

International Journal of Group Theory
ISSN (print): 2251-7650, ISSN (on-line): 2251-7669
Vol. 2 No. 2 (2013), pp. 25-33.
© 2013 University of Isfahan



ON SOME SUBGROUPS ASSOCIATED WITH THE TENSOR SQUARE OF A GROUP

M. M. NASRABADI^{*}, A. GHOLAMIAN AND M. J. SADEGHIFARD

Communicated by Alireza Moghaddamfar

ABSTRACT. In this paper we present some results about subgroup which is generalization of the subgroup $R_2^{\otimes}(G) = \{a \in G | [a,g] \otimes g = 1_{\otimes}, \forall g \in G\}$ of right 2_{\otimes} -Engel elements of a given group G. If p is an odd prime, then with the help of these results, we obtain some results about tensor squares of p-groups satisfying the law $[x, g, y] \otimes g = 1_{\otimes}$, for all $x, g, y \in G$. In particular p-groups satisfying the law $[x, g, y] \otimes g = 1_{\otimes}$, for all $x, g, y \in G$. In particular p-groups satisfying the law $[x, g, y] \otimes g = 1_{\otimes}$ have abelian tensor squares. Moreover, we can determine tensor squares of two-generator p-groups of class three satisfying the law $[x, g, y] \otimes g = 1_{\otimes}$.

1. Introduction

Throughout this paper p is an odd prime and a p-group is a group in which every element has order a power of p.

For any group G, the non-abelian tensor square $G \otimes G$ is a group generated by the symbols $g \otimes h$, subject to the relations,

$$gg' \otimes h = (g^{g'} \otimes h^{g'})(g' \otimes h),$$

$$g \otimes hh' = (g \otimes h')(g^{h'} \otimes h^{h'}),$$

for all $g, g', h, h' \in G$, where $g^h = h^{-1}gh$ is conjugation on the right. The non-abelian tensor square is a special case of the non-abelian tensor product which has its origins in homotopy theory. It was introduced by R. Brown and J. L. Loday in [3] and [4], extending ideas of J. H. C. Whitehead in [13]. In [2], R. Brown, D. L. Johnson, and E. F. Robertson start the investigation of non-abelian tensor squares as group theoretical objects.

MSC(2010): Primary: 20F45; Secondary: 20F99

Keywords: Non-abelian tensor square, Engel elements of a group, p-groups.

Received: 7 May 2012, Accepted: 13 October 2012.

^{*}Corresponding author.

The set of all elements a in G such that $a \otimes g = 1_{\otimes}$, for all g in G, is called the tensor centre of G and denoted by $Z^{\otimes}(G)$. This concept was introduced by G. J. Ellis [5]. Following [0] The set of right n - Encod elements of a group G is defined as

Following [9] The set of right n_{\otimes} -Engel elements of a group G is defined as

$$R_n^{\otimes}(G) = \{ a \in G : [a_{n-1}x] \otimes x = 1_{\otimes} \quad \forall x \in G \}.$$

Here commutators are denoted by $[x, y] = x^{-1}y^{-1}xy$ and $[x_1, \ldots, x_n, x_{n+1}] = [[x_1, \ldots, x_n], x_{n+1}]$. Also for $x \in G$, the set of all x^g for $g \in G$ is called, the conjugacy class of x in G and is denoted by x^G . The set of right n_{\otimes} -Engel elements in groups have been studied by several authors (see for example [1], [9], [10]). One of the results of [1] shows that $R_2^{\otimes}(G)$ is always a characteristic subgroup of G. Moravec in [9] investigated the properties of right 2_{\otimes} -Engel elements of a group G. The subgroup

 $B^{\otimes}(G)$ studied in this paper is a generalization of $R_2^{\otimes}(G)$.

Definition 1.1. Let G be a group. We define $C^{\otimes}(G) = \{a \in G | [ay, g, x] \otimes g = [y, g, x] \otimes g \ \forall g, y, x \in G\},\$ $B^{\otimes}(G) = \{a \in G | [a, g, x] \otimes g = 1_{\otimes} \ \forall g, x \in G\}.$

One can easily check that $C^{\otimes}(G)$ is a characteristic subgroup of G. It will be shown that $B^{\otimes}(G) = C^{\otimes}(G)$. Thus $B^{\otimes}(G)$ is a characteristic subgroup of G.

