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ON THE VALUES OF INDEPENDENCE AND DOMINATION POLYNOMIALS AT SPECIFIC POINTS

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Abstract. Let G be a simple graph of order n. We consider the independence polynomial and the domination polynomial of a graph G. The value of a graph polynomial at a specific point can give sometimes a very surprising information about the structure of the graph. In this paper we investigate independence and domination polynomial at -1 and 1.

1. Introduction

Let $G = (V, E)$ be a simple graph. For any vertex $v \in V$, the *open neighborhood* of v is the set $N(v) = \{u \in V | uv \in E\}$ and the *closed neighborhood* is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. The *corona* of two graphs G_1 and G_2, as defined by Frucht and Harary in [12], is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2, where the ith vertex of G_1 is adjacent to every vertex in the ith copy of G_2. The corona $G \circ K_1$, in particular, is the graph constructed from a copy of G, where for each vertex $v \in V(G)$, a new vertex v' and a pendant edge vv' are added. The *join* of two graphs G_1 and G_2, denoted by $G_1 + G_2$ is a graph with vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2) \cup \{uv | u \in V(G_1) \text{ and } v \in V(G_2)\}$.
Graph polynomials are a well-developed area useful for analyzing properties of graphs. The value of a graph polynomial at a specific point can give sometimes a very surprising information about the structure of the graph. Balister, et al. in [9] proved that for any graph G, $|q(G, -1)|$ is always a power

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of 2, where $q(G,x)$ is interlace polynomial of a graph G. Stanley in [14] proved that $(-1)^n P(G,-1)$ is the number of acyclic orientations of G, where $P(G,\lambda)$ is the chromatic polynomial of G and $n=|V(G)|$.

In this paper we study independence and domination polynomial of a graph at -1 and 1. For convenience, the definition of the independence and domination polynomial of a graph will be given in the next sections.

We denote the path of order n, the cycle of order n, and the wheel of order n, by P_n, C_n, and W_n, respectively.

2. The independence polynomial of a graph at -1 and 1

An *independent set* of a graph G is a set of vertices where no two vertices are adjacent. The *independence number* is the size of a maximum independent set in the graph and denoted by $\alpha(G)$. For a graph G, let i_k denote the number of independent sets of cardinality k in G ($k=0,1,\ldots,\alpha$). The *independence polynomial* of G,

$$I(G,x)=\sum_{k=0}^{\alpha} i_k x^k,$$

is the generating polynomial for the independent sequence $(i_0,i_1,i_2,\ldots,i_\alpha)$.
$I(G;1)=i_0+i_1+i_2+\ldots+i_\alpha$ equals the number of independent sets of G, and $I(G;-1)=i_0-i_1+i_2-\ldots+(-1)^\alpha i_\alpha=f_0(G)-f_1(G)$, where $f_0(G)=i_0+i_2+i_4+\ldots$, $f_1(G)=i_1+i_3+i_5+\ldots$ are equal to the numbers of independent sets of even size and odd size of G, respectively. $I(G;-1)$ is known as the alternating number of independent sets.
We need the following theorem:

Theorem 2.1. ([13]) *If $w\in V(G)$ and $uv\in E(G)$, then the following equalities hold:*

(i) $I(G;x)=I(G-w;x)+xI(G-N[w];x)$,
(ii) $I(G;x)=I(G-uv;x)-x^2I(G-N(u)\cup N(v);x)$.

A vertex v is pendant if its neighborhood contains only one vertex; an edge $e=uv$ is pendant if one of its endpoints is a pendant vertex.

Theorem 2.2. *If $u\in V(G)$ is a pendant vertex of G and $v\in N(u)$, then*

$$I(G;-1)=-I(G-N[v];-1).$$

Proof. Since $u\in V(G)$ is a pendant vertex of G and $v\in N(u)$, Theorem 2.1 assures that $I(G;x)=I(G-v;x)+xI(G-N[v];x)=(1+x)I(G-\{u,v\};x)+xI(G-N[v];x)$. Therefore $I(G;-1)=-I(G-N[v];-1)$. □

Theorem 2.3. *For $n \geq 1$, the following hold:*

(i) $I(P_{3n-2}, -1) = 0$ *and* $I(P_{3n-1}, -1) = I(P_{3n}, -1) = (-1)^n$*;*

(ii) $I(C_{3n}, -1) = 2(-1)^n$, $I(C_{3n+1}, -1) = (-1)^n$ *and* $I(C_{3n+2}, -1) = (-1)^{n+1}$*;*

(iii) $I(W_{3n+1}, -1) = 2(-1)^n - 1$ *and* $I(W_{3n}, -1) = I(W_{3n+2}, -1) = (-1)^n - 1$*.*

Proof.

