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Hydrodynamic damped pitch motion of tension leg platforms

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ABSTRACT: Because of fluctuation in leg tension, pitch motion is very effective fatigue and life safety of leg elements in tension leg structures (TLSs). In this paper an exact solution for pitch vibration of a TLS interacting with ocean wave is presented. The legs of TLP are considered as elastic springs. The flow is assumed to be irrotational and single-valued velocity potentials are defined. The effects of radiation and scattering are considered in the boundary value problem. Because of linear behavior of legs during wave excitation, ignoring coupling effects with other degrees of freedom, the analytical solution of pitch response has good agreement with the real behavior of the structure.

Keywords: *Hydrodynamic; Tension Leg Structure (TLS); Pitch; Wave; Radiation; Scattering*

INTRODUCTION

The TLS is a kind of compliant-type structure vertically moored in the ocean. It can be a system for oil exploration in deep water and moored reservoir as well as a wave breaker in shallow water. The structure is considered as a rigid body having six degrees-of-freedom, namely surge, sway, heave, roll, pitch and yaw. The Tension Leg Platform is a hybrid structure with respect to horizontal degrees-of-freedom, it is compliant and behaves like a floating structure, whereas with respect to the vertical degrees-of-freedom, it is stiff and resembles a fixed structure and is not allowed to float freely. Also among the various degrees of freedom, vertical and rotational motion (heave and pitch) are very important because of the direct effect on the stress fluctuation that leads to fatigue and fracture of the legs. Therefore conceptual studies to understand the dynamic vertical response of TLS can be useful for designers.

Liu et al. described an analysis of the non-linear effects and identification of non-linear pitch motion on tension leg platforms (Jui-Jung *et al.,* 2004). The purpose of their paper was to accurately identify pitch motion on the tension leg platform and to interpret the non-linear effects using statistical methods, the NARMAX methodology, and the higher order frequency response functions.

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An analytical solution for surge motion of tension leg platform (TLP) was proposed and demonstrated (Lee and Lee, 1993; Lee, 1994; Lee, 1999), in which the surge motion of a platform with pre-tensioned tethers was calculated. In that study, however, the elasticity of tethers was only implied and the motion of tethers was also simplified as on-line rigid-body motion proportional to the top platform. Thus, both the material property and the mechanical behavior for the tether incorporated in the tension leg platform system were ignored. When this simplification was applied, no matter what the material used was or what the dimension of tethers was, the dynamic response of the platform would remain the same in terms of the vibration mode, periods and the vibration amplitude. An important point in that study was linearization of the surge motion. But it is obvious that the structural behavior in the surge motion is highly nonlinear because of large deformation of TLP in the surge motion degree of freedom (geometric nonlinearity) and nonlinear drag forces of Morison equation. Therefore the obtained solution is not true for the actual engineering application. For pitch degree of freedom the structural behavior is linear, because there is not geometric nonlinearity in the pitch motion degree of freedom and drag forces on legs have no vertical component. In this paper the analytical solution of pitch vibration is presented.

A continues model for vertical motion of TLP considering the effect of continues foundation has been reported (Tabeshpour *et al.,* 2004). The effect of added mass fluctuation on the pitch response of tension leg platform has been investigated by using perturbation method both for discrete and continues models (Tabeshpour *et al.,* 2006a). An analytical heave vibration of TLP with radiation and scattering effects for undamped system has been presented (Tabeshpour *et al.,* 2006b). Similar method is presented for hydrodynamic pitch response of the structure (Tabeshpour *et al.,* 2006).

In this study the equation of the motion, and the corresponding solution for pitch motion of the tension leg platform system subjected to sea wave, is derived and solved analytically. Based on Lee and Lee (1993) results, first the scattering problem is solved and the results were used to calculate the forcing function for the radiation problem and then both solutions were used for the solution of the pitch motion. The structural model is very simple but several complicated factors such as buoyancy, scattering, radiation and simulated ocean wave load are considered.

MATERIALS AND METHODS

General Wave Theory

For the inviscid and incompressible fluid and irrotational flow, a single-valued velocity potential ϕ can be defined as:

$$u=\{u_x,u_z\}^T=-\left\{\frac{\partial\phi}{\partial x},\frac{\partial\phi}{\partial z}\right\}^T \quad (1)$$

where u is the flow velocity vector. The velocity potential satisfies the Laplace equation:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=0 \quad (2)$$

and the Bernoulli equation

$$-\frac{\partial\phi}{\partial t}+\frac{p}{\rho_w}+gz=0 \quad (3)$$

in the flow field, where p is the pressure and ρ_w is the water density.

