs and outputs of Decision Making Units (DMUs) and the efficient frontier, also known as production function, expresses the relationship between the inputs utilized and the outputs produced, namely, which inputs can produce outputs.

3. Cost Efficiency of Buyer-Supplier

The most important key to creating a Buyer-Supplier capable of this rapid response and high level of adaptability is integration of effective management capabilities. Regarding this subject, there are two different situations for performance evaluation: one with common unit prices and costs for all Buyer-Suppliers and the other with different prices and costs from Buyer-Supplier to Buyer-Supplier.

3.1 *Common Unit Prices and Costs*

Suppose there is n Buyer-Suppliers (BS) as shown in (Figure. 1) [7]. So that, each BS_j, $(j = 1, 2, \cdots, n)$, has P inputs to the Buyer B_j, $x_{pj}(p = 1, 2, \cdots, P)$, and K outputs from this B_j, $i_{kj}, (k = 1, 2, \cdots, K)$. On the other hand, these K outputs then become the inputs to the Supplier S_j, so that, are referred to as intermediate products. The outputs from the S_j are denoted $y_{qj}(q = 1, 2, \cdots, Q)$, and also $c = (c_1, \cdots, c_p)$ and $z = (z_1, \cdots, z_k)$ are the common unit input-cost vector for buyer and supplier, respectively.

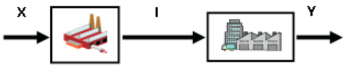

Figure 1. Buyer-Supplier process of supply chain

Therefore, by the structure of the supposed production possibility set T_{BS} as defined by, [9]:

$$\begin{aligned} T_{BS} = \{x_p, y_q)| \ & \sum_{j=1}^{n} \lambda_j^B x_{pj} \leqslant x_p, \ \ p = 1, \cdots, P \\ & \sum_{j=1}^{n} \lambda_j^B x_{kj} \geqslant i_k, \ \ k = 1, 2, \cdots, K. \\ & \sum_{j=1}^{n} \lambda_j^S i_{kj} \leqslant i_k, \ \ k = 1, 2, \cdots, K. \\ & \sum_{j=1}^{n} \lambda_j^S y_{qj} \geqslant y_q, \ \ q = 1, 2, \ldots, Q. \\ & \lambda_j^S, \lambda_j^B \geqslant 0, \ \ j = 1, 2, \cdots, N.\} \end{aligned} \tag{1}$$

Using T_{BS}, we propose the following models to find optimal value of X and I i.e. X^* and I^*:

$$\begin{aligned} & \min \sum_{p=1}^{P} c_p x_{pd} \\ & \text{s.t.} \\ & \quad \sum_{j=1}^{n} \lambda_j^B x_{pj} \leqslant x_{pd}, \ p = 1, \cdots, P \\ & \quad \sum_{j=1}^{n} \lambda_j^B i_{kj} \geqslant i_{kd}, \ k = 1, \cdots, K \\ & \quad \sum_{j=1}^{n} \lambda_j^S i_{kj} \leqslant i_{kd}, \ k = 1, \cdots, K \\ & \quad \sum_{j=1}^{n} \lambda_j^S y_{qj} \geqslant y_{qd}, \ q = 1, \cdots, Q \\ & \quad \lambda_j^B \geqslant 0, \lambda_j^S \geqslant 0, x_{pd} \geqslant 0, i_{kd} \geqslant 0 \end{aligned} \tag{2}$$

And

$$\begin{aligned}
&\min \theta_d \\
&\text{s.t.} \\
&\quad \sum_{j=1}^{n} \lambda_j^B x_{pj} \leqslant \theta_d x_{pd},\ p = 1, \cdots, P \\
&\quad \sum_{j=1}^{n} \lambda_j^B i_{kj} \geqslant i_{kd},\ k = 1, \cdots, K \\
&\quad \sum_{j=1}^{n} \lambda_j^S i_{kj} \leqslant i_{kd},\ k = 1, \cdots, K \\
&\quad \sum_{j=1}^{n} \lambda_j^S y_{qj} \geqslant y_{qd},\ q = 1, \cdots, Q \\
&\quad \lambda_j^B \geqslant 0, \lambda_j^S \geqslant 0, x_{pd} \geqslant 0, i_{kd} \geqslant 0
\end{aligned} \tag{3}$$

