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Analysis of Renewal Input State Dependent Vacation Queue with N-Policy

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Abstract. This paper analyzes renewal input state dependent queue with N- policy wherein the server takes exactly one vacation. Using the supplementary variable technique and recursive method, we derive the steady state system length distributions at various epochs. Various performance measures has been presented. Finally, some numerical computations in the form of graphs are presented to show the parameter effect on various performance measures.

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1. Introduction

In real life queueing situations, it is observed that the server may become unavailable for a certain period of time due to many reasons. This period of server absence from the system is known as vacation. Server vacation queues are utilizable in situations particularly when there is no customer in the queue so that he can stop the regular service and attend some secondary jobs like maintenance works, taking rest, etc. An excellent and comprehensive survey on this topic can be found in [3], [9], etc. In a single vacation (SV) queue the server takes exactly one vacation whenever the system becomes empty. If there are waiting customers at a vacation completion epoch, the server begins to serve them; otherwise he will wait for customers to arrive. A $GI/M/1$ vacation queue has been analyzed in [2].

Queueing systems with N policy are useful to provide basic framework for efficient modeling and analysis of several practical situations. The concept of N policy

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was introduced in [11], which turns the server on when the number of customers in the system reaches a certain number N (≥ 1), and turns the server off when there are less than N in the system. A recursive method for the $GI/M/1/K$ queue with N-policy has been presented in [6] using embedded Markov chain method. A finite buffer $GI/M/1$ queue with SV under N-policy is studied in [7]. A recent study on discrete time $GI/D-MSP/1/K$ queue with N-policy have been discussed in [4] by using supplementary variable technique.

Including various dependencies in a queueing model makes it more practical. The batch service queue with change over times and Bernoulli schedule vacation interruption has been studied by Vijaya Laxmi and Seleshi [10]. Further, it is often observed that arrivals and their service times depend on the system state which is termed as state dependent queues. An iterative algorithm for numerically computing the stationary queue length distributions of $M(k)/G/1/N$ and $GI/M(k)/1/N$ queues has been developed in [12]. A computational algorithm of $GI/M(n)/1/K$ queue with state dependent vacations and N-policy is analyzed in [1]. The stationary distribution and the expected values of the first passage times from one level to other levels are obtained using an algorithm presented in [5].

A state dependent arrival and second optional vacation of $M^X/G/1$ queueing model has been developed in [8]. The present literature shows that so far the analysis of finite buffer $GI/M(n)/1/SV$ queue with N-policy has not been done, to the best of our knowledge. This motivated us to study the finite capacity $GI/M(n)/1/SV$ queue under N-policy.

In this paper, we focus on a finite buffer $GI/M(n)/1/SV$ queue with N-policy. The service times and vacation times are assumed to be exponentially distributed and are state dependent. Using the supplementary variable technique and recursive method, the steady state probabilities at various epochs are obtained. Some performance measures such as blocking probability, the expected queue length, the expected waiting time, etc. have been evaluated. Numerical results have been illustrated in the form of graphs.

2. Description of the Model

Let us consider a $GI/M(n)/1/K$ SV queue with N-policy. The inter-arrival times of successive arrivals are assumed to be independent and identically distributed random variables with cumulative distribution function $A(x)$, probability density function $a(x)$, $x \geq 0$, Laplace-Stiletjes transform (L.-S.T.) $A^*(\theta)$ with mean inter-arrival time $1/\lambda = -A^{*(1)}(0)$, where $h^{(1)}(0)$ denotes the first derivative of $h(\theta)$ evaluated at $\theta = 0$. Customers are served on a first come first served (FCFS) queue discipline. The service rates and vacation times are exponentially distributed with rates μ_n $(1 \leq n \leq K)$ and $\gamma_n(0 \leqslant n \leqslant K)$, respectively, when there are n customers in the system. The mean service rate μ and mean vacation rate γ are given by $\mu = \sum_{n=1}^{K} \mu_n/K,\ \gamma = \sum_{n=0}^{K} \gamma_n/K$. The traffic intensity is given by $\rho = \lambda/\mu$.

