Adaptive neural network observer based synchronization control of uncertain chaotic system

Milad Malekzadeh[⊠1],Esmaeil salahshour[1]

1) Young Researchers and Elite Club, Ayatollah Amoli Branch, Islamic Azad University, Amol, Iran

Milad.malekzade@gmail.com; esmaeil.salahshoor@yahoo.com

Abstract

This paper addresses a nonlinear observer based control scheme to synchronize chaotic systems subject to uncertainties and external disturbances. It is assumed that the dynamic of slave system is not completely known. In order to compensate for the system perturbation resulting from parameter variations and mismodeling phenomena, an adaptive neural network observer is employed to handle this problem. A nonlinear observer for a class of nonlinear systems is proposed based on a generalized dynamic recurrent neural network. The weights of the proposed neural network in the observer are tuned on-line with no off-line learning phase required. Also, no exact information of the nonlinear term of the system is required and this important characteristic compensates considerable part of uncertainty. To realize control purpose, two controllers are considered. At first, PID controller is combined with proposed observer and then 2nd order sliding mode controller called twisting algorithm is applied to synchronize systems. This method is implemented on the Duffing chaotic systems and simulation results confirm the effectiveness of the proposed method.

Keywords: adaptive observer, neural network, chaos, synchronization.

1. Introduction

Synchronization of chaotic systems has attracted the attention of scientists from different fields in the last decade. Nowadays, this topic is used in many useful practical projects such as secure communication [1]. During recent years different methods were used to synchronize chaos, such as PC method [2], active control [3], adaptive control [4], time-delay feedback approach [5], backstepping design method [6], sampled-data feedback synchronization method [7] and etc.

Usually synchronization methods use master-slave configuration. This means a specific chaotic system is called master whilst the other under-controlled dynamic establishes the slave. The aim of the synchronization is forcing the output of the slave system to track the output of the master asymptotically. This will be achieved using the error between outputs those two systems, as a control signal to the slave one. In the synchronization task, the aim is to stabilize the error such that difference between the states of the two master and slave systems converges to zero asymptotically. But this definition will be feasible when the dynamics of master and slave system are completely known. While enough information of slave system is not available, the designer should add a nonlinear observer to solve this problem [8-12]. In this article, we assumed that the identification of slave system is incomplete and no exact information of the

nonlinear term of the system is attainable. In order to unravel this problem, an adaptive neural network observer is considered to estimate the states of systems. This observer can estimate the nonlinear term of the systems without exact information of them and this characteristic will compensate mismodeling. Also the neural network can estimate any nonlinear system. The training process of neural network is implemented online and offline, but online weight adjusting is so useful and can be implemented in experimental projects. The performance of proposed observer is online [13].

To realize control scheme, we consider two controllers. At first, PID controller is combined with observer in close loop structure and then a 2nd order sliding mode controller called twisting algorithm [14] will be substitute instead of PID.
This paper is organized as follows:

In section 2, basic definition of adaptive neural network observer is presented. The structure of twisting algorithm is rendered in section 3. Then in section 4, the proposed method is applied to slave system to synchronize with master system and then the simulation results will be depicted. Conclusion of paper is given in section 5.

2. Adaptive Neural Network Observer

Consider the following single input-single output system assuming that pair of (A, C) is observable i.e. of a canonical observer form [13]:

$$
\dot{x} = Ax + b[f(x) + g(x)u + d(t)]
$$
$$
y = C^T x
$$

(1)

That $x \in R^n, y \in R, u \in R, b \in R^n$ and $d(t)$ is the unknown disturbance with known upper bound and $f, g : R^n \rightarrow R$ unknown smooth function.

The linear system is defined as an observer canonical form if A and C are given as follows:
$$
\dot{x} = Ax
$$
$$
y = C^T x
$$

$$
A = \begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ & & & & \\ 0 & 0 & ... & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 \\ 0 \\ . \\ . \\ 0 \\ 0 \end{bmatrix}
$$

(2)

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However there is no restriction on the input matrix coefficient i.e. b. The observer dynamic as a replica of the system states but includes a correction term which is considered as follows:

$$\dot{\hat{x}} = A\hat{x} + b[\hat{f}(\hat{x}) + \hat{g}(\hat{x})u - v(t)] + k[y - C^T \hat{x}]$$
$$\hat{y} = C^T x \tag{3}$$

That $\hat{x}$ denotes the estimates of state x and $K = [k_1 k_2 ....k_n]^T$ is the observer gain chosen where $(A - KC^T)$ is strictly Hurwitz. $v(t)$ is robustifying term to control disturbance.