So that, model (3) to calculate the technical efficiency of d^{th} Buyer-Supplier, [2]. Therefore, we define cost efficiency and allocative efficiency of Buyer-Supplier chain using models (4) and (5), respectively as follows:

$$\gamma_d^* = \frac{\sum_{p=1}^{P} c_p x_{pd}^*}{\sum_{p=1}^{P} c_p x_{pd}} \tag{4}$$

$$\alpha_d^* = \frac{\gamma_d^*}{\theta_d^*} \tag{5}$$

3.2 *Different Prices and Costs*

The common price and cost assumption is not always valid in performance evaluation of Buyer-Supplier and it is demonstrated that efficiency measures based on this assumption can be misleading [9]. So we have to introduce a new cost-efficiency. Let us define cost-based production possibility set, T_{NEW-BS}. For this propose we as follows:

$$\begin{aligned}
T_{BS} = \{(c'_p x_p, y_q)|\ & \sum_{j=1}^{n} \lambda_j^B c'_{pd} x_{pj} \leqslant x_p, && p = 1, \cdots, P \\
& \sum_{j=1}^{n} \lambda_j^B i_{kj} \geqslant i_{kd}, && k = 1, 2, \cdots, K. \\
& \sum_{j=1}^{n} \lambda_j^S z'_{kj} i_{kj} \leqslant z'_{kd} i_{kd}, && k = 1, 2, \cdots, K. \\
& \sum_{j=1}^{n} \lambda_j^S y_{qj} \geqslant y_q, && q = 1, 2, \cdots, Q. \\
& \lambda_j^S, \lambda_j^B \geqslant 0, && j = 1, 2, \cdots, N.\}
\end{aligned} \tag{6}$$

Based on this production possibility set T_{NEW-BS}, a new technical efficiency measure,θ'_d is obtained for Buyer-Supplier as follows:

$$
\begin{aligned}
&\min \theta'_d \\
&\text{s.t.} \\
&\quad \sum_{j=1}^{n} \lambda_j^B c'_p x_{pj} \leqslant \theta'_d c'_p x_{pd},\ p = 1, \dots, P \\
&\quad \sum_{j=1}^{n} \lambda_j^B i_{kj} \geqslant i_{kd},\ k = 1, \dots, K \\
&\quad \sum_{j=1}^{n} \lambda_j^S z'_k i_{kj} \leqslant z'_k i_{kd},\ k = 1, \dots, K \\
&\quad \sum_{j=1}^{n} \lambda_j^S y_{qj} \leqslant y_{qd},\ q = 1, \cdots, Q \\
&\quad \lambda_j^B \geqslant 0, \lambda_j^S \geqslant 0, x_{pd} \geqslant 0, i_{kd} \geqslant 0
\end{aligned}
\tag{7}
$$

To obtainX^* as optimal value we consider of X the following LP model:

$$
\begin{aligned}
&\min \sum_{p=1}^{P} c'_{pd} x_{pd} \\
&\text{s.t.} \\
&\quad \sum_{j=1}^{n} \lambda_j^B c'_{pj} x_{pj} \leqslant c'_{pd} x_{pd},\ p = 1, \dots, P \\
&\quad \sum_{j=1}^{n} \lambda_j^B i_{kj} \geqslant i_{kd},\ k = 1, \dots, K \\
&\quad \sum_{j=1}^{n} \lambda_j^S z'_{kj} i_{kj} \leqslant z'_{kj} i_{kd},\ k = 1, \dots, K \\
&\quad \sum_{j=1}^{n} \lambda_j^S y_{qj} \leqslant y_{qd},\ q = 1, \dots, Q \\
&\quad \lambda_j^B \geqslant 0, \lambda_j^S \geqslant 0, x_{pd} \geqslant 0, i_{kd} \geqslant 0
\end{aligned}
\tag{8}
$$

Using $x^*$and I^*, the new cost efficiency is defined as:

$$
\gamma'^*_d = \frac{\sum_{p=1}^{P} c'_{pd} x^*_{pd}}{\sum_{p=1}^{P} c'_{pd} x_{pd}} \tag{9}
$$