A two-dimensional tension leg platform interacting with a long crested linear wave propagating in the x-direction is considered here as is shown in Fig. 1. The wave form and the associated velocity potential are given accordingly as,

$$\eta_i=-iA_i\exp[-(K_1x+i\omega t)] \quad (4)$$

and

$$\phi_i=\frac{A_ig}{\omega}\frac{\cos[K_1(z+h)]}{\cos(K_1h)}\exp[-(K_1x+i\omega t)] \quad (5)$$

where A_i is the wave amplitude, g is the gravitational constant, h is the water depth, $\omega=2\pi/T$ is the angular frequency with T as the period, and $K_1=-ik$, where $k=2\pi/L$ is the wave number with L as the wave length. K_1 satisfies the dispersion relation

$$\omega^2=-gK_1\tan(K_1h) \quad (6)$$

RESULTS AND DISCUSSION

Boundary Value Problem

In the platform system, the motion of the structure induced by the small amplitude incident wave is assumed to be small. The wave induced structural motion can be solved from the imposed boundary problem. Because of the linearity of the problem, the problem can be incorporated into a scattering and a radiation problem. The wave force calculated from the scattering problem provides the force function in the radiation problem, and the forced oscillation then generates outgoing waves.

A tension leg platform system is illustrated in Fig. 1, where the flow field is divided into three regions with two artificial boundaries at $x=-b$ and $x=b$. In region **I**, the total velocity potential ϕ_I consists of incident waves ϕ_i, scattered waves ϕ_{Is}, and radiated waves ϕ_{Iw}.

$$\phi_I=\phi_i+\phi_{Is}+\phi_{Iw} \quad (7)$$

In region **II** and **III**, the total velocity potential ϕ_{II} and ϕ_{III} consists of scattered waves ϕ_{IIs} and ϕ_{IIIs}, and radiated waves ϕ_{IIw} and ϕ_{IIIw}. The subscript s denotes the scattering problem and w denotes the radiation (wave making) problem

$$\phi_{II}=\phi_{IIs}+\phi_{IIw} \quad (8)$$

$$\phi_{III}=\phi_{IIIs}+\phi_{IIIw} \quad (9)$$

All of the velocity potentials satisfy the Laplace equation. Furthermore, Sommerfeld's radiation condition is satisfied at the infinity of region **I** and **III** to secure unique solutions

$$\lim_{x\to\pm\infty}\left[\frac{\partial\phi}{\partial x}\pm\frac{1}{C_w}\frac{\partial\phi}{\partial t}\right]=0 \quad (10)$$

where C_w is the wave celerity.

