International Journal of
Mathematical Modelling & Computations
Vol. 05, No. 03, Summer 2015, 267- 275

Optimum Generalized Compound Linear Plan for Multiple-Step Step-Stress Accelerated Life Tests

N. Chandra[a*] and Mashroor Ahmad Khan[b]

[a,b]*Department of Statistics Ramanujan School of Mathematical Sciences, Pondicherry University (A Central University), Pondicherry-605 014, India.*

Abstract. In this paper, we consider an $M-$step($M \geqslant 3$) i.e., multiple step-stress accelerated life testing (ALT) experiment with unequal duration of time $\tau_i(\tau_1 < \tau_2 < \cdots < \tau_{m-1})$. It is assumed that the time to failure of a product follows Rayleigh distribution with a log-linear relationship between stress and lifetime and also we assume a generalized Khamis-Higgins model for the effect of changing stress levels. Taking into account that the problem of choosing the optimal time for 3-step step-stress tests under compound linear plan was initially attempted by [17]. We ever first have developed a generalized compound linear plan for multiple-step step-stress setting using variance-optimality criteria. Some numerical examples are discussed to illustrate the proposed procedures.

Received: 17 November 2014, Revised: 15 April 2015, Accepted: 8 September 2015.

Keywords: Accelerated life testing; Rayleigh distribution; Cumulative exposure model; Maximum likelihood estimate; Generalized compound linear plan.

Index to information contained in this paper

1 **Introduction**
2 **Model Description and Basic Assumptions**
3 **Maximum Likelihood Estimation (MLE) of Parameters**
4 **Optimum criterion**
5 **Numerical study**
6 **Conclusion**

1. Introduction

The accelerated life test (ALT) is frequently used to obtain information about system or product life distribution more quickly than under normal operating conditions. Then the data from such tests will be transformed to estimate the distribution of failure time under usual conditions. A special case of ALT is the step-stress, [21] described that a step-stress test which allows the stress of a unit to be changed at pre-specified times. In step-stress test, an initial low stress is

*Corresponding author. Email: nc.stat@gmail.com

applied to all test units. If a unit does not fail in a pre-specified time, the stress is increased. There, can be more than one change of stress level. For more details, see [3, 4, 6, 10, 15, 20, 27] and [24].

During the past two decades, the problem of determination of the optimal duration of stress changing of the step-stress test has attracted great attention in the reliability literature. The development of optimal test plan under step-stress model has been attempted by many researchers and they commonly used to design an optimal plan by minimizing the asymptotic variances of the maximum likelihood estimator of the log of mean life or some percentile of life at a specified stress level. [20] initiated research in this area by assuming complete failure data under two stress levels (simple step-stress) with the assumption of exponential lifetimes. Extension of the above results to the time-censored and the three stress level cases were obtained by [2] and [17], respectively. For the general M-level and k-variable case, some numerical investigations were under taken by [16]. For an M-step step-stress ALT with equal duration steps, [12], [5] and [25] and [26] tackled the problem of determining the optimal stress change points when the available data are exponential with progressively Type-I censored. For more details, see [13, 18, 22]. The problem of optimal time issues with the cumulative exposure model for two stress levels under different lifetime distributions has been studied by several authors such as [1, 7, 8, 11, 14, 23]. Recently, [9] studied a problem for 3-step, step-stress ALT by assuming the existence of linear as well as quadratic relationship between the log mean failure time and stress, and also they developed an optimum plan under compound linear plan.

Although much work has been done on the determination of optimal simple step-stress ALT plans, attention on the general $M-\text{step}(\geqslant 3)$ step-stress testing with unequal test length under the Rayleigh distributions has not been paid in the literature. While most of models use simple step-stress plans that use only two stresses. It has practical limitations; they highly depend upon the assumption of a linear relationship between time to failure and stress and also use two extreme stresses that can cause irrelevant failure modes, Khamis and Higgins [16].

In practice, the assumption of equal duration steps may be not the best for M-step (or higher steps) step-stress life test planning. [6] also pointed out in his challenging open problem 6 that the equi-spaced time intervals are quite appealing in the framework of a step-stress test, it will be of interest to consider a general setting in which we allow unequal steps and develop the corresponding inferential procedures. It will then be useful to assess the loss (in precision or information) incurred in adopting an equi-spaced step-stress test. Needless to say, the determination of optimal step-stress tests in this general setting (i.e., finding the optimal time points $\tau_1 < \tau_2 < \cdots < \tau_{m-1}$) will be of great practical value.

