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Flexible Beam Robust H_∞ Loop Shaping Controller Design Using Particle Swarm Optimization

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Abstract

In this paper a new strategy is proposed to design a fixed-structure robust controller for a flexible beam. Robust controller designed by the conventional $\boldsymbol{H_\infty}$ loop shaping method is not appropriate for a beam because of its high order and complicated form. Fixed-structure $\boldsymbol{H_\infty}$ loop shaping control in conjunction with particle swarm optimization (PSO)algorithm is used to overcome this drawback. The performance and robust stability conditions of the $\boldsymbol{H_\infty}$ loop shaping controller are formulated as the cost function in the optimization problem. PSO is adopted to optimize the parameters and cost function. The proposed control design and $\boldsymbol{H_\infty}$ loop shaping method are successfully applied on the flexible beam, and results of the two approaches are compared. Simulation results show the superiorities of the proposed controller in terms of having a lower order and simple structure; besides the beam stability and robust performance are retained as well. Also in comparison to the solutions based on genetic algorithms, the use of PSO shows better efficiency in terms of computational time.

Keywords: *Fixed-structure, Robust controller, Flexible beam, $\boldsymbol{H_\infty}$loop shaping, Particle swarm optimization*

1. Introduction

Since the beam is a fundamental element in many structures, its control design has been considered as the main subject of many researches. The main class of controllers for these systems are linear feedback controllers designed using robust controller design techniques [1-5]. Also, a large number of papers have been published on the related problem of designing a robust feedback controller for a slewing beam, such as flexible robot arm [6-10]. This paper will be concerned with the problem of robust control of a flexible beam against uncertainties caused by parameters and modes variations of the system model. The $\boldsymbol{H_\infty}$ loopshaping control is an effective robust control technique that is most suitable for the systems with unstructured uncertainties. It was firstly developed by [11], and has been widely used for practical applications [12-15].

However, the controllers which are developed based on the conventional H_∞ loop shaping control design normally have the high order, and it is difficult to implement in reality. Basically, the lower order H_∞ loop shaping controllers can be synthesized by either the order reduction or the structure-specified design method. The parameters of a lower order structure-specified controller are determined such that the controller is admissible and the H_∞ norm from the exogenous inputs to controlled outputs is minimized [15]. The structure-specified design method generates a complex and non-convex optimization problem, which is difficult to solve analytically. Genetic algorithm (GA) and PSO are efficient in solving such multi-objective problems. GA was applied for tuning the parameters of the structure-specified controller before [15], however its computational time was considerable. PSO is one of the most efficient techniques for adapting parameters of fixed order controllers [17]. This method is faster than GA based algorithms in such problems because of its simple calculation [18,19].

In this paper, PSO is implemented to optimize the parameters of structure-specified H_∞ loop shaping controller of a flexible beam. A nominal model of the beam is firstly shaped by a pre-compensator and a post-compensator in order to achieve a desired open loop shape. A structure-specified controller is then defined. Finally, PSO is used to search for parameters of the controller such that the cost function is minimized. Outcomes of the proposed method have been finally compared with the conventional H_∞ loop shaping approach. The comparison about the computational time of GA based and PSO based algorithms is also included.

The next sections of the paper are organized as follows: Dynamics of the flexible beam system is described in section 2. Section 3 contains a brief presentation about the H_∞ loop shaping controller design, and the proposed method is introduced completely in section 4. Results are shown in section 5. Finally section 6 presents the conclusion.

2. Dynamics of flexible beam

In this section, a formulation as a model for the experimental flexible cantilever beam located at the Australian Defence Force Academy (ADFA) is described. In this flexible beam, the control actuator is a piezoceramic patch bonded to the beam. When a voltage is applied to the patch, the resulting piezoceramic stress produces a bending moment in the beam proportional to the applied voltage. Details of the modeling process is included here since this modeling process is an integral part of our overall controller design. The approach taken in modeling the beam is an assumed modes approach. The parameters of the ADFA beam are shown in Table 1. For this beam, we consider the transfer function from v_a to $y(r,t)$. Here $v_a(t)$ is the voltage applied to the piezoceramic patch and $y(r,t)$ is the deflection of the beam at position r. Using the assumed modes approach, the following formula can be obtained:

$$\frac{y(r,s)}{v_a(s)}=\sum_{i=1}^{\infty}\frac{C_a\varphi_i(r)[\varphi_i'(r_1)-\varphi_i'(r_2)]}{\rho AL^3(s^2+\omega_i^2)} \tag{1}$$

here

$$\begin{aligned} &C_a=0.5\,E_a\,d_{31}w(t_a+t_b),\\ &\varphi_i(r)=L(\cosh\lambda_i-\cos\lambda_i r-k_i(\sinh\lambda_i r-\sin\lambda_i r)),\\ &k_i=\frac{cos\;\lambda_i L+\cosh\lambda_i L}{sin\;\lambda_i L+sin\,\mathrm{h}\,\lambda_i L}. \end{aligned} \tag{2}$$

where the quantities λ_i are the real roots of the equation:

$$1+\cos\lambda_i L \cosh\lambda_i L = 0 \tag{3}$$

These quantities determine the natural frequencies of the beam as follows:

$$\omega_i = \sqrt{\frac{EI}{\rho A}}\lambda_i^2\ ,\ I = \frac{wt_b^3}{12}\ ,\ A = wt_b. \tag{4}$$

Also, note that the notation $\varphi_i'(r)$ refers to derivative of the function $\varphi_i(r)$.

By specifying r to be the location of the sensor, Equation (1) defines the transferring function of the beam. In practice, rather than using the infinite dimensional transfer function (1), we truncate this series after a finite number of modes *N*. In particular, in this paper we choose *N=2*. This meant that we begin solving the problem of designing a robust controller against beam un-modeled dynamics considering this assumption.

Transfer function of the model can be derived as follows:

$$P(s) = K\frac{s}{(s+a)}\frac{(s-b)}{(s+b)}\sum_{i=1}^{2}\frac{C_a\varphi_i(a_1)[\varphi_i'(r_1)-\varphi_i'(r_2)]}{\rho AL^3(s^2+2\zeta_i\ \omega_i s+\omega_i^2)} \tag{5}$$

where K is a gain parameter and the parameter ζ_i represents the damping of each mode. These parameters were chosen as:

$$K = \frac{1}{3.3\times10^{-4}}, a=10, b=3, \zeta_1 = 0.007, \zeta_2 = 0.002. \tag{6}$$

Table 1. ADFA beam parameters value

Parameter	Value
Beam length, L	*1.023 m*
Beam width, w	*0.04995 m*
Beam thickness, t_b	*0.00285 m*
Beam density, ρ	2712.6 kg/m^3
Beam Young's modulus, E	6.94× 10^{10}N/m^2
Piezoceramic position, r_1	*0.03765 m*
Piezoceramic position, r_2	*0.10771 m*
Accelerometer 1 position, a_1	*0.902 m*
Accelerometer 2 position, a_2	*0.783 m*
Accelerometer 3 position, a_3	*0.540 m*
Accelerometer 4 position, a_4	*0.233 m*
Charge Constant, d_{31}	*-210*× 10^{-12} *m/V*
Voltage Constant, g_{31}	*-11.5*× 10^{-3} *Vm/N*
Coupling Coefficient, k_{31}	*-0.34*
Beam Young's modulus, E_a	*6.9*× 10^{10} *N/*m^2
Capacitance per unit area, *C*	*68.35* μ*F/*m^2
Piezoceramic width, h_p	*0.02581 m*
Piezoceramic thickness, t_a	*0.01*

3. H_∞ loop shaping control design

The H_∞ loop shaping controller design is based on the configuration shown in Figure 1. The nominal model of the system is defined as *P*, and the shaped plant with a pre-compensator *W1* and a post-compensator $W2$ *W2* is defined as *Ps*. Equation (7) presents the relationship of these variants as follows:

$$P_s = W2\ P\ W1 = \tilde{M}^{-1}\tilde{N} = \begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} \tag{7}$$

where A_s, B_s, C_s, C_s and D_s are the matrices of the shaped plant in the state-space representation. $\tilde{M}$ and $\tilde{N}$ are the normalized left co prime factors of *Ps*.

