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DIRECTIONALLY n-SIGNED GRAPHS-III: THE NOTION OF SYMMETRIC BALANCE

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(DEDICATED TO HONORABLE SHRI DR. M. N. CHANNABASAPPA ON HIS 82^{ND} BIRTHDAY)

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Abstract. Let $G = (V, E)$ be a graph. By *directional labeling (or d-labeling)* of an edge $x = uv$ of G by an ordered n-tuple $(a_1, a_2, \dots, a_n)$, we mean a labeling of the edge x such that we consider the label on uv as $(a_1, a_2, \dots, a_n)$ in the direction from u to v, and the label on x as $(a_n, a_{n-1}, \dots, a_1)$ in the direction from v to u. In this paper, we study graphs, called *(n, d)-sigraphs*, in which every edge is d-labeled by an n-tuple $(a_1, a_2, \dots, a_n)$, where $a_k \in \{+, -\}$, for $1 \le k \le n$. In this paper, we give different notion of balance: symmetric balance in a (n, d)-sigraph and obtain some characterizations.

1. Introduction

For graph theory terminology and notation in this paper we follow the book [3]. All graphs considered here are finite and simple.

There are two ways of labeling the edges of a graph by an ordered n-tuple $(a_1, a_2, \dots, a_n)$ (See [12]).
1. *Undirected labeling* or *labeling*. This is a labeling of each edge uv of G by an ordered n-tuple $(a_1, a_2, \dots, a_n)$ such that we consider the label on uv as $(a_1, a_2, \dots, a_n)$ irrespective of the direction from u to v or v to u.

2. *Directional labeling* or *d-labeling*. This is a labeling of each edge uv of G by an ordered n-tuple $(a_1, a_2, \dots, a_n)$ such that we consider the label on uv as $(a_1, a_2, \dots, a_n)$ in the direction from u to v,

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and $(a_n, a_{n-1}, \ldots, a_1)$ in the direction from v to u.

Note that the d-labeling of edges of G by ordered n-tuples is equivalent to labeling the symmetric digraph $\overrightarrow{G} = (V, \overrightarrow{E})$, where uv is a symmetric arc in $\overrightarrow{G}$ if, and only if, uv is an edge in G, so that if $(a_1, a_2, \ldots, a_n)$ is the d-label on uv in G, then the labels on the arcs $\overrightarrow{uv}$ and $\overrightarrow{vu}$ are $(a_1, a_2, \ldots, a_n)$ and $(a_n, a_{n-1}, \ldots, a_1)$ respectively.

Let H_n be the *n-fold sign group*,

$$H_n = \{+, -\}^n = \{(a_1, a_2, \ldots, a_n) : a_1, a_2, \ldots, a_n \in \{+, -\}\}$$

with co-ordinate-wise multiplication. Thus, writing $a = (a_1, a_2, \ldots, a_n)$ and $t = (t_1, t_2, \ldots, t_n)$ then $at := (a_1t_1, a_2t_2, \ldots, a_nt_n)$. For any $t \in H_n$, the *action* of t on H_n is $a^t = at$, the co-ordinate-wise product.

Let $n \geq 1$ be a positive integer. An *n-signed graph (n-signed digraph)* is a graph $G = (V, E)$ in which each edge (arc) is labeled by an ordered n-tuple of signs, i.e., an element of H_n. A *signed graph* $G = (V, E)$ is a graph in which each edge is labeled by $+$ or $-$. Thus a 1-signed graph is a signed graph. Signed graphs are well studied in literature (See for example [1, 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32]).

In this paper, we study graphs in which each edge is labeled by an ordered n-tuple $a = (a_1, a_2, \ldots, a_n)$ of signs (i.e, an element of H_n) in one direction but in the other direction its label is the reverse: $a^r = (a_n, a_{n-1}, \ldots, a_1)$, called *directionally labeled n-signed graphs* (or *(n, d)-signed graphs*).

Note that an n-signed graph $G = (V, E)$ can be considered as a symmetric digraph $\overrightarrow{G} = (V, \overrightarrow{E})$, where both $\overrightarrow{uv}$ and $\overrightarrow{vu}$ are arcs if, and only if, uv is an edge in G. Further, if an edge uv in G is labeled by the n-tuple $(a_1, a_2, \ldots, a_n)$, then in $\overrightarrow{G}$ both the arcs $\overrightarrow{uv}$ and $\overrightarrow{vu}$ are labeled by the n-tuple $(a_1, a_2, \ldots, a_n)$.