The neural network equation that used in this observer is as follows:

$$f(x) = W_f^T \sigma_f(x) + \varepsilon_f$$
$$g(x) = W_g^T \sigma_g(x) + \varepsilon_g \tag{4}$$

Where consist of two layers, the weight of first layer will be $V = I$ but the second layer weight must be tuned. The architecture of the proposed neural network is shown in Figure1:
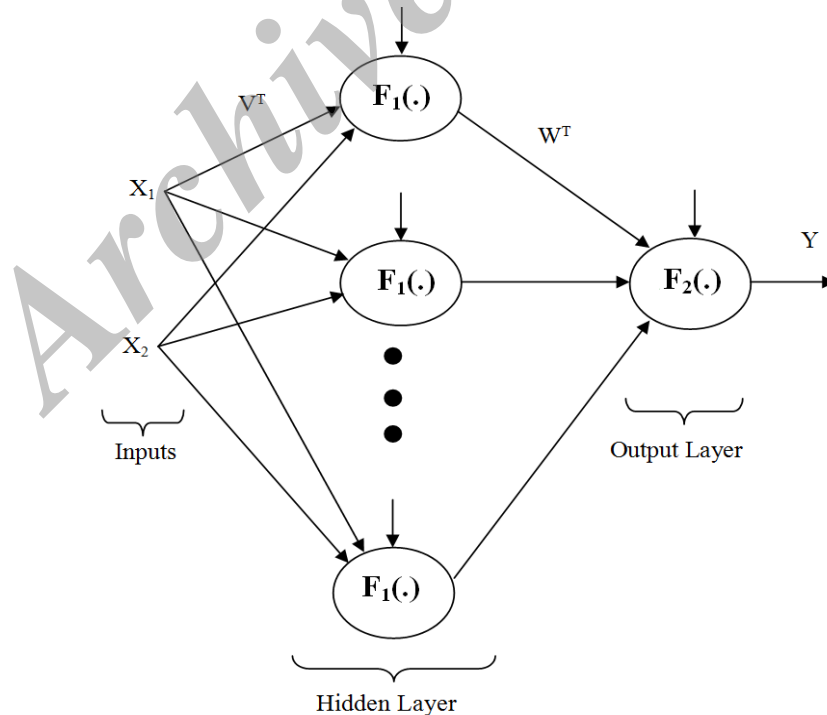

Figure 1. architecture of the proposed neural network

One of the advantages of neural network that is used in this paper is that there is no need to data for training. In fact, in [13]; an equation is proposed for training the

network using the equations correspondent to the stability of system and provides the value of network weight for system at each moment. Training of this neural network for two nonlinear terms is achieved from the following differential equation:

$$\dot{\hat{W}}_f = F_f \ \hat{\sigma}_f \ \tilde{y} - k_f \ F_f \ |\tilde{y}| \hat{W}_f$$
$$\dot{\hat{W}}_g = F_g \ \hat{\sigma}_g \ \tilde{y} \ u - k_g \ F_g \ |\tilde{y}| \hat{W}_g$$

(5)