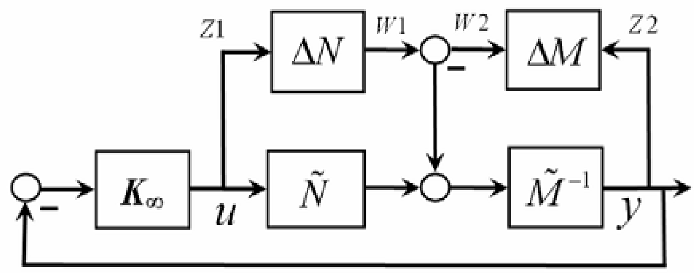

Figure 1. Robust stabilization with respect to the co prime factor uncertainties

By assuming that the shaped plant is perturbed by unstructured uncertainties $\Delta M$and ΔN, the perturbed plant P_Δ is presented in Equation (8).

$$P_\Delta = (\tilde{M} + \Delta M)^{-1}(\tilde{N} + \Delta N) \tag{8}$$

It is proved from the small gain theorem that the shaped plant *Ps* is stable with all the unknown but bounded uncertainties $\|\Delta M \quad \Delta N\|_\infty \prec \varepsilon$ $\|\Delta M \Delta N\|_\infty <$ if and only if there exists an admissible controller K_∞ such that Equation (9) is obtained.

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + P_s K_\infty)^{-1} \tilde{M}^{-1} \right\|_\infty \le \gamma = 1/\varepsilon \tag{9}$$

The minimization of γ (the maximization of ε)ε) results in the maximization of the robustness of the system. A procedure, called the H_∞ loop shaping controller design, was proposed in [11] and further developed in [20]. The block diagram of the H_∞ loop shaping control is shown in Figure 2. The H_∞ loop shaping controller design procedure is summarized as follows:

Step 1: The nominal plant *P* is shaped using a pre-compensator *W2* to achieve a desired open loop shape. *W1* is used to achieve the tracking performance and disturbance attenuation and *W2* is used to attenuate the sensor noise. *W1* and *W2* are selected so that *Ps* contains no hidden modes, and has the following properties.

(i) To achieve the good tracking performance and good disturbance rejection. The large open loop gain at a low frequency range is required.

(ii) To achieve the good robust stability and sensor noise rejection. The small open loop gain at a high frequency range is required.

When *W1* and *W2* are selected, the value of γ_{opt} is evaluated using Equation (10) where λ_{max} is the maximum eigenvalue.

$$\gamma_{opt} = [1 + \lambda_{\max}(Z, X)]^{1/2} \tag{10}$$

where Z and X are the solutions of the two following Riccati equations:

$$(A_s - B_s S^{-1} D_s{}^T C_s)Z + Z(A_s - B_s S^{-1} D_s{}^T C_s)^T - ZC_s{}^T R^{-1} C_s Z + B_s S^{-1} B_s{}^T = 0 \tag{11}$$

$$(A_s - B_s S^{-1} D_s{}^T C_s)^T X + X(A_s - B_s S^{-1} D_s{}^T C_s)^T - XB_s S^{-1} B_s{}^T X + C_s{}^T R^{-1} C_s = 0 \tag{12}$$

where

$$R = I + D_s D_s{}^T \; R = I + D_s D_s^T \text{ and } S = I + D_s{}^T D_s \tag{13}$$

W1 and *W2* are adjusted until a satisfied γ_{opt} is achieved. If γ_{opt} is too large ($\gamma_{opt} \succ 4$), *W1* and *W2* are incompatible and should be adjusted.