Therefore, this nice suggestion has motivated to authors to consider unequal duration steps problem for products with Rayleigh distribution. The main focus of this paper, to develop generalized compound linear plan for $M-\text{step}(M \geqslant 3)$ step-stress ALT and to investigate the choice of optimal change points at different stress levels. Under complete data, the optimum time points are obtained by using V-optimality criterion. To the best of our knowledge, either a theoretical or a numerical verification of generalized compound linear plan for $M-\text{step}(M \geqslant 3)$ has not been made in the literature.

In the subsequent sections, the proposed model and assumptions are discussed in section 2. In section 3, we present the expected Fisher information matrix of the MLEs of the unknown parameters. The optimality criteria and the generalized compound linear plan are given in section 4. Some simulation results are illustrated in section 5. A conclusion of the proposed study is summarized in section 6.

Notations

x_0 Design (use) stress
x_i i^{th} test-stress, $i = 1, 2, \cdots, m$; $x_1 < x_2 < \cdots < x_m$
m Number of stress level
ξ Extrapolation amount where $\xi = (x_1 - x_0)/(x_m - x_1)$
n Total number of units placed on test
n_i Number of failed units at stress
$t_{i,j}$ Ordered failure time of test unit at stress $x_i; i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, n_i$
θ_i Mean life at stress $x_i, i = 0, 1, 2, \cdots, m$
τ_i Time of stress changing at x_i
$F_i(t)$ CDF of Rayleigh distribution with mean θ_i
$F(t)$ CDF of a test unit under M-step step-stress
β_0, β_1 Unknown parameters of log-linear relationship between stress and life.

2. Model Description and Basic Assumptions

Let us assumed $x_1 < x_2 < \cdots < x_m$ that are the ordered stress level to be used in the life test. For $i = 1, 2, \cdots, m$, let n_i denote the number of units failed at stress level x_i, and $t_{i,j}$ denote the j^{th} ordered failure time among the n_i failed units at x_i. Therefore, a step-stress model with unequal step duration τ_i proceeds as follows: Initially n experimental units are placed on a life test at stress x_1, and run until a pre-specified time τ_1; at that point, the stress is changed to x_2 and the test is continued on the remaining $n - n_1$ units until τ_2, and so on. Finally, at time τ_k, all surviving $n - \sum_{i=1}^{m} n_i$ items are failed or censored, thereby terminating the life test.

At stress level $x_i, i = 1, 2, \cdots, m$, the lifetime T of a test unit is assumed to follow a Rayleigh distribution with cumulative distribution function (CDF)

$$F_i(t; \theta_i) = 1 - \exp\left(-\frac{t^2}{2\theta_i^2}\right), \qquad t > 0, \theta > 0 \tag{1}$$

Basic Assumptions

(i). Under any constant stress, the time to failure of a test unit follows a Rayleigh distribution with distribution function is given in equation (1).

(ii). At any stress level x_i, the mean time to failure θ_i, of a test unit is a log-linear function of stress, i.e.,

$$\log(\theta_i) = \beta_0 + \beta_1 x_i, i = 1, 2, \cdots, m. \tag{2}$$

where, β_0, β_1 are the unknown parameters depending on the nature of the product and method of the test.

(iii). The lifetimes of test units are independent and identically distributed.

Under the assumption (i) and (ii), the CDF of the lifetime of a test unit under M-step step-stress ALT is given by

$$F_i(t) = F_i(t - \tau_{i-1} + S_{i-1}; \theta_i) \quad \text{if} \quad \begin{cases} \tau_{i-1} \leqslant \tau_i & \text{for } i=1,2,\cdots,m-1 \\ \tau_{k-1} < \infty & \text{for } i=m \end{cases} \tag{3}$$

where, $S_0 = 0, \tau_0 = 0$ and $S_{i-1} = \frac{\theta_i}{\theta_{i-1}}(\tau_{i-1} + S_{i-2} - \tau_{i-2})$ is the solution of $F_i(S_{i-1}; \theta_i) = F_{i-1}(\tau_{i-1} + S_{i-2} - \tau_{i-2}; \theta_{i-1}), i = 1, 2, \cdots, m$.