Let the state of the system at time t be described by the random variables $N_s(t)$ denoting the system size, $U(t)$ the remaining inter-arrival time for the next arrival and $\zeta(t)$ the state of the server, which is defined as

$$\zeta(t) = \begin{cases} 0, & \text{if the server is on vacation,} \\ 1(2), & \text{if the server is working (dormant).} \end{cases}$$

The joint probabilities are defined as

$$\pi_{n,j}(x,t) = Pr(N_s(t) \leqslant n,\ x \leqslant U(t) \leqslant x + dx,\ \zeta(t) = j),\ x \geqslant 0,\ 0 \leqslant n \leqslant K.$$

At steady state as $t \to \infty$, the above probabilities are given by $\pi_{n,j}(x),\ j = 0, 1, 2$.

2.1 *Steady state equations and solution*

To obtain the system length distributions at arbitrary epoch, we first develop the differential difference equations, using the inter-arrival time as the supplementary variable, as follows.

$$
\begin{aligned}
-\pi_{0,0}^{(1)}(x) &= \mu_1\pi_{1,1}(x) - \gamma_0\pi_{0,0}(x),\\
-\pi_{n,0}^{(1)}(x) &= -\gamma_n\pi_{n,0}(x) + a(x)\pi_{n-1,0}(0),\ 1 \leqslant n \leqslant K-1,\\
-\pi_{K,0}^{(1)}(x) &= -\gamma_K\pi_{K,0}(x) + a(x)\Big(\pi_{K-1,0}(0) + \pi_{K,0}(0)\Big),\\
-\pi_{0,2}^{(1)}(x) &= \gamma_0\pi_{0,0}(x),\\
-\pi_{n,2}^{(1)}(x) &= \gamma_n\pi_{n,0}(x) + a(x)\pi_{n-1,2}(x),\ 1 \leqslant n \leqslant N-1,\\
-\pi_{1,1}^{(1)}(x) &= -\mu_1\pi_{1,1}(x) + \mu_2\pi_{2,1}(x),\\
-\pi_{n,1}^{(1)}(x) &= -\mu_n\pi_{n,1}(x) + \mu_{n+1}\pi_{n+1,1}(x) + a(x)\pi_{n-1,1}(0),\ 2 \leqslant n \leqslant N-1,\\
-\pi_{N,1}^{(1)}(x) &= -\mu_N\pi_{N,1}(x) + \mu_{N+1}\pi_{N+1,1}(x) + \gamma_N\pi_{N,0}(x) + a(x)\pi_{N-1,1}(0)\\
&\quad + a(x)\pi_{N-1,2}(0),\\
-\pi_{n,1}^{(1)}(x) &= -\mu_n\pi_{n,1}(x) + \mu_{n+1}\pi_{n+1,1}(x) + \gamma_n\pi_{n,0}(x) + a(x)\pi_{n-1,1}(0),\\
&\quad N+1 \leqslant n \leqslant K-1,\\
-\pi_{K,1}^{(1)}(x) &= -\mu_K\pi_{K,1}(x) + \gamma_K\pi_{K,0}(x) + a(x)\Big(\pi_{K-1,1}(0) + \pi_{K,1}(0)\Big),
\end{aligned}
$$

where $\pi_{n,j}(0)$ are the respective rates of arrivals. Let $\pi^*_{n,j}(\theta)$ be the L.-S.T. of $\pi_{n,j}(x)$ with $\pi_{n,j} \equiv \pi^*_{n,j}(0)$, where $\pi_{n,j} \equiv \pi^*_{n,j}(0)$, where $\pi_{n,j}$ are the joint probabilities that there are n customers in the system and the server is in state j at an arbitrary epoch. Multiplying the above equations by $e^{-\theta x}$ and integrating with respect to x from 0 to ∞ yields

