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A Taylor Series Approach for Solving Linear Fractional Decentralized Bi-Level Multi-Objective Decision-Making Under Fuzziness

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Abstract. This paper presents a Taylor series approach for solving linear fractional decentralized bi-level multi-objective decision-making (LFDBL-MODM) problems with a single decision maker at the upper level and multiple decision makers at the lower level. In the proposed approach, the membership functions associated with each objective(s) of the level(s) of LFDBL-MODM are transformed to a linear form by using a Taylor series and then they are unified. On using the Kuhn-Tucker conditions, the problem is finally reduced to a single objective. Numerical example is given in order to illustrate the efficiency and superiority of the proposed approach.

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Index to information contained in this paper

1 **Introduction**
2 **Problem Formulation**
3 **Fuzzy Decision Models for LFDBL-MODM Problem**
4 **The Taylor Series for Solving LFDBL-MODM Problem**
5 **Numerical Example**
6 **Conclusion**

1. Introduction

Bilevel decentralized decision-making(BLDDM)problems with single decision maker at the upper level(ULDM) and multiple decision makers at the lower levels(LLDMs) are frequently encountered in hierarchical organization of large companies such as government, agencies, profit or non-profit organizations, manufacturing plants, and logistic companies. Solution technique is explicitly assigning to each decision maker(DM), a unique objective, a set of decision variables, and a set

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of common constraints which affect all DMs. Each unit or department independently seeks its own interest, but it is affected by the actions of the other units [1]. Most of the developments in BLDDM focus on single(linear, linear fractional, or non-linear)objective programming problems to each DM at the upper and the lower levels [1, 2, 4, 7, 9, 10, 14]. In a hierarchical DM context, it has been realized that each DM should have a motivation to cooperate with the others, and a minimum level of satisfaction of the DMs at a lower level must be considered for the overall benefit of the organization. The use of the concept of membership function of fuzzy set theory to multi-level programming problems for satisfactory decisions was first introduced by Lai [5]. There after, Lai satisfactory solution concept was extended by Shih et al [13] with a supervised search procedure with the use of maxmin operator was stuied by Bellman and Zadeh [3]. Abo- Sinna extended the fuzzy approach for multi-level programming problems of Shih et al in [13] for solving bi-level and three-level non-linear multi-objective programming problems. The basic concept of these fuzzy programming(FP)approaches is the same as that of each lower level DMs optimizes his/her objective function, taking a goal or preference of the first level DMs into consideration. In the decision process by considering the membership functions of the fuzzy goals for the decision variables of all the DMs,we solve a FP problem with a set of an overall satisfactory degree to any of the upper levels. If the proposed solution is not satisfactory to any upper levels, the solution search is continued by redefining the elicited membership functions until a satisfactory solution is reached [7].

In this paper, we consider LFDBL-MODM problems in which there are a single DM at the upper level and two or more DMs at the lower level, and objective functions of the DMs and constraint functions are linear functions. The membership functions, which are associated with each objective(s) of the level(s) LFDBL-MODM problems are transformed to linear form by using first-order Taylor polynomial series. Here, the obtained Taylor series which has polynomial membership functions is equivalent to fractional membership functions which is associated to each objective(s) of each level(s), which reduce the LFDBL-MODM problem into a single objective.

In other words, suitable transformation can be applied to formulate an equivalent LFDBL-MODM problem. The performance of the proposed method will be experimentally validated by example.

2. Problem Formulation

Assume that there are two levels in a hierarchy structure with ULDM or DM_0 and p DMs at the $LLDM_i$ or DM_i , $i = 1, 2, \ldots, p$. Let the vector of decision variables $x = (x_0, x_1, \ldots, x_p) \in R^n$ be partitioned between the upper and lower DMs. The ULDM has control over the vector $x_0 \in R^{n_0}$, and $LLDM_k$, $k = 1, 2, \ldots, p$, has control over the vector $x_k \in R^{n_k}$, where $n = n_0 + n_1 + \cdots + n_p,\ x_k = (x_{k1}, x_{k2}, \ldots, x_{kn_k}), k = 0, 1, \ldots, p$. Furthermore,assume that

$$F_i(x_0, x_1, \ldots, x_p) = F_i(x) : R^{n_0} \times R^{n_1} \times \cdots \times R^{n_p} \longrightarrow R^{m_i}, \quad i = 0, 1, 2 \ldots, p$$

is the vector of objective functions to the $DM_i, i = 0, 1, \ldots, p$. So the LFDBL-MODM problem of maximization type may be formulated as follows [10]:

(upper level)

$$\underset{x_0}{\mathrm{Max}}\ F_0(x) = \underset{x_0}{\mathrm{Max}}\ (f_{01}(x), f_{02}(x), \ldots, f_{0m_0}(x)) \qquad [DM_0]$$

where $x_1, x_2, \ldots, x_p$ solves

(lower level)

$$\underset{x_1}{\text{Max}}\ F_1(x) = \underset{x_1}{\text{Max}}\ (f_{11}(x), f_{12}(x), \ldots, f_{1m_1}(x)) \qquad [DM_1] \tag{1}$$

$$\underset{x_2}{\text{Max}}\ F_2(x) = \underset{x_2}{\text{Max}}\ (f_{21}(x), f_{22}(x), \ldots, f_{2m_2}(x)) \qquad [DM_2]$$

$$\vdots$$

$$\underset{x_p}{\text{Max}}\ F_p(x) = \underset{x_p}{\text{Max}}\ (f_{p1}(x), f_{p2}(x), \ldots, f_{pm_p}(x)) \qquad [DM_p]$$

subject to

$$x \in G = \left\{ x \in R^n \mid A_0x_0 + A_1x_1 + \cdots + A_px_p \leqslant b, x \geqslant 0, b \in R^m \right\} \neq \varnothing$$

Where

$$f_{ij}(x) = \frac{c_{ij}x + \alpha_{ij}}{d_{ij}x + \beta_{ij}}$$

for i= 0, we have j = 1, 2, . . . , m_0, for ULDM objective functions,
for i=1,2,. . . ,p , we have j = 1, 2, . . . , m_i, for LLDM objective functions, where $m_i, i = 1, 2, \ldots, p$ is the number of DM_i s LLDM objective functions, m is the number of the constraints, A_i is the coefficients of matrices of size $m \times n_i$, $c_{ij}, d_{ij} \in R^n, d_{ij}x + \beta_{ij} > 0$ for all $x \in G$ and α_{ij}, β_{ij} are constants(for i=0, 1,. . . , p & j = 1, 2, . . . , m_i).

Definition 1 For any $x_0(x_0 \in G_0 = \{x_0|(x_0, x_1, \ldots, x_p) \in G\})$ given by ULDM, if the decision-making variables $x_k(x_k \in G_k = \{x_k|(x_0, x_1, \ldots, x_p) \in G\})$ is the Pareto optimal solution of the LLDM$_k$, $k = 1, 2, \ldots, p$, then $(x_0, x_1, \ldots, x_p)$ is a feasible solution of LFDBL-MODM problem [6].

Definition 2 If $(x_0^*, x_1^*, \ldots, x_p^*)$ is a feasible solution of the LFDBL-MODM problem; no other feasible solution $(x_0, x_1, \ldots, x_p) \in G$ exists, such that $f_{0j}(x_0^*, x_1^*, \ldots, x_p^*) \leqslant f_{0j}(x_0, x_1, \ldots, x_p)$; so $(x_0^*, x_1^*, \ldots, x_p^*)$, $j = 1, 2, \ldots, m_0$ is the Pareto optimal solution of the LFDBL-MODM problem [6].

