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ON THE VALUES OF INDEPENDENCE AND DOMINATION POLYNOMIALS AT SPECIFIC POINTS

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ABSTRACT. Let G be a simple graph of order n. We consider the independence polynomial and the domination polynomial of a graph G. The value of a graph polynomial at a specific point can give sometimes a very surprising information about the structure of the graph. In this paper we investigate independence and domination polynomial at -1 and 1.

1. Introduction

Let G = (V, E) be a simple graph. For any vertex $v \in V$, the open neighborhood of v is the set $N(v) = \{u \in V | uv \in E\}$ and the closed neighborhood is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. The corona of two graphs G_1 and G_2 , as defined by Frucht and Harary in [12], is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 , where the *i*th vertex of G_1 is adjacent to every vertex in the *i*th copy of G_2 . The corona $G \circ K_1$, in particular, is the graph constructed from a copy of G, where for each vertex $v \in V(G)$, a new vertex v' and a pendant edge vv' are added. The *join* of two graphs G_1 and G_2 , denoted by $G_1 + G_2$ is a graph with vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2) \cup \{uv | u \in V(G_1)$ and $v \in V(G_2)$.

Graph polynomials are a well-developed area useful for analyzing properties of graphs. The value of a graph polynomial at a specific point can give sometimes a very surprising information about the structure of the graph. Balister, et al. in [9] proved that for any graph G, |q(G, -1)| is always a power

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of 2, where q(G, x) is interlace polynomial of a graph G. Stanley in [14] proved that $(-1)^n P(G, -1)$ is the number of acyclic orientations of G, where $P(G, \lambda)$ is the chromatic polynomial of G and n = |V(G)|.

In this paper we study independence and domination polynomial of a graph at -1 and 1. For convenience, the definition of the independence and domination polynomial of a graph will be given in the next sections.

We denote the path of order n, the cycle of order n, and the wheel of order n, by P_n , C_n , and W_n , respectively.

2. The independence polynomial of a graph at -1 and 1

An independent set of a graph G is a set of vertices where no two vertices are adjacent. The independence number is the size of a maximum independent set in the graph and denoted by $\alpha(G)$. For a graph G, let i_k denote the number of independent sets of cardinality k in G ($k = 0, 1, ..., \alpha$). The independence polynomial of G,

$$I(G, x) = \sum_{k=0}^{\alpha} i_k x^k,$$

is the generating polynomial for the independent sequence $(i_0, i_1, i_2, \ldots, i_{\alpha})$.

 $I(G;1) = i_0 + i_1 + i_2 + \ldots + i_\alpha$ equals the number of independent sets of G, and $I(G;-1) = i_0 - i_1 + i_2 - \ldots + (-1)^\alpha i_\alpha = f_0(G) - f_1(G)$, where $f_0(G) = i_0 + i_2 + i_4 + \ldots, f_1(G) = i_1 + i_3 + i_5 + \ldots$ are equal to the numbers of independent sets of even size and odd size of G, respectively. I(G;-1) is known as the alternating number of independent sets.

We need the following theorem:

Theorem 2.1. ([13]) If $w \in V(G)$ and $uv \in E(G)$, then the following equalities hold:

(i)
$$I(G; x) = I(G - w; x) + xI(G - N[w]; x),$$

(ii) $I(G; x) = I(G - uv; x) - x^2I(G - N(u) \cup N(v); x).$

A vertex v is pendant if its neighborhood contains only one vertex; an edge e = uv is pendant if one of its endpoints is a pendant vertex.

Theorem 2.2. If $u \in V(G)$ is a pendant vertex of G and $v \in N(u)$, then

$$I(G; -1) = -I(G - N[v]; -1).$$

Proof. Since $u \in V(G)$ is a pendant vertex of G and $v \in N(u)$, Theorem 2.1 assures that $I(G;x) = I(G - v; x) + xI(G - N[v]; x) = (1 + x)I(G - \{u, v\}; x) + xI(G - N[v]; x)$. Therefore I(G; -1) = -I(G - N[v]; -1).