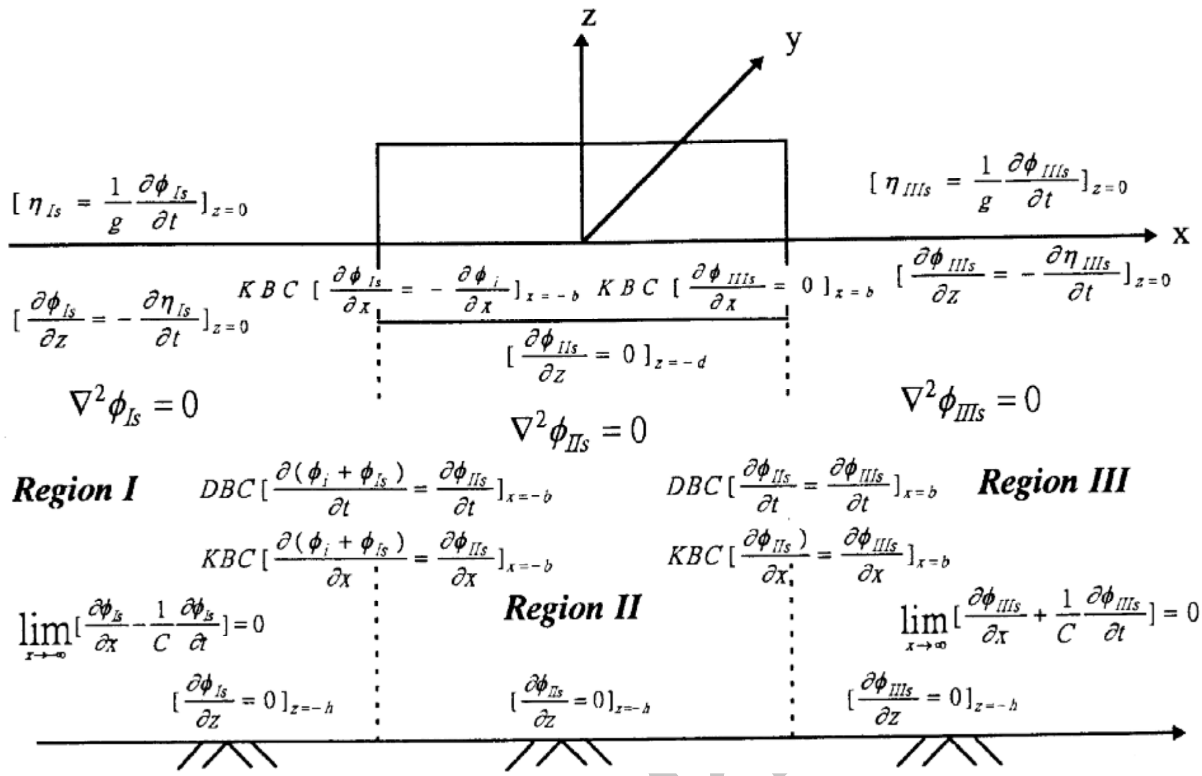

Fig. 1: Illustration for the scattering problem (Lee et al. 1999)

Scattering and Radiation Problems

In the scattering (diffraction) problem, the incident wave is considered to be diffracted by a fixed structure. The corresponding boundary value problem was also shown in Fig. 1. In the radiation problem the structure is considered to be forced into motion by the wave force induced by incident waves and scattered waves. The corresponding boundary value problem is illustrated in Fig. 2. The displacement of the dragged pitch motion is given by

$$\theta = S\exp(-i\omega t) \quad (11)$$

where S is the unknown amplitude of the pitch motion.

Applying the method of the separation of variables, matching the horizontal boundary conditions in each region, and applying Sommerfeld's condition to regions **I** and **III**, the corresponding surface elevation and velocity potential of scattering (s) and radiation (w) problems can be found as follows (Lee and Lee, 1993):

In region **I**:

$$\phi_{Is/w} = \sum_{j=1}^{\infty} \frac{A_{Is/wj}\,g}{\omega} \frac{\cos[K_j(z+h)]}{\cos(K_j h)} \exp[-(K_j(x+b)-i\omega t)] \quad (12)$$

and

$$\eta_{Is/w} = -i\sum_{j=1}^{\infty} A_{Is/wj} \exp[-(K_j(x+b)-i\omega t)] \quad (13)$$

In region **III**:

$$\phi_{IIIs/w} = \sum_{j=1}^{\infty} \frac{A_{IIIs/wj}\,g}{\omega} \frac{\cos[K_j(z+h)]}{\cos(K_j h)} \exp[-(K_j(x-b)+i\omega t)] \quad (14)$$

and

$$\eta_{IIIs/w} = -i\sum_{j=1}^{\infty} A_{IIIs/wj} \exp[-(K_j(x-b)+i\omega t)] \quad (15)$$

where the eigenvalues K_j can be solved from the dispersion equation

$$\omega^2 = gK_j \tan(K_j h) \quad (16)$$

with

$K_j = -ik$, for $j = 1$;