3. Fuzzy Decision Models for LFDBL-MODM Problem

To solve the LFDBL-MODM by adopting, the leader-follower Stakelberg and the well-known fuzzy decision model of Sakawa [11, 12] each of the objectives in each level to build membership function, goals and tolerances should be determined first. However, they could hardly be determined without meaningful supporting data. We should first find the individual best solution (f_{ij}^+) and individual worst solution (f_{ij}^-) , where

$$f_{ij}^+ = \text{Max} \quad f_{ij}(x)\ , \qquad\qquad f_{ij}^- = \text{Min} \quad f_{ij}(x)$$
$$s.t \qquad\qquad\qquad\qquad s.t \tag{2}$$
$$x \in G \qquad\qquad\qquad\qquad x \in G$$

Goals and tolerances can then be reasonably set for individual solution and the difference of the best and worst solution, respectively. This data can then be formulated as the following membership function of fuzzy set theory [6]:

$$\mu_{f_{ij}(x)} f_{ij}(x) = \begin{cases} 1 & f_{ij}(x) \geqslant f_{ij}^+ \\ \dfrac{f_{ij}(x) - f_{ij}^-}{f_{ij}^+ - f_{ij}^-} & f_{ij}^- \leqslant f_{ij}(x) \leqslant f_{ij}^+ \\ 0 & f_{ij}(x) \leqslant f_{ij}^- \end{cases} \tag{3}$$

4. The Taylor Series for Solving LFDBL-MODM Problem

In the LFDBL-MODM problems, membership functions associated to each of the objectives in each level are firstly transformed by using Taylor series and then a satisfactory value(s) for the variable(s) of the model is obtained by solving the fuzzy model, which has a single objective function. Here, the fractional linear membership functions from each objectives of each levels is converted to a linear polynomial on using Taylor series . Then, the LFDBL-MODM on using Kuhn-Tucker conditions can be reduced to a single objective. The proposed approach can be explained as following in four steps.

Step 1. Determine $x_{ij}^* = (x_{ij}^{0*}, x_{ij}^{1*}, \ldots, x_{ij}^{p*}), (i = 0, 1, \ldots, p \& j = 1, 2, \ldots, m_i)$ which is the value(s) that is used to maximize each of the objectives in upper level and lower level membership Function $\mu_{f_{ij}}(x)$ associated to the upper level and lower level $f_{ij}(x_1, x_2)(i = 0, 1, \ldots, p \& j = 1, 2, \ldots, m_i)$ respectively, where n is the number of the variables.

Step 2. Transform membership functions by using first-order Taylor polynomial series

$$\mu_{f_{ij}}(f_{ij}(x)) \cong \widehat{\mu_{f_{ij}}}(f_{ij}(x)) = \mu_{f_{ij}}(f_{ij}(x_{ij}^*)) + \Big((x_0 - x_{ij}^{0*})\frac{\partial}{\partial x_0} + (x_1 - x_{ij}^{1*})\frac{\partial}{\partial x_1} + \cdots + (x_p - x_{ij}^{p*})\frac{\partial}{\partial x_p}\Big)\mu_{f_{ij}}(f_{ij}(x_{ij}^*))$$

$$\mu_{f_{ij}}(f_{ij}(x)) \cong \widehat{\mu_{f_{ij}}}(f_{ij}(x)) = \mu_{f_{ij}} f_{ij}(x_{ij}^*) + \sum_{k=0}^{p}(x_k - x_{ij}^{k*})\frac{\partial \mu_{f_{ij}} f_{ij}(x_{ij}^*))}{\partial x_k} \tag{4}$$

Step 3. Sum of the membership functions associated to the upper level is shown by P(x) as below. Note that the problem is solved by assuming that weights of the objectives in upper level are equal.

$$P(x) = \sum_{j=1}^{m_0}\Big(\mu_{f_{0j}} f_{0j}(x_{0j}^*) + \sum_{k=0}^{p}(x_k - x_{0j}^{k*})\frac{\partial \mu_{f_{0j}} f_{0j}(x_{0j}^*)}{\partial x_k}\Big) \tag{5}$$

