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CONNECTED COTOTAL DOMINATION NUMBER OF A GRAPH

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Abstract. A dominating set $D \subseteq V$ of a graph $G = (V, E)$ is said to be a connected cototal dominating set if $\langle D \rangle$ is connected and $\langle V - D \rangle \neq \varnothing$, contains no isolated vertices. A connected cototal dominating set is said to be minimal if no proper subset of D is connected cototal dominating set. The connected cototal domination number $\gamma_{ccl}(G)$ of G is the minimum cardinality of a minimal connected cototal dominating set of G. In this paper, we begin an investigation of connected cototal domination number and obtain some interesting results.

1. Introduction

All graphs considered here are simple, finite, connected and nontrivial. Let $G = (V(G), E(G))$ be a graph, where $V(G)$ is the *vertex set* and $E(G)$ be the *edge set* of G. The vertex $v \in V$ is called a *pendant vertex*, if $deg_G(v) = 1$ and an *isolated vertex* if $deg_G(v) = 0$, where $deg_G(x)$ is the *degree of a vertex* $x \in V(G)$. A vertex which is adjacent to a pendant vertex is called a *support vertex*. We denote $\delta(G)(\Delta(G))$ as the *minimum(maximum) degree* and $p = |V(G)|$, $q = |E(G)|$ the *order* and *size* of G respectively. A *spanning subgraph* is a subgraph containing all the vertices of G. A shortest $u-v$ path is often called a *geodesic*. The *diameter* $diam(G)$ of a connected graph G is the length of any longest geodesic. The *neighborhood* of a vertex u in V is the set $N(u)$ consisting of all vertices v which are adjacent with u. The *closed neighborhood* is $N[u] = N(u) \cup \{u\}$. The *corona* of two graphs G and H is the graph $G \circ H$ formed from one copy of G and $|V(G)|$ copies of H where i^{th} vertex of G is adjacent to every vertex in the i^{th} copy of H. Any undefined term in this paper may be found in [5] or [6].

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A set $D \subseteq V$ of a graph $G = (V, E)$ is a *dominating set* if for every vertex $v \in V - D$ there exists a vertex $u \in D$ such that v is adjacent to u. A dominating set D is said to be *minimal* if no proper subset of D is a dominating set. The minimum cardinality of a minimal dominating set of G is called a *domination number* $\gamma(G)$ of G. A dominating set D is said to be *connected dominating set* if $\langle D \rangle$ is connected. The *connected domination number* $\gamma_c(G)$ of G is the minimum cardinality of a minimal connected dominating set of G [10]. A dominating set D is said to be a *cototal dominating set* if $\langle V - D \rangle$ contains no isolated vertices. The *cototal domination number* $\gamma_{cl}(G)$ of G is the minimum cardinality of a minimal cototal dominating set of G [8]. This concept was also studied as restrained domination in graphs by G. S. Domke [4] and M.A. Henning [7] as follows:

A set $S \subseteq V$ is a *restrained dominating set* if every vertex in $V - S$ is adjacent to a vertex in S and to another vertex in $V - S$. Let $\gamma_r(G)$ denote the size of a smallest restrained dominating set with cardinality $\gamma_r(G)$.

In [1], H. Chen et.al., has been studied the concept of k- connected restrained domination in graphs as follows:

Let $G = (V, E)$ be a graph. A *k-connected restrained dominating set* is a set $S \subseteq V$ where S is a restrained dominating set and $G[S]$ has at most k-components. The *k-connected restrained domination number* of G is denoted by $\gamma_r^k(G)$ is the smallest cardinality of a k- connected restrained dominating set of G.

Our aim in this paper is to introduce a new domination parameter in the field of domination in theory of graphs which is as follow:

A dominating set $D \subseteq V$ of a graph $G = (V, E)$ is said to be a *connected cototal dominating set* if $\langle D \rangle$ is connected and $\langle V - D \rangle \neq \varnothing$, contains no isolated vertices. A connected cototal dominating set is said to be *minimal* if no proper subset of D is connected cototal dominating set. The *connected cototal domination number* $\gamma_{ccl}(G)$ of G is the minimum cardinality of a minimal connected cototal dominating set of G.