$(2j-3)\frac{\pi}{2} < K_j h < (j-1)\pi$, for $j \geq 2$

In region **II**:

$$\phi_{IIs} = \frac{ig}{\omega}\left[\left(A_{IIsP1}\frac{x}{b} + A_{IIsN1}\right)\cos K_{II1}(z+h) + \sum_{j=2}^{\infty}(-1)^{j-1}\left(A_{IIsPj}\exp(-K_{IIj}(x+b)) + A_{IIsNj}\exp(K_{IIj}(x-b))\right)\cos K_{IIj}(z+h)\right]\exp(-i\omega t) \quad (17)$$

$$\phi_{IIw} = \begin{cases} \dfrac{-x(z+h)^2 + x^3/3}{2(h-d)} + \dfrac{ig}{\omega} \\ \left[\left(A_{IIwP1}\dfrac{x}{b} + A_{IIwN1}\right)\cos K_{II1}(z+h) + \sum_{j=2}^{\infty}(-1)^{j-1}\left(A_{IIwPj}\exp(-K_{IIj}(x+b)) + A_{IIwNj}\exp(K_{IIj}(x-b))\right)\cos K_{IIj}(z+h)\right] \end{cases}\exp(-i\omega t) \quad (18)$$

where the eigenvalues K_{IIj} can be solved from the dispersion equation

$$K_{IIj} = \frac{(j-1)\pi}{h-d}, \qquad j \geq 1$$

The series of four unknowns $A_{Is/wj}$, $A_{IIs/wPj}$, $A_{IIIs/wj}$ and $A_{IIs/wNj}$ can further be solved from the following four equations derived from the four boundary conditions on the two vertical boundaries of region **II**. They are, for $\alpha \geq 1$

$$\frac{K_\alpha\langle Z_\alpha Z_\alpha\rangle}{\cos K_\alpha h}A_{Is/w\alpha} - i\left[\frac{1}{b}\langle Z_{II1}Z_\alpha\rangle^d A_{IIs/wP1} + \sum_{j=2}^{\infty}(-1)^{j-1}K_{IIj}\langle Z_{IIj}Z_\alpha\rangle^d\left(-A_{IIs/wPj} + \exp(-2K_{IIj}b)A_{IIs/wNj}\right)\right] = \begin{cases}\delta_{\alpha 1}\exp(K_1 b)\dfrac{K_1\langle Z_1 Z_1\rangle}{\cos K_1 h}A_i & s \\ \dfrac{i\omega^2}{g}\langle Z_\alpha^o\rangle S & w\end{cases} \quad (19)$$

$$-i\sum_{j=1}^{\infty}\frac{\langle Z_{II\alpha}Z_j\rangle^d}{\cos(K_j h)}A_{Is/wj} - \langle Z_{II\alpha}Z_{II\alpha}\rangle^d \times \left\{\left[(1-\delta_{\alpha 1})(-1)^{\alpha-1} - \delta_{\alpha 1}\right]A_{IIs/wP\alpha} + \left[(1-\delta_{\alpha 1})(-1)^{\alpha-1} - \delta_{\alpha 1}\right]\exp(-2K_{II\alpha}b)A_{IIs/wN\alpha}\right\} = \begin{cases} i\exp(K_1 b)\dfrac{\langle Z_{II\alpha}Z_1\rangle^d}{\cos(K_1 h)}A_i & s \\ 0 & w\end{cases} \quad (20)$$

$$i\frac{\langle Z_{II1}Z_\alpha\rangle^d}{b}A_{Is/wP1} + i\sum_{j=2}^{\infty}\left[(-1)^{j-1}K_{IIj}\langle Z_{IIj}Z_\alpha\rangle^d \times \left(\exp(-2K_{IIj}b)A_{IIs/wPj} + A_{IIIs/wNj}\right)\right] = \begin{cases}0 & s \\ -\dfrac{i\omega^2}{g}\langle Z_\alpha^0\rangle S & w\end{cases} \quad (21)$$

$$i\sum_{j=1}^{\infty}\frac{\langle Z_{II\alpha}Z_j\rangle^d}{\cos(K_j h)}A_{IIIs/wj} - \langle Z_{II\alpha}Z_{II\alpha}\rangle^d\left[(1-\delta_{\alpha 1})(-1)^{\alpha-1} - \delta_{\alpha 1}\right] \times \left(\exp(-2K_{II\alpha}b)A_{IIs/wP\alpha} + A_{IIs/wN\alpha}\right) = \begin{cases}0 & s \\ 0 & w\end{cases} \quad (22)$$

where δ is the Kronecker delta, and the notations of $\langle Z_* Z_*\rangle^d$, $\langle Z_* Z_*\rangle$, and $\langle Z_\alpha^o\rangle$ are defined in the Appendix (A). It is clear that equations (19) and (21) are obtained from the kinematic boundary conditions, and (20) and (22) from the dynamic boundary conditions.