Where σ denotes the activation function and $\tilde{y} = y - \hat{y}$. Activation function is applied in the neural networks in different ways. In this observer the activation function for first layer is chosen sigmoid and for second layer is chosen purline. Also the number of nodes in second layer is chosen 20. Figure2 presents the block diagram of this observer:
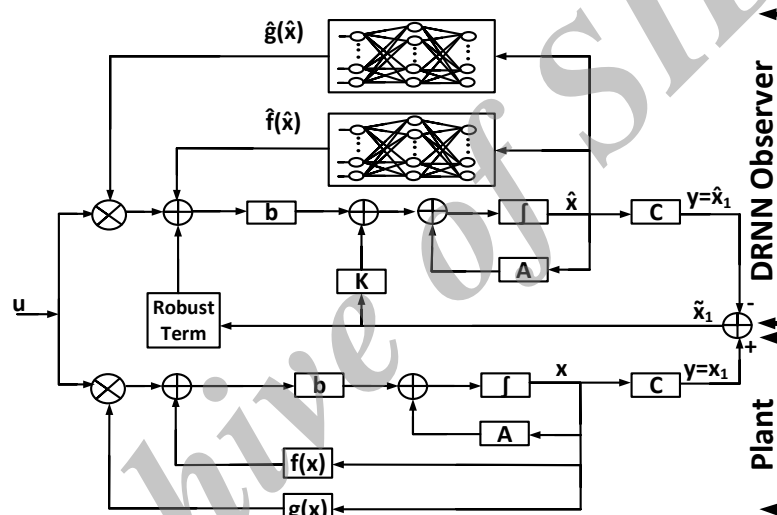
Figure 2.block diagram of adaptive neural network observer [13]

3. Twisting Algorithm

This algorithm is a systematic algorithm to apply for dynamic with relative degree one and two. The dynamic is represented in the following state space format:

$$\dot{x} = Ax(t) + Bu(t), \qquad y = Cx, \qquad e = r - y$$

(6)

where $x \in R^n$, $u \in R^1$, r are state, control effort and the command signal respectively. The output is denoted by y and e stands for the error signal. The control signal u of the twisting algorithm for system with relative degree two is presented as follows [14]:

$$u(t) = c_1.\text{sgn}(e) + c_2 \, \text{sgn}(\dot{e})$$

(7)

Where c_1, c_2 are positive coefficients. The block diagram of twisting algorithm is presented in Figure3:
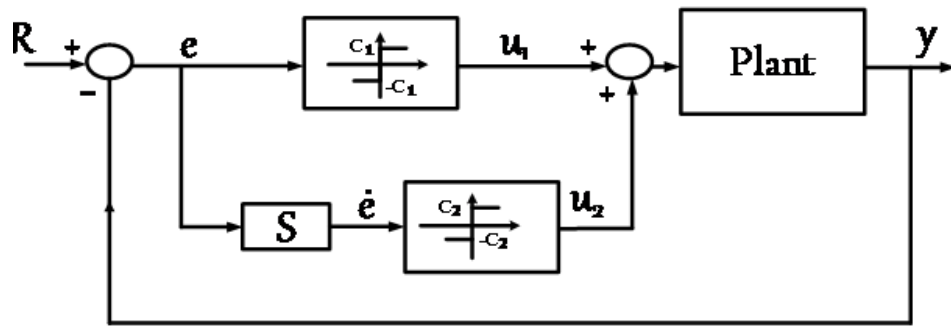
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Figure 3.block diagram of twisting algorithm [14]

4. Simulation

This section presents the simulation of the proposed method with Matlab software. As mentioned, two Duffing chaotic systems with different initial condition are considered to synchronize by proposed method while it is assumed the equation of slave system is not accurate. So the observer is employed to estimate the trajectory of slave systems. The equation of Duffing system is presented in the following equation [15]:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos t \tag{8}$$
$$y = x_1$$

The initial condition of master and slave systems are considered $x_m(0) = [3 \quad 1]$, $x_s(0) = [2 \quad 2]$ respectively.

The chaotic behavior of Duffing system is depicted on Figure 4:
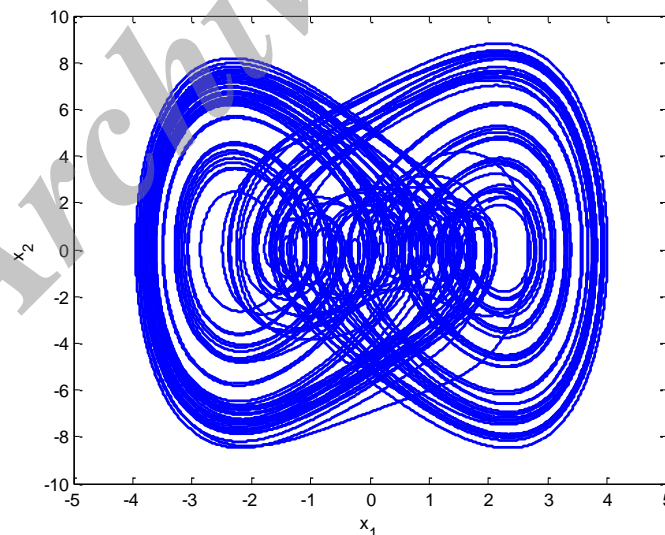

