Evaluating of Feasible Solutions on Parallel Scheduling Tasks with DEA Decision Maker

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Abstract

This paper surveys parallel scheduling problems and metrics correlated to and then applys metrics to make decision in comparison to other policy schedulers. Parallel processing is new trend in computer science especially in embedded and multicore systems whereas needs more power consumption to reach speed up. The QOS requirement for users is to have good responsiveness and for service providers or system owners to have high throughput and low power consumption in parallel processing or embedded multicore systems. Moreover, fairness is vital issue to make decision wether the scheduler is good or not. Using the metrics is very intricate because misleadling metrics will cause to lose performance and system utility that is why the metrics has been opted cautiously in this paper. However, satisfying all of the objects in which have potentially conflicts is computationally NP-Hard. So, tradeoff between metrics is needed. This paper indicates DEA FDH model based on linear programming that will select the optimal scheduling near to exact solution.

***Keywords*:** *Data Envelopement Anlysis, Linear Programming, Multicore System, Scheduling.*

1. Introduction

The scheduling is1sue is vital problem in both multiprocessor and embedded mulicore systems[1] especially encountering with some constraints like power limitation, bandwith boundaries and minimum throughputs.The goal is to use resources efficiently for instance the user of the multiprocessor,multicore systems or systems in the internet providing services such as clouds[2] needs reasonable responsiveness and on the other hand, service providers need to have high throughputs and lower power consumption simultaneously and if the users do not observe good QOS they may change the providers[3]. Poor QOS is directly related to trivial scheduling that is why the scheduling is very important issue in aforementioned areas. Finding optimal scheduling in this circumstance is computationally NP-Hard[4] especially to fulfill multi objectives that have potentially conflicts for instance if the object is to rise throughput it definitely imposes more energy consumption or shrinking the makesapn makes some tasks violate their due dates. Service providers and their users consider different goals therefore many papers with heuristic approach have been published to reach these objects[5,6,7]. There are a lot of objectives in both users' point of view and system owners' prespective such as flow, peak in flight, Mean stretch, Mean SLR, Average Utilisation and so on [3]. For real time circumstance such as cloud the majority of aforementioned metrics is misleading and selecting the exact metrics have drastic affection on performance

analysis. In real time cloud the tasks have deadline and the vital metric that user observes is total execution time or makespan and for system owners the turnaround time is very eminent represented by cumulative completion time that's why in this paper, the scheduling problem with satisfying to reduce makespan and to minimize tardy tasks for users and to ameliorate throughputs and to minimize cumulative completion time for service providers have been taken into consideration.The rest of the paper organized as follows.In section 2, DEA FDH literature will be introduced and section 3 discusses about scheduling problem and application of DEA to select optimal solution and afterward remarkable outcomes will be brought in result section, i.e., section 4.

2. Literature of DEA FDH

For the sake of evaluation of feasible solutions, a mathematical model has been proposed. Itshows that how the science of data envelopement analysis, DEA in short, based on linear programming can help us as decision making tools to excerpt the optimal scheduling solution according to our objective functions[8].

DEA as a strong tool thathas had great achievement in recent years for decision making scopes in some counteries whereas it has been applied in variety fields [15]. One of the reasons of DEA's prosperity is to have complex relation between multi inputs and multi outputs activities in intricate systems. In other words whenever there is any discussion about system performance all analytical method befor DEA have had drawbacks that could not represent an exact assessment. Nowadays, application of DEA is being widely used for evaluation of systems in which have multi inputs and multi outputs such as broad range of systems like schools, universities, hospitals through robots, software systems, airplanes, nuclear syatems and etc. It had triumph and good outcomes for example in the united states airports for opimal finding of existing capacity it has been applied for awhile and it has made money saving and has reduced expenditures. The lifetime of DEA dated to 30 recent years, but it has had drastic affection on the affairs in which these have correlation to decision making. By referring to authentic and well known magazines,many marvellousapplication of DEA can be seen. Assume there are$n$ decision making units, $DMUs$, in which each of them consumes m inputs and produces s outputs. EachDMU_j, jth decision making unit, so that ($j=1,...,n$) is a bidimentional vector $DMU_j = (x_j, y_j)$ whereas $x_j = (x_{1j},...,x_{mj})$is the input vector and $y_j = (y_{1j},...y_{sj})$ is the output vector. To evaluate each of n units' performance, the unknown weight for every input and output must be found whereas the division of output'swighted sum to input'swighted sum to be maximized in eachDMU_k. It means below:

$$\max \frac{u_1 y_{1k} + ... u_s y_{sk}}{v_1 x_{1k} + ... v_m x_{mk}} \tag{1}$$

Note that, maximum value of relation (1) is calculated according to selected weights on (u,v) whereas $u = (u_1,...,u_s)$ and $v = (v_1,...,v_m)$are the weights given to output and input of system respectively. One of the reasons that made DEA prosperous is not to