(i) We prove by induction on n. For $n = 1$, since $I(P_1, x) = 1 + x$, $I(P_2, x) = 1 + 2x$ and $I(P_3, x) = 1 + 3x + x^2$, we have $I(P_1, -1) = 0$ and $I(P_2, -1) = I(P_3, -1) = -1$. Suppose that the result is true for any $k \leq 3n$. By Theorem 2.1(i), $I(P_{k+1}, x) = I(P_k, x) + xI(P_{k-1}, x)$, therefore

$$I(P_{3n+1}, -1) = I(P_{3n}, -1) - I(P_{3n-1}, -1) = (-1)^n - (-1)^n = 0;$$
$$I(P_{3n+2}, -1) = I(P_{3n+1}, -1) - I(P_{3n}, -1) = 0 - (-1)^n = (-1)^{n+1};$$
$$I(P_{3n+3}, -1) = I(P_{3n+2}, -1) - I(P_{3n+1}, -1) = (-1)^{n+1} - 0 = (-1)^{n+1}$$

(ii) Since $I(C_3, x) = 1 + 3x$, $I(C_4, x) = 1 + 4x + 2x^2$, $I(C_5, x) = 1 + 5x + 5x^2$ and $I(C_6, x) = 1 + 6x + 9x^2 + 2x^3$, we have $I(C_3, -1) = 2(-1)$, $I(C_4, -1) = -1$ and $I(C_5, -1) = (-1)^2$. By Theorem 2.1 and Part (i), $I(C_n, x) = I(P_{n-1}, x) + xI(P_{n-3}, x)$. Therefore $I(C_n, -1) = I(P_{n-1}, -1) - I(P_{n-3}, -1)$. Now it is easy to see that $I(C_{3k}, -1) = 2(-1)^k$, $I(C_{3k+1}, -1) = (-1)^k$ and $I(C_{3k+2}, -1) = (-1)^{k+1}$.

(iii) By Theorem 2.1(i), $I(W_n, x) = I(C_{n-1}, x) + x$. Now by part (ii) we have the result. □

In the following theorem we compute the number of independent sets of paths, cycles and wheels:

Theorem 2.4. (i) *For any positive integer n,*

$$I(P_n, 1) = \prod_{s=1}^{\lfloor \frac{n+1}{2} \rfloor} \left(1 + 4\cos^2 \frac{s\pi}{n+2}\right).$$

(ii) *For any integer $n \geq 3$,*

$$I(C_n, 1) = \prod_{s=1}^{\lfloor \frac{n}{2} \rfloor} \left(1 + 4\cos^2 \frac{(s - \frac{1}{2})\pi}{n}\right).$$

(iii) *For any integer $n \geq 4$,*

$$I(W_n, 1) = 1 + \prod_{s=1}^{\lfloor \frac{n-1}{2} \rfloor} \left(1 + 4\cos^2 \frac{(s - \frac{1}{2})\pi}{n-1}\right).$$

Proof. We know that (see [8]), $I(P_n, x) = \prod_{s=1}^{\lfloor \frac{n+1}{2} \rfloor} \left(2x + 1 + 2x\cos \frac{2s\pi}{n+2}\right)$, $I(C_n, x) = \prod_{s=1}^{\lfloor \frac{n}{2} \rfloor} \left(2x + 1 + 2x\cos \frac{(2s-1)\pi}{n}\right)$ and $I(W_n, x) = I(C_{n-1}, x) + x$. By substituting $x = 1$, we have the results. □

We recall the following theorem:

Theorem 2.5. ([3, 11]) *If G has t connected components $G_1, \ldots, G_t$, then* $I(G, x) = \prod_{i=1}^{t} I(G_i, x)$.