Equations (19) - (22) can then be solved for the four series of the unknowns A_{Isj}, A_{IIsPj}, A_{IIsNj} and A_{IIIsj} in the scattering problem and substituted into the corresponding equations to calculate the follow properties. However, for the radiation problem equations (20) and (22) involve the unknown S, and therefore an additional equation is required to resolve all unknowns A_{Iwj}, A_{IIwPj}, A_{IIwNj} and A_{IIIwj}. More calculations are presented in Appendix (B).

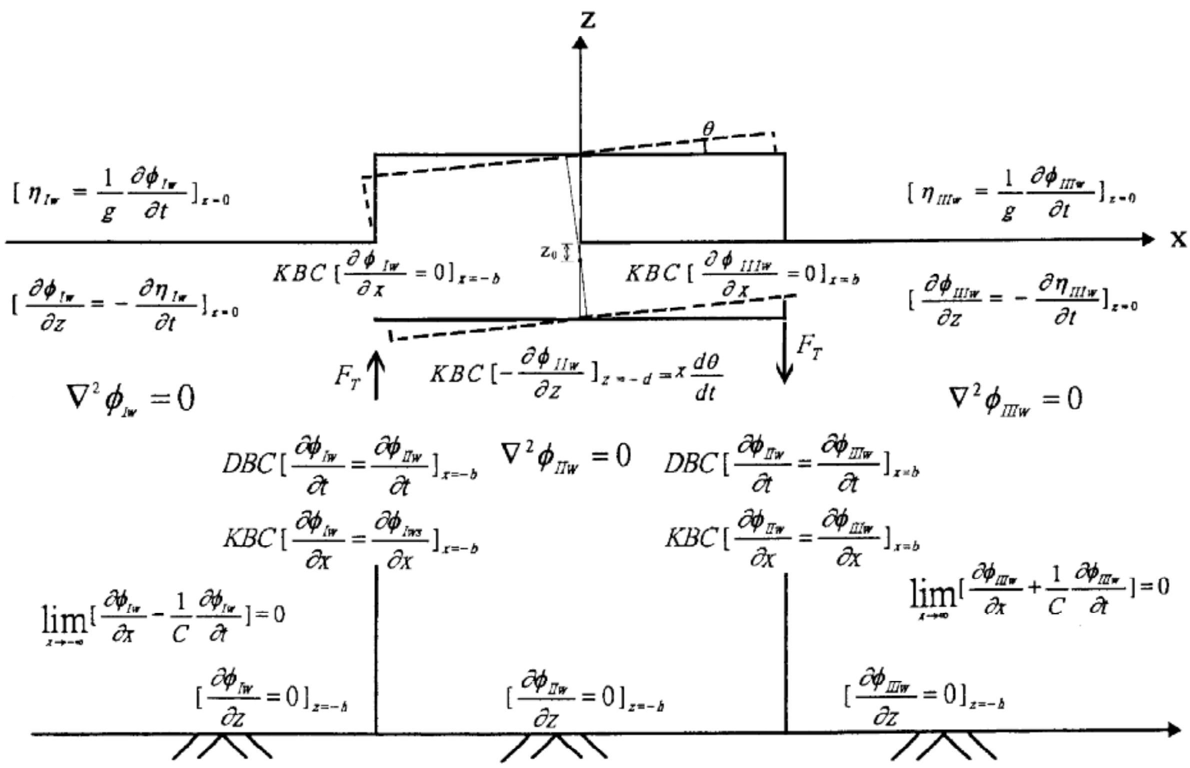

Fig. 2: Illustration for the radiation problem

Simple Model for Motion of the Platform

The equation of motion of the platform ignoring structural damping of tethers is as follows (see Fig. 3)

$$I_0 \frac{d^2\theta}{dt^2} + C_{er}\frac{d\theta}{dt} + K_{er}\theta = M_{py} + M_{am} + M_{rd} \quad (23)$$

where

I_0 : is the moment of inertia of the platform structure,

C_{er} : is the equivalent viscous structural damping,

K_{er} : is the equivalent stiffness of the platform,

M_{py} is the moment wave force acting on pitch direction of the structure,

$M_{am} = -I_a \frac{d^2\theta}{dt^2}$ is the moment related to the added mass and

$M_{rd} = -C_{rp}\frac{d\theta}{dt}$ is the radiation damping moment force from fluid-structure interaction.