Hence, the Rayleigh cumulative distribution function for M-step, step-stress ALT using K-H model is given by

$$F(t) = \begin{cases} 1 - \exp\left(-\frac{t^2}{2\theta_1^2}\right), & \text{for } 0 < t \leqslant \tau_1 \\ 1 - \exp\left(-\frac{t^2 + S_{i-1}^2 - \tau_{i-2}^2}{2\theta_i^2}\right), & \text{for } \tau_{i-1} \leqslant t \leqslant \tau_i \text{ if } i = 2, \cdots, m-1, \\ & \text{and for } \tau_i \leqslant t < \infty \text{ if } i = m. \end{cases} \tag{4}$$

The corresponding probability density function (PDF) of the lifetime of a test unit is given by

$$f(t) = \begin{cases} \frac{t}{\theta_1^2} \exp\left(-\frac{t^2}{2\theta_1^2}\right), & \text{for } 0 < t \leqslant \tau_1 \\ \frac{t}{\theta_i^2} \exp\left(-\frac{t^2 + S_{i-1}^2 - \tau_{i-2}^2}{2\theta_i^2}\right), & \text{for } \tau_{i-1} \leqslant t \leqslant \tau_i \text{ if } i = 2, \cdots, m-1, \\ & \text{and for } \tau_i \leqslant t < \infty \text{ if } i = m. \end{cases} \tag{5}$$

3. Maximum Likelihood Estimation (MLE) of Parameters

The MLE method is used for parameters estimation and analysis of failure time data. From equation (5), the joint p.d.f. of observed data $n = (n_1, n_2, \cdots, n_m)$ and $t = (t_1, t_2, \cdots, t_m)$ with $t_i = (t_{i,1}, t_{i,2}, \cdots, t_{i,n_i})$ is given by

$$L(\mathbf{t}, \mathbf{n}) = \prod_{i=1}^{m} \left[\prod_{j=1}^{n_i} f_i(t_{ij}) \right] \tag{6}$$

Hence, by using the assumption (ii) the log-likelihood function of β_0 and β_1 is given by

$$\log L(\beta_0, \beta_1) = -2 \sum_{i=1}^{m} n_i \log(\beta_0 + \beta_1 x_i) + \sum_{i=1}^{m} \sum_{j=1}^{n_i} \log(t_{ij}) - \sum_{i=1}^{m} \sum_{j=1}^{n_i} \log(Z_{ij}) \tag{7}$$

where, $Z_{ij} = \frac{t_{ij}^2 + S_{i-2}^2 - \tau_{i-1}^2}{2e^{2(\beta_0 + \beta_1 x_i)}}, i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n_i$. Note that the MLEs of β_0 and β_1 exist only if $n_i > 0$, in equation (7). By using the following expressions:

$$\frac{\partial S_{i-1}}{\partial \beta_1} = \sum_{h=2}^{i} (x_h - x_{h-1}) S_{h-1} e^{\beta_1 (x_i - x_h)} \tag{8}$$

$$\frac{\partial Z_{ij}}{\partial \beta_0} = -Z_{ij}, \quad \frac{\partial Z_{ij}}{\partial \beta_1} = -2x_i Z_{ij} + \left(\frac{\sum_{h=2}^{i} (x_h - x_{h-1}) S_{h-1} e^{\beta_1 (x_i - x_h)}}{2e^{2(\beta_0 + \beta_1 x_i)}} \right) \tag{9}$$
$$\text{for } i = 1, 2, \cdots, m \text{ and } j = 1, 2, \cdots, n_i$$

The MLEs $\hat{\beta}_0$ and $\hat{\beta}_1$ can be obtained by solving the following two likelihood equations:

$$\frac{\partial \log L}{\partial \beta_0} = -2 \sum_{i=1}^{m} n_i + \sum_{i=1}^{m} \sum_{j=1}^{n_i} Z_{ij} = 0 \tag{10}$$

$$\frac{\partial \log L}{\partial \beta_1} = -2\sum_{i=1}^{m} n_i x_i + \sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(2x_i Z_{ij} - \frac{\sum_{h=2}^{i}(x_h - x_{h-1})S_{h-1}e^{\beta_1(x_i-x_h)}}{2e^{2(\beta_0+\beta_1 x_i)}}\right) = 0 \tag{11}$$

The likelihood equations (10) and (11) has no close form solution for β_0 and β_1 and cannot be solved analytically. Thus, MLEs $(\hat{\beta}_0,\ \hat{\beta}_1)$ of β_0 and β_1 can be obtained through numerical methods such as Newton-Raphson method. The Newton–Raphson algorithm implemented in R package *maxLik* is used to maximize the log-likelihood in equation (7).