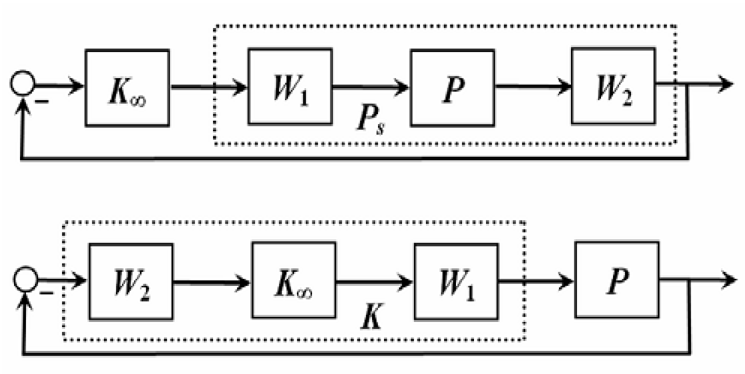

Figure 2. Block diagram of the H_∞ loop shapingcontrol

Step 2: Select $\varepsilon \prec \varepsilon_{opt} = \gamma_{opt}{}^{-1}$, and then synthesis a sub-optimal controller K_∞ as shown in (14):

$$K_\infty = \begin{bmatrix} A_s + B_s F + \gamma^2 (Q^T)^{-1} Z C_s{}^T (C_s + D_s F) & \gamma^2 (Q^T)^{-1} Z C_s{}^T \\ B_s{}^T X & -D_s{}^T \end{bmatrix} \tag{14}$$

where

$$F = -S^{-1}(D_s{}^T C_s + B_s{}^T X) \text{ and } Q = (I - \gamma^2) I + XZ. \tag{15}$$

Step 3: The final controller is calculated as Equation (16):

$$K = W1\, K_\infty\, W2 \tag{16}$$

4. Structure-specified H_∞ loop shaping controller design based on PSO

The H_∞ loop shaping controller design procedure presented in section 3 is straightforward; and it is useful for the beam systems with the unstructured uncertainties. However, the obtained final controller has the high order which leads to difficulties when implemented in practice. The procedure for the lower order robust controller design needs to be investigated. A new method for designing the structure-specified H_∞ loop shaping controllers based on the PSO was introduced by [16]. The design procedure is described as follows:

4.1 Selection of weighting functions

Since the algorithm is based on the H_∞ loop shaping method, the plant is firstly shaped by using the pre-compensator and post-compensator. In this paper, the lead/lag type compensators are used for weighting functions presented in Equations (17) and (18). The shaped plant is thus described as Equation (19):

$$W1 = K1\frac{s+\alpha 1}{s+\beta 1} \tag{17}$$

$$W2 = K2\frac{s+\alpha 2}{s+\beta 2} \tag{18}$$

$$P_s = W2\ P\ W1 \tag{19}$$

4.2. Structure-specified controller design

The structure-specified controller, *K(s)* is defined as Equation (20).

$$K(s) = \frac{N_k(s)}{D_k(s)} = \frac{a_m s^m + a_{m-1}s^{m-1} + \ldots + a_0}{s^n + b_{n-1}s^{n-1} + \ldots + b_0} \tag{20}$$

The structure-specified controller can be in any forms such as first order, PID, second order controllers etc. by selecting suitable values of m and n.

4.3. Cost function definition

The structure-specified H_∞ loop shaping controller design problem can be defined as the problem of finding the parameters of all admissible controllers represented by Equation (20) such that the H_∞ norm ($\|T_{zw}\|_\infty$) presented by Equation (9)$\|T_{zw}\|_\infty$ is minimized.

Since $K(s) = W1KW2$, therefore $K_\infty = W1^{-1}K(s)W2^{-1}$ and the following equations can be derived:

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + Ps\ K_\infty)^{-1}\tilde{M}^{-1} \right\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + Ps\ K_\infty)^{-1} \begin{bmatrix} I & Ps \end{bmatrix} \right\|_\infty \tag{21}$$

$$J_{\cos t} = \|T_{zw}\|_\infty = \left\| \begin{matrix} \begin{bmatrix} I \\ W1^{-1}K(s)W2^{-1} \end{bmatrix} \times \\ (I + P_s W_1^{-1}K(s)W_2^{-1})^{-1}\begin{bmatrix} I & P_s \end{bmatrix} \end{matrix} \right\|_\infty \tag{22}$$

Equation (22) is defined as the objective function of the optimization problem and it can be evaluated using the robust control toolbox in MATLAB.