I_a and C_{rp} will be determined in this paper. The equivalent stiffness of the platform system is presented, when the material property and the tether dimension are taken into account, as

$$K_{re} = K_{rt} + K_{rb} \quad (24)$$

where

$$K_{rt} = 2b \times \frac{A_t E}{l} = \frac{2A_t Eb}{h-d} \quad (25)$$

and

$$K_{rb} = \frac{4}{3}\rho_w g b^2 \quad (26)$$

A_t is the total area of the tethers cross section, E is the Young modulus of the tether material and l is the length of the tether.

The wave force moment M_{py} can be obtained through the integration of the total hydrodynamic pressure over the surfaces of the structure

$$M_{py} = M_{py}^0 \exp(-i\omega t) \quad (27)$$

where

$$M_{py}^0 = -F_{wz}^0 \times \overline{X} + F_{wx}^0 \times \overline{Z} \quad (28)$$

in which

F_{wz}^0 is the wave force in z direction,

F_{wx}^0 is the wave force in x direction,

$\overline{X}$ is the distance between the center of mass (C.M.) and center of stiffness (C.S.) in x direction and

$\overline{Z}$ is the distance between the center of mass (C.M.) and center of stiffness (C.S.) in z direction.
Structural modeling and generated loads are shown in Fig. 3.

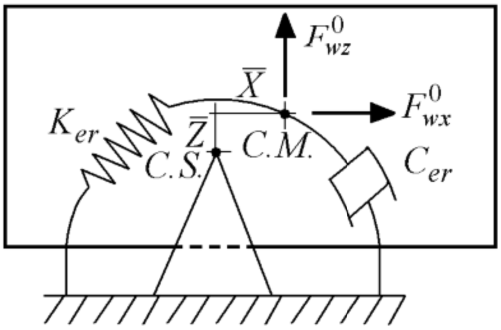

Fig. 3: Structural modeling and generated loads

In order to determine the vertical and horizontal forces, one can integrate the hydrodynamic pressure on the bottom and lateral surface of the structure respectively. After some calculation these forces are determined as

$$F_{wz}^{0} = \rho i\omega\left\{2A_{IIsN1}b + \sum_{j=2}^{\infty}(-1)^{j-1}\left(A_{IIsPj}+A_{IIsNj}\right)\times \cos K_{IIj}(h-d)\frac{\exp(2K_{IIj}b)-1}{K_{IIj}}\right\} \tag{29}$$

and [1]

$$F_{wx}^{0} = \rho g A_i \frac{\exp(-iK_1b)-\exp(iK_1b)}{\cos(K_1h)}\times \frac{\sin(K_1h)-\sin\left[K_1(h-d)\right]}{K_1} + \rho i\omega\sum\left\{(-1)^{j-1}\left(A_{IsPj}-A_{IIIsPj}\right)\times \frac{\sin(K_jh)-\sin\left[K_j(h-d)\right]}{K_j}\right\} \tag{30}$$

Added mass and radiation damping

Added mass and radiation damping are obtained as follows

$$I_a = \mathrm{Re}(M_0) \tag{31}$$

$$C_{rp} = \mathrm{Im}(\omega M_0) \tag{32}$$

in which

$$M_0 = \int_{-d}^{0}\left(\phi_{IIIw}\big|_{x=-b}-\phi_{Iw}\big|_{x=b}\right)(z-z_0)\,dz - \int_{-b}^{b}\phi_{IIw}\big|_{z=-d}\,x\,dx \tag{33}$$

The above integral can be calculated considering equations (28), (29) and (30) as follows

$$M_0 = -F_{0z}\times\overline{X} + F_{0x}\times\overline{Z} \tag{34}$$

where

$$F_{0z} = \rho\left\{\left[(h-d)+2A_{IIwN1}\right]b - \frac{b^3}{3(h-d)} + \sum_{j=2}^{\infty}(-1)^{j-1}\left(A_{IIwPj}+A_{IIwNj}\right)\times \cos K_{IIj}(h-d)\frac{\exp(2K_{IIj}b)-1}{K_{IIj}}\right\} \tag{35}$$

and

$$F_{0x} = \sum_{j=1}^{\infty}\left\{(-1)^{j-1}\left(A_{IsPj}-A_{IIIsPj}\right)\times \frac{\sin(K_jh)-\sin\left[K_j(h-d)\right]}{K_j}\right\} \tag{36}$$