In [1], the authors study voltage graph defined as follows: A *voltage graph* is an ordered triple $\overrightarrow{G} = (V, \overrightarrow{E}, M)$, where V and $\overrightarrow{E}$ are the vertex set and arc set respectively and M is a group. Further, each arc is labeled by an element of the group M so that if an arc $\overrightarrow{uv}$ is labeled by an element $a \in M$, then the arc $\overrightarrow{vu}$ is labeled by its inverse, a^{-1}.

Since each n-tuple $(a_1, a_2, \ldots, a_n)$ is its own inverse in the group H_n, we can regard an n-signed graph $G = (V, E)$ as a voltage graph $\overrightarrow{G} = (V, \overrightarrow{E}, H_n)$ as defined above. Note that the d-labeling of edges in an (n, d)-signed graph considering the edges as symmetric directed arcs is different from the above labeling. For example, consider a $(4, d)$-signed graph in **Figure 1**. As mentioned above, this can also be represented by a symmetric 4-signed digraph. Note that this is not a voltage graph as defined in [1], since for example; the label on $\overrightarrow{v_2v_1}$ is not the (group) inverse of the label on $\overrightarrow{v_1v_2}$.

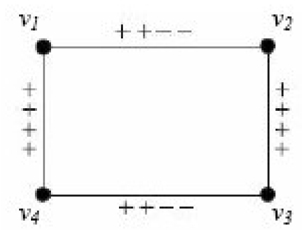

FIGURE 1.

In [10, 11], the authors initiated a study of $(3, d)$ and $(4, d)$-Signed graphs. Also, discussed some applications of $(3, d)$ and $(4, d)$-Signed graphs in real life situations.

In [12], the authors introduced the notion of complementation and generalize the notion of balance in signed graphs to the directionally n-signed graphs. In this context, the authors look upon two kinds of complementation: complementing some or all of the signs, and reversing the order of the signs on each edge. Also given some motivation to study (n, d)-signed graphs in connection with relations among human beings in society.

In [12], the authors defined complementation and isomorphism for (n, d)-signed graphs as follows: For any $t \in H_n$, the *t-complement* of $a = (a_1, a_2, \ldots, a_n)$ is: $a^t = at$. The *reversal* of $a = (a_1, a_2, \ldots, a_n)$ is: $a^r = (a_n, a_{n-1}, \ldots, a_1)$. For any $T \subseteq H_n$, and $t \in H_n$, the *t-complement* of T is $T^t = \{a^t : a \in T\}$.

For any $t \in H_n$, the *t-complement* of an (n, d)-signed graph $G = (V, E)$, written G^t, is the same graph but with each edge label $a = (a_1, a_2, \ldots, a_n)$ replaced by a^t. The *reversal* G^r is the same graph but with each edge label $a = (a_1, a_2, \ldots, a_n)$ replaced by a^r.

Let $G = (V, E)$ and $G' = (V', E')$ be two (n, d)-signed graphs. Then G is said to be *isomorphic* to G' and we write $G \cong G'$, if there exists a bijection $\phi : V \to V'$ such that if uv is an edge in G which is d-labeled by $a = (a_1, a_2, \ldots, a_n)$, then $\phi(u)\phi(v)$ is an edge in G' which is d-labeled by a, and conversely.

For each $t \in H_n$, an (n, d)-signed graph $G = (V, E)$ is *t-self complementary*, if $G \cong G^t$. Further, G is *self reverse*, if $G \cong G^r$.

Proposition 1.1. (**E. Sampathkumar et al.** [12]) *For all $t \in H_n$, an (n, d)-signed graph $G = (V, E)$ is t-self complementary if, and only if, G^a is t-self complementary, for any $a \in H_n$.*

Let $v_1, v_2, \ldots, v_m$ be a cycle C in G and $(a_{k1}, a_{k2}, \ldots, a_{kn})$ be the n-tuple on the edge $v_kv_{k+1}, 1 \leq k \leq m-1$, and $(a_{m1}, a_{m2}, \ldots, a_{mn})$ be the n-tuple on the edge v_mv_1.

For any cycle C in G, let $P(\overrightarrow{C})$ denotes the product of the n-tuples on C given by: $(a_{11}, a_{12}, \ldots, a_{1n})(a_{21}, a_{22}, \ldots, a_{2n}) \ldots (a_{m1}, a_{m2}, \ldots, a_{mn})$ and $P(\overleftarrow{C}) =$ $(a_{mn}, a_{m(n-1)}, \ldots, a_{m1})(a_{(m-1)n}, a_{(m-1)(n-1)}, \ldots, a_{(m-1)1}) \ldots (a_{1n}, a_{1(n-1)}, \ldots, a_{11})$.