At first, we determine some information about $B^{\otimes}(G)$. Next we show that every p-group satisfying the law $[x, g, y] \otimes g = 1_{\otimes}$ has an abelian tensor square and $\gamma_3(G) \leq Z^{\otimes}(G)$. With the help of this we can compute tensor squares of two-generator p-groups of class 3 satisfying the law $[x, g, y] \otimes g = 1_{\otimes}$.

2. Preliminary Results

In this section we summarize some basic results which will be used in the proof of our main results.

Lemma 2.2. ([2]) Let $g, g', h, h' \in G$. The following relations hold in $G \otimes G$: $(g^{-1} \otimes h)^g = (g \otimes h)^{-1} = (g \otimes h^{-1})^h$. (2.2.1) $(g' \otimes h')^{g \otimes h} = (g' \otimes h')^{[g,h]}$. (2.2.2) $[g,h] \otimes g' = (g \otimes h)^{-1}(g \otimes h)^{g'}$. (2.2.3) $g' \otimes [g,h] = (g \otimes h)^{-g'}(g \otimes h)$. (2.2.4) $[g,h] \otimes [g',h'] = [g \otimes h, g' \otimes h']$. (2.2.5)

Proposition 2.3. ([2]) For a given group G, there exists a homomorphism $\lambda: G \otimes G \to G'$ such that $\lambda(g \otimes h) = [g, h]$, for all $g, h \in G$.

www.SID.ir

Lemma 2.4. ([1,9]) Let G be a group, $a \in R_2^{\otimes}(G)$ and $x, y, z \in G$. Then $[a, x] \otimes y = ([a, y] \otimes x)^{-1}$. (2.4.1) $[a, x]^z \otimes x = 1_{\otimes}$. (2.4.2) $[a, x] \in R_2^{\otimes}(G)$. (2.4.3) $[a, x] \otimes [y, z] = 1_{\otimes}$. (2.4.4) $[x, y] \otimes a = ([x, a] \otimes y)^2$ and $a \otimes [x, y] = ([a, x] \otimes y)^2$. (2.4.5) $([a, x, y] \otimes z)^2 = 1_{\otimes}$. (2.4.6)

Theorem 2.5. ([7]) The variety of groups which are nilpotent of class n or less (where n is a fixed integer greater than 2) may be defined by the law

$$[x, y_1, \dots, y_{n-1}, x] = 1$$

3. Results for $B^{\otimes}(G)$

The goal of this section is to prove the following results for $B^{\otimes}(G)$, which are tensor analogues to the work of Kappe in [6].

Lemma 3.1. Let G be a group, $g, x, d, d', d'' \in G$, $i, j \in \mathbb{Z}$ and $a \in B^{\otimes}(G)$. Then $[a, g, x]^{d} \otimes g^{d'} = 1_{\otimes}$. (3.1.1) $[a, g^{i}, x]^{d} \otimes g^{d'} = 1_{\otimes}$. (3.1.2) $[[a, g^{i}]^{d}, x]^{d'} \otimes (g^{j})^{d''} = 1_{\otimes}$. (3.1.3) $[a, g, x]^{d} \otimes a^{d'} = 1_{\otimes}$. (3.1.4)

Proof. Commutator expansion gives,

$$1_{\otimes} = [a, g, xd] \otimes g$$

= $[a, g, d][a, g, x]^d \otimes g$
= $([a, g, d] \otimes g)^{[a, g, d]^d}([a, g, x]^d \otimes g)$
= $[a, g, x]^d \otimes g$,

lead to (3.1.1).

To prove (3.1.2) for i > 0, we use induction on i. (3.1.1) is as the basis of an inductive proof. Now assume that (3.1.2) follows when i is a fixed positive k, we prove that (3.1.2) is true for i = k + 1.

$$\begin{split} [a, g^{k+1}, x]^d \otimes g^{d'} &= [a, g^k. g, x]^d \otimes g^{d'} \\ &= [[a, g][a, g^k]^g, x]^d \otimes g^{d'} \\ &= ([a, g, x]^w [[a, g^k]^g, x])^d \otimes g^{d'} \\ &= ([a, g, x]^{wd} \otimes g^{d'})^{w'} ([[a, g^k]^g, x]^d \otimes g^{d'}) \\ &= [a, g^k, x^{g^{-1}}]^{gd} \otimes g^{d'} \\ &= 1_{\otimes}, \end{split}$$

www.SID.ir

where $w = [a, g^k]^g$ and $w' = [[a, g^k]^g, x]^d$. If i < 0, then put t = -i. Hence,