Theorem 2.3. For $n \ge 1$, the following hold:

- (i) $I(P_{3n-2}, -1) = 0$ and $I(P_{3n-1}, -1) = I(P_{3n}, -1) = (-1)^n$;
- (ii) $I(C_{3n}, -1) = 2(-1)^n$, $I(C_{3n+1}, -1) = (-1)^n$ and $I(C_{3n+2}, -1) = (-1)^{n+1}$;
- (iii) $I(W_{3n+1}, -1) = 2(-1)^n 1$ and $I(W_{3n}, -1) = I(W_{3n+2}, -1) = (-1)^n 1$.

Proof.

(i) We prove by induction on *n*. For n = 1, since $I(P_1, x) = 1 + x$, $I(P_2, x) = 1 + 2x$ and $I(P_3, x) = 1 + 3x + x^2$, we have $I(P_1, -1) = 0$ and $I(P_2, -1) = I(P_3, -1) = -1$. Suppose that the result is true for any $k \leq 3n$. By Theorem 2.1(i), $I(P_{k+1}, x) = I(P_k, x) + xI(P_{k-1}, x)$, therefore

$$I(P_{3n+1}, -1) = I(P_{3n}, -1) - I(P_{3n-1}, -1) = (-1)^n - (-1)^n = 0;$$

$$I(P_{3n+2}, -1) = I(P_{3n+1}, -1) - I(P_{3n}, -1) = 0 - (-1)^n = (-1)^{n+1};$$

$$I(P_{3n+3}, -1) = I(P_{3n+2}, -1) - I(P_{3n+1}, -1) = (-1)^{n+1} - 0 = (-1)^{n+1}$$

- (ii) Since $I(C_3, x) = 1 + 3x$, $I(C_4, x) = 1 + 4x + 2x^2$, $I(C_5, x) = 1 + 5x + 5x^2$ and $I(C_6, x) = 1 + 6x + 9x^2 + 2x^3$, we have $I(C_3, -1) = 2(-1)$, $I(C_4, -1) = -1$ and $I(C_5, -1) = (-1)^2$. By Theorem 2.1 and Part (i), $I(C_n, x) = I(P_{n-1}, x) + xI(P_{n-3}, x)$. Therefore $I(C_n, -1) = I(P_{n-1}, -1) - I(P_{n-3}, -1)$. Now it is easy to see that $I(C_{3k}, -1) = 2(-1)^k$, $I(C_{3k+1}, -1) = (-1)^k$ and $I(C_{3k+2}, -1) = (-1)^{k+1}$.
- (iii) By Theorem 2.1(i), $I(W_n, x) = I(C_{n-1}, x) + x$. Now by part (ii) we have the result.

In the following theorem we compute the number of independent sets of paths, cycles and wheels:

Theorem 2.4. (i) For any positive integer n,

$$I(P_n, 1) = \prod_{s=1}^{\lfloor \frac{n+1}{2} \rfloor} \left(1 + 4\cos^2 \frac{s\pi}{n+2} \right).$$

(ii) For any integer $n \ge 3$,

$$I(C_n, 1) = \prod_{s=1}^{\lfloor \frac{n}{2} \rfloor} \left(1 + 4\cos^2 \frac{(s - \frac{1}{2})\pi}{n} \right).$$

(iii) For any integer $n \ge 4$,

$$I(W_n, 1) = 1 + \prod_{s=1}^{\lfloor \frac{n-1}{2} \rfloor} \left(1 + 4\cos^2 \frac{(s-\frac{1}{2})\pi}{n-1} \right).$$

Proof. We know that (see [8]), $I(P_n, x) = \prod_{s=1}^{\lfloor \frac{n+1}{2} \rfloor} \left(2x + 1 + 2x \cos \frac{2s\pi}{n+2} \right)$, $I(C_n, x) = \prod_{s=1}^{\lfloor \frac{n}{2} \rfloor} \left(2x + 1 + 2x \cos \frac{(2s-1)\pi}{n} \right)$ and $I(W_n, x) = I(C_{n-1}, x) + x$. By substituting x = 1, we have the results. \Box

We recall the following theorem:

Theorem 2.5. ([3, 11]) If G has t connected components G_1, \ldots, G_t , then $I(G, x) = \prod_{i=1}^t I(G_i, x)$.