An n-tuple $(a_1, a_2, \ldots, a_n)$ is *identity n-tuple*, if each $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. Further an n-tuple $a = (a_1, a_2, \ldots, a_n)$ is *symmetric*, if $a^r = a$, otherwise it is a *non-symmetric n-tuple*. In (n, d)-sigraph $G = (V, E)$ an edge labeled with the identity n-tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Note that the above products $P(\overrightarrow{C})$ as well as $P(\overleftarrow{C})$ are n-tuples. In general, these two products need not be equal. However, the following holds.

Proposition 1.2. (**E. Sampathkumar et al.** [12])
For any cycle C of an (n, d)-sigraph $G = (V, E)$, $P(\overleftarrow{C}) = P(\overrightarrow{C})^r$.

Corollary 1.3. (**E. Sampathkumar et al.** [12])
For any cycle C, $P(\overleftarrow{C}) = P(\overrightarrow{C})$ if, and only if, $P(\overrightarrow{C})$ is a symmetric n-tuple. Furthermore, $P(\overrightarrow{C})$ is the identity n-tuple if, and only if, $P(\overleftarrow{C})$ is.

2. Balance in an (n, d)-Signed Graph

In [12], the authors defined two notions of balance in an (n, d)-signed graph $G = (V, E)$ as follows:

Definition . Let $G = (V, E)$ be an (n, d)-sigraph. Then,
(i) G is *identity balanced* (or *i-balanced*), if $P(\overrightarrow{C})$ on each cycle of G is the identity n-tuple, and
(ii) G is *balanced*, if every cycle contains an even number of non-identity edges.

Note: An i-balanced (n, d)-sigraph need not be balanced and conversely. For example, consider the $(4, d)$-sigraphs in Figure.2. In Figure.2(a) G is an i-balanced but not balanced, and in Figure.2(b) G is balanced but not i-balanced.

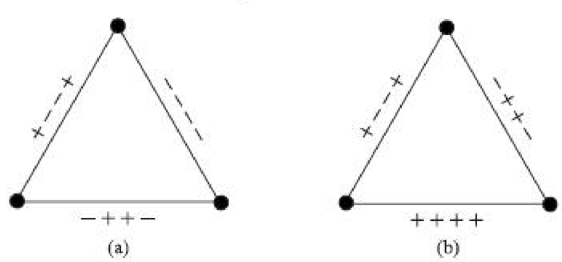

Figure 2.

An (n, d)-signed graph $G = (V, E)$ is i-balanced if each non-identity n-tuple appears an even number of times in $P(\overrightarrow{C})$ on any cycle of G.

However, the converse is not true. For example see Figure.3(a). In Figure.3(b), the number of non-identity 4-tuples is even and hence it is balanced. But it is not i-balanced, since the 4-tuple $(++--)$ (as well as $(--++)$) does not appear an even number of times in $P(\overrightarrow{C})$ of 4-tuples.

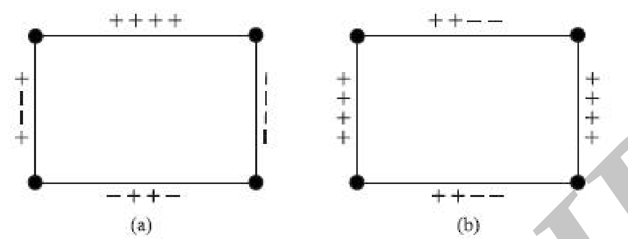

Figure 3.

In [12], the authors obtained some characterizations of balanced and i-balanced (n, d)-sigraphs.

In [13], E. Sampathkumar et al. defined the path balance in an (n, d)-signed graphs as follows: Let $G = (V, E)$ be an (n, d)-sigraph. Then G is

(1) *Path i-balanced*, if any two vertices u and v satisfy the property that for any $u - v$ paths P_1 and P_2 from u to v, $\mathcal{P}(\overrightarrow{P}_1) = \mathcal{P}(\overrightarrow{P}_2)$.
(2) *Path balanced* if any two vertices u and v satisfy the property that for any $u - v$ paths P_1 and P_2 from u to v have same number of non identity n-tuples.

Clearly, the notion of path balance and balance coincides. That is an (n, d)-signed graph is balanced if, and only if, G is path balanced. If an (n, d) signed graph G is i-balanced then G need not be path i-balanced and conversely. In [13], the authors obtained the characterization path i-balanced (n, d)-signed graphs as follows:

Theorem 2.1. (Characterization of Path i-balanced (n, d)-Signed Graphs)
An (n, d)-signed graph is path i-balanced if, and only if, any two vertices u and v satisfy the property that for any two vertex disjoint $u - v$ paths P_1 and P_2 from u to v, $\mathcal{P}(\overrightarrow{P}_1) = \mathcal{P}(\overrightarrow{P}_2)$.