$$\begin{split} [a,g^{i},x]^{d} \otimes g^{d'} &= [a,g^{-t},x]^{d} \otimes g^{d'} \\ &= [[a,g^{t}]^{-g^{-t}},x]^{d} \otimes g^{d'} \quad by(2.1.3) \\ &= [[a,g^{t}]^{-1},x^{g^{t}}]^{g^{-t}d} \otimes g^{d'} \\ &= [a,g^{t},x^{g^{t}}]^{-wd} \otimes g^{d'} \quad by(2.1.3) \\ &= ([a,g^{t},x^{g^{t}}]^{wd} \otimes g^{d'})^{-w'} \quad by(2.2.1) \\ &= 1 \otimes . \end{split}$$

where $w = [a, g^t]^{-1}g^{-t}$ and $w' = [a, g^t, x^{g^t}]^{-wd}$. These imply (3.1.2).

(3.1.3) is proved in a way similar to (3.1.2).

For the proof of (3.1.4) note that,

$$1_{\otimes} = [a, ag, x]^{d} \otimes (ag)^{d'}$$
$$= [[a, ag], x]^{d} \otimes a^{d'}g^{d'}$$
$$= ([a, g, x]^{d} \otimes g^{d'})([a, g, x]^{d} \otimes a^{d'})^{g^{d'}}$$
$$= ([a, g, x]^{d} \otimes a^{d'})^{g^{d'}}, \quad by(3.1.1)$$

proving (3.1.4).

Theorem 3.2. Let G be a group. Then we have $B^{\otimes}(G) = C^{\otimes}(G). \quad (3.2.1)$ $R_2^{\otimes}(G) \subseteq B^{\otimes}(G). \quad (3.2.2)$ $[a, g, x, h] \otimes g = 1_{\otimes}, \text{ for } a \in B^{\otimes}(G) \text{ and } g, x, h \in G. \quad (3.2.3)$ $[a, g, x, h] \otimes h = 1_{\otimes}, \text{ for } a \in B^{\otimes}(G) \text{ and } g, x, h \in G. \quad (3.2.4)$ $([a, g, x, b, c] \otimes h)^2 = 1_{\otimes}, \text{ for } a \in B^{\otimes}(G) \text{ and } g, x, b, c, h \in G. \quad (3.2.5)$ $[a, g, x] \otimes h = ([a, h, x] \otimes g)^{-1}, \text{ for } a \in B^{\otimes}(G) \text{ and } g, x, h \in G. \quad (3.2.6)$

Proof. To prove (3.2.1) let $a \in B^{\otimes}(G)$. Then $[[a,g]^y,x]^{[y,g]} \otimes g = 1_{\otimes}$ by (3.1.3). Hence,

$$\begin{split} [ay,g,x]\otimes g &= ([[a,g]^y,x]^{[y,g]}\otimes g)^{[y,g,x]}([y,g,x]\otimes g) \\ &= [y,g,x]\otimes g \end{split}$$

i.e., $B^{\otimes}(G) \subseteq C^{\otimes}(G)$ and hence (3.2.1) follows since clearly $C^{\otimes}(G) \subseteq B^{\otimes}(G)$. To prove (3.2.2), let $a \in R_2^{\otimes}(G)$. Then by proposition 2.3, [a, g, g] = 1. Hence, by (2.4.3) and (2.4.1) *www.SID.ir* Int. J. Group Theory 2 no. 2 (2013) 25-33

we have

$$[a, g, x] \otimes g = ([[a, g], g] \otimes x)^{-1}$$
$$= (1 \otimes x)^{-1}$$
$$= 1_{\otimes},$$

proving $R_2^{\otimes}(G) \subseteq B^{\otimes}(G)$. To prove (3.2.3), we have

$$[a, g, x, h] \otimes g = [[a, g]^{-1}[a, g]^{x}, h] \otimes g$$

= $[a, g, h]^{-w}[[a, g]^{x}, h] \otimes g$
= $([a, g, h]^{w} \otimes g)^{-w'}([[a, g]^{x}, h] \otimes g)$
= $1_{\otimes}, \quad by(3.1.3)$

where $w = [a, g]^{-1}[a, g]^x$ and $w' = [a, g, h]^{-w}[[a, g]^x, h]$. To prove (3.2.4), since $a \in B^{\otimes}(G)$, then by proposition 2.3 we have 1 = [a, gh, x, gh]. Hence, commutator expansion gives