Figure 4.chaotic behavior of Duffing system in 250 seconds

According to proposed method, the block diagram of the control scheme is rendered in Figure 5:
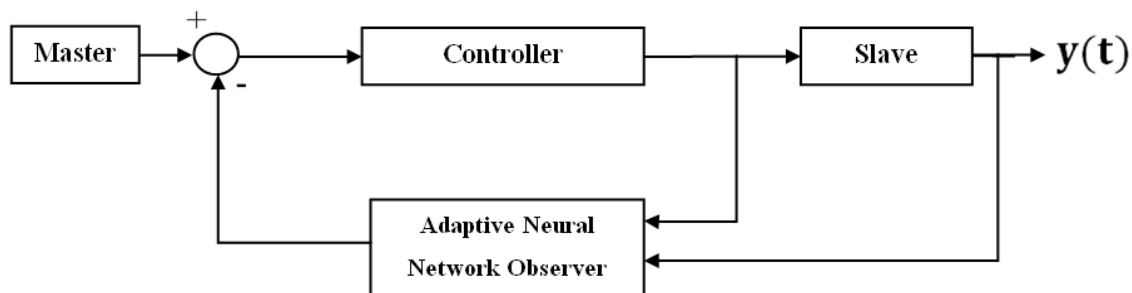
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Figure 5.block diagram of control scheme

Preliminary, PID controller is implemented to synchronize these systems. The equation of controller is defined as follows:

$$u(t) = (k_p + \frac{k_i}{s} + k_d s) e \tag{9}$$

The error of system is set the difference between first state of master and slave system. So we have:

$$u(t) = k_p(e) + \frac{k_i}{s} e + k_d(\dot{e}) = k_p(x_{1m} - \hat{x}_{1s}) + \frac{k_i}{s}(x_{1m} - \hat{x}_{1s}) + k_d(x_{2m} - \hat{x}_{2s}) \tag{10}$$

Where k_p, k_i, k_d is constant.

By applying control effort of PID to slave system, the synchronization will be happened. The control input of PID controller is shown as Figure 6 and the synchronizations of states are depicted on Figure 7 and Figure 8:
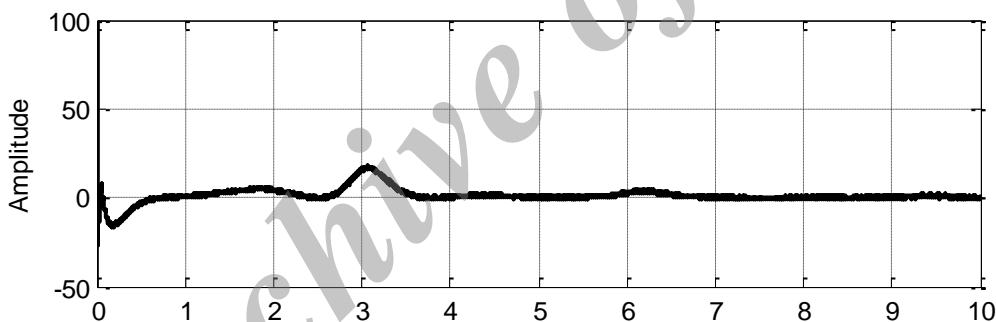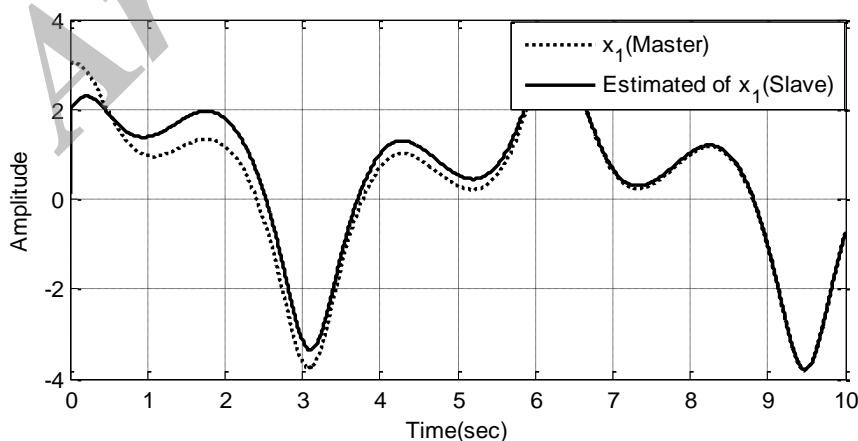