Now the equation of motion is fully determined as

$$(I_0+I_a)\frac{d^2\theta}{dt^2}+(C_{er}+C_{rp})\frac{d\theta}{dt}+K_{er}\theta = M_{py} \tag{37}$$

Substituting equation (12) into (36) one obtains

$$\left[-(I_0+I_a)\omega^2 - i(C_{er}+C_{rp})\omega + K_{er}\right]S = M_{py}^{0} \tag{38}$$

Defining

$$\zeta = \frac{C_{rp}}{2(I_0+I_a)\omega_s} = \frac{\mathrm{Im}(\omega M_0)}{2\left[I_0+\mathrm{Re}(M_0)\right]\omega_s} \tag{39}$$

and

$$\omega_s = \sqrt{\frac{K_{er}}{I_0+I_a}} \tag{40}$$

and considering equations (31) and (32) one obtains

$$\left\{-\left[I_0+\mathrm{Re}(M_0)\omega^2\right] - i\left(\begin{matrix}2\left[I_0+\mathrm{Re}(M_0)\right]\omega_s\zeta \\ +\mathrm{Im}(M_0)\omega\end{matrix}\right)\omega + K_{er}\right\}S = M_{py}^{0} \tag{41}$$

Or

$$S = \frac{\left\| M_{py}^{0} \right\|}{\sqrt{\begin{aligned}&\left(K_{er} - \left[I_0 + \mathrm{Re}(M_0)\right]\omega^2\right)^2 + \\ &\left(2\left[I_0 + \mathrm{Re}(M_0)\right]\zeta\omega_s + \mathrm{Im}(M_0)\omega\right)^2 \omega^2\end{aligned}}} \tag{42}$$

CONCLUSION

The analytical solution of pitch response of TLP was presented for a simple model. The effects of radiation and scattering were considered in the boundary value problem. A set of equations to describe the motion of the platform subjected to the wave-induced pitch motion and the flow-induced drag motion were derived, and the corresponding close form analytical solution was presented as an infinite series form for the dynamic behavior of the platform utilized for the tension leg platform structural system.

Appendix (A)

$$\left\langle Z_j Z_\alpha \right\rangle = \int_{-h}^{0} \cos[K_j(z+h)]\cos[K_\alpha(z+h)]\mathrm{d}z = \begin{cases} 0 & \text{if } j \neq \alpha \\ \dfrac{h}{2}\left[1 + \dfrac{\sin(2K_\alpha h)}{2K_\alpha h}\right] & \text{if } j = \alpha \end{cases} \tag{A1}$$

$$\left\langle Z_{IIj} Z_\alpha \right\rangle^d = \int_{-h}^{-d} \cos[K_{IIj}(z+h)]\cos[K_\alpha(z+h)]\mathrm{d}z = \begin{cases} \dfrac{1}{2}\left\{\begin{aligned}&\dfrac{\sin(K_{IIj}+K_\alpha)(h-d)}{K_{IIj}+K_\alpha} + \\ &\dfrac{\sin(K_{IIj}-K_\alpha)(h-d)}{K_{IIj}-K_\alpha}\end{aligned}\right\} & \text{if } K_{IIj} \neq K_\alpha \\ \dfrac{h}{2}\left[1 + \dfrac{\sin[2K_\alpha(h-d)]}{2K_\alpha h}\right] & \text{if } K_{IIj} = K_\alpha \end{cases} \tag{A2}$$

$$\left\langle Z_{IIj} Z_{II\alpha} \right\rangle^d = \int_{-h}^{-d} \cos[K_{IIj}(z+h)]\cos[K_{II\alpha}(z+h)]\mathrm{d}z = \begin{cases} 0 & \text{if } j \neq \alpha \\ h-d & \text{if } j = \alpha = 1 \\ \dfrac{h-d}{2} & \text{if } j = \alpha \neq 1 \end{cases} \tag{A3}$$

$$\left\langle Z_j^o \right\rangle = \int_{-d}^{0} \cos[K_j(z+h)]\mathrm{d}z = \frac{1}{K_j}[\sin K_j h - \sin K_j(h-d)] \tag{A4}$$

Appendix (B)

The complete solution of ϕ_{IIw} is

$$\phi_{IIw} = \phi_{IIw}^{h} + \phi_{IIw}^{p} \tag{B1}$$

where ϕ_{IIw}^{h} and ϕ_{IIw}^{p} are the homogeneous and particular parts of the ϕ_{IIw} respectively.