$$(\gamma_0 - \theta)\pi^*_{0,0}(\theta) = \mu_1\pi^*_{1,1}(\theta) - \pi_{0,0}(0), \tag{1}$$
$$(\gamma_n - \theta)\pi^*_{n,0}(\theta) = A^*(\theta)\pi_{n-1,0}(0) - \pi_{n,0}(0),\ 1 \leqslant n \leqslant K-1, \tag{2}$$
$$(\gamma_K - \theta)\pi^*_{K,0}(\theta) = A^*(\theta)\Big(\pi_{K-1,0}(0) + \pi_{K,0}(0)\Big) - \pi_{K,0}(0), \tag{3}$$
$$-\theta\pi^*_{0,2}(\theta) = \gamma_0\pi^*_{0,0}(\theta) - \pi_{0,2}(0), \tag{4}$$
$$-\theta\pi^*_{n,2}(\theta) = \gamma_n\pi^*_{n,0}(\theta) + A^*(\theta)\pi_{n-1,2}(0) - \pi_{n,2}(0),\ 1 \leqslant n \leqslant N-1, \tag{5}$$
$$(\mu_1 - \theta)\pi^*_{1,1}(\theta) = \mu_2\pi^*_{2,1}(\theta) - \pi_{1,1}(0), \tag{6}$$

(7)

$$(\mu_n-\theta)\pi^*_{n,1}(\theta)=\mu_{n+1}\pi^*_{n+1,1}(\theta)+A^*(\theta)\pi_{n-1,1}(0)-\pi_{n,1}(0),\quad 2\leqslant n\leqslant N-1, \tag{8}$$

$$(\mu_N-\theta)\pi^*_{N,1}(\theta)=\mu_{N+1}\pi^*_{N+1,1}(\theta)+\gamma_N\pi^*_{N,0}(\theta)+A^*(\theta)\pi_{N-1,1}(0)+A^*(\theta)\pi_{N-1,2}(0)-\pi_{N,1}(0), \tag{9}$$

$$(\mu_n-\theta)\pi^*_{n,1}(\theta)=\mu_{n+1}\pi^*_{n+1,1}(\theta)+\gamma_n\pi^*_{n,0}(\theta)+A^*(\theta)\pi_{n-1,1}(0)-\pi_{n,1}(0),\quad N+1\leqslant n\leqslant K-1, \tag{10}$$

$$(\mu_K-\theta)\pi^*_{K,1}(\theta)=\gamma_K\pi^*_{K,0}(\theta)+A^*(\theta)\Big(\pi_{K-1,1}(0)+\pi_{K,1}(0)\Big)-\pi_{K,1}(0). \tag{11}$$

Adding equations (1) - (11), taking limit as $\theta\to 0$ and using the normalization condition $\sum_{n=0}^{K}\pi_{n,0}+\sum_{n=1}^{K}\pi_{n,1}+\sum_{n=0}^{N-1}\pi_{n,2}=1$, we obtain $\sum_{n=0}^{K}\pi_{n,0}(0)+\sum_{n=1}^{K}\pi_{n,1}(0)+\sum_{n=0}^{N-1}\pi_{n,2}(0)=\lambda$. The left hand side denotes the mean number of entrances into the system per unit time and is equal to mean arrival rate λ.

2.2 *Derivation of rate probabilities $\pi_{n,j}(0)$ and $\pi_{n,2}(0)$*

To obtain the steady state distribution of number of customers in the system at pre-arrival epochs, we first evaluate the rate probabilities $\pi_{n,j}(0),\ j\le n\le K; j=0,1$ and $\pi_{n,2}(0),\ 0\le n\le N-1$.
Substituting $\theta=\gamma_K$ in (3), we get

$$\pi_{K-1,0}(0)=\left(\frac{1-A^*(\gamma_K)}{A^*(\gamma_K)}\right)\pi_{K,0}(0)$$

From (3), we have

$$\pi^*_{K,0}(\theta)=\frac{A^*(\theta)-A^*(\gamma_K)}{(\gamma_K-\theta)A^*(\gamma_K)}\pi_{K,0}(0)$$