Figure 6.control input (PID)

Figure 7.synchronization of x_1
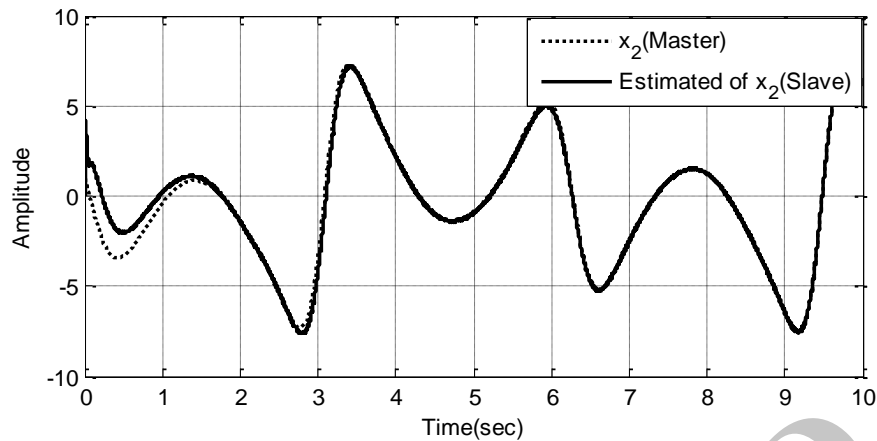
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Figure 8.synchronization of x_2

The estimation errors of states are depicted in Figure 9& Figure 10:
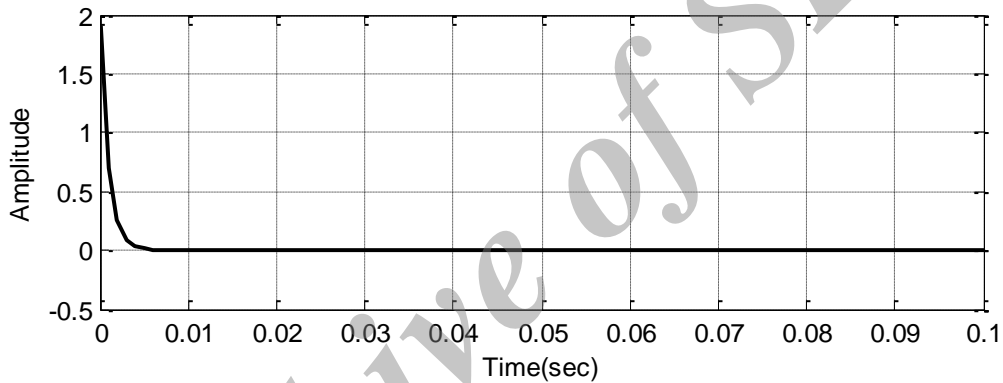

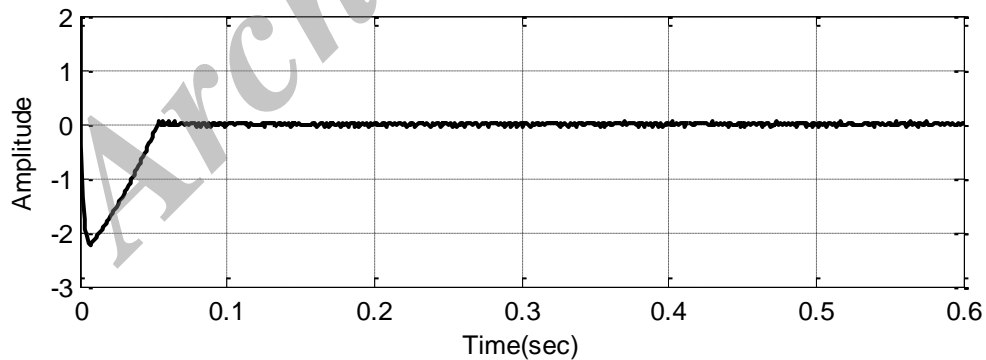*Figure 9.stimation error of* x_1

Figure 10.stimation error of x_2

Now the performance of twisting algorithm will be evaluated. According to () the control effort of this controller is made as follows:

$$u(t) = c_1.\text{sgn}(e) + c_2\,\text{sgn}(\dot{e}) = c_1.\text{sgn}(\text{x}_{1m} - \hat{\text{x}}_{1s}) + c_2\,\text{sgn}(\text{x}_{2m} - \hat{\text{x}}_{2s}) \tag{11}$$

By applying control effort of twisting algorithm to slave system, the synchronization will be happened. The control input of twisting algorithm is shown as Figure 11 and the synchronizations of states are depicted on Figure 12 and Figure 13:
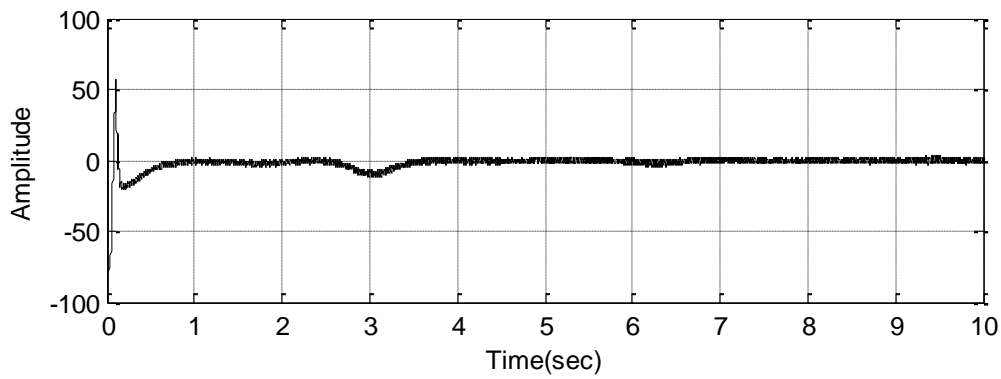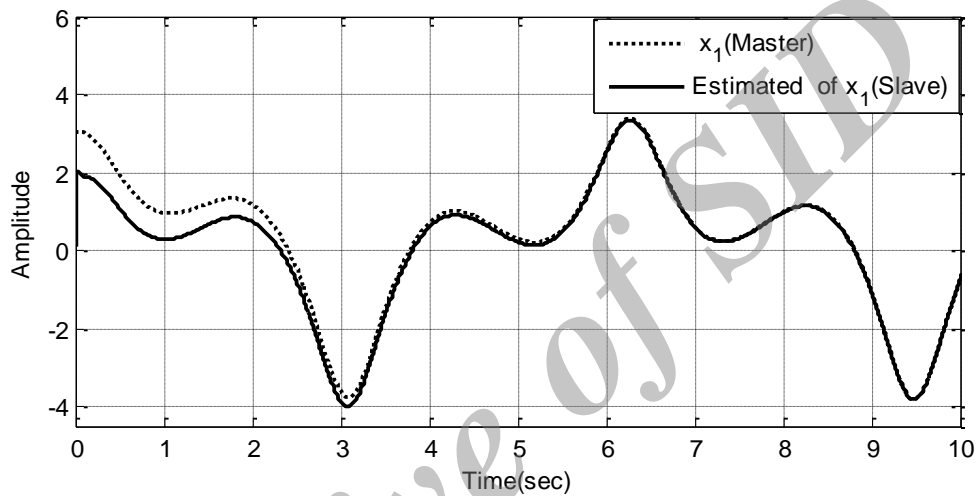
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Figure 11.control input (Twisting algorithm)