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intervene to select weights, i.e., weights will be attained via model solution, but relation (1) has the constraints below:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 , \quad j = 1,...,n \tag{2}$$

It means with considering high boundary for objective function, weights can be found whereas the abovementioned forlmula will be maximized for DMU_k. Therefore it can be transformed to new mathematic model:

$$\max \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$$

$$s.t.$$

$$\begin{cases} \dfrac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 , j = 1,...,n \\ u_r \geq 0, r = 1,...s \\ v_i \geq 0, i = 1,...m \end{cases} \tag{3}$$

Model (3) is corresponded to model (4) [8].

$$\theta_k^* = \max \ \theta_k = \mu_1 y_{1k} + ... \mu_s y_{sk}$$

$$s.t.$$

$$\begin{cases} \delta_1 x_{1k} + ... + \delta_m x_{mk} = 1 \\ (\mu_1 y_{1j} + ... + \mu_s y_{sj}) - (\delta_1 x_{1j} + ... + \delta_m x_{mj}) \leq 0 , \ j = 1,...,n \\ \mu_r \geq 0, r = 1,...,s \\ \delta_i \geq 0, i = 1,...,m \end{cases} \tag{4}$$

Optimal solution of Model (4) is the performance of DMU_k. Indeed, fair comparison between all *DMUs*indicates the maximum value of performance of*k*th decision making unit is θ_k^* that is the optimal solution of Model(3). Dual of Model(4) can be presented as Model(5):

$$\theta_k^* = \min \ \theta_k$$
$$s.t.$$

$$\begin{cases} \sum_{j=1}^{n} \lambda_j x_j - x_k \theta_k \leq o_m \\ \sum_{j=1}^{n} \lambda_j y_j \geq y_k \\ \lambda_j \geq 0, j = 1,...,n \end{cases} \tag{5}$$

By solving the Model (4) and Model (5) the efficient-one decision making unit will be determined [9]. By solving Model (4) for DMU_k, it gives optimal solution in (λ^*, θ^*) whereas:

$$E_k = \{ \ j : \lambda_j^* > 0 \ \} \tag{6}$$

The set E_k includes all indices of efficient units which is the reference set of DMU_k for instance if the units A and B are efficient and in solving the Model (5) for unit of C there are $\lambda_A^* = 0.5$, $\lambda_B^* = 0.7$ then virtual unit $(\lambda_A^* x_A + \lambda_B^* x_B, \lambda_A^* y_A + \lambda_B^* y_B)$ is constructed so that the considered unit DMU_k will be compared with it. As Model(5) has a drawback, that is, it may happen an inefficient unit will be compared with composition of real efficient units in which none of them is not observable that is why Model(7) is proposed so that the only difference is to have binary values for λ_j.

$$\theta_k^* = \min \ \theta_k$$
$$s.t.$$

$$\begin{cases} \sum_{j=1}^{n} \lambda_j x_j - x_k \theta_k \leq o_m \\ \sum_{j=1}^{n} \lambda_j y_j \geq y_k \\ \lambda_j \in \{0,1\}, j = 1,...,n \end{cases} \tag{7}$$

3. Applying DEA FDH on Multi objective Scheduling

As been mentioned, scheduling problem is crucial both for multiprocessor and multicore systems.Our problem attributes are brought in Table 1. Each task has execution timeand due dates in which scheduling tasks after due date is not beneficial that is why in some feasible schedulingdepicted hatching in figure1 through figure 3. Our objective functions are to minimize makespan and to maximize the cumulative completion.Moreover, $C_{\max} = \max\{ C_j \mid T_j \in \tau \}$ is makespan whereas C_j is completition

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timefor task T_j , throughput is the number of tasks executing per time tick and cumulative completion is calculated via $\sum_{j=1}^{n} (1 + C_{max} - t_j) * t_{exec}^{j}$ for each task $t_j \in \tau$ [10, 11, 12, and 13]. In addition, cumulative completion gives partial insight into throughput and system utilization.Because high value of this metric shows that more tasks finished sooner[14].

Table1. Task Charactristics

Task No.	t_j	Duedate(t_j)
t_1	1	2
t_2	1	2
t_3	2	3
t_4	3	4

The feasible solutions for given set of tasks are illustrated in figure1 through figure3. To find optimal solution in which fulfill objective functions the method DEA FDH model has been applied and each feasible solution is used as *DMU* thereon. In this model, two inputs have been consumed as tardy tasks, makespan respectively and it produces two outputs as throughput and cumulative completion respectively as in Table2.