Step 4. After applying the Kuhn-Tucker conditions to the objective function of the lower level, we find satisfactory $x^* = (x_1^*,\ x_2^*, \ldots,\ x_p^*)$ by solving the reduced problem to a single objective. Therefore FLFBP is now converted into a new mathematical model as follows:

$$\begin{aligned} Max \quad & P(x) \\ s.t \quad & \\ & A_0x_0 + A_1x_1 + \cdots + A_px_p + u = b \\ & wA_i - \nu_i = \sum_{j=1}^{m_i}\frac{\partial \mu_{ij}(x_{ij}^*)}{\partial x_i} \\ & wu = 0,\ x_i\nu_i = 0 \\ & x_i,\ w,\ u,\ \nu_i \geqslant 0,\ i = 1, 2, \ldots, p \end{aligned} \tag{6}$$

In this method, zero-one variables η *and* ξ_i, is added to each constraint $wu = 0$ and $x_i\nu_i = 0$, respectively. In addition, each of these constraints is replaced by two linear inequalities involving η *and* ξ_i and M, a large positive constant. The auxiliary formulation now becomes

$$\begin{aligned} Max \quad & P(x) \\ s.t \quad & \\ & A_0x_0 + A_1x_1 + \cdots + A_px_p + u = b \end{aligned}$$

$$wA_i - \nu_i = \sum_{j=1}^{m_i} \frac{\partial \mu_{ij}(x_{ij}^*)}{\partial x_i} \tag{7}$$

$$\begin{aligned}
&w \leqslant M\eta,\ u \leqslant M(1-\eta)\\
&x_i \leqslant M\xi_i,\ \nu_i \leqslant M(1-\xi_i)\\
&\eta,\ \xi_i \in \{0,1\}\\
&x_i,\ w,\ u,\ \nu_i \geqslant 0,\ i = 1,2,\ldots,p
\end{aligned}$$

5. Numerical Example

To demonstrate the method for LFDBL-MODM problem, let us consider the following example:

$$\begin{aligned}
&\underset{x_0}{\text{Max}}\ (f_{01} = \frac{-x_0 - 4x_1 + x_2 + 1}{2x_0 + 3x_1 + x_2 + 2},\ f_{02} = \frac{-2x_0 + x_1 + 3x_2 + 4}{2x_0 - x_1 + x_2 + 5})\\
&\text{where } x_1, x_2 \text{ solves}\\
&\underset{x_1}{\text{Max}}\ (f_{11} = \frac{3x_0 - 2x_1 + 2x_2}{x_0 + x_1 + x_2 + 3},\ f_{12} = \frac{-7x_0 - 2x_1 + x_2 + 1}{5x_0 + 2x_1 + x_2 + 1})\\
&\underset{x_2}{\text{Max}}\ (f_{21} = \frac{x_0 + x_1 + x_2 - 4}{x_0 - 2x_1 + 10x_2 + 6},\ f_{22} = \frac{2x_0 - x_1 + x_2 + 4}{-x_0 + x_1 + x_2 + 10})\\
&\text{subject to}\\
&\quad x_0 + x_1 + x_2 \leqslant 5 \qquad -x_0 + x_1 + x_2 \leqslant 1\\
&\quad x_0 + x_1 - x_2 \leqslant 2 \qquad x_0 - x_1 + x_2 \leqslant 4\\
&\quad x_0 + x_1 + x_2 \geqslant 1 \qquad x_0 + 2x_2 \leqslant 4\\
&\quad x_0, x_1, x_2 \geqslant 0
\end{aligned} \tag{8}$$

We first obtain the f_{ij}^+ and f_{ij}^- for each objective(s) of each level(s), then $f^+ = (f_{01}^+, f_{02}^+, f_{11}^+, f_{12}^+, f_{21}^+, f_{22}^+) = (0.67, 1.25, 1.47, 1, 0.02, 1.25)$ and $f^- = (f_{01}^-, f_{02}^-, f_{11}^-, f_{12}^-, f_{21}^-, f_{22}^-) = (-0.73, 0, -0.5, -1.11, -0.75, 0.27)$ are obtained. The membership functions from each objective(s) of each level(s) are obtained as follows:

$$\mu_{f_{01}} f_{01}(x) = \begin{cases} 1 & f_{01}(x) \geqslant 0.67 \\ \dfrac{\dfrac{-x_0 - 4x_1 + x_2 + 1}{2x_0 + 3x_1 + x_2 + 2} - (-0.73)}{0.67 - (-0.73)} & -0.73 \leqslant f_{01}(x) \leqslant 0.67 \\ 0 & f_{01}(x) \leqslant -0.73 \end{cases}$$

$$\mu_{f_{01}} f_{01}(x) = \begin{cases} 1 & f_{01}(x) \geqslant 0.67 \\ \dfrac{0.46x_0 - 1.81x_1 + 1.73x_2 + 2.46}{2.8x_0 + 4.2x_1 + 1.4x_2 + 2.8} & -0.73 \leqslant f_{01}(x) \leqslant 0.67 \\ 0 & f_{01}(x) \leqslant -0.73 \end{cases} \tag{9}$$

In the same way, the other membership functions are formed as

$$\mu_{f_{02}} f_{02}(x) = \begin{cases} 1 & f_{02}(x) \geqslant 1.25 \\ \dfrac{-2x_0 + x_1 + 3x_2 + 4}{2.5x_0 - 1.25x_1 + 1.25x_2 + 6.25} & 0 \leqslant f_{02}(x) \leqslant 1.25 \\ 0 & f_{02}(x) \leqslant 0 \end{cases} \tag{10}$$

$$\mu_{f_{11}} f_{11}(x) = \begin{cases} 1 & f_{11}(x) \geqslant 1.47 \\ \dfrac{3.5x_0 - 1.5x_1 + 2.5x_2 + 1.5}{1.97x_0 + 1.97x_1 + 1.97x_2 + 5.91} & -0.5 \leqslant f_{11}(x) \leqslant 1.47 \\ 0 & f_{11}(x) \leqslant -0.5 \end{cases} \quad (11)$$

$$\mu_{f_{12}} f_{12}(x) = \begin{cases} 1 & f_{12}(x) \geqslant 1 \\ \dfrac{-1.45x_0 + 0.22x_1 + 2.11x_2 + 2.11}{10.55x_0 + 4.22x_1 + 2.11x_2 + 2.11} & -1.11 \leqslant f_{12}(x) \leqslant 1 \\ 0 & f_{12}(x) \leqslant -1.11 \end{cases} \quad (12)$$

$$\mu_{f_{21}} f_{21}(x) = \begin{cases} 1 & f_{21}(x) \geqslant 0.02 \\ \dfrac{1.75x_0 - 0.5x_1 + 8.5x_2 + 0.5}{0.77x_0 - 1.54x_1 + 7.7x_2 + 4.62} & -0.75 \leqslant f_{21}(x) \leqslant 0.02 \\ 0 & f_{21}(x) \leqslant -0.75 \end{cases} \quad (13)$$

$$\mu_{f_{22}} f_{22}(x) = \begin{cases} 1 & f_{22}(x) \geqslant 1.25 \\ \dfrac{2.27x_0 - 1.27x_1 + 0.73x_2 + 1.3}{-0.98x_0 + 0.98x_1 + 0.98x_2 + 9.8} & 0.27 \leqslant f_{22}(x) \leqslant 1.25 \\ 0 & f_{22}(x) \leqslant 0.27 \end{cases} \quad (14)$$

If the problem is solved for each of the membership functions, then $\mu^*_{f_{01}}(f_{01}(0,0,1))$, $\mu^*_{f_{02}}(f_{02}(0,\ 1,\ 0))$,$\mu^*_{f_{11}}(f_{11}(2.67, 0, 0.67))$, $\mu^*_{f_{12}}(f_{12}(0,\ 0,\ 1))$ $\mu^*_{f_{21}}(f_{21}(1.67,\ 1.5,\ 1.17))$ and $\mu^*_{f_{22}}(f_{22}(2.67,\ 0,\ 0.67))$ are obtained, and the associated membership functions are then transformed by using first-order Taylor polynomial series as below.