4.4. PSO based design

Once an objective function and a structure of the controller are defined, the procedure for solving the optimization problem based on the PSO algorithm is described in 5 steps as follows:

Step 1: Set $x_i = (x_{i1}, x_{i2}, ..., x_{iN}) = (a_0, a_1, ..., b_0, b_1, ...)$. The number of parameters of the controller in Equation (20) is the particle dimension $N=m+n+1$. The maximum number of iterations is defined as *GenMax*.

Step 2: When the swarm size is *H*, initialize a random swarm of *H* particles as $[x_1 \; x_2 \; ... \; x_H]$.

Step 3: The fitness of particles is evaluated by the objective functions of optimization problem. For each generation of particles, evaluate the objective function for each particle using the cost function shown by Equation (22). Furthermore, the best previously visited position of particle *i* is noted as the individual best position $P_i = (P_{i1}, P_{i2}, ..., P_{iN})$. The position of the best individual of whole swarm is noted as the global best position $G = (g_1, g_2, ..., g_N)$. Determine the individual best P_i and global best *G(k)*.

Step 4: Update the particle velocity $v_i = (v_{i1}, v_{i2}, ..., v_{ik})$, and its new position using Equations (23) and (24).

$$v_i(k+1) = \omega \; v_i(k) + c_1 \; r_1 \; (P_i(k) - x_i(k)) + c_2 \; r_2 \; (G(k) - x_i(k)) \tag{23}$$

$$x_i(k+1) = x_i(k) + v_i(k) \tag{24}$$

where ω ω, called inertia weight. r_1, r_2 are the random variables in the range of [0, 1]. c_1 and c_2 are the positive constant acceleration coefficients.x_i is the updated position and $v_i(k+1)$ is the updated velocity of every particle. Velocity is also limited to the range of $[-v_{\max}, v_{\max}]$.

Step 5: When the maximum number of iterations is obtained, the algorithm is ended. If the maximum number of iterations is not obtained, go back to Step3.

5. Results

5.1. H_∞ loop shaping results

The proposed algorithms and procedures presented in section 4 were used to design a controller to control the flexible beam deflection. The algorithms were developed and implemented in MATLAB.

By substitution of the parameters in Table 1 into Equations (1) and (3) to (5), the nominal transfer function of the flexible beam can be described as follows:

$$P = \frac{-6.475s^2 + 4.0302s + 175.77}{5s^4 + 3.5682s^3 + 139.5021s^2 + 0.0929s + 10^{-6}} \tag{25}$$

The weighting function *W1* is selected by some trials for shaping the plant. *W2* is selected as the identity matrix with an assumption that the sensor noise is negligible.

$$W1 = \frac{s+0.5}{s^2 + 2000s + 999998}, \qquad W2 = I \tag{26}$$

By substitution of *W1* and *W2* into Equation (7), and using (10) and (13), finally $\gamma_{opt} = 1.9741$ is obtained. The stability margin is $\varepsilon_{opt} = 0.5066$, and $\varepsilon = 0.5065 \prec \varepsilon_{opt}$ is selected. Using Equations (14) to (16), the full order controller can be described in (27).

$$\begin{aligned} K(s) = {} & (3346s^6 + 6.697\,e6s^5 + 3.335\,e9s^4 + 4.508\,e9s^3 + 9.49\,e10s^2 + 5.943\,e10s + 6.34\,e9)/(s^8 + 5968s^7 \\ & + 1.387\,e7s^6 + 1.582\,e10s^5 + 8.893\,e12s^4 + 1.98\,e15s^3 + 4.687\,e15s^2 + 5.748\,e16s + 2.159\,e16) \end{aligned} \tag{27}$$

The full order controller represented above is of eighth order. Therefore, it is difficult to be implemented in reality. In the next section, implementation of the first order controller design is demonstrated and compared with this one. Figure 3 shows the bode plot of the H_∞ loop shaping controller and Figure 4 shows step response of the closed loop system and its characteristics using the full order controller.