Theorem 2.6. *For any tree T, $I(T; -1) \in \{-1, 0, 1\}$, i.e., the number of independent sets of even size varies by at most one from the number of independent sets of odd size.*

Proof. By induction on $n = |V(T)|$. It is not hard to see that for any tree with order $1, 2, 3, 4, 5$ we have the result. Now suppose that $|V(T)| = n + 1 \geq 5$, v is a pendant vertex of T, $N(v) = \{u\}$ and $T_1, T_2, \ldots, T_k$ are the trees of the forest $T - N[u]$. According to Theorem 2.1(ii), we have:

$$\begin{aligned} I(T; x) &= I(T - uv; x) - x^2 I(T - N(u) \cup N(v); x) = \\ & (1 + x) I(T - v; x) - x^2 I(T - N[u]; x) = \\ & (1 + x) I(T - v; x) - x^2 I(T_1; x) I(T_2; x) \ldots I(T_k; x) \end{aligned}$$

Therefore $I(T; -1) = -I(T_1; -1) I(T_2; -1) \ldots I(T_k; -1) \in \{-1, 0, 1\}$, since every tree T_i has less than n vertices, by the induction hypothesis, $I(T_i; -1) \in \{-1, 0, 1\}$. □

Theorem 2.7. *If F is a forest, then $I(F; -1) \in \{-1, 0, 1\}$.*

Proof. If $T_1, T_2, \ldots, T_k$, $k \geq 1$, are the connected components of F, then $I(F; x) = I(T_1; x) I(T_2; x) \ldots I(T_k; x)$, because $F = T_1 \cup T_2 \cup \ldots \cup T_k$. Now, the conclusions follow from Theorem 2.6. □

Theorem 2.8. ([3, 11]) $I(G_1 + G_2; x) = I(G_1; x) + I(G_2; x) - 1$.

Theorem 2.9. *For any integer k there is some connected graph G such that $I(G; -1) = k$.*

Proof. First suppose that k is a negative integer. It is easy to check that $I(K_{a,a,\ldots,a}, x) = n(1 + x)^a - (a - 1)$, and therefore $I(K_{a,a,\ldots,a}, -1) = 1 - a$ i.e., for any negative integer k there is some connected graph G such that $I(G; -1) = k$. Now suppose that k is a positive integer. Let G_i, $1 \leq i \leq k$, be k graphs with $I(G_i; -1) = 2$ for every $i \in \{1, 2, \ldots, k\}$ (for example we can put $G_i = C_6$ for every $i \in \{1, 2, \ldots, k\}$) and $H = G_1 + G_2 + \ldots + G_k$. Then

$$I(H, x) = I(G_1, x) + I(G_2, x) + \ldots + I(G_k, x) - (k - 1)$$

and consequently $I(H, -1) = k + 1$. The case of $k = 0$ follows from Theorem 2.6. □

3. The domination polynomial at −1 and 1

A set $S \subseteq V$ is a *dominating set* if $N[S] = V$, or equivalently, every vertex in $V \backslash S$ is adjacent to at least one vertex in S. An *i-subset* of $V(G)$ is a subset of $V(G)$ of cardinality i. Let $\mathcal{D}(G,i)$ be the family of dominating sets of G which are i-subsets and let $d(G,i) = |\mathcal{D}(G,i)|$. The polynomial $D(G,x) = \sum_{i=1}^{|V(G)|} d(G,i)x^i$ is defined as *domination polynomial* of G ([1, 4, 5]). Every root of $D(G,x)$ is called a *domination root* of G. The set of all roots of $D(G,x)$ is denoted by $Z(D(G,x))$. Let $A \subseteq B \subseteq V$. Define $\mathcal{D}_{A,B}(G,i)$ as follows

$$\mathcal{D}_{A,B}(G,i) = \{S \in \mathcal{D}(G,i) \mid S \cap B = A\}.$$

Let $d_{A,B}(G,i) = |\mathcal{D}_{A,B}(G,i)|$ and define $D_{A,B}(G,x) = \sum_{i=1}^{|V|} d_{A,B}(G,i)x^i$.
We shall consider the value of domination polynomial at −1 and 1.
Note that $D(G;1) = d(G,1) + d(G,2) + \ldots + d(G,n)$ equals the number of dominating sets of G, and $D(G;-1) = -d(G,1) + d(G,2) - d(G,3) + \ldots + (-1)^n d(G,n) = g_0(G) - g_1(G)$, where $g_0(G) = d(G,2) + d(G,4) + d(G,6) + \ldots, g_1(G) = d(G,1) + d(G,3) + d(G,5) + \ldots$ are equal to the numbers of dominating sets of even size and odd size of G, respectively. We call $D(G;-1)$ as the alternating number of dominating sets.
The following theorem gives the value of domination polynomial of complete graphs, complete bipartite graphs and stars at 1 and −1:

Theorem 3.1. (i) $D(K_n,-1) = 1$, $D(K_{m,n},-1) \in \{\pm 1, 3\}$ *and* $D(K_{1,n},-1) \in \{-1,1\}$.
(ii) $D(K_n,1) = D(K_{1,n},1) = 2^n + 1$ *and* $D(K_{m,n},1) = (2^m-1)(2^n-1)+2$.