For simplicity, the minimal connected cototal dominating set is denoted by $\gamma_{ccl}-$set and in a graph G, $\gamma_{ccl}-$set contains every pendant vertex(if any) and its support vertex in G.

Example

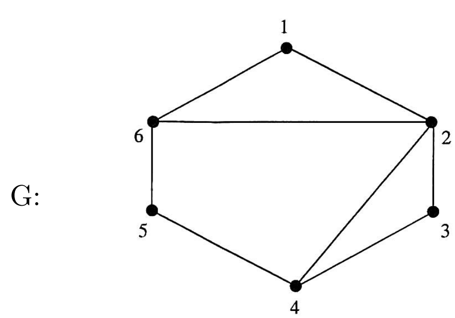

Figure 1

In Figure 1, $V(G) = \{1, 2, 3, 4, 5, 6\}$.

The minimal connected dominating sets are $D_1 = \{2, 6\}$ $D_2 = \{2, 4\}$. Therefore $\gamma_c(G) = |D_1| = |D_2| = 2$.

The minimal cototal dominating sets are $D_1 = \{2, 5\}$, $D_2 = \{3, 6\}$ and $D_3 = \{1, 4\}$. Therefore $\gamma_{cl}(G) = |D_1| = |D_2| = |D_3| = 2$.

The minimal connected cototal dominating sets are, $D_1 = \{1, 2, 6\}$ and $D_2 = \{2, 3, 4\}$. Therefore $\gamma_{ccl}(G) = |D_1| = |D_2| = 3$.

The following observations are immediate.

Observations

(i) $\gamma(G) \leq \gamma_{cl}(G)$ and $\gamma_c(G) \leq \gamma_{ccl}(G)$.

(ii) Let D be a $\gamma_{ccl}-$ set of G, then $\langle D \rangle$ is a tree.

(iii) In a graph G, γ_c- set contains no pendant vertex(if any) in G.

Note: The connected cototal domination number and $k-$connected restrained numbers are same if and only if $k = 1$ and $V - D \neq \emptyset$.

There are certain class of graphs in which the difference between $\gamma_{ccl}(G)$ and γ_r^k is very large. For example trees. Such that $\gamma_r^1(T) = p$ where as $\gamma_{ccl}(T)$ does not exists.

2. Characterization of connected cototal dominating sets

Obviously, we ask the natural question regarding the existence of connected cototal dominating sets. Our first theorem gives the characterization of the existence of connected cototal dominating sets in a graph G.

Theorem 2.1. *A graph G has a connected cototal dominating set if and only if it satisfies the following conditions.*

(i) $|V(G)| \geq 3$

(ii) *G is not a tree*

(iii) *Let $u \in V(G)$ and D be a $\gamma_{ccl}-$ set. Then $V - D \neq \{u\}$.*

The following theorem gives the relationship between $\gamma_{ccl}(G)$ and $\gamma_{ccl}(H)$, where H is a spanning subgraph of G.

Theorem 2.2. *For any graph G, $\gamma_{ccl}(G) \leq \gamma_{ccl}(H)$. Further, the equality holds if and only if $\gamma_{ccl}(G) = p - 2$ and H is unicyclic.*

Proof. Let D be a $\gamma_{ccl}-$set of G and H be any spanning subgraph of G. Let D' be the $\gamma_{ccl}-$set of H. By Theorem 2.1, H must contain at least one cycle. Obviously, $|D| \leq |D'|$. Hence, $\gamma_{ccl}(G) \leq \gamma_{ccl}(H)$.

For equality, suppose $\gamma_{ccl}(G) = p - 2$ and H is unicyclic. Let $v_iv_jv_kv_i$ be a cycle in H. Since $\gamma_{ccl}(G) = p-2$, therefore $D = \{v_1, v_2, ..., v_{p-2}\}$ is a minimal connected dominating set of H, such that $V - D = \{v_i, v_j\}$ and the induced subgraph of $\langle V - D \rangle$ will form K_2. Hence $\langle V - D \rangle$ contains no isolated vertex. Therefore,

$$\begin{aligned}\gamma_{ccl}(H) &= |D| \\ &= |V| - |\{v_i, v_j\}| \\ &= p - 2 \\ &= \gamma_{ccl}(G).\end{aligned}$$

The converse is obvious. □

In the next theorem, we calculate the $\gamma_{ccl}(G)$ of some standard class of graphs.