Substituting $\theta=\gamma_n$ for $n=K-1,\dots,1$ in (2), we get

$$\pi_{n-1,0}(0)=\frac{\pi_{n,0}(0)}{A^*(\gamma_n)},\ n=K-1,\dots,1.$$

From (2), we obtain

$$\pi^*_{n,0}(\theta)=\frac{A^*(\theta)\pi_{n-1,0}(0)-\pi_{n,0}(0)}{(\gamma_n-\theta)},\quad n=K-1,\dots,1.$$

Substituting $\theta = \mu_n$ $(2 \le n \le K-1)$ in (11)–(6) we get

$$\pi_{K-1,1}(0) = \frac{1 - A^*(\mu_K)}{A^*(\mu_K)}\pi_{K,1}(0) - \frac{\gamma_K}{A^*(\mu_K)}\pi_{K,0}(0),$$

$$\pi_{n-1,1}(0) = \frac{\pi_{n,1}(0)}{A^*(\mu_n)} - \frac{\mu_{n+1}\pi^*_{n+1,1}(\mu_n)}{A^*(\mu_n)} - \frac{\gamma_n\pi^*_{n,0}(\mu_n)}{A^*(\mu_n)}, \quad n = K-1, \ldots, N+1,$$

$$\pi_{N-1,1}(0) = \frac{\pi_{N,1}(0)}{A^*(\mu_N)} - \frac{\mu_{N+1}\pi^*_{N+1,1}(\mu_N)}{A^*(\mu_N)} - \frac{\gamma_N\pi^*_{N,0}(\mu_N)}{A^*(\mu_N)} - \pi_{N-1,2}(0),$$

$$\pi_{n-1,1}(0) = \frac{\pi_{n,1}(0)}{A^*(\mu_n)} - \frac{\mu_{n+1}\pi^*_{n+1,1}(\mu_n)}{A^*(\mu_n)}, \quad n = N-1, \ldots, 2,$$

where $\pi^*_{n,1}(\theta)$ are given by the following:

$$\pi^*_{K,1}(\theta) = \frac{\gamma_K\pi^*_{K,0}(\theta) + A^*(\theta)(\pi_{K-1,1}(0) + \pi_{K,1}(0)) - \pi_{K,1}(0)}{(\mu_K - \theta)},$$

$$\pi^*_{n,1}(\theta) = \frac{\gamma_n\pi^*_{n,0}(\theta) + \mu_{n+1}\pi^*_{n+1,1}(\theta) + A^*(\theta)\pi_{n-1,1}(0) - \pi_{n,1}(0)}{(\mu_n - \theta)},$$
$$n = K-1, \ldots, N+1,$$

$$\pi^*_{N,1}(\theta) = \frac{\gamma_N\pi^*_{N,0}(\theta) + \mu_{N+1}\pi^*_{N+1,1}(\theta) + A^*(\theta)\pi_{N-1,1}(0)}{(\mu_N - \theta)}$$
$$\frac{+A^*(\theta)\pi_{N-1,2}(0) - \pi_{N,1}(0)}{(\mu_N - \theta)},$$

$$\pi^*_{n,1}(\theta) = \frac{\mu_{n+1}\pi^*_{n+1,1}(\theta) + A^*(\theta)\pi_{n-1,1}(0) - \pi_{n,1}(0)}{(\mu_n - \theta)}, \ n = K-1, \ldots, 2,$$

For $\theta = \gamma_n$ $(0 \le n \le K)$, $\pi^*_{n,0}(\theta)$ are given by

$$\pi^*_{K,0}(\theta) = -A^{*(1)}(\theta)\left(\pi_{K-1,0}(0) + \pi_{K,0}(0)\right),$$
$$\pi^*_{n,0}(\theta) = -A^{*(1)}(\theta)\pi_{n-1,0}(0), \quad 1 \le n \le K-1.$$