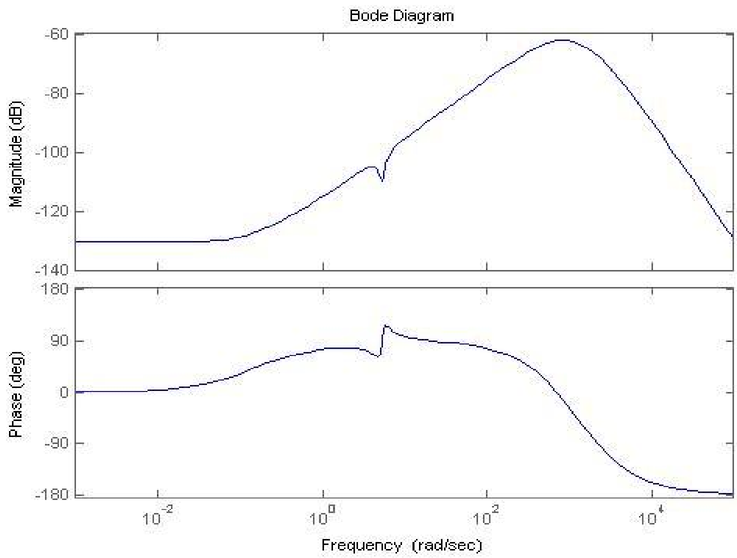

Figure 3. Bode diagram of full order controller

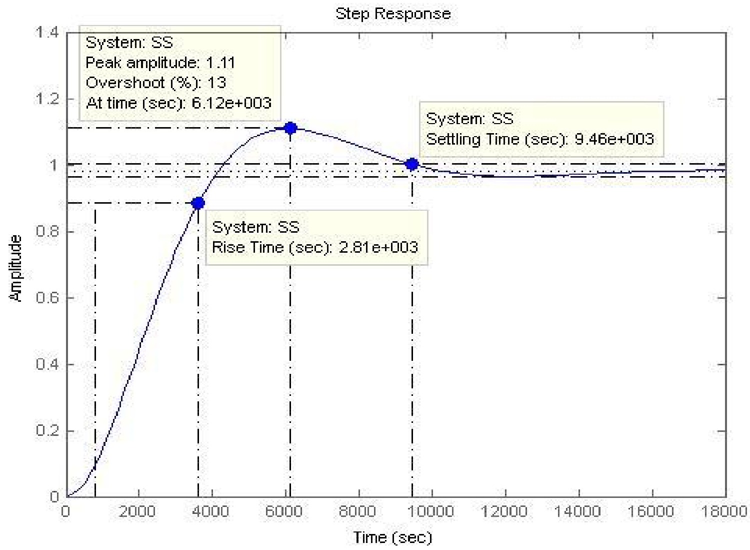

Figure 4. Step response of full order controller and its characteristics

5.2. First order H_∞ loop shaping controller design based on PSO

The first order controller is a structure-specified controller which is represented in Equation (28). The following parameters are selected as: (i) The swarm size is 40; (ii) The dimension of each solution candidate in the first order controller is 2 (a_0, b_0); (iii) $c_1 = c_2 = 2$; and (iv) The maximum number of iterations *GenMax* is 50. The PSO algorithm is used to search for parameters of the controller (a_0, b_0).

$$K_1(s) = a_0 / (s + b_0) \tag{28}$$

In the PSO algorithm, the weight ω is automatically changed so that the algorithm converges slowly to the optimal solution at the end of the searching progress in order to avoid the premature convergence. The initial weight is set to $\omega = 0.9\,\omega$, and the final weight is set to $\omega = 0.4\,\omega$. The value of cost function is obtained as follows: $J_{\cos t} = \gamma_{opt} = 1(\varepsilon_{opt} = 1)$ .The obtained controller is shown by Equation (29).

$$K_1(s) = 346.8327 / (s + 0.1) \tag{29}$$

Figure 5 shows the bode diagram of full order controller and the first order one in two different colors. Figure 6 presents the step response of the closed loop system and its characteristics using the obtained first order controller.