Theorem 2.3. (i) *For any cycle* C_p; $p \geq 3$, $\gamma_{ccl}(C_p) = p - 2$.
(ii) *For any wheel* W_p; $p \geq 4$, $\gamma_{ccl}(W_p) = 1$.
(iii) *For any complete graph* K_p; $p \geq 3$, $\gamma_{ccl}(K_p) = 1$.
(iv) *For any graph* $H = G + K_1$, $\gamma_{ccl}(H) = 1$.
(v) *For any complete bipartite graph* $K_{m,n}$; $2 \leq m \leq n$, $\gamma_{ccl}(K_{m,n}) = 2$.
(vi) *For any grid graph* $P_2 \times P_k$; $k \geq 2$, $\gamma_{ccl}(P_2 \times P_k) = 2\lceil \frac{k}{3} \rceil$.
(vii) *For any grid graph* $C_3 \times C_k$; $k \geq 3$, $\gamma_{ccl}(C_3 \times C_k) = 3\lceil \frac{k}{3} \rceil$.

The following result is immediate from Theorem 2.3.

Theorem 2.4. *For any graph* G, $1 \leq \gamma_{ccl}(G) \leq p-2$. *Further, the equality of lower bound is attained if and only if* $\delta(G) \geq 2$ *and* $\Delta(G) = p - 1$ *and the equality of an upper bound holds if* $G = C_p$, $p \geq 3$ *or unicyclic.*

Proof. Let G be any nontrivial connected graph of order at least three. Suppose $\gamma_{ccl}(G) = p-1$. Let D be a minimal connected cototal dominating set of G, then $\gamma_{ccl}(G) = |D| = p-1$. Then $\langle V - D\rangle = \{v_i\}$ is an isolated vertex, a contradiction. Hence $\gamma_{ccl}(G) \leq |D| - 1 = p - 2$.
For the equality of lower bound, suppose $\delta(G) \geq 2$ and $\Delta(G) = p - 1$. Let v be a vertex of maximum degree. Then $D = \{v\}$ and such that $\langle V - D\rangle$ has no isolated vertex. Therefore D is a minimal connected cototal dominating set of G. Hence $\gamma_{ccl}(G) = |D| = |\{v\}| = 1$.
Converse is easy to follow.
Equality of an upper bound can be easily verified. □

To prove the next theorem we need the following result.
Theorem A [6] For any graph G, $\lceil \frac{p}{1+\Delta(G)} \rceil \leq \gamma(G)$.

Theorem 2.5. *For any graph* G, $\lceil \frac{p}{1+\Delta(G)} \rceil \leq \gamma_{ccl}(G) \leq 2q - p$. *Further, the equality of a lower bound is attained if* $\Delta(G) = p-1$ *and* $\delta(G) \geq 2$ *and equality of an upper bound is attained if* $\gamma_{ccl}(G) = p-2$.

Proof. The lower bound follows from Theorem A and Observation (i). Further if $\Delta(G) = p - 1$ and $\delta(G) \geq 2$, then equality of lower bound can be easily verified.
Now, for upper bound, we have by Theorem 2.4,

$$\begin{aligned}\gamma_{ccl}(G) &\leq p - 2 \\ &\leq 2(p - 1) - p \\ &\leq 2q - p.\end{aligned}$$

If $\gamma_{ccl}(G) = p - 2$, then equality of an upper bound can be easily verified. □

Theorem 2.6. *Let G_1 and G_2 be two connected graphs with $\delta(G_1) \geq 2$ and $\delta(G_2) \geq 2$. Then $\gamma_{ccl}(G_1 \circ G_2) = |V(G_1)|(1 + \gamma_{ccl}(G_2))$.*