3. Symmetric Balance in an (n, d)-Signed Graph

Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, \ldots, a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let

$$H_n = \{(a_1, a_2, \ldots, a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$$

be the set of all symmetric n-tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m, where $m = \lceil n/2 \rceil$.

We now define a new notion of balance in (n,d)-sigraphs as follows:

Definition. Let $G=(V,E)$ be an (n,d)-sigraph. Then G is *symmetric balanced* or *s-balanced* if $P(\overrightarrow{C})$ on each cycle C of G is symmetric n-tuple.

Note.

1. If an (n,d)-sigraph $G=(V,E)$ is i-balanced then clearly G is s-balanced. But a s-balanced (n,d)-sigraph need not be i-balanced. For example, the $(4,d)$-sigraphs in Figure 4. G is an s-balanced but not i-balanced.
2. A s-balanced (n,d)-sigraph need not be balanced and conversely.
3. In view of Corollary 1.3, the notion of s-balance is well defined since if $\mathcal{P}(\overrightarrow{C})$ is symmetric n-tuple then $\mathcal{P}(\overleftarrow{C})$ is also symmetric.

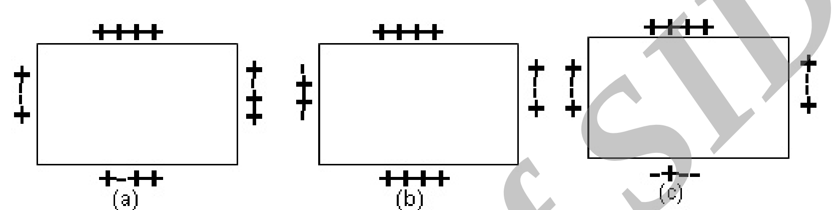

FIGURE 4.

4. Criteria for s-Balance

In this section, we obtain some characterizations for s-balanced (n,d)-sigraphs:

Theorem 4.1. *An (n,d)-sigraph is s-balanced if, and only if, every cycle of G contains an even number of non-symmetric n-tuples.*

Proof. (Necessary) Suppose that G is s-balanced. We first note that product any two non-symmetric n-tuples is symmetric, it follows that product of an even number of non-symmetric n-tuples is symmetric. Suppose that there exists a cycle C in G containing odd number of non-identity n-tuple. Since product of odd number of non-symmetric n tuples is non-symmetric, and product of symmetric n-tuples is symmetric, $\mathcal{P}(\overrightarrow{C})$ is non-symmetric n-tuple, a contradiction.

(Sufficiency) Suppose that every cycle C of G contains even number of non-symmetric n-tuples. Then $\mathcal{P}(\overrightarrow{C})$ is symmetric and hence G is s-balanced. □

The following result gives a necessary and sufficient condition for a balanced (n,d)-sigraph to be s-balanced.

Theorem 4.2. *A balanced (n,d)-sigraph $G=(V,E)$ is s-balanced if and only if every cycle of G contains even number of non identity symmetric n tuples.*

Proof. Suppose G is balanced and every cycle of G contains even number of non identity symmetric n-tuples. Let C be a cycle in G. Since G is balanced, C contains an even number of non identity n-tuples and so number of non-symmetric n tuples in C is even. Hence $\mathcal{P}(\overrightarrow{C})$ is symmetric n tuple. Hence G is s-balanced.

Conversely suppose that G is balanced and s-balanced. Then the number of non-identity n-tuples as well as the number of non-symmetric n-tuples on any cycle C of G is even. Hence the number of every cycle of G contains an even number of non-identity symmetric n-tuples. □

The following result is well known (see [4]).

Theorem 4.3. (Harary [4]).
A sigraph $G = (V, E)$ is balanced, if, and only if, its vertex set V can be partitioned into two sets V_1 and V_2 such that every negative edge joins a vertex in V_1 and a vertex in V_2, and every positive edge joins two vertices in V_1 or in V_2.

Let $G = (V, E)$ be an (n, d)-sigraph. An edge in G labelled by a symmetric edge is called *symmetric edge*. Otherwise it is called *non-symmetric edge*. We now give another characterization of s-balanced (n, d)-sigraph, which is analogous to the partition criteria for balance in signed graph due to Harary [4].