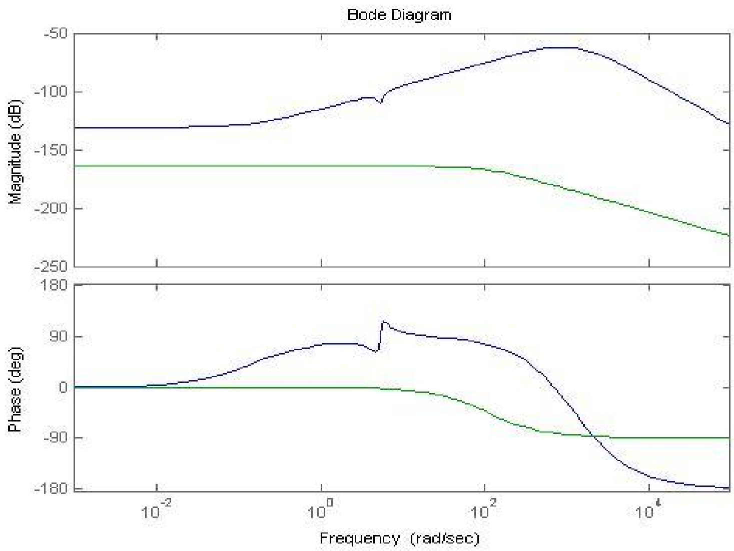

Figure 5. Bode diagram of full order controller and first order one

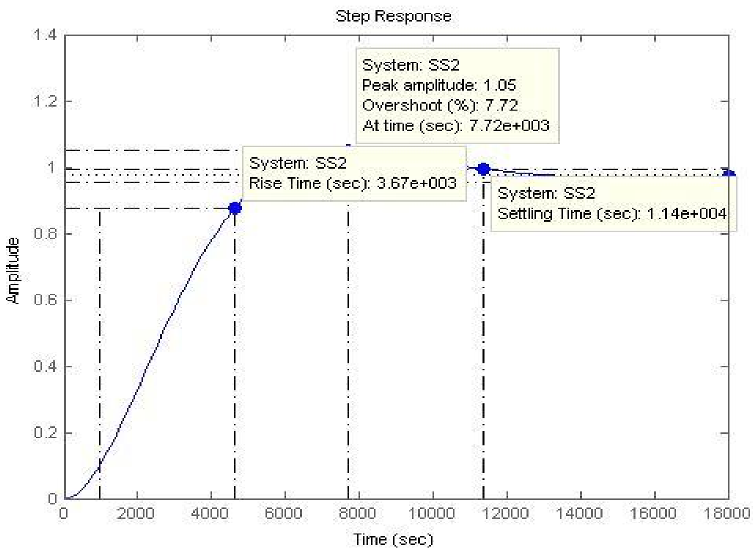

Figure 6. Step response of first order controller and its characteristics

Convergence of the structure-specified algorithm can be seen in Figure 7.