Proof. Let G_1 and G_2 be any two nontrivial connected graphs of order at least two. Let us construct a minimal connected cototal dominating set D of $G_1 \circ G_2$, such that, $|D| = |V(G_1)| + |V(G_1)|\gamma_{ccl}(G_2)$. Let H be the spanning subgraph of G_1, then clearly $|H| \subseteq D$. By Theorem 2.4, $\gamma_{ccl}(G_1) \leq p - 2$ and $\gamma_{ccl}(G_2) \leq p' - 2$, where p' is the $|V(G_2)|$. By the definition of corona of a graph, each vertex of G_1 is attached with a vertex of G_2. Therefore $|D| = |H| + |H|(p - 2)$ is the minimal connected cototal dominating set of $G_1 \circ G_2$. Therefore,

$$\begin{aligned}\gamma_{ccl}(G_1 \circ G_2) &= |D| \\ &= |H| + |H|(p - 2) \\ &= |V(G_1)| + |V(G_1)|\gamma_{ccl}(G_2) \\ &= |V(G_1)|(1 + \gamma_{ccl}(G_2)).\end{aligned}$$ □

Corollary 1. Let G_1 be any graph and G_2 any complete graph of order at least three. Then $\gamma_{ccl}(G_1 \circ G_2) = |V(G_1)|$.

3. Particular Values of $\gamma_{ccl}(G)$

Theorem 3.1. *Let G be any nontrivial connected graph of order at least three. Then $\gamma_{ccl}(G) = 1$ if and only if $\delta(G) \geq 2$ and $\gamma(G) = 1$.*

Proof. Let G be any graph of order at least three with $\gamma_{ccl}(G) = 1$. We consider the following cases:.
Case 1. Suppose $\delta(G) = 1$ and $\gamma(G) = 1$. Let $\{v_i\}$; $1 \leq i \leq p - 1$ be the set of vertices of degree $p - 1$ and $\{u_i\}$ be the set of its neighbors. Then by the definition of connected cototal dominating set, $\gamma_{ccl}(G) = |\{v_i\} \cup \{u_i\}| \geq 2$, a contradiction.
Case 2. Suppose $\delta(G) = 2$, then there exists a vertex u of maximum degree less than or equal to $p - 2$. Let v be a vertex of minimum degree, then by the definition of connected cototal dominating set, $P = v_0, v_1 \cdots, u$ is a path in G. If $\langle V - P \rangle$ has no isolated vertices, then $|P|$ is a connected cototal dominating set. If P is minimal then obviously, $|P| \geq 2$.
Hence $\gamma_{ccl}(G) = |P| \geq 2$, a contradiction.
Conversely, suppose $\delta(G) \geq 2$ and $\gamma(G) = 1$, then there exists a vertex u of degree $p - 1$. Since $\delta(G) \geq 2$, therefore $\langle V(G) - \{u\} \rangle$ contains no isolated vertices. Therefore, $\{u\}$ is a minimal connected cototal dominating set. Hence $\gamma_{ccl}(G) = |\{u\}| = 1$. □

Theorem 3.2. *Let G be any graph with at least three vertices. Then $\gamma_{ccl}(G) = 2$ if and only if there exists at least two adjacent vertices of degree $p - 2$ and $\langle V - D \rangle$ has no isolated vertices.*

Proof. Let G be any graph of order at least three with $\gamma_{ccl}(G) = 2$. Suppose G does not contain two adjacent vertices of degree $p - 2$, then we consider the following cases:
Case 1. Suppose there exists a vertex of degree $p - 1$. Then by Theorem 3.1, $\gamma_{ccl}(G) = 1$.
Case 2. Suppose there exists exactly one vertex of degree $p - 2$. Then we consider the following subcases:

Subcase 2.1. If $\delta(G) = u$ and $\Delta(G) = p - 2 = v$, then there exists a vertex x which is nonadjacent to v but adjacent to a vertex y such that $deg(y) \geq 2$. Therefore $D = \{u, v, y, x\}$ and $\langle V - D \rangle$ contains no isolated vertices. Therefore D is a minimal connected cototal dominating set of G. Therefore,

$$\begin{aligned}\gamma_{ccl}(G) &= |D| \\ &= |\{u, v, y, x\}| \\ &= 4, \text{ a contradiction.}\end{aligned}$$