ϕ_{IIw} satisfies the Laplace equation

$$\frac{\partial^2 \phi_{IIw}}{\partial x^2} + \frac{\partial^2 \phi_{IIw}}{\partial z^2} = 0 \tag{B2}$$

with the following boundary conditions

$$\left.\frac{\partial \phi_{IIw}}{\partial z}\right|_{z=-d} = f(x), \left.\frac{\partial \phi_{IIw}}{\partial z}\right|_{z=-h} = 0 \tag{B3}$$

In order to solve the eq. (B2) the following relation is assumed

$$\phi_{IIw}(x,z) = \psi_{IIw}(x,z) + f(x)\frac{(z+h)^2}{2(h-d)} + g(x)h(z) \tag{B4}$$

The homogeneous parts of ϕ_{IIw}^{h} and ψ_{IIw}^{h} are the same

$$\phi_{IIw}^{h} = \psi_{IIw}^{h} \tag{B5}$$

and the particular part of the ϕ_{IIw} is as follows

$$\phi_{IIw}^{p} = \psi_{IIw}^{p} + f(x)\frac{(z+h)^2}{2(h-d)} + g(x)h(z) \tag{B6}$$

Now considering the boundary condition as $\left.\frac{\partial \phi_{IIw}}{\partial z}\right|_{z=-h} = 0$, one obtains

$$h(z) = cte = a$$

$$\phi_{IIw}^{p} = \psi_{IIw}^{p} + f(x)\frac{(z+h)^2}{2(h-d)} + ag(x) \tag{B7}$$

Substituting eq. (B7) into the eq.(B2), one obtains

$$\frac{\partial^2 \phi_{IIw}}{\partial x^2} + \frac{\partial^2 \phi_{IIw}}{\partial z^2} = \frac{\partial^2 \psi_{IIw}}{\partial x^2} + \frac{\partial^2 \psi_{IIw}}{\partial z^2} + f''(x)\frac{(z+h)^2}{2(h-d)} + \frac{f(x)}{(h-d)} + ag''(x) = 0 \tag{B8}$$

or

$$g''(x) = -f''(x)\frac{(z+h)^2}{2a(h-d)} - \frac{f(x)}{a(h-d)} \tag{B9}$$

Because of rigid body rotation around point $(0, z_0)$, f(x) should be considered as a linear function $f(x) = f_0 x$

then

$$g''(x) = \frac{f_0 x}{a(h-d)} \text{ and } g(x) = \frac{f_0}{2a(h-d)} \frac{x^3}{3} \text{ or}$$

$$\phi^p_{IIw} = \psi^p_{IIw} + f_0 \frac{-(z+h)^2 x + x^3/3}{2(h-d)} \qquad \text{(B10)}$$

and

$$\frac{\partial^2 \psi_{IIw}}{\partial x^2} + \frac{\partial^2 \psi_{IIw}}{\partial z^2} = 0 \qquad \text{(B11)}$$

with the following boundary conditions

$$\left.\frac{\partial \psi_{IIw}}{\partial z}\right|_{z=-h} = 0 \qquad \text{(B12)}$$

$$\left.\frac{\partial \psi_{IIw}}{\partial z}\right|_{z=-d} = 0 \qquad \text{(B13)}$$

The homogeneous part of solution of the eq. (B11) is as follows

$$\psi^h_{IIw} = \frac{ig}{\omega}\left[\left(A_{IIwP1}\frac{x}{b} + A_{IIwN1}\right)\cos K_{II1}(z+h) + \sum_{j=2}^{\infty}(-1)^{j-1}\left(A_{IIwPj}\exp(-K_{IIj}(x+b)) + A_{IIwNj}\exp(K_{IIj}(x-b))\right)\cos K_{IIj}(z+h)\right]\exp(-i\omega t) \qquad \text{(B14)}$$

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