Theorem 2.6. For any tree T, $I(T; -1) \in \{-1, 0, 1\}$, i.e., the number of independent sets of even size varies by at most one from the number of independent sets of odd size.

Proof. By induction on n = |V(T)|. It is not hard to see that for any tree with order 1, 2, 3, 4, 5 we have the result. Now suppose that $|V(T)| = n + 1 \ge 5$, v is a pendant vertex of T, $N(v) = \{u\}$ and T_1, T_2, \ldots, T_k are the trees of the forest T - N[u]. According to Theorem 2.1(*ii*), we have:

$$I(T;x) = I(T - uv;x) - x^2 I(T - N(u) \cup N(v);x) =$$

(1+x)I(T - v;x) - x²I(T - N[u];x) =
(1+x)I(T - v;x) - x²I(T_1;x)I(T_2;x) ... I(T_k;x)

Therefore $I(T; -1) = -I(T_1; -1)I(T_2; -1) \dots I(T_k; -1) \in \{-1, 0, 1\}$, since every tree T_i has less than n vertices, by the induction hypothesis, $I(T_i; -1) \in \{-1, 0, 1\}$.

Theorem 2.7. If F is a forest, then $I(F; -1) \in \{-1, 0, 1\}$.

Proof. If T_1, T_2, \ldots, T_k , $k \ge 1$, are the connected components of F, then $I(F; x) = I(T_1; x)I(T_2; x) \ldots I(T_k; x)$, because $F = T_1 \cup T_2 \cup \ldots \cup T_k$. Now, the conclusions follow from Theorem 2.6. \Box

Theorem 2.8. ([3, 11]) $I(G_1 + G_2; x) = I(G_1; x) + I(G_2; x) - 1.$

Theorem 2.9. For any integer k there is some connected graph G such that I(G; -1) = k.

Proof. First suppose that k is a negative integer. It is easy to check that $I(K_{a,a,\dots,a}, x) = n(1+x)^a - (a-1)$, and therefore $I(K_{a,a,\dots,a}, -1) = 1 - a$ i.e., for any negative integer k there is some connected graph G such that I(G; -1) = k. Now suppose that k is a positive integer. Let G_i , $1 \le i \le k$, be k graphs with $I(G_i; -1) = 2$ for every $i \in \{1, 2, \dots, k\}$ (for example we can put $G_i = C_6$ for every $i \in \{1, 2, \dots, k\}$) and $H = G_1 + G_2 + \ldots + G_k$. Then

$$I(H, x) = I(G_1, x) + I(G_2, x) + \ldots + I(G_k, x) - (k - 1)$$

and consequently I(H, -1) = k + 1. The case of k = 0 follows from Theorem 2.6.

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3. The domination polynomial at -1 and 1

A set $S \subseteq V$ is a dominating set if N[S] = V, or equivalently, every vertex in $V \setminus S$ is adjacent to at least one vertex in S. An *i*-subset of V(G) is a subset of V(G) of cardinality i. Let $\mathcal{D}(G,i)$ be the family of dominating sets of G which are *i*-subsets and let $d(G,i) = |\mathcal{D}(G,i)|$. The polynomial $D(G,x) = \sum_{i=1}^{|V(G)|} d(G,i)x^i$ is defined as domination polynomial of G ([1, 4, 5]). Every root of D(G,x)is called a *domination root* of G. The set of all roots of D(G,x) is denoted by Z(D(G,x)). Let $A \subseteq B \subseteq V$. Define $\mathcal{D}_{A,B}(G,i)$ as follows

$$\mathcal{D}_{A,B}(G,i) = \{ S \in \mathcal{D}(G,i) \mid S \cap B = A \}.$$

Let $d_{A,B}(G,i) = |\mathcal{D}_{A,B}(G,i)|$ and define $D_{A,B}(G,x) = \sum_{i=1}^{|V|} d_{A,B}(G,i)x^i$. We shall consider the value of domination polynomial at -1 and 1.