Subcase 2.2. If $\delta(G) = 2$ and $\Delta(G) = p-2$. Let u be a vertex of minimum degree, which is adjacent to v and x, such that $deg(v) = p - 2$ and $deg(x) = 2$. Suppose there exists a vertex z which is not adjacent to x then by Subcase 2.1, $\gamma_{ccl}(G) \geq 3$, a contradiction. Therefore, either $D_1 = \{v, x\}$ or $D_2 = \{u, v\}$ is a minimal connected cototal dominating set of G. If D_1 is a minimal connected cototal dominating set, then $\langle V(G) - D_1 \rangle = \{u\}$ contains an isolated vertex, a contradiction. If D_2 is a minimal connected cototal dominating set, then $\langle V(G) - D_2 \rangle = \{x\}$ contains an isolated vertex, a contradiction. Let $\langle V(G) - D_3 \rangle = \{u, v, x\}$ be a connected dominating set of G. If $\langle V(G) - D_3 \rangle$ does not contain an isolated vertex, then D_3 is a minimal connected cototal dominating set of G. Therefore,

$$\begin{aligned}\gamma_{ccl}(G) &= |D_3| \\ &= |\{u, v, x\}| \\ &= 3, \text{ a contradiction.}\end{aligned}$$

Case 3. Suppose there exists two nonadjacent vertices u and v such that $deg(u) = deg(v) = p-2$, then there exists a vertex x such that x is nonadjacent to both u and v. Now to dominate x, we consider a vertex z such that $z \in N[x]$ and which is adjacent to both u and v. Then clearly, $\gamma_{ccl}(G) \geq 3$, a contradiction.

Conversely, let u and v be any two adjacent vertices in G, such that $deg(u) = deg(v) = p - 2$. Let $D = \{u, v\}$. Since $\delta(G) \geq 2$, therefore $\langle V(G) - D \rangle$ contains no isolated vertices. Then clearly D is a minimal connected cototal dominating set of G. Hence,

$$\begin{aligned}\gamma_{ccl}(G) &= |D| \\ &= |\{u, v\}| \\ &= 2.\end{aligned}$$

□

Theorem 3.3. *Let G be a graph of order at least four with $\delta(G) \geq 2$ and $diam(G) = 2$. Then $2 \leq \gamma_{ccl}(G) \leq 3$.*

Proof. If G satisfies the hypothesis of the Theorem 3.3, then clearly $\gamma_{ccl}(G) \geq 2$.

For upper bound, let $\gamma_{ccl}(G) \leq 3$ and $diam(G) \neq 2$. Let D be a minimal connected cototal dominating set of G. We consider the following cases:

Case 1. If $diam(G) = 1$ then $G = K_p$, by Theorem 2.3, $\gamma_{ccl}(G) = 1$, a contradiction.

Case 2. If $diam(G) \geq 3$ then clearly $|D| \geq 4$. Hence $\gamma_{ccl}(G) = |D| \geq 4$, a contradiction. Hence $diam(G) = 2$.

□

4. Comparison of $\gamma_{ccl}(G)$ with other domination parameters

A dominating set D is said to be a *nonsplit dominating set* if the induced subgraph $\langle V - D\rangle$ is connected. The *nonsplit domination number* $\gamma_{ns}(G)$ of a graph G is the minimum cardinality of a nonsplit dominating set of G [9].
In the following theorem we give the relationship between $\gamma_{ccl}(G)$ and $\gamma_{ns}(G)$.

Theorem 4.1. *For any minimal connected cototal dominating set D of G, $V - D$ is a nonsplit dominating set of G if and only if for each $v \in D$, the following conditions holds:*
(i) *there exists a cycle in G containing v*
(ii) *for some cycle C_p, $\langle D\rangle$ contains a path $P_{p-1} < C_p$ with v as a pendant vertex of P_{p-1}*

Proof. First we prove the necessity.
Suppose $V - D$ is a dominating set of G and let some vertex $v \in D$. Suppose one of the given condition is not satisfied, then $N[v] \subseteq D$. This implies that v is nonadjacent to any vertex of $V - D$, a contradiction to the hypothesis that $V - D$ is a dominating set. Hence in $\langle V - D\rangle$ there exists a cycle containing v, which proves (i).
Similarly we can prove (ii).
Sufficiency is obvious. □

Next theorem gives the relationship between $\gamma_{ccl}(G)$ and $\gamma_c(G)$.