$$\mu_{f_{01}}(f_{01}(x)) \cong \widehat{\mu_{f_{01}}}(f_{01}(x)) = \mu_{f_{01}}(f_{01}(0,0,1)) + \Big((x_0 - 0)\frac{\partial}{\partial x_0} + (x_1 - 0\frac{\partial}{\partial x_1} + (x_2 - 1) \frac{\partial}{\partial x_2}\Big)\mu_{f_{01}}(f_{01}(0,0,1)) = 0.56 + 0.11x_0 - 2x_1 + 0.11x_2 \quad (15)$$

In the same way, the other membership functions are transformed on using first-order Taylor polynomial series.

$$\mu_{f_{02}}(f_{02}(x)) = 0.87 - 1.12x_0 + 0.56x_1 + 0.44x_2 \quad (16)$$

$$\mu_{f_{11}}(f_{11}(x)) = 1.02 + 0.24x_0 - 0.55x_1 - 0.07x_2 \quad (17)$$

$$\mu_{f_{12}}(f_{12}(x)) = 1 - 6x_0 - 2x_1 \quad (18)$$

$$\mu_{f_{21}}(f_{21}(x)) = -0.22 + 0.06x_0 + 0.06x_1 + 0.05x_2 \quad (19)$$

$$\mu_{f_{22}}(f_{22}(x)) = 0.17 + 0.41x_0 - 28x_1 - 0.03x_2 \quad (20)$$

The P(x) is obtained by adding (15) and (16) as follows:

$$P(x) = \mu_{f_{11}}(f_{11}(x)) + \mu_{f_{12}}(f_{12}(x)) = 1.43 - 1.01x_0 - 1.44x_1 + 0.55x_2 \quad (21)$$

After applying the Kuhn-Tucker conditions to the obective function of the lower level , a new auxiliary problem is to be solved as follows:

$$\begin{array}{ll} Max & P(x) = 1.43 - 1.01x_0 - 1.44x_1 + 0.55x_2 \\ s.t & \\ & x_0 + x_1 + x_2 + u_1 = 5 \qquad -x_0 + x_1 + x_2 + u_2 = 1 \\ & x_0 + x_1 - x_2 + u_3 = 2 \qquad x_0 - x_1 + x_2 + u_4 = 4 \\ & x_0 + x_1 + x_2 + u_5 = 1 \qquad x_0 + 2x_2 + u_6 = 4 \\ & w_1 + w_2 + w_3 - w_4 + w_5 - \nu 1 = -0.55 - 2 \\ & w_1 + w_2 - w_3 + w_4 + w_5 + 2w_6 - \nu 2 = 0.05 - 0.03 \end{array} \quad (22)$$

$$w_j \leqslant M\eta_j, \quad u_j \leqslant M(1-\eta_j)$$
$$x_i \leqslant M\xi_i, \quad \nu_i \leqslant M(1-\xi_i)$$
$$\eta_j, \ \xi_i \in \{0,1\}$$
$$x_0, x_1, x_2, w_j, u_j, \nu_i \geqslant 0 \quad i=1,2, \ j=1,\ldots,6$$

The problem with M = 1000, is solved and the Pareto optimal solution of the above problem is obtained as follows:

$$x_0^* = 0, \qquad x_1^* = 0, \qquad x_2^* = 1,$$

$$f_{01}(x) = 0.67, \quad f_{02}(x) = 1.17, \quad f_{11}(x) = 0.5, \quad f_{12}(x) = 1,$$

$$f_{21}(x) = -0.19, \quad f_{22}(x_1, x_2) = 0.45.$$

6. Conclusion

In this paper, for solving LFDBL-MODM problems a powerful and robust method which is based on Taylor series and Kuhn-Tucker conditions is proposed. Membership functions associated to each objectives of each levels are transformed by using Taylor series, that is the LFDBL-MODM problem is reduced to an equivalent single objective linear programming problem by using the first-order Taylor polynomial series and Kuhn-Tucker conditions was applied as a new approach to bilevel fractional programming problems compare to the previous method in the literature. Proposed method is applied to a numerical example to test the effect on the performance. The result indicates that the proposed method is very simple, efficient and robust.

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