For $\theta = \mu_n$ $(1 \le n \le K)$, $\pi^*_{n,1}(\theta)$ are given by

$$\pi^*_{K,1}(\theta) = -\left(\gamma_K\pi^{*(1)}_{K,0}(\theta) + A^{*(1)}(\theta)(\pi_{K-1,1}(0) + \pi_{K,1}(0))\right),$$

$$\pi^*_{n,1}(\theta) = -\left(\gamma_n\pi^{*(1)}_{n,0}(\theta) + \mu_{n+1}\pi^{*(1)}_{n+1,1}(\theta) + A^{*(1)}(\theta)\pi_{n-1,1}(0)\right),$$
$$N+1 \le n \le K-1,$$

$$\pi^*_{N,1}(\theta) = -\Bigg(\gamma_N\pi^{*(1)}_{N,0}(\theta) + \mu_{N+1}\pi^{*(1)}_{N+1,1}(\theta)$$
$$+A^{*(1)}(\theta)\pi_{N-1,2}(0) + A^{*(1)}(\theta)\pi_{N-1,1}(0)\Bigg),$$

$$\pi^*_{n,1}(\theta) = -\left(\mu_{n+1}\pi^{*(1)}_{n+1,1}(\theta) + A^{*(1)}(\theta)\pi_{n-1,1}(0)\right), \ 2 \le n \le N-1.$$

For $\theta = 0$, from (5) we obtain $\pi^{*}_{n,2}(\theta) = (\pi_{n-1,2}(0) - \gamma_n \pi^{*(1)}_{n,0}(\theta))/\lambda,\ 1 \le n \le N-1.$

2.3 *Relation between steady state distribution at pre-arrival and arbitrary epochs*

Let $\pi^{-}_{n,j}, j \le n \le K, j = 0,1;\ \pi^{-}_{n,2}, 0 \le n \le N-1$ denote the pre-arrival epoch probabilities, that is, an arrival finds n customers in the system and the server is in state j at an arrival epoch. Applying Bayes' theorem, we have

$$\pi^{-}_{n,j} = \pi_{n,j}(0)/\lambda, j \le n \le K, j = 0,1;\ \pi^{-}_{n,2} = \pi_{n,2}(0))/\lambda, 0 \le n \le N-1. \quad (12)$$

Setting $\theta = 0$ in (3), (2), (11) - (6) and using (12), the relations between pre-arrival and arbitrary epoch probabilities are obtained as

$$\pi_{0,0} = \frac{\lambda}{\mu_1}\left[\pi^{-}_{N,0} + \pi^{-}_{K,1} + \pi^{-}_{N-1,1} + \pi^{-}_{0,0} + \sum_{n=2}^{N-2}\pi^{-}_{n,0} - \sum_{n=1}^{N-3}\pi^{-}_{n,1}\right],$$

$$\pi_{n,0} = \frac{\lambda}{\gamma_n}\left[\pi^{-}_{n-1,0} - \pi^{-}_{n,0}\right], \quad 1 \leqslant n \leqslant K-1,$$

$$\pi_{K,0} = \left(\frac{\lambda}{\gamma_K}\right)\pi^{-}_{K-1,0},$$

$$\pi_{1,1} = \frac{\lambda}{\mu_1}\left[\pi^{-}_{N,0} + \pi^{-}_{K,1} + \pi^{-}_{N-1,1} + \sum_{n=3}^{N-2}\pi^{-}_{n,0} - \sum_{n=1}^{N-3}\pi^{-}_{n,1}\right]$$

$$\pi_{n,1} = \frac{\lambda}{\mu_n}\left[\pi^{-}_{N,0} + \pi^{-}_{K,1} - \pi^{-}_{N-1,1} + \sum_{n=n+1}^{N-2}\pi^{-}_{n,0} - \sum_{n=n}^{N-3}\pi^{-}_{n,1}\right],\ 2 \le n \le N-3,$$

$$\pi_{N-2,1} = \frac{\lambda}{\mu_{N-2}}\left[\pi^{-}_{N,0} + \pi^{-}_{K,1} - \pi^{-}_{N-1,1}\right],$$

$$\pi_{N-1,1} = \frac{\lambda}{\mu_{N-1}}\left[\pi^{-}_{N,0} + \pi^{-}_{K,1} + \pi^{-}_{N-1,1} - \pi^{-}_{N-2,1}\right],$$