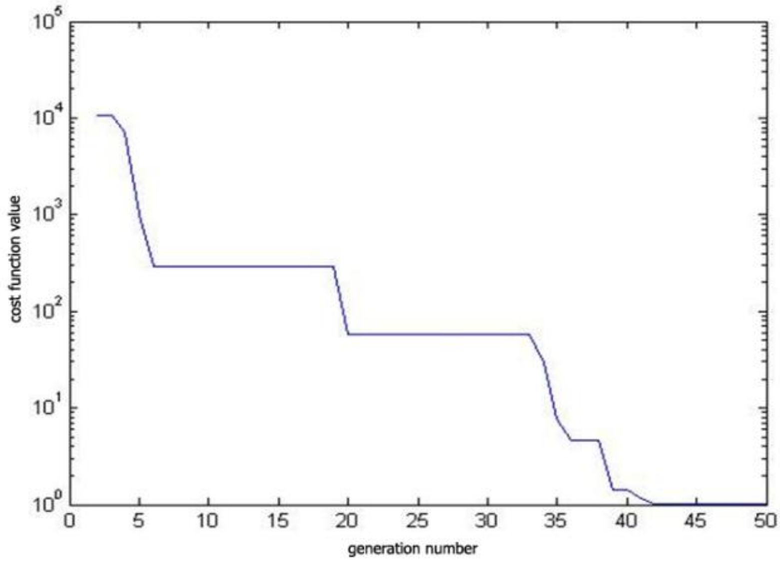

Figure 7. The cost function value versus the generation number of simulation

5.3. Comparison between the full order and first order controller

Bode diagrams of the two controllers show the stability of them. Step responses of the two systems which correspondingly use the conventional H_∞ loop shaping controller and the proposed structure-specified H_∞ loop shaping one are almost similar, but first order controller performance is better than the conventional full order one because of these reasons: the first order controller has a smaller maximum overshoot (7.72%), whereas this value is 13% using full order controller; and the overshoot is very important because the system will be oscillated and become unstable if the overshoot is too large. In addition, stability margin of the conventional controller is 0.5065, while the same value for the structure-specified controller is 1. Another significant superiority of the proposed approach over the conventional method is having lower order controller.

5.4. Comparison between PSO based algorithm and GA based one

In order to compare the computational time of PSO based and GA based algorithms, the MATLAB Genetic Algorithm and its Direct Search Toolbox were used for adapting the first order controller with the following set up: (i) The population size is 40, (ii) The crossover fraction is 0.8, (iii) The Gaussian mutation is used (iv) The programs were run for ten trials on a core i2 CPU, 2.4 GHz, 4GB RAM computer. For PSO the average computational time of ten trials was 200 seconds, and it was 440 seconds for GA based approach.

5.5. Robustness of the proposed controller

Robustness of the proposed controller is tested against uncertainties of the system transfer function. Inaccurate measurement, external disturbances and un-modeled dynamics may lead to model uncertainty. As a test case, damping of two modes, ζ_i are increased to the values of 0.012 and 0.009. Gain parameter K is also increased to 10^4 [9]. Transfer function with uncertainties is shown in Equation (30).

$$P_\Delta = \frac{-5.1s^2 + 5.021s + 201.01}{5.061s^4 + 4.4412s^3 + 127s^2 + 0.1s + 0.005} \quad (30)$$

The step response of the closed loop system using the proposed first order controller for this test case is shown in Figure 8. The results prove that the designed first order controller is robust to the parameter variations, so that the system stability is attained through the experiment.

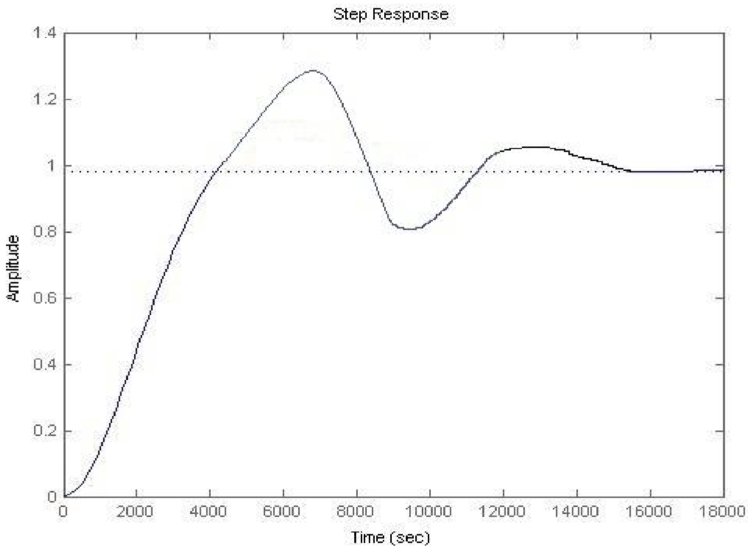

Figure 8. Step response of first order controller under uncertainties

6. Conclusion

Full order H_∞ loop shaping controller and structure-specified controller are designed for the flexible beam system in this paper, and results are finally compared to prove the effectiveness and superiorities of the proposed method for the beam system. According to the results, stability margin resulted from the proposed controller is better than the same value from the old method. It is revealed that the designed controller guarantees robustness as well. On the other hand, novel H_∞ loop shaping controller has lower order and less complicated structure. Also, comparing the maximum overshoots, rise time and settling time, the structure-specified design showed better results than the conventional method. PSO algorithm simplified solving non-convex nonlinear optimization problem associated with fixed-structure controller in this process. Collectively, the proposed approach is more suitable to implement for practical beam control.

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