Theorem 4.2. *A connected dominating set D is a connected cototal dominating set if and only if $\langle V - D\rangle \neq \varnothing$ has no isolated vertices.*

Proof. Proof follows from the definition of connected cototal dominating set of G. □

A dominating set D is said to be a *total dominating set* if the induced subgraph $\langle D\rangle$ has no isolated vertices. The *total domination number* $\gamma_t(G)$ of G is the minimum cardinality of a total dominating set of G [3].
The following theorem gives the relationship between $\gamma_{ccl}(G)$ and $\gamma_t(G)$

Theorem 4.3. *Let G be any graph and D be a minimal connected cototal dominating set of G. Then $V - D$ is a total dominating set if and only if G satisfies the following conditions:*
(i) $\delta(G) \geq 2$
(ii) $N(v) \cap (V - D) \neq \varnothing$ *for all $v \in D$.*

Proof. Let D be a minimal connected cototal dominating set of G for which $V - D$ is a total dominating set. We consider the following cases:
Case 1. Let v be a vertex of minimum degree. Suppose $\delta(G) = 1$. Then $u \in D$. Since $V - D$ is a total dominating set, therefore u must be adjacent to at least one vertex of $V - D$, a contradiction.
Case 2. Since every vertex of G is adjacent to at least one vertex of $V - D$, therefore $N(v) \cap (V - D) \neq \varnothing$ for all $v \in D$.
Conversely, suppose the given conditions are satisfied. Then obviously every vertex in V is adjacent to some vertex in $V - D$ and hence $V - D$ is a total dominating set of G. □

In the next theorem we characterize the graphs which have equal connected domination and connected cototal domination number.

Theorem 4.4. *For any graph G, $\gamma_c(G) = \gamma_{ccl}(G)$ if and only if G satisfies the following conditions:*
(i) $\delta(G) \geq 2$
(ii) *G contains C_3 as an induced subgraph having the vertex set $\{x, y, z\}$, and*
(a) *If $deg(x) = 2$ then $deg(y) = 2$ and $deg(z) = 2$*
(b) *If $deg(x) \geq 3$ then $deg(y) = 2$ and $deg(z) = 2$*

Proof. Let G be any graph of order at least three with $\gamma_c(G) = \gamma_{ccl}(G)$. Let D be a minimal connected cototal dominating set of G. Then we consider the following cases:
Case 1. Suppose $\delta(G) = 1$ and G satisfies the condition (ii). Let v be a vertex of minimum degree, such that $deg(v) = 1$. By Observation (i), $\{v\} \subseteq D$. Also by Observation (ii), $v \notin D'$, where D' is a minimal connected dominating set of G. Hence $|D| + 1 = |D'|$. Therefore,

$$\begin{aligned}\gamma_{ccl}(G) &= |D| \\ &= |D'| - 1 \\ &= \gamma_c(G) - 1, \text{ a contradiction.}\end{aligned}$$

Case 2. If $\delta(G) = 1$ and does not satisfies the condition (ii). Then by Case 1, $\gamma_{ccl}(G) > \gamma_c(G)$, a contradiction.
Conversely, suppose G satisfies the conditions (i) and (ii). Then one can easily observe that $\gamma_c(G) = \gamma_{ccl}(G)$. □

Nordhaus-Gaddum Type Results

Theorem 4.5. *Let G be any graph such that both G and $\overline{G}$ are connected, then*
(i) $\gamma_{ccl}(G) + \gamma_{ccl}(\overline{G}) \leq 2(p-1)$
(ii) $\gamma_{ccl}(G) \cdot \gamma_{ccl}(\overline{G}) \leq (p-1)^2$.

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