Note that $D(G;1) = d(G,1) + d(G,2) + \ldots + d(G,n)$ equals the number of dominating sets of G, and $D(G;-1) = -d(G,1) + d(G,2) - d(G,3) + \ldots + (-1)^n d(G,n) = g_0(G) - g_1(G)$, where $g_0(G) = d(G,2) + d(G,4) + d(G,6) + \ldots, g_1(G) = d(G,1) + d(G,3) + d(G,5) + \ldots$ are equal to the numbers of dominating sets of even size and odd size of G, respectively. We call D(G;-1) as the alternating number of dominating sets.

The following theorem gives the value of domination polynomial of complete graphs, complete bipartite graphs and stars at 1 and -1:

Theorem 3.1. (i)
$$D(K_n, -1) = 1$$
, $D(K_{m,n}, -1) \in \{\pm 1, 3\}$ and $D(K_{1,n}, -1) \in \{-1, 1\}$
(ii) $D(K_n, 1) = D(K_{1,n}, 1) = 2^n + 1$ and $D(K_{m,n}, 1) = (2^m - 1)(2^n - 1) + 2$.

Proof. The results follow easily from $D(K_n, x) = (1+x)^n + 1$ and $D(K_{m,n}, x) = ((1+x)^m - 1)((1+x)^n - 1) + x^m + x^n$ (see [4]).

Theorem 3.2. Let G be a graph. If G has a vertex v of degree k such that N[v] is a clique, then

$$D(G, x) = xD(G - N[v], x) + (1 + x) \sum_{\emptyset \neq A \subseteq N(v)} D_{A, N(v)}(G - v, x).$$

where N_2, \ldots, N_{2^k} are all nonempty subsets of N(v). Moreover, D(G, -1) = -D(G - N[v], -1).

Proof. Suppose that $N(v) = \{v_1, \ldots, v_k\}$. Let $N_1 = \emptyset, N_2, \ldots, N_{2^k}$ be all subsets of N(v). Let A be a dominating set of G with cardinality i. We have the following cases:

- 1) $v \in A$ and $A \cap N(v) = N_1$, then $A \{v\}$ is a dominating set of G N[v] with cardinality i 1.
- 2) $v \in A$ and there exists an index $t, 2 \leq t \leq 2^k$ such that $A \cap N(v) = N_t$. Since N[v] is a clique, $A - \{v\}$ is a dominating set of G - v with cardinality i - 1 and $(A - \{v\}) \cap N(v) = N_t$.
- 3) $v \notin A$. Therefore $A \cap N(v) \neq \emptyset$ and there exists an index $l, 2 \leq l \leq 2^k$ such that $A \cap N(v) = N_l$. Therefore A is a dominating set of G - v with cardinality i and $A \cap N(v) = N_l$.

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By considering the above cases we obtain that

$$d(G,i) = d(G - N[v], i - 1) + \sum_{r=2}^{2^k} d_{N_r, N(v)}(G - v, i - 1) + \sum_{r=2}^{2^k} d_{N_r, N(v)}(G - v, i)$$

Therefore we have

$$D(G, x) = xD(G - N[v], x) + (1 + x)\sum_{r=2}^{2^{k}} D_{N_{r}, N(v)}(G - v, x).$$

Now, by putting x = -1 in the above equality the proof is complete. \Box

As a consequence of the previous theorem we determine the exact value of the domination polynomial of a forest at -1. We recall the following theorem:

Theorem 3.3. ([4]) If G has t connected components G_1, \ldots, G_t , then $D(G, x) = \prod_{i=1}^t D(G_i, x)$.