Theorem 4.4. (Characterization of s-balanced (n, d)-sigraph)
An (n, d)-sigraph $G = (V, E)$ is s-balanced if and only if the vertex set $V(G)$ of G can be partitioned into two sets V_1 and V_2 such that each symmetric edge joins the vertices in the same set and each non-symmetric edge joins a vertex of V_1 and a vertex of V_2.

Proof. We associate a sigraph G' with G on the same vertex set V and the edge set E of G as follows: an edge ab in G' is lableled $+$ or $-$ according as ab is a symmetric edge or non-symmetric edge in G. Clearly, the (n, d)-sigraph G is s-balanced if, and only if, the sigraph G' is balanced, and the result follows from Theorem 4.3. □

An (n, d)-sigraph is said to be *complete* if the underlying graph of G is complete.The *s-balance base* with axis a of a complete (n, d)-sigraph $G = (V, E)$ consists list of the product of the n-tuples on the triangles containing a.

Theorem 4.5. *A complete (n, d)-sigraph is s-balanced if, and only if, all the triangles of a base are s-balanced.*

Proof. Suppose all the triangles a base are s-balanced. Indeed, for any triangle (bed) not appearing in the base with axis a, we have
$\mathcal{P}(\overrightarrow{bcd}) = \mathcal{P}(\overrightarrow{abc}).\ \mathcal{P}(\overrightarrow{abd}).\ \mathcal{P}(\overrightarrow{acd})$=symmetric n-tuple.

Conversely, if the (n, d)-sigraph is s-balanced, all these triangles are symmetric and particular those of a base. □

5. Locally s-Balanced (n, d)-Signed Graph

The notion of local balance in signed graph was introduced by F. Harary [5]. A signed graph $G = (V, E)$ is locally at a vertex v, or G is *balanced at* v, if all cycles containing v are balanced. A cut point in a connected graph G is a vertex whose removal results in a disconnected graph. The following result due to Harary [5] gives interdependence of local balance and cut vertex of a signed graph.

Theorem 5.1. (**F. Harary** [5])
If a connected signed graph $G = (V, E)$ is balanced at a vertex u. Let v be a vertex on a cycle C passing through u which is not a cut point, then G is balanced at v.

In [13], the authors extend the notion of local balance in signed graph to (n, d)-signed graphs as follows: Let $G = (V, E)$ be a (n, d)-signed graph. Then for any vertices $v \in V(G)$, G is *locally i-balanced at v* (*locally balanced at v*) if all cycles in G containing v is i-balanced (balanced.)

Analogous to the above result, in [13], the authors obtained the following for an (n, d)-signed graphs:

Theorem 5.2. *If a connected (n, d)-signed graph $G = (V, E)$ is locally i-balanced (locally balanced) at a vertex u and v be a vertex on a cycle C passing through u which is not a cut point, then S is locally i-balanced (locally balanced) at v.*

By the motivation of the above locally i-balanced (*locally balanced*) in an (n, d)-signed graph introduced by E. Sampathkumar et al. [13], in this section, we define locally s-balanced for an (n, d)-signed graphs:

Definition. Let $G = (V, E)$ be a (n, d)-sigraph. Then for any vertices $v \in V(G)$, G is *locally s-balanced at v* if all cycles in G containing v is s-balanced.

Theorem 5.3. *If a connected (n, d)-signed graph $G = (V, E)$ is locally s-balanced at a vertex u and v be a vertex on a cycle C passing through u which is not a cut point, then S is locally s-balanced at v.*

Proof. Suppose that G is s-balanced at u and v be a vertex on a cycle C passing through u which is not a cut point. Assume that G is not s-balanced at v. Then there exists a cycle C_1 in G which is not s-balanced. Since G is s-balanced at u, the cycle C is s-balanced.

With out loss of generality we may assume that $u \notin C$ for if u is in C and G is s-balanced at u C is s-balanced. Let $e = uw$ be an edge in C. Since v is not a cut point there exists a cycle C_0 containing e and v. Then C_0 consists of two paths P_1 and P_2 joining u and v.

Let v_1 be the first vertex in P_1 and v_2 be a vertex in P_2 such that $v_1 \neq v_2 \in C$, such points do exist since v is not a cut point and $v \in C$. Since $u, v \in C_0$. Let P_3 be the path on C_0 from v_1 and v_2, P_4 be a path in C containing v and P_5 is the path from v_1 to v_2. Then $P_5 \cup P_4$ and $P_3 \cup P_5$ are cycles containing u and hence are s-balanced, since they contain u. That is $\mathcal{P}(P_3)$ and $(\mathcal{P}(P_5))$ are either symmetric or non-symmetric so that $C = P_3 \cup P_5$ is s-balanced. This completes the proof. □

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