$$1 = [[a, h][a, g]^h, x, gh]$$

= $[[a, h, x]^{[a,g]^h}[[a, g]^h, x], gh]$
= $[y_1y_2, gh],$

where $y_1 = [a, h, x]^{[a,g]^h}$ and $y_2 = [[a, g]^h, x]$. Now by (3.1.3), $y_1 \otimes h = 1_{\otimes}$ and $y_2 \otimes g = 1_{\otimes}$. Hence, by proposition 2.3 we have $[y_1, h] = 1$ and $[y_2, g] = 1$. Then the commutator expansion of $1 = [y_1y_2, gh]$ gives

$$1 = [y_1, gh]^{y_2}[y_2, gh] = [y_1, h]^{y_2}[y_1, g]^{hy_2}[y_2, h][y_2, g]^h.$$

Hence, $1 = [y_1, g]^{hy_2}[y_2, h]$ and so

$$1_{\otimes} = [y_1, g]^{hy_2} [y_2, h] \otimes h$$
$$= ([y_1, g]^{hy_2} \otimes h)^{[y_2, h]} ([y_2, h] \otimes h).$$

Now we prove $[y_1,g]^{hy_2} \otimes h = 1_{\otimes}$. We have

$$1_{\otimes} = y_1^{ghy_2} \otimes h \quad by(3.1.3)$$

= $(y_1[y_1, g])^{hy_2} \otimes h$
= $(y_1^{hy_2} \otimes h)^{[y_1, g]^{hy_2}}([y_1, g]^{hy_2} \otimes h)$
= $[y_1, g]^{hy_2} \otimes h. \quad by(3.1.3)$

Hence, we obtain

$$1_{\otimes} = [y_2, h] \otimes h = [[a, g]^h, x, h] \otimes h$$

www.SID.ir

30 Int. J. Group Theory 2 no. 2 (2013) 25-33

M.M. Nasrabadi, A. Gholamian and M.J. Sadeghifard

The substitution of x^h for x finally gives

$$1_{\otimes} = [[a,g]^h, x^h, h] \otimes h = ([a,g,x,h] \otimes h)^h,$$

proving (3.2.4).

To prove (3.2.5), $[a, g, x] \in R_2^{\otimes}(G)$ by (3.2.4). Hence, (3.2.5) follows by (2.4.6). Finally for (3.2.6), expansion of $1_{\otimes} = [a, gh, x] \otimes gh$, as in the proof (3.2.4), leads to

$$1_{\otimes} = (y_1 \otimes g)^{hy_2} (y_2 \otimes h)$$

where $y_1 = [a, h, x]^{[a,g]^h}$ and $y_2 = [[a,g]^h, x]$. Now by (3.1.4) we have $[a, h, x] \otimes [a,g]^h = 1_{\otimes}$, since $[a,g]^h \in \langle a^G \rangle$. Hence, $[[a, h, x], [a,g]^h] = 1$ by proposition 2.3. Therefore $y_1 = [a, h, x]$. Now we have

$$(y_1 \otimes g)^{hy_2} = ([a, h, x] \otimes g)([[a, h, x], g] \otimes hy_2) \quad \text{by (2.2.3)} \\ = ([a, h, x] \otimes g)([[a, h, x], g] \otimes y_2)([[a, h, x], g] \otimes h)^{y_2} \\ = ([a, h, x] \otimes g)([[a, h, x], g] \otimes y_2) \quad \text{by (3.2.3)} \\ = [a, h, x] \otimes g \quad \text{by (3.1.4), since } y_2 \in \langle a^G \rangle.$$

To simplify $y_2 \otimes h$, write $[a,g]^h = [a,g][a,g,h]$ and expand

$$y_{2} \otimes h = [a, g, x]^{w_{0}}[a, g, h, x] \otimes h$$

= $([a, g, x]^{w_{0}} \otimes h)^{w_{1}}([a, g, h, x] \otimes h)$
= $([a, g, x]^{w_{0}} \otimes h)^{w_{1}}$ since $[a, g] \in B^{\otimes}(G)$
= $([a, g, x]^{w_{0}} \otimes h)([a, g, x]^{w_{0}}, h] \otimes w_{1})$, by (2.2.3)

where $w_0 = [a, g, h]$ and $w_1 = [w_0, x]$. Now $[a, g, x] \otimes w_0 = 1_{\otimes}$ by (3.1.4), since $w_0 \in \langle a^G \rangle$. Hence, $[[a, g, x], w_0] = 1$ by proposition 2.3. Therefore $[a, g, x]^{w_0} = [a, g, x]$. Also $([a, g, x]^{w_0}, h] \otimes w_1) = 1_{\otimes}$ by (3.1.4), since $w_1 \in \langle a^G \rangle$. Hence,

$$y_2 \otimes h = [a, g, x] \otimes h.$$

Now, altogether we have

$$1_{\otimes} = (y_1 \otimes g)^{hy_2}(y_2 \otimes h) = ([a, h, x] \otimes g)([a, g, x] \otimes h)$$

proving (3.2.6).