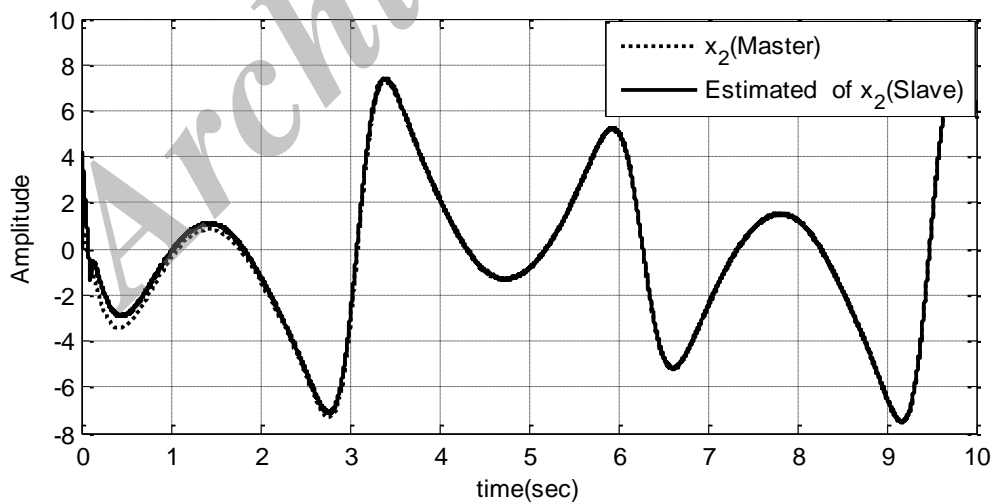

Figure 12.synchronization of x_1

Figure 13.synchronization of x_2

The estimation errors of states are depicted in Figure 14 and Figure 15:
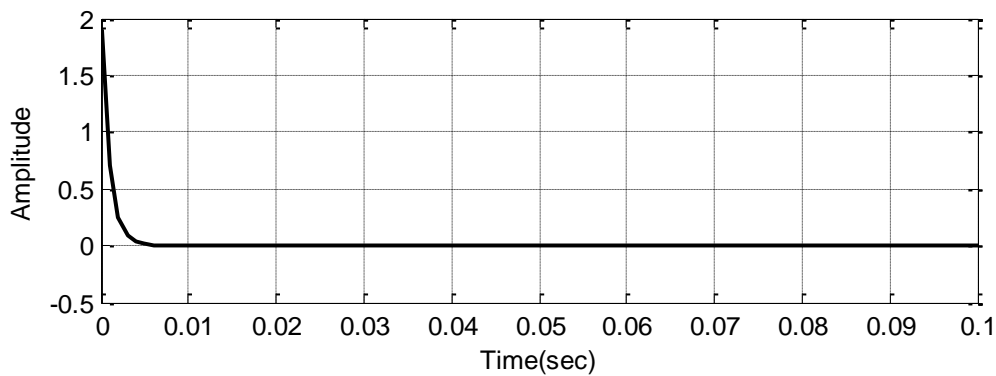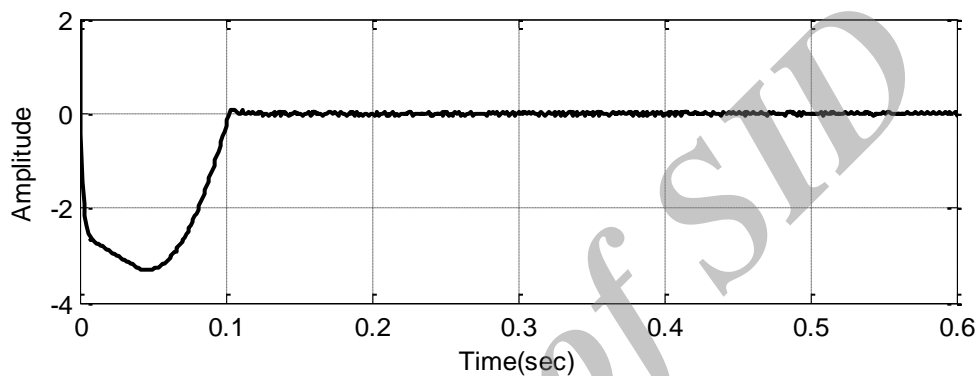
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Figure 14.stimation error of x_1