Let $\theta = (\theta_1, \theta_2)' = (\beta_0, \beta_1)'$. Under some mild regularity conditions, for a large sample size n, the vector of the MLEs $\hat{\theta}$ is approximately distributed as bivariate normal with mean vector θ and variance-covariance matrix $I^{-1}(\theta)$, which is the inverse of the Fisher information matrix given by

$$I(\theta) = -E\left\{\frac{\partial^2 \log(\theta)}{\partial\theta_i\partial\theta_j}\right\} = n\begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22}\end{bmatrix} \tag{12}$$

The double and mixed partial derivatives of equation (7) are given as

$$\frac{\partial^2 \log L}{\partial \beta_0^2} = -4\sum_{i=1}^{m}\sum_{j=1}^{n_i} Z_{ij}$$

$$\frac{\partial^2 \log L}{\partial \beta_0, \partial \beta_1} = -4\sum_{i=1}^{m}\sum_{j=1}^{n_i} x_i Z_{ij}$$

$$\frac{\partial^2 \log L}{\partial \beta_1^2} = -4\sum_{i=1}^{m}\sum_{j=1}^{n_i} x_i^2 Z_{ij} + 2\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{h=2}^{i} x_i \frac{(x_h - x_{h-1})S_{h-1}e^{\beta_1(x_i-x_h)}}{2e^2(\beta_0+\beta_1 x_i)}$$
$$-\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{h=2}^{i}\sum_{l=2}^{h}(x_h - x_{h-1})\frac{(x_l - x_{l-1})S_{l-1}e^{\beta_1(x_i-x_h)}}{2e^2(\beta_0+\beta_1 x_i)}$$

Now, the elements of the expected Fishers Information matrix are given as:

$$I_{11} = -E\left\{\frac{\partial^2 \log L}{\partial \beta_0^2}\right\} = \sum_{i=1}^{n_i} A_i(\tau)$$

$$I_{12} = I_{21} = -E\left\{\frac{\partial^2 \log L}{\partial \beta_0, \partial \beta_1}\right\} = \sum_{i=1}^{n_i} x_i A_i(\tau)$$

$$I_{22} = -E\left\{\frac{\partial^2 \log L}{\partial \beta_1^2}\right\} = \sum_{i=1}^{n_i} x_i^2 A_i(\tau)$$

where

$$A_1 = 1 - exp\left\{-\frac{\tau_1^2}{2\theta_1^2}\right\}$$

$$A_2 = exp\left\{-\frac{\tau_1^2}{2\theta_1^2}\right\}\left\{1 - exp\left[-\frac{\tau_2^2 - \tau_1^2}{2\theta_2^2}\right]\right\}$$

....

$$A_m = exp\left\{-\frac{\tau_{m-1}^2 - \tau_{m-2}^2}{2\theta_{m-1}^2} - \frac{\tau_{m-2}^2 - \tau_{m-3}^2}{2\theta_{m-2}^2} - \cdots - \frac{\tau_2^2 - \tau_1^2}{2\theta_2^2} - \frac{\tau_1^2}{2\theta_1^2}\right\}$$

Our main objective in this paper is to explore the choice of $\tau_i,\ i = 1, 2, \cdots, m-1$, the time of the stress change. We investigated the selection of the optimal time point according to the variance-optimality criteria mainly based on equation (12) and are discussed in the next section.