4. The variety with the tensor square

A variety is a class of groups defined by laws, or identical relations.

Lemma 4.1. Every group G of the variety with the law

$$[x, g, y] \otimes g = 1_{\otimes}, \qquad (4.1.1)$$

for all $x, g, y \in G$ is nilpotent of class at most 3.

www.SID.ir

Proof. Expansion of $1_{\otimes} = [x, xg, y] \otimes xg$ implies that

$$1_{\otimes} = [x, g, y] \otimes x.$$

Thus G is nilpotent of class at most 3 by proposition 2.3 and theorem 2.5.

Proposition 4.2. Let G be a group satisfying the law (4.1.1) and $a, b, x, y \in G$. Then $G' \subseteq R_2^{\otimes}(G).$ (4.2.1) $[a, b] \otimes [x, y] = ([a, b, x] \otimes y)^2$ and $[a, b] \otimes [x, y] = ([a, [x, y]] \otimes b)^2$. (4.2.2) $([a, b] \otimes [x, y])^2 = 1_{\otimes}.$ (4.2.3)

Proof. To prove (4.2.1), by (2.1.4) and lemma 4.1 we have

 $[x^{-1},y^{-1},z][y,z^{-1},x^{-1}][z,x,y]=1,\\$

for all $x, y, z \in G$. Hence, $1_{\otimes} = [x^{-1}, y^{-1}, z][y, z^{-1}, x^{-1}][z, x, y] \otimes y^{-1}$. The equation yields

$$1_{\otimes} = [z, x, y] \otimes y,$$

proving (4.2.1).

Next, (4.2.2) follows from (4.2.1) and (2.4.5).

Finally for (4.2.3), we have

$$[a,b] \otimes [x,y] = ([x,y,a]^{-1} \otimes b)^{2} \text{ by } (4.2.2)$$

$$= ([x,y,a] \otimes b)^{-2} \text{ by } (2.2.1), (2.2.3) \text{ and } 4.1$$

$$= ([x,y,b] \otimes a)^{2} \text{ by } (4.2.1) \text{ and } (2.4.1)$$

$$= ([x,a,b] \otimes y)^{-2} \text{ by } (3.2.6)$$

$$= ([a,x,y] \otimes b)^{-2} \text{ by } (4.2.1) \text{ and } (2.4.1)$$

$$= ([a,b,y] \otimes x)^{2} \text{ by } (4.2.1) \text{ and } (2.4.1)$$

$$= ([a,b,y] \otimes x)^{2} \text{ by } (3.2.6)$$

$$= ([a,b,x] \otimes y)^{-2} \text{ by } (4.2.1) \text{ and } (2.4.1)$$

$$= ([a,b,x] \otimes y)^{-2} \text{ by } (4.2.1) \text{ and } (2.4.1)$$

$$= ([a,b] \otimes [x,y])^{-1}, \text{ by } (4.2.2)$$

proving, (4.2.3)

Theorem 4.3. A group G has a finite covering by subgroups satisfying the law (4.1.1) if and only if $|G: B^{\otimes}(G)| < \infty$.

Proof. Suppose that $G = \bigcup_{i=1}^{n} H_i$ where H_i are subgroups of G satisfying the law (4.1.1). The result of B. H. Neumann [11], shows that we may assume that $|G:H_i| < \infty$ for every *i*. Hence, the subgroup $D = \bigcap_{i=1}^{n} H_i$ has a finite index in G. It is clear that $D \leq B^{\otimes}(G)$ and hence, $|G:B^{\otimes}(G)| < \infty$.

Conversely, let $\{g_1, \ldots, g_n\}$ be a transversal of $B^{\otimes}(G)$ in G and let $H_i = \langle g_i \rangle B^{\otimes}(G)$. We have $G = \bigcup_{i=1}^n H_i$, hence it suffices to prove that each H_i satisfying the law (4.1.1).