Therefore, the new allocative efficiency of γ'^*_d Buyer-Supplier is defined as the ratio of to θ'^*_d , i.e.

$$
\alpha'^*_d = \frac{\gamma'^*_d}{\theta'^*_d} \tag{10}
$$

4. Illustrative Example

In this section we apply the proposed approach to appraise the performance of 8 Buyer-Supplier, and the data are shown in Table 1 and 2. So that, Table 1 shows the common unit costs for all Buyer-Suppliers and Table 2 shows the different costs from Buyer-supplier to Buyer-Supplier.

Table 1. Data set (common unit input-cost)

NO.	X_1	C_1	X_2	C_2	X_3	C_3	I_1	Z_1	I_2	Z_2	Y_1	Y_2
BS1	1.017	0.645	1.221	0.432	1.222	0.371	166.976	0.163	8.310	0.724	122.195	3.757
BS2	0.592	0.645	0.611	0.432	1.222	0.371	166.976	0.163	8.310	0.724	122.195	0.660
BS3	0.724	0.645	0.645	0.432	0.606	0.371	48.283	0.163	3.410	0.724	34.412	0.771
BS4	0.515	0.645	0.486	0.432	0.376	0.371	49.917	0.163	2.348	0.724	15.280	0.320
BS5	0.478	0.645	0.526	0.432	0.385	0.371	49.917	0.163	5.461	0.724	34.990	0.843
BS6	0.613	0.645	0.407	0.432	0.341	0.371	23.105	0.163	1.241	0.724	32.578	0.462
BS7	0.791	0.645	0.708	0.432	0.441	0.371	39.459	0.163	1.149	0.724	30.233	0.673
BS8	1.236	0.645	0.713	0.432	0.555	0.371	37.495	0.163	4.083	0.724	20.601	0.486

Table 2. Data set (different input-cost)

NO.	X_1	C_1	X_2	C_2	X_3	C_3	I_1	Z_1	I_2	Z_2	Y_1	Y_2
BS1	1.017	0.446	1.221	0.443	1.222	0.342	166.976	0.985	8.310	0.690	122.195	3.757
BS2	0.592	0.248	0.611	0.638	1.222	0.457	166.976	0.051	1.763	0.724	122.195	0.660
BS3	0.724	0.705	0.645	0.575	0.606	0.404	48.283	0.163	3.410	0.249	34.412	0.771
BS4	0.515	0.645	0.486	0.432	0.376	0.401	49.917	0.168	2.348	0.335	15.280	0.320
BS5	0.478	0.724	0.526	0.510	0.385	0.371	49.917	0.539	5.461	0.342	34.990	0.843
BS6	0.613	0.554	0.407	0.442	0.341	0.356	23.105	0.209	1.241	0.989	32.578	0.462
BS7	0.791	0.336	0.708	0.322	0.441	0.233	39.459	0.124	1.149	0.489	30.233	0.673
BS8	1.236	0.668	0.713	0.423	0.555	0.347	37.495	0.185	4.083	0.177	20.601	0.486

Table 3 and 4 report the technical, cost and allocative efficiency scores of Buyer-Supplier chains, so that, there are two different situations, first with common unit costs for all Buyer-Suppliers and second with different costs for each Buyer-Supplier. On the other hand, Table 3 and 4 reports the cost efficiency is not greater than the technical efficiency.

Table 3. Cost efficiency & Allocative efficiency (common input-cost)

NO.	θ_d^*	γ_d^*	α_d^*	X_1^*	X_2^*	X_3^*	I_1^*	I_2^*
BS1	1.000	1.000	1.000	1.017	1.221	1.222	166.976	8.310
BS2	0.451	0.350	0.777	0.179	0.214	0.215	29.399	1.460
BS3	0.431	0.363	0.842	0.220	0.263	0.261	35.634	1.822
BS4	0.291	0.217	0.746	0.092	0.111	0.1109	14.936	0.769
BS5	0.734	0.562	0.766	0.237	0.284	0.283	38.576	1.959
BS6	0.498	0.333	0.668	0.146	0.174	0.170	23.105	1.241
BS7	0.517	0.315	0.608	0.192	0.230	0.228	31.133	1.593
BS8	0.294	0.168	0.571	0.137	0.164	0.163	22.302	1.135

Cost efficiency has the advantage of allowing for Buyer-Supplier. However, according to the results, the allocative inefficiency is much worse than the more modest technical inefficiency.