Figure 15.stimation error of x_2

The performances of two controllers with observer are acceptable and the simulation results confirm this claim. The comparative analysis of two controllers with observer is presented in the table 1:

Table 1.comparison study between Twisting algorithm and PID controller with proposed observer

RMSE	Control input	Estimation(x_1)	Estimation(x_2)	Synchronization(x_1)	Synchronization(x_2)
twisting	7.1613	0.020432	0.2687	3.3937	7.2335
PID	7.8531	0.20432	0.053902	3.5797	7.2789

5. Conclusion

In this paper, an observer-based control scheme has been presented to synchronize chaotic systems. It has been supposed that the nonlinear term of the salve system is not known. In order to access the dynamic of slave system, an adaptive neural network observer has been employed. Also PID controller and Twisting algorithm have been combined by proposed observer respectively, to realize synchronization purposes. The performances of these controllers have been depicted in simulation results and accuracy of them has been demonstrated by numerical analysis. Simulation results present the capability of proposed method.

References

[1] L.Kocarev, U.Parlitz "General approach for chaotic synchronization with applications to communication", Physical review letters, vol. 74, pp 5028-5031, .1995.

[2] Pecora, L.M., Carroll, T.L, 'Synchronization in Chaotic Systems', Physical Review Letters, 64, pp. 821-824(1990).

[3] Chen, H. K., 'Global Chaos Synchronization of New Chaotic Systems via Nonlinear Control', Chaos, Solitons and Fractals, 23, pp. 1245-1251, (2005).

[4] Zhi-Yuliu, Chia-Juliu, 'Synchronization of Uncertain Hyperchaotic and Chaotic Systems by Adaptive Control', International Journal of Bifurcation and Chaos, Vol. 18, No. 12 (2008) 3731–3736.

[5] Park, J. H., Kwon, O. M., 'A Novel Criterion for Delayed Feedback Control of Time-Delay Chaotic Systems', Chaos, Solitons and Fractals, 17, pp. 709-716, (2003).

[6] Wu, X., Lü, J. 'Parameter Identification and Backstepping Control of Uncertain Lü System', Chaos, Solitons and Fractals, pp. 18, (2003).

[7] Murali, K., Lakshmanan, M. 'Secure Communication Using a Compound Signal Using Sampled-Data Feedback', Applied Mathematics and Mechanics, 11, pp. 1309-1315, (2003).

[8] W.Jiang, QiaoGuo-Dong, Deng Bin "Observer-based robust adaptive variable universe Fuzzy control for chaotic system", chaos , solitons and fractals, 2005,.Vol. 23, pp. 1013–1032.

[9] Z.Zhang, H.Shao, Z.Wang, H.Shen "Reduced-order observer design for the synchronization of the generalized Lorenz chaotic systems", applied mathematics and computation , 2012,.Vol. 218, pp. 7614–7621.

[10] B.Sharna, I.N.Kar "Observer-based synchronization scheme for a class of chaotic systems using contraction theory", nonlinear dynamics, 2011,.Vol. 63, pp. 429–445.

[11] M.A.Khanesar, M.Teshnehlab, O.Kaynak "Observer-based indirect model reference fuzzy control system with application to control of chaotic systems", journal of franklin institute , 2013,.Vol. 350, pp. 419–436.

[12] H.Adloo, N.Noroozi, P.Karimaghaee "Observer-based model reference adaptive control for unknown time-delay chaotic systems with input nonlinearity", nonlinear dynamics, 2012,.Vol. 67, pp. 1337–1356.

[13] Young H.Kim,Frank L.Lewis and Chaouki T.Abdallahs "A dynamic recurrent neural network based adaptive observer for a class of nonlinear systems", Automatica,Vol 33,no 8,pp 1539-1543,1997.

[14] Igor Boiko, Ivan Castellanos, and Leonid Fridman, " Analysis of response of second-order sliding mode controllers to external inputs in frequency domain", Int, J. Robust nonlinear control, Vol.18, pp 502-514, 2008.

[15] Wang Jiang,QiaoGuo-Dong,Deng Bin, "Observer-based robust adaptive variable universe fuzzy control for chaotic system", Chaos, Solitons and Fractals. , Vol 23, pp 1013-1032,2005.