Let $x = g^i a, y = g^j b$ and z be arbitrary elements of $\langle g \rangle B^{\otimes}(G)$, where $i, j \in \mathbb{Z}$ and $a, b \in B^{\otimes}(G)$. Since $a, b \in B^{\otimes}(G)$, using (3.1.4), we obtain,

$$\begin{split} [x,y,z]\otimes y &= [g^ia,g^jb,z]\otimes g^jb\\ &= [[g^i,g^jb]^a[a,g^jb],z]\otimes g^jb \end{split}$$

www.SID.ir

31

$$= [[g^{i}, g^{j}b]^{a}, z]^{w}[a, g^{j}b, z] \otimes g^{j}b$$

$$= ([[g^{i}, g^{j}b]^{a}, z]^{w} \otimes g^{j}b)^{w'}([a, g^{j}b, z] \otimes g^{j}b)$$

$$= ([[g^{i}, b]^{a}, z]^{w} \otimes b)^{w'}([[g^{i}, b]^{a}, z]^{w} \otimes g^{j})^{bw'} \quad a \in B^{\otimes}(G)$$

$$= ([[g^{i}, b]^{a}, z]^{w} \otimes g^{j})^{bw'} \quad b \in B^{\otimes}(G)$$

$$= ([[b, g^{i}]^{a}, z]^{v} \otimes g^{j})^{-v'}$$

$$= 1_{\otimes}, \quad b \in B^{\otimes}(G)$$
^aw, v' = [[b, q^{i}]^{a}, z]^{-v}bw', as required. \square

where $w = [a, g^j b], w' = [a, g^j b, z], v = [b, g^i]^{-a} w, v' = [[b, g^i]^a, z]^{-v} bw'$, as required.

Theorem 4.4. Let G be a p-group satisfying the law (4.1.1). Then $G \otimes G$ is abelian. (4.4.1) $\gamma_3(G) \leq Z^{\otimes}(G)$. (4.4.2)

Proof. If G is an abelian p-group, then the result is obvious. Otherwise, $G \otimes G$ has no element of order 2, since G is a p-group and p is an odd prime. Hence, $[a, b] \otimes [x, y] = 1_{\otimes}$ by (4.2.3). So (2.2.5) implies (4.4.1). Next we have $([a, b, x] \otimes y)^2 = 1_{\otimes}$ by (4.2.2) since $[a, b] \otimes [x, y] = 1_{\otimes}$. Thus $1_{\otimes} = [a, b, x] \otimes y$, proving (4.4.2).

Corollary 4.5. Let G be a two-generator p-group of class 3 satisfying the law (4.1.1). Then we can determine tensor square of G.

Proof. If G is a p-group satisfying the law (4.1.1), then by (4.4.2), $\gamma_3(G) \leq Z^{\otimes}(G)$. Now using the result of G. J. Ellis [5], we see that $G \otimes G \cong \frac{G}{\gamma_3(G)} \otimes \frac{G}{\gamma_3(G)}$, hence the calculations of tensor squares reduce to the calculations of tensor squares of class 2 groups. From [8], we obtain the complete classification of tensor square of two-generator p-groups of class 2. Hence by Theorem 36 from [8]

$$G \otimes G \cong \begin{cases} C_{p^{\alpha}} \times C_{p^{\beta}} \times C_{p^{\beta}} \times C_{p^{\beta}} \times C_{p^{\rho}} \times C_{p^{\rho}} & if\rho \leq \sigma \\ C_{p^{\alpha}} \times C_{p^{\beta}} \times C_{p^{\beta}} \times C_{p^{\beta+\tau}} \times C_{p^{\sigma}} \times C_{p^{\sigma}} & if\rho > \sigma \end{cases}$$

where $\tau = \min(\alpha - \beta, \rho - \sigma)$ and $\alpha, \beta, \gamma, \rho$ and σ are defined integers as in Theorem 1 from [8](see [8] for more information).

Acknowledgments

The authors would like to thank the referee for his/her helpful comments.

References

- [1] D. P. Biddle and L-C. Kappe, On subgroups related to the tensor center, *Glasgow Math. J.*, **45** (2003) 323–332.
- [2] R. Brown, D. L. Johnson and E. F. Robertson, Some computations of nonabelian tensor products of groups, J. Algebra, 111 (1987) 177–202.
- [3] R. Brown and J. L. Loday, Excision homotopique en basse dimension, C. R. Acad. Sci. Ser. I Math. Paris, 298 (1984) 353–356.
- [4] R. Brown and J. L. Loday, Van Kampen theorems for diagrams of spaces, *Topology*, 26 (1987) 311–335.