4. Optimum criterion

4.1 *Variance-(V) optimality*

The mean of the failure time distribution is an important characteristic and imperative in reliability analysis. In step-stress scheme, the researchers often wish to estimate the mean lifetime at the use stress with maximum precision and minimum variability possible. We can use the asymptotic variance of the log of the mean lifetime θ_0 at use stress as the objective function for selecting the optimal stress change time. For this purpose, we consider an objective function from equation (12) as

$$\begin{aligned}\Phi(\tau_1, \tau_2, \cdots, \tau_{m-1}) &= nAVar(\log\hat{\theta}_0)\\ &= nAVar\left(\hat{\beta}_0 + \hat{\beta}_1 x_0\right)\\ &= n(1,\ x_0) I_n^{-1}(\hat{\beta}_0, \hat{\beta}_1)\begin{pmatrix}1\\ x_0\end{pmatrix}\end{aligned} \quad (13)$$

$$\Phi(\tau_1, \tau_2, \cdots, \tau_{m-1}) = \frac{\sum_{i=1}^m A_i(\tau)(x_i - x_0)^2}{4\sum_{i=1}^m \sum_{l=1}^m A_i(\tau)A_l(\tau)(x_i - x_l)^2} \quad (14)$$

where, AVar Stands for asymptotic-variance and x_0 is the design (use) stress. The V-optimality is the one that minimizes $\Phi(\tau_1, \tau_2, \cdots, \tau_{m-1})$ in equation (14).

4.2 *Generalized Compound Linear Plan*

The $M-$ step($M \geqslant 3$) step-stress optimum linear plan (14) by minimizing $\Phi(\tau_1, \tau_2, \cdots, \tau_{m-1})$, occurs when $\tau_1 = \tau_2 = \cdots = \tau_{m-1}$ so that only two extreme stresses x_1 and x_m are used in testing. Hence, the optimal stress change point at

$\tau_1 = \tau_2 = \cdots = \tau_{m-1}$ is given as:

$$\tau^* = \theta\sqrt{2\log\left(\frac{1+2\xi}{\xi}\right)}, \text{ where } \xi = \frac{x_1 - x_0}{x_m - x_1} \tag{15}$$

According to [17] only two extreme stresses are used and that can cause irrelevant failure modes. So, they had given another plan, called compound linear plan for 3-step SSALT. In which they uses the optimum simple step-stress plan twice. Therefore, the generalized compound linear plan for $M-\text{step}(M \geqslant 3)$ is given as

$$\tau'_{i-1} = \theta_{i-1}\sqrt{2\log\left(\frac{1+2\xi_{i-1}}{\xi_{i-1}}\right)}, \text{ with stresses } x_0, x_{i-1}, x_i,\ i = 2, 3, ..m. \text{ Then}$$

$$\tau^*_{i-1} = \tau^*_{i-2} + \tau'_{i-1} \tag{16}$$

4.3 *The case for 3-step step-stress*

Let us consider the case $m = 3$; to investigate the solution for proposed model given in equation (16).

$$\tau_1^* = \theta_1\sqrt{2\log\left(\frac{1+2\xi_1}{\xi_1}\right)}, \text{ with stresses } x_0, x_1, x_2 \text{ and } \xi_1 = \frac{x_1 - x_0}{x_2 - x_1}.$$

$$\tau'_2 = \theta_2\sqrt{2\log\left(\frac{1+2\xi_2}{\xi_2}\right)}, \text{ with stresses } x_0, x_2, x_3 \text{ and } \xi_2 = \frac{x_2 - x_0}{x_3 - x_1}, \text{ then}$$

$$\tau_2^* = \tau_1^* + \tau'_2.$$

Similarly, we can obtain for $m = 4, 5, \ldots, \infty$. The numerical values of the optimum stress changing times are calculated by considering some selected value of $m = (4, 6)$ in the formula (16), are tabulated in the tables 1-3 given in next section.

5. Numerical study

The main purpose of this paper is to determine the optimal unequal time points $\tau_i(\tau_1, \tau_2, \cdots, \tau_{m-1})$ that minimize the asymptotic variance of the MLEs of the log mean life at the normal-use stress x_0 under V-optimality criteria. For numerical investigation of generalized compound linear plan (16), we conducted a small numerical study. The optimum times $\tau_i^*(\tau_1^*, \tau_2^*, \cdots, \tau_{m-1}^*)$ of changing stress levels are presented in Tables 1-3.