Theorem 3.4. Let *F* be a forest. Then $D(F, -1) = (-1)^{\alpha(F)}$.

Proof. By Theorem 3.3 it is sufficient to prove the theorem for trees. We prove this theorem by induction on n, where n is the order of tree T. For n = 1, 2 there is nothing to prove. Now, Let $n \ge 3$. Let u_0 be an arbitrary vertex of T. Assume that w is a vertex with maximum distance of u_0 . Clearly d(w)=1. Let $vw \in E(T)$. Obviously, at most one of the neighbors of v has degree more than one. Since $n \ge 3$, then $d(v) \ge 2$. Assume that d(v) = t and $\{u_1, \ldots, u_{t-1}\} \subseteq N(v)$ and $d(u_i)=1$, for $i = 1, \ldots, t-1$. Let $H = T - \{v, u_1, \ldots, u_{t-1}\}$. Clearly, H is a tree and $\alpha(T) = \alpha(H) + t - 1$. Now, By Theorems 3.3 and 3.2 and induction hypothesis we have

$$D(T, -1) = -D(T - N[u_1], -1) = -D(H, -1)(-1)^{t-2} = (-1)^{\alpha(H)}(-1)^{t-1}.$$

Therefore $D(T, -1) = (-1)^{\alpha(T)}$ and the proof is complete. \Box

By Theorem 3.4 we have the following corollary:

Corollary 3.5. For any forest F, $D(F; -1) \in \{-1, 1\}$, i.e., the number of dominating sets of even size varies by at most one from the number of dominating sets of odd size.

Lemma 3.6. ([2]) If n is a positive integer, then

$$D(C_n, -1) = \begin{cases} 3 & \text{if } n \equiv 0 \pmod{4}; \\ -1 & \text{otherwise.} \end{cases}$$

Corollary 3.7. For any $n \ge 4$, $D(W_n, -1) \in \{-1, 3\}$.

Graph polynomial at 1 and -1

Proof. Since for every $n \ge 4$, $D(W_n, x) = x(1+x)^{n-1} + D(C_{n-1}, x)$ ([4]), the result follows from Lemma 3.6.

Since $D(P_n, x) = x[D(P_{n-1}, x) + D(P_{n-2}, x) + D(P_{n-3}, x)]$ and $D(C_n, x) = x[D(C_{n-1}, x) + D(C_{n-2}, x) + D(C_{n-3}, x)]$ (see [6, 7]) we have the following theorem:

- **Theorem 3.8.** (i) For every $n \ge 4$, $D(P_n, 1) = D(P_{n-1}, 1) + D(P_{n-2}, 1) + D(P_{n-3}, 1)$ with initial values $D(P_1, 1) = 1$, $D(P_2, 1) = 3$ and $D(P_3, 1) = 5$.
 - (ii) For every $n \ge 4$, $D(C_n, 1) = D(C_{n-1}, 1) + D(C_{n-2}, 1) + D(C_{n-3}, 1)$ with initial values $D(C_1, 1) = 1, D(C_2, 1) = 3$ and $D(C_3, 1) = 7$.
 - (iii) For any $n \ge 4$, $D(W_n, 1) = D(C_{n-1}, 1) + 2^{n-1}$.

Theorem 3.9. Let G be a unicycle graph. Then $D(G, -1) \in \{\pm 1, \pm 3\}$.

Proof. We prove the theorem by induction on n = |V(G)|. If n = 3, then $G = K_3$ and there is nothing to prove. Now, let $n \ge 4$. If $G = C_n$, then by Lemma 3.6, $D(G, -1) \in \{-1, 3\}$. If $G \ne C_n$, then there exists a vertex u such that d(u) = 1. Suppose that $N(u) = \{v\}$. Clearly, at most one of the components of G - N[u] is a unicycle graph. Now, by Theorems 3.3, 3.2 and 3.4 and induction hypothesis the proof is complete. \Box

Theorem 3.10. ([10]) For every graph, the number of dominating sets is odd.