Table 4. Cost efficiency & Allocative efficiency (different input-cost)

NO.	$\theta_d'^*$	$\gamma_d'^*$	$\alpha_d'^*$	X_1^*	X_2^*	X_3^*	I_1^*	I_2^*
BS1	0.547	0.446	0.816	0.646	0.504	0.348	41.551	4.546
BS2	0.927	0.562	0.606	0.549	0.254	0.274	50.116	2.494
BS3	0.385	0.346	0.898	0.220	0.248	0.226	34.389	2.585
BS4	0.249	0.198	0.794	0.075	0.120	0.094	14.720	0.848
BS5	0.377	0.377	0.999	0.180	0.198	0.145	18.810	2.058
BS6	0.480	0.309	0.644	0.115	0.171	0.164	23.368	1.163
BS7	0.914	0.533	0.583	0.304	0.378	0.403	37.543	1.868
BS8	0.373	0.241	0.646	0.217	0.266	0.173	20.953	2.292

Table 3 report that BS1 is technical, cost and allocative efficient. The table also reports improve inefficient Buyer-Suppliers. Based on the efficiency scores we can compare 8 Buyer-Suppliers as shown in Figure 2 and 3.

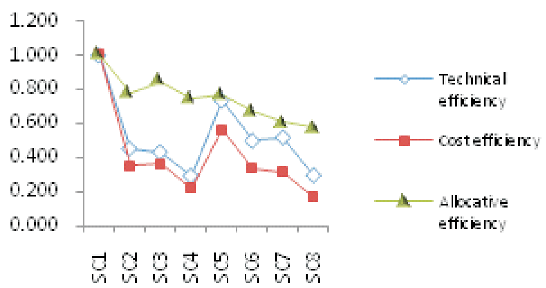

Figure 2. Common costs

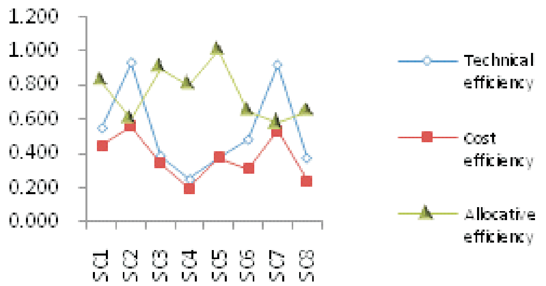

Figure 3. Different costs

We can conclude from the results that:

1. The technical efficiency from our models (Column 2 in Table 3 and 4) is larger than the cost efficiency (Column 3 in Table 3 and 4).
2. Technical efficiency model to determine the production frontier.
3. Cost efficiency model minimum cost solution subject to feasibility.
4. Allocative efficiency refers to the ability to choose optimum input levels for given factor prices. Also allocative efficiency was found to be generally higher in DEA.

5. Conclusions

The central aim of Buyer-Supplier, to have minimal cost and maximal profit. Unfortunately, many times, looking at these business functions in Buyer-Supplier and getting them to work together is the only way to solve complex Buyer-Supplier problems, which ignore the cost and allocative efficiency in performance evaluation of Buyer-Supplier. Therefore, this paper advances cost, allocative and technical efficiency of Buyer-Supplier. So that, cost information are available and also there are two situations, namely, common unit costs and other different costs for all Buyer-Suppliers. Meanwhile, competitive pressures are also driving companies to develop very lean Buyer-Supplier to reduce costs and improve productivity. In addition, the inherent characters of Buyer-Supplier should be paid more attention in the cost efficiency, so that, cost efficiency is defined as a measure of how well the companys resources in a whole Buyer-Supplier field are utilized to achieve its specific goals. Further researches will shed light on the cost efficiency and allocative efficiency of network process.

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