Proof. The results follow easily from $D(K_n,x) = (1+x)^n + 1$ and $D(K_{m,n},x) = ((1+x)^m - 1)((1+x)^n - 1) + x^m + x^n$ (see [4]). □

Theorem 3.2. *Let G be a graph. If G has a vertex v of degree k such that $N[v]$ is a clique, then*

$$D(G,x) = xD(G-N[v],x) + (1+x)\sum_{\emptyset \neq A \subseteq N(v)} D_{A,N(v)}(G-v,x).$$

where $N_2, \ldots, N_{2^k}$ are all nonempty subsets of $N(v)$. Moreover, $D(G,-1) = -D(G-N[v],-1)$.

Proof. Suppose that $N(v) = \{v_1, \ldots, v_k\}$. Let $N_1 = \emptyset, N_2, \ldots, N_{2^k}$ be all subsets of $N(v)$. Let A be a dominating set of G with cardinality i. We have the following cases:

1) $v \in A$ and $A \cap N(v) = N_1$, then $A - \{v\}$ is a dominating set of $G - N[v]$ with cardinality $i-1$.
2) $v \in A$ and there exists an index t, $2 \leq t \leq 2^k$ such that $A \cap N(v) = N_t$. Since $N[v]$ is a clique, $A - \{v\}$ is a dominating set of $G - v$ with cardinality $i-1$ and $(A - \{v\}) \cap N(v) = N_t$.
3) $v \notin A$. Therefore $A \cap N(v) \neq \emptyset$ and there exists an index l, $2 \leq l \leq 2^k$ such that $A \cap N(v) = N_l$. Therefore A is a dominating set of $G - v$ with cardinality i and $A \cap N(v) = N_l$.

By considering the above cases we obtain that

$$d(G,i) = d(G-N[v], i-1) + \sum_{r=2}^{2^k} d_{N_r,N(v)}(G-v, i-1) + \sum_{r=2}^{2^k} d_{N_r,N(v)}(G-v, i).$$

Therefore we have

$$D(G,x) = xD(G-N[v], x) + (1+x)\sum_{r=2}^{2^k} D_{N_r,N(v)}(G-v, x).$$

Now, by putting $x=-1$ in the above equality the proof is complete. □

As a consequence of the previous theorem we determine the exact value of the domination polynomial of a forest at -1. We recall the following theorem:

Theorem 3.3. ([4]) *If G has t connected components $G_1,\ldots,G_t$, then $D(G,x)=\prod_{i=1}^{t} D(G_i,x)$.*

Theorem 3.4. *Let F be a forest. Then $D(F,-1)=(-1)^{\alpha(F)}$.*

Proof. By Theorem 3.3 it is sufficient to prove the theorem for trees. We prove this theorem by induction on n, where n is the order of tree T. For $n=1,2$ there is nothing to prove. Now, Let $n\geq 3$. Let u_0 be an arbitrary vertex of T. Assume that w is a vertex with maximum distance of u_0. Clearly $d(w)$=1. Let $vw\in E(T)$. Obviously, at most one of the neighbors of v has degree more than one. Since $n\geq 3$, then $d(v)\geq 2$. Assume that $d(v)=t$ and $\{u_1,\ldots,u_{t-1}\}\subseteq N(v)$ and $d(u_i)$=1, for $i=1,\ldots,t-1$. Let $H=T-\{v,u_1,\ldots,u_{t-1}\}$. Clearly, H is a tree and $\alpha(T)=\alpha(H)+t-1$. Now, By Theorems 3.3 and 3.2 and induction hypothesis we have

$$D(T,-1) = -D(T-N[u_1],-1) = -D(H,-1)(-1)^{t-2} = (-1)^{\alpha(H)}(-1)^{t-1}.$$

Therefore $D(T,-1)=(-1)^{\alpha(T)}$ and the proof is complete. □

By Theorem 3.4 we have the following corollary:

Corollary 3.5. *For any forest F, $D(F;-1)\in\{-1,1\}$, i.e., the number of dominating sets of even size varies by at most one from the number of dominating sets of odd size.*

Lemma 3.6. ([2]) *If n is a positive integer, then*

$$D(C_n,-1)=\begin{cases} 3 & \textit{if } n\equiv 0\,(mod\,4); \\ -1 & \textit{otherwise.}\end{cases}$$

Corollary 3.7. *For any $n\geq 4$, $D(W_n,-1)\in\{-1,3\}$.*

Proof. Since for every $n \geq 4$, $D(W_n, x) = x(1+x)^{n-1} + D(C_{n-1}, x)$ ([4]), the result follows from Lemma 3.6. □

Since $D(P_n, x) = x[D(P_{n-1}, x) + D(P_{n-2}, x) + D(P_{n-3}, x)]$ and $D(C_n, x) = x[D(C_{n-1}, x) + D(C_{n-2}, x) + D(C_{n-3}, x)]$ (see [6, 7]) we have the following theorem:

Theorem 3.8. (i) *For every $n \geq 4$, $D(P_n, 1) = D(P_{n-1}, 1) + D(P_{n-2}, 1) + D(P_{n-3}, 1)$ with initial values $D(P_1, 1) = 1, D(P_2, 1) = 3$ and $D(P_3, 1) = 5$.*

(ii) *For every $n \geq 4$, $D(C_n, 1) = D(C_{n-1}, 1) + D(C_{n-2}, 1) + D(C_{n-3}, 1)$ with initial values $D(C_1, 1) = 1, D(C_2, 1) = 3$ and $D(C_3, 1) = 7$.*

(iii) *For any $n \geq 4$, $D(W_n, 1) = D(C_{n-1}, 1) + 2^{n-1}$.*

Theorem 3.9. *Let G be a unicycle graph. Then $D(G, -1) \in \{\pm 1, \pm 3\}$.*

Proof. We prove the theorem by induction on $n = |V(G)|$. If $n = 3$, then $G = K_3$ and there is nothing to prove. Now, let $n \geq 4$. If $G = C_n$, then by Lemma 3.6, $D(G, -1) \in \{-1, 3\}$. If $G \neq C_n$, then there exists a vertex u such that $d(u) = 1$. Suppose that $N(u) = \{v\}$. Clearly, at most one of the components of $G - N[u]$ is a unicycle graph. Now, by Theorems 3.3, 3.2 and 3.4 and induction hypothesis the proof is complete. □

Theorem 3.10. ([10]) *For every graph, the number of dominating sets is odd.*

Theorem 3.11. *Let G be a graph. Then $D(G, r)$ is odd for every odd integer r. In particular $D(G, -1)$ is odd.*

Proof. Suppose that r, s are two odd integers. Clearly, for every graph G, $D(G, r) \equiv D(G, s) \pmod 2$. Therefore $D(G, r) \equiv D(G, 1) \pmod 2$. So by Theorem 3.10, we conclude $D(G, r)$ is odd. Therefore $D(G, -1)$ is odd. □

The following corollary is an immediate consequence of Theorem 3.11.

Corollary 3.12. *Every integer domination root of a graph G is even.*

We need the following theorem to prove another results:

Theorem 3.13. ([4]) *Let G_1 and G_2 be graphs of orders n_1 and n_2, respectively. Then*

$$D(G_1 + G_2, x) = \Big((1+x)^{n_1} - 1\Big)\Big((1+x)^{n_2} - 1\Big) + D(G_1, x) + D(G_2, x).$$

The following corollary is an immediate consequence of above theorem:

Corollary 3.14. *For any two graphs G_1 and G_2*

$$D(G_1 + G_2, -1) = D(G_1, -1) + D(G_2, -1) + 1.$$

Theorem 3.15. *For any odd integer $n = 2k - 1$ ($k \in \mathbb{Z}$), there is some connected graph G such that $D(G; -1) = n$.*