$$\pi_{N,1} = \frac{\lambda}{\mu_N}\left[\pi^{-}_{N-1,0} + \pi^{-}_{N-1,1} - \pi^{-}_{K-2,1} + \pi^{-}_{K,1} + \pi^{-}_{N-1,2}\right],$$

$$\pi_{n,1} = \frac{\lambda}{\mu_n}\left[\pi^{-}_{n-1,0} + \pi^{-}_{n-1,1} - \pi^{-}_{K-2,1} + \pi^{-}_{K,1})\right],\ N+1 \le n \le K-1,$$

$$\pi_{K,1} = \frac{\lambda}{\mu_K}\left[\pi^{-}_{K-1,0} + \pi^{-}_{K-1,1}\right].$$

Differentiating (5) and setting $\theta = 0$, we obtain $\pi_{n,2} = \pi^{-}_{n-1,2} - \gamma_n \pi^{*(1)}_{n,0}(0), 1 \le n \le N-1$. Finally using the normalization condition, we have $\pi_{0,2} = 1 - \sum_{n=0}^{K}\pi_{n,0} - \sum_{n=1}^{K}\pi_{n,1} - \sum_{n=1}^{N-1}\pi_{n,2}$.

Table 1. The mean queue length and blocking probability for different values of K and λ in the single vacation $E_5/M(n)/1/K$ system. $N = 3$.

	$K=5$		$K=10$		$K=15$	
λ	L_q	P_{loss}	L_q	P_{loss}	L_q	P_{loss}
2.0	2.97929	0.28151	5.58354	0.05726	6.48208	0.00540
2.1	3.05245	0.30598	5.93068	0.07550	7.16482	0.01002
2.2	3.11753	0.32940	6.24198	0.09556	7.84477	0.01703
2.3	3.17558	0.35173	6.51762	0.11691	8.50055	0.02676
2.4	3.22751	0.37298	6.75960	0.13904	9.11369	0.03925
2.5	3.27409	0.39316	6.97092	0.16150	9.67141	0.05429

3. Performance Measures

Once the state probabilities at various epochs are known, one can evaluate various performance measures of the model. The average queue L_q, blocking probability P_{loss}, the average waiting time in the queue W_q of a customer using Little's rule can be obtained as:

$$L_q = \sum_{n=1}^{K}(n-1)\pi_{n,0} + \sum_{n=1}^{K}(n-1)\pi_{n,1} + \sum_{n=1}^{N-1}(n-1)\pi_{n,2};$$

$$P_{loss} = \pi^{-}_{K,0} + \pi^{-}_{K,1}; \quad W_q = L_q/\hat{\lambda},$$

where $\hat{\lambda} = \lambda(1 - P_{loss})$ is the effective arrival rate.

4. Numerical Results

In this section, we study the effect of model parameters on the system performance measures. The capacity of the system is fixed at $K = 10$ and the threshold value at $N = 5$. The traffic intensity is taken as $\rho = 0.5$ and for $1 \leq n \leq N$ the various parameters of the model are assumed to be $\mu_n = ln[n + 0.4]$, $\gamma_n = ln[n + 0.1]$ and $\gamma_0 = 0.1$ with means $\mu = 1.61722$, $\gamma = 1.40818$, respectively. For HE_2 distribution we have taken $\lambda_1 = 0.385845$, $\lambda_2 = 3.0$, $\sigma_1 = 0.4$ and $\sigma_2 = 0.6$.