Table2. DEA FDH parameters correspond to our problem

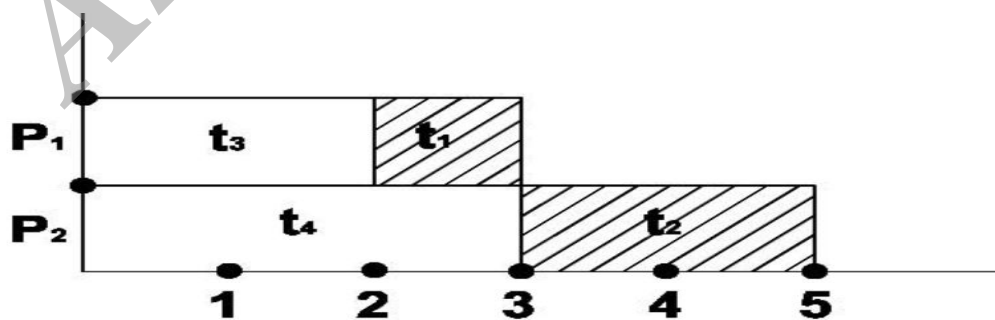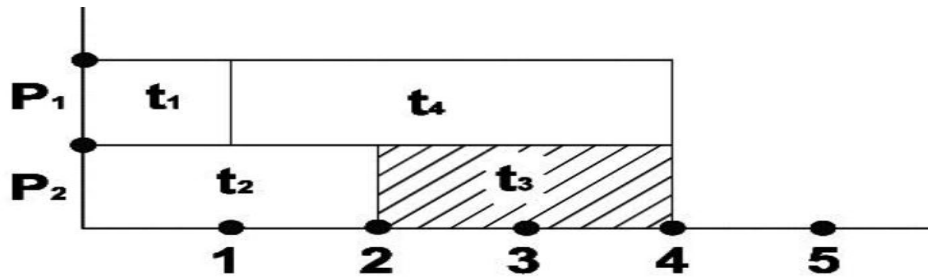| DMU No.	Tardy tasks (I_1)	Makespan (I_2)	Throughput (O_1)	Cumulative completion (O_2)
Feasible solution1	2	5	40(percent)	14
Feasible solution2	1	4	75(percent)	15
Feasible solution3	2	5	40(percent)	14

Figure1. Feasible solution1
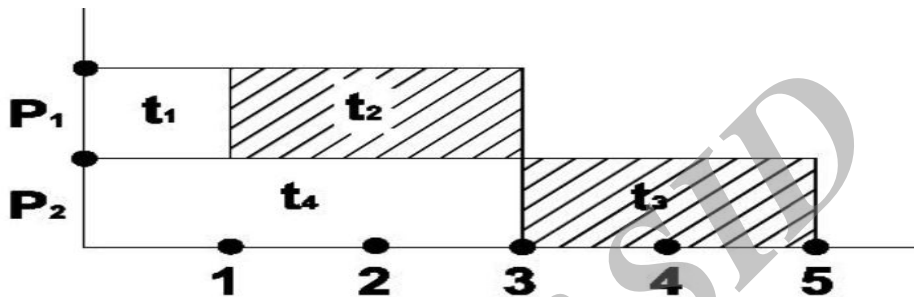
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Figure2. Feasible solution2

Figure3. Feasible solution3

It can be seen after modeling feasible solutions in DEA FDH. For instance, the first unit is modeled as below to be solved.

$$\theta_1^* = \min \theta$$

$$s.t.$$
$$-2\lambda_1 - \lambda_2 - 2\lambda_3 + 2\theta \geq 0$$
$$-5\lambda_1 - 4\lambda_2 - 5\lambda_3 + 5\theta \geq 0$$
$$-40\lambda_1 - 75\lambda_2 - 40\lambda_3 + 40\theta \geq 0$$
$$14\lambda_1 + 15\lambda_2 + 14\lambda_3 \geq 14$$
$$\lambda_1, \lambda_2, \lambda_3 \in \{0,1\}$$

After executing the aforementioninequitions in DEA-solver based on mathematical definition and designed by [9] the reference set of unit1 is unit2 because $(\lambda_1^*, \lambda_2^*, \lambda_3^*, \theta_1^*) = (0,1,0,1)$. If this method is used for unit3 it will also refer to unit2. Consequently, the optimal solution is unit2 or the second feasible solution.

4. Conclusion

To sum up:scheduling in multiprocessor and multicore systems for satisfying multi objectives is NP-Complete problem because some of metrics have potentially conflicts and needs to be compromised. Moreover, selecting the exact metrics is very crucial issue in both users' and service provider's prespective to assess the system performance accurately. To reach the goal selecting tardy tasks and makespan to be minimized for user, utilization and cumulative completion time to be optimized for service providers have been considered. For evaluating and selecting the optimal solution in which near to

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exact QOS requirement for both user and service provider point of view the method DEA FDH model has been applied and the result illustrates this technique excerpts the best one.

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