Proof. Let G_i ($1 \le i \le k$) be k graphs with $D(G_i, -1) = 1$ for every $i \in \{1, 2, \ldots, k\}$ and $H = G_1 + G_2 + \ldots + G_k$. Then

$$D(H, x) = D(G_1, x) + \ldots + D(G_k, x) + (k - 1)$$

and consequently, $D(H, -1) = k + (k - 1) = 2k - 1 = n$. So the result is true for positive odd integer. Now we consider negative odd integer. Let G_i ($1 \le i \le k$) be k graphs with $D(G_i, -1) = -3$ for every $i \in \{1, 2, \ldots, k\}$ and $H = G_1 + G_2 + \ldots + G_k$. Then

$$D(H, x) = D(G_1, x) + \ldots + D(G_k, x) + (k - 1)$$

and consequently, $D(H, -1) = -3k + (k - 1) = -2k - 1$. □

As an example for positive odd integer (in above theorem) we consider complete n-partite graphs. It is easy to check that for the complete n-partite graph $K_{m_1,m_2,\ldots,m_n}$ we have

$$D(K_{m_1,m_2,\ldots,m_n}, x) = \sum_{i=m_1}^{m_n} x^i + \sum_{i=2}^{n} ((1 + x)^{m_i} - 1)((1 + x)^{m_1+\ldots+m_{i-1}} - 1).$$

Suppose that for every $1 \le i \le n$ the number m_i is even. By above formula we have

$$D(K_{m_1,m_2,\ldots,m_n}, -1) = \sum_{i=m_1}^{m_n} (-1)^i + \sum_{2}^{n} 1$$

Since all of m_i are even, we have $D(K_{m_1,\ldots,m_n}, -1) = 2n - 1$.

Now we give an example for negative odd integer (in above theorem). Suppose that G is a unicycle graph G with $D(G, -1) = -3$ (see Theorem 3.9). Consider $H = \underbrace{G + G + \ldots + G}_{n-times}$. It is easy to see that $D(H, -1) = -2k + 1$.

In this paper we studied independence and domination polynomial at -1 and 1. There are many graphs which have -2 as domination roots (as an example all graphs of the form $H \circ K_1$ have this property, see [1]). We think that the study of the value of domination polynomial of a graph at -2 is important, because we have the following conjecture:

Conjecture 3.16. *If r is an integer domination root of a graph, then $r = 0$ or $r = -2$.*

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References

[1] S. Akbari, S. Alikhani, and Y. H. Peng, Characterization of graphs using domination polynomial, *European J. Combin.*, **31** (2010) 1714-1724.

[2] S. Akbari and M. R. Oboudi, Cycles are determined by their domination polynomials, *Ars Combin.*, to appear.

[3] Y. Alavi, P. J. Malde, A. J. Schwenk and P. Erdös, The vertex independence sequence of a graph is not constrained, *Congr. Numer.*, **58** (1987) 15-23.

[4] S. Alikhani and Y. H. Peng, Introduction to domination polynomial of a graph, *Ars Combin.*, in press. Available at `http://arxiv.org/abs/0905.2251`.

[5] S. Alikhani, Y. H. Peng, Domination polynomials of cubic graphs of order 10, *Turkish J. Math.*, **35** (2011) 355-366.

[6] S. Alikhani and Y. H. Peng, Dominating sets and domination polynomial of paths, *Int. J. Math. Math. Sci.*, **2009**, Article ID 542040, 10 pages doi:10.1155/2009/542040.

[7] S. Alikhani and Y. H. Peng, Dominating sets and domination polynomials of certain graphs, II, *Opuscula Math.*, **30** no. 1 (2010) 37-51.

[8] S. Alikhani and Y. H. Peng, Independence roots and independence fractals of certain graphs, *J. Appl. Math. Comput.*, **36** (2011) 89-100.

[9] P. N. Balister, B. Bollobas, J. Cutler, and L. Pebody, The interlace polynomial of graphs at -1, *European J. Combin.*, **23** (2002) 761-767.

[10] A. E. Brouwer, *The number of dominating sets of a finite graph is odd*, preprint.

[11] J. I. Brown, K. Dilcher and R. J. Nowakowski, Roots of independence polynomials of well covered graphs, *J. Algebraic Combin.*, **11** (2000) 197-210.

[12] R. Frucht and F. Harary, On the corona of two graphs, *Aequationes Math.*, **4** (1970) 322-325.

[13] I. Gutman and F. Harary, Generalizations of the matching polynomial, *Utilitas Math.*, **24** (1983) 97-106.

[14] R. P. Stanley, Acyclic orientations of graphs, *Discrete Math.*, **5** (1973) 171-178.

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