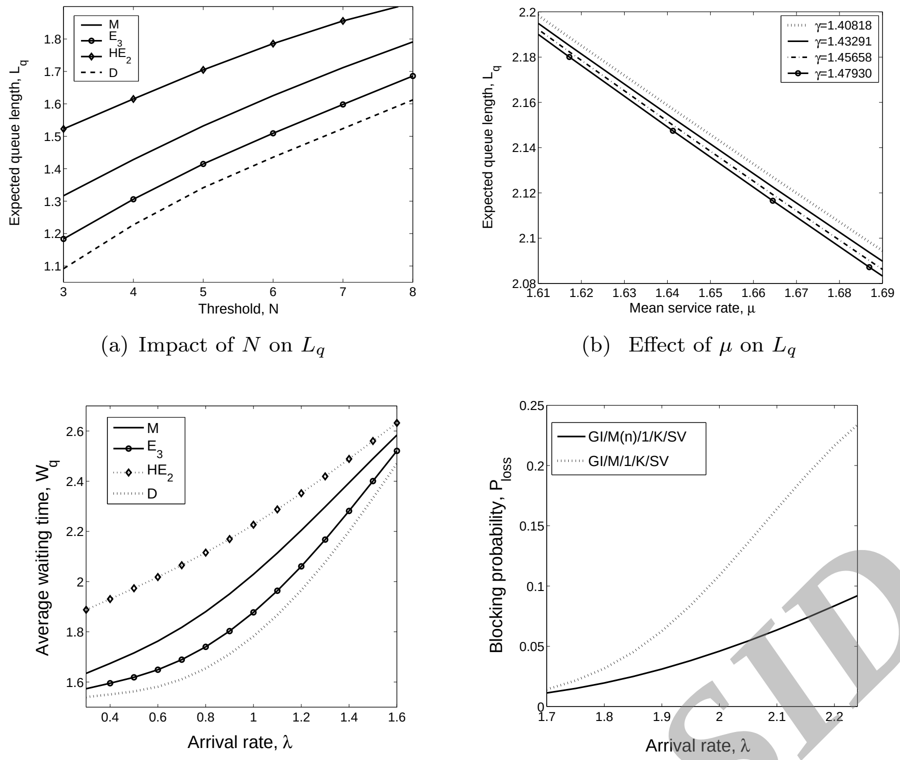
Table 1 shows the effect of buffer content K and the mean arrival rate λ on the various performance measures. The inter-arrival distribution and the threshold values are assumed to be E_5 and $N = 3$, respectively. One can observe that L_q and P_{loss} increase as λ increases. Furthermore, as K increases L_q increases and the loss probability decreases, this fact being common in practical experience.

Figure (a) presents the effect of N on expected queue length L_q for various inter-arrival time distributions. From the figure it can be observed that as N increases, L_q increases. Further, for fixed N, HE_2 distribution yields highest queue lengths whereas deterministic distribution yields the lowest.

Figure (b) demonstrates the effect of mean service rate μ on the expected queue length L_q for various values of γ when the inter-arrival times follow HE_2 distribution. For fixed γ, the expected queue length decreases with the increase of μ. Moreover, L_q decreases with the increase of mean vacation rate γ.

The impact of arrival rate λ on W_q for various inter-arrival time distributions is presented in Figure (c). It can be observed that as λ increases, the expected queue lengths increase, as intuitively expected. Further, for a fixed λ, among all the distributions considered, deterministic distribution yields the least queue lengths whereas HE_2 distribution the highest queue lengths.

The effect of λ on P_{loss} with state dependent and constant service rate models is demonstrated in Figure (d) when the inter-arrival times are assumed to be de-



(a) Impact of N on L_q

(b) Effect of μ on L_q

(c) Effect of λ on W_q

(d) Effect of λ on P_{loss}

terministic. One may observe that the blocking probability in case of models with state dependent services are lower when compared to models with constant service rates. Hence, for better utilization of the server, one may consider queueing models with state dependent services which reduces the blocking of customers effectively.

5. Conclusions

This paper presents a $GI/M(n)/1/K/$SV queue with N-policy. The inter-arrival times of customers are arbitrarily distributed while the service rates and vacation rates are exponentially distributed. A recursive method has been developed to obtain the steady state system length distributions at pre-arrival and arbitrary epochs. Various performance measures of the model have been presented. Computational experiences are demonstrated with a variety of numerical results in the form of graphs. The recursive method developed in this paper is easy to implement and can be adopted to analyze more complex models such as $GI^{[X]}/M(n)/1/K$ working vacation queue with N-policy, $GI^{[X]}/M(n)/1/K$ queue with Bernoulli scheduled vacation interruption, etc., which are left for future investigation.

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