Theorem 3.11. Let G be a graph. Then D(G, r) is odd for every odd integer r. In particular D(G, -1) is odd.

Proof. Suppose that r, s are two odd integers. Clearly, for every graph $G, D(G, r) \equiv D(G, s) \pmod{2}$. Therefore $D(G, r) \equiv D(G, 1) \pmod{2}$. So by Theorem 3.10, we conclude D(G, r) is odd. Therefore D(G, -1) is odd. \Box

The following corollary is an immediate consequence of Theorem 3.11.

Corollary 3.12. Every integer domination root of a graph G is even.

We need the following theorem to prove another results:

Theorem 3.13. ([4]) Let G_1 and G_2 be graphs of orders n_1 and n_2 , respectively. Then

$$D(G_1 + G_2, x) = \left((1+x)^{n_1} - 1 \right) \left((1+x)^{n_2} - 1 \right) + D(G_1, x) + D(G_2, x).$$

The following corollary is an immediate consequence of above theorem:

Corollary 3.14. For any two graphs G_1 and G_2

$$D(G_1 + G_2, -1) = D(G_1, -1) + D(G_2, -1) + 1$$

Theorem 3.15. For any odd integer n = 2k - 1 ($k \in \mathbb{Z}$), there is some connected graph G such that D(G; -1) = n.

Proof. Let G_i $(1 \le i \le k)$ be k graphs with $D(G_i, -1) = 1$ for every $i \in \{1, 2, ..., k\}$ and $H = G_1 + G_2 + ... + G_k$. Then

$$D(H, x) = D(G_1, x) + \ldots + D(G_k, x) + (k - 1)$$

and consequently, D(H, -1) = k + (k - 1) = 2k - 1 = n. So the result is true for positive odd integer. Now we consider negative odd integer. Let G_i $(1 \le i \le k)$ be k graphs with $D(G_i, -1) = -3$ for every $i \in \{1, 2, ..., k\}$ and $H = G_1 + G_2 + ... + G_k$. Then

$$D(H, x) = D(G_1, x) + \ldots + D(G_k, x) + (k - 1)$$

and consequently, D(H, -1) = -3k + (k - 1) = -2k - 1.

As an example for positive odd integer (in above theorem) we consider complete *n*-partite graphs. It is easy to check that for the complete *n*-partite graph $K_{m_1,m_2,...,m_n}$ we have

$$D(K_{m_1,m_2,\dots,m_n},x) = \sum_{i=m_1}^{m_n} x^i + \sum_{i=2}^n ((1+x)^{m_i} - 1)((1+x)^{m_1+\dots+m_{i-1}} - 1).$$

Suppose that for every $1 \le i \le n$ the number m_i is even. By above formula we have

$$D(K_{m_1,m_2,\dots,m_n},-1) = \sum_{i=m_1}^{m_n} (-1)^i + \sum_{i=m_1}^n (-1)^i$$

Since all of m_i are even, we have $D(K_{m_1,...,m_n}, -1) = 2n - 1$.

Now we give an example for negative odd integer (in above theorem). Suppose that G is a unicycle graph G with D(G, -1) = -3 (see Theorem 3.9). Consider $H = \underbrace{G + G + \ldots + G}_{n-times}$. It is easy to see that D(H, -1) = -2k + 1.

In this paper we studied independence and domination polynomial at -1 and 1. There are many graphs which have -2 as domination roots (as an example all graphs of the form $H \circ K_1$ have this property, see [1]). We think that the study of the value of domination polynomial of a graph at -2 is important, because we have the following conjecture:

Conjecture 3.16. If r is an integer domination root of a graph, then r = 0 or r = -2.

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