6. Conclusion

This paper has presented the optimum $M-\text{Step}(M \geqslant 3)$ i.e., multiple-step step-stress accelerated life test under generalized compound linear plan, which assume

Table 1. Optimal change times and the associated asymptotic variance according to the V-optimality and generalize compound linear plan (16) under 3-step step-stress setting based on complete data in the Rayleigh case

x_1	x_2	x_3	τ_1^*	τ_2^*	nAVar
0.2	0.3	0.8	8.189	17.02	3.022
	0.6	0.8	10.07	15.35	1.074
	0.5	1.0	9.576	16.22	0.987
	0.8	1.0	10.85	15.08	0.699
0.4	0.5	0.8	6.308	12.50	9.650
	0.7	0.8	7.045	11.58	3.270
	0.8	1.0	7.341	11.57	2.192
	0.9	1.0	7.605	11.28	1.682

Table 2. Optimal change times and the associated asymptotic variance according to the V-optimality and generalize compound linear plan (16) under 4-step step-stress setting based on complete data in the Rayleigh case

x_1	x_2	x_3	x_4	τ_1^*	τ_2^*	τ_3^*	nAVar
0.2	0.3	0.7	1.0	8.19	16.68	21.57	3.349
	0.4	0.6	0.8	8.97	15.67	20.95	2.065
	0.5	0.7	0.8	9.58	15.501	20.04	1.318
	0.6	0.8	1.0	10.1	15.35	19.58	1.071
0.4	0.5	0.6	0.8	6.31	11.94	17.22	15.84
	0.6	0.7	0.8	6.71	11.75	16.28	5.982
	0.5	0.8	1.0	6.31	12.50	16.73	9.623
	0.7	0.9	1.0	7.05	11.76	15.45	3.174

Table 3. Optimal change times and the associated asymptotic variance according to the V-optimality and generalize compound linear plan (16) under 6-step step-stress setting based on complete data in the Rayleigh case

x_1	x_2	x_3	x_4	x_5	x_6	τ_1^*	τ_2^*	τ_3^*	τ_4^*	τ_5^*	nAVar
0.2	0.3	0.4	0.6	0.7	0.9	8.19	15.32	22.02	27.06	31.78	6.265
	0.3	0.6	0.7	0.9	1.0	8.19	16.32	21.35	26.07	29.74	3.927
	0.4	0.5	0.7	0.8	1.0	8.97	15.28	21.21	25.74	29.94	2.407
0.4	0.5	0.6	0.7	0.8	0.9	6.31	11.94	16.98	21.51	25.59	15.91
	0.6	0.7	0.8	0.9	1.0	6.71	11.75	16.28	20.36	24.03	5.761

that a linear relationship exists between the time to failure and the stress. A Rayleigh distribution and a generalized Khamis-Higgins model were assumed. We have obtained optimal choice of $(\tau_1, \tau_2, \cdots, \tau_{m-1})$ in general $M-$Step$(M \geqslant 3)$ step-stress setting, particularly for $m = 3, 4, 6$, by minimizing the asymptotic variance of the maximum likelihood estimator of the log of mean life time of the distribution at the design stress, given in Table 1,2,3, respectively. The numerical study shows that generalized compound linear plan working well for multiple-step step-stress test.

Some more interesting optimum plans for $M -$ Step$(M \geqslant 3)$ step-stress under quadratic relationship between life and stress can be developed by using both classical and Bayesian techniques, for other life time distributions also.

References

[1] Al-Haj Ebrahem, M. and Al-Masri, A., *Optimum simple step-stress plan for log-logistic cumulative exposure model*, Metron International Journal of Statistics, LXV(1) (2007) 23-34.

[2] Bai, D. S., Kim, M. S. and Lee, S. H., *Optimum Simple Step-Stress Accelerated Life Tests with Censoring*, IEEE transactions on Reliability, 38(5) (1989) 528-532.

[3] Balakrishnan, N. and Xie, Q., *Exact inference for a simple step-stress model with type-II hybrid censored data from the exponential distribution*, Journal of Statistical Planning and Inference, 137 (2007a) 2543-2563.

[4] Balakrishnan, N. and Xie, Q., *Exact inference for a simple step-stress model with type-I hybrid censored data from the exponential distribution*, Journal of Statistical Planning and Inference, 137 (2007b) 3268-3290.
[5] Balakrishnan, N. and Han, D., *Optimal step-stress testing for progressively Type-I censored data from exponential distribution*, Journal of Statistical Planning and Inference, 139 (2008) 1782-1798.
[6] Balakrishnan, N., *A synthesis of exact inferential results for exponential step-stress models and associated optimal accelerated life-tests*, Metrika, 69 (2009) 351-396.
[7] Chandra, N. and Khan, M. A., *A New Optimum Test Plan for Simple Step-Stress Accelerated Life Testing*, Applications of Reliability Theory and Survival Analysis, Bonfring Publication, Coimbatore, India (2012) 57-65.
[8] Chandra, N. and Khan, M. A., *Optimum plan for step-stress accelerated life testing model under type-I censored samples*, Journal of modern mathematics and statistics, 7(5) (2013) 58-62.
[9] Chandra, N., Khan, M. A. and Pandey, M., *Optimum test plan for 3-step, step-stress accelerated life tests*, International Journal of Performability Engineering, 10(1) (2014) 03-14.
[10] Fan, T. H., Wang, W. L. and Balakrishnan, N., *Exponential progressive step-stress life-testing with link function based on Box-Cox transformation*, Journal of Statistical Planning and Inference, 138 (2008) 2340-2354.
[11] Fard, N. and Li, C., *Optimal simple step stress accelerated life test design for reliability prediction*, Journal of statistical planning and inference, 139(5) (2009) 1799-1808.
[12] Gouno, E., Sen, A. and Balakrishnan, N., *Optimal step-stress test under progressive Type-I censoring*, IEEE Transactions on Reliability, 53 (2004) 383-393.
[13] Guan, Q. and Tang, Y., *Optimal step-stress test under Type-I censoring for multivariate exponential distribution*, Journal of statistical planning and inference, 142(7) (2012) 1908-1923.
[14] Hassan, A. S. and Al-Ghamdi, A. S., *Optimum step-stress accelerated life testing for Lomax distribution*, Journal of Applied Sciences Research, 5 (2009) 2153-2164.
[15] Kamal, M., Zarrin, S. and Islam, A., *Design of Accelerated Life Testing Using Geometric Process for Type-II Censored Pareto Faiure Data*, International Journal of Mathematical Modelling & Computations, 4(2) (2014) 125-134.
[16] Khamis, I. H., *Optimum M-step, step-stress test with k stress variables*, Communications in Statistics-Simulation and Computation, 26 (1997) 1301-1313.
[17] Khamis, I. H. and Higgins, J. J., *Optimum 3-step step-stress tests*, IEEE Transactions on Reliability, 45 (1996) 341-345.
[18] Lin, C. T., Chou, C. C. and Balakrishnan, N., *Planning step-stress test plans under Type-I censoring for the log-location-scale case*, Journal of statistical computation and simulations, 83(10) (2013) 1852-1867.
[19] Meeker, W. Q. and Escobar, L. A., *Statistical Methods for Reliability Data*, John Wiley, New York (1998).
[20] Miller, R. and Nelson, W. B., *Optimum simple step-stress plans for accelerated life testing*, IEEE transactions on reliability, R-32(1) (1983) 59-65.
[21] Nelson, W. B., *Accelerated life testing step-stress models and data analysis*, IEEE Trans on Reliability, 29(1980) 103-108.
[22] Shen, K.F., Shen, Y.J. and Leu, L.Y., *Design of optimal stepstress accelerated life tests under progressive type-I censoring with random removals*, Quality and Quantity, 45(3) (2011) 587-597.
[23] Srivastava, P. W. and Shukla, R., *Optimum Log-Logistic Step-Stress Model with Censoring*, International Journal of Quality and Reliability Management, 25(9) (2008) 968-976.
[24] Wang, B.X., *Interval estimation for exponential progressive Type-II censored step-stress accelerated life-testing*, Journal of statistical planning and inference, 140 (2010) 2706-2718.
[25] Wu, S. J., Lin, Y. P. and Chen, Y. J., *Planning step-stress life test with progressively Type-I group-censored exponential data*. Statistica Neerlandica, 60 (2006) 46-56.
[26] Wu, S. J., Lin, Y. P. and Chen, Y. J., *Optimal step-stress test under Type-I progressive group-censoring with random removals*, Journal of Statistical Planning and Inference, 138 (2008) 817-826.
[27] Xiong, C., *Inferences on a simple step-stress model with type-II censored exponential data*, IEEE Transactions on Reliability, 47 (1998) 142-146.