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AN IMPROVED CHARGED SYSTEM SEARCH FOR STRUCTURAL DAMAGE IDENTIFICATION IN BEAMS AND FRAMES USING CHANGES IN NATURAL FREQUENCIES

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ABSTRACT

It is well known that damaged structural members may alter the behavior of the structures considerably. Careful observation of these changes has often been viewed as a means to identify and assess the location and severity of damages in structures. Among the responses of a structure, natural frequencies are both relatively easy to obtain and independent from external excitation, and therefore, could be used as a measure of the structure's behavior before and after an extreme event which might have lead to damage in the structure.

Inverse problem of detection and assessment of structural damage using the changes in natural frequencies is addressed in this paper. This can be considered as an optimization problem with the location and severity of the damages being its variables. The objective is to set these variables such that the natural frequencies of the finite element model correspond to the experimentally measured frequencies of the actual damaged structure.

In practice, although the exact number of damaged elements is unknown, it is usually believed to be small compared to the total number of elements of the structure. In beams and frames particularly, the necessity to divide the structural members into smaller ones in order to detect the location of the cracks more accurately, deepens this difference. This can significantly improve the performance of the optimization algorithms in solving the inverse problem of damage detection.

In this paper, the Charged System Search algorithm developed by Kaveh and Talatahari [1] is improved to comprise the above mentioned point. The performance of the improved algorithm is then compared to the standard one in order to emphasize the efficiency of the

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proposed algorithm in damage detection inverse problems.

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1. INTRODUCTION

One of the most important aspects of evaluation of structural systems and ensuring their lifetime safety is structural damage detection [2]. Different occurrences ranging from manufacturing defects in structural materials to deterioration under service loads may result in structural damages. Accurately detecting these damages is obviously a critical task so as to retain the structure's integrity and functionality.

Damage causes changes in structural parameters (e.g., the stiffness of a structural member), which in turn, alter the dynamic properties (such as natural frequencies and mode shapes) [3]. Among the responses of a structure, natural frequencies are both relatively easy to obtain and independent from external excitation, and therefore, could be used as a measure of the structure's behavior before and after an extreme event which might have lead to damage in the structure. So, they have been used extensively in the formulation of inverse problems of damage detection. An inverse problem may be defined as determination of the internal structure of a physical system from the system's measured behavior or identification of the unknown input that gives rise to a measured output signal [4].

One of the earliest uses of natural frequencies for structural damage detection is due to Cawley and Adams [5]. Hassiotis and Jeong [6] used an observation of the sensitivity of eigen-frequencies to local stiffness reduction to detect the reduction in stiffness. Nikolakopoulos et al. [7] used contour graph forms to show the dependency of the first two structural eigen-frequencies on crack depth and location. Ruotolo and Surace [8] utilized a genetic algorithm to address the problem of non-destructive location and depth measurement of cracks in beams formulated as an inverse optimization. Cerri and Vestroni [9] investigated the problem of finding damaged zones in beam models using the reduction of the stiffness occurring in the damaged region. They used natural frequencies to measure this stiffness reduction. Liu and Chen [10] explored the problem in frequency domain introducing a computational inverse technique for identifying stiffness distribution on structures using structural dynamics response. Maity and Tripathy [11] used a genetic algorithm for the detection of structural damage by the use of changes in natural frequencies. Liszkai and Raich [12] used some advanced genetic algorithm representations for the structural damage identification of beams and frames. Sahoo and Maity [13] proposed a hybrid neuro-genetic algorithm and considered both natural frequencies and strains as input parameters to address the problem of damage detection. Mehrjoo et al. [3] used artificial neural networks for the damage detection of truss bridge joints using both natural frequencies and mode shapes.

Charged System Search (CSS) is a population based meta-heuristic optimization algorithm which has been proposed recently by Kaveh and Talatahari [2]. In the CSS each solution candidate is considered as a charged sphere called a Charged Particle (CP). The electrical load of a CP is determined considering its fitness. Each CP exerts an electrical force on all the others according to the Coulomb and Gauss laws from electrostatics. Then the new positions of all the CPs are calculated utilizing Newtonian mechanics, based on the acceleration produced by the electrical force, the previous velocity and the previous position of each CP. Many different structural optimization problems have been successfully solved by the CSS [14-17].

In this paper an improved Charged System Search is introduced and utilized for solving the inverse problem of damage identification in beams and frames. The improvements allow the algorithm to change the number of variables (number of damaged members) dynamically as the optimization process proceeds.

The remainder of this paper is organized as follows: Section 2 briefly represents the formulation of the problem under consideration. The optimization algorithm is introduced in Section 3. A brief background of the standard CSS is also represented. Numerical examples are studied in Section 4. Finally, the concluding remarks are provided in Section 5.

2. PROBLEM FORMULATION

In this section the inverse problem of structural damage identification using changes in natural frequencies is briefly offered. Required finite element equations are reviewed first.

2.1. Finite element equations

From finite elements theory, the stiffness and mass matrices of a beam element can be expressed as [18]:

$$[\mathrm{K}] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{1}$$

$$[\mathrm{M}] = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{2}$$

Where A, E, L, I, and ρ are cross-sectional area, modulus of elasticity, length, second moment of inertia, and density of the member, respectively.

Similarly, the stiffness and mass matrices of a planar frame element in local coordinate system can be expressed as [18]:

$$[k]=\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \tag{3}$$

$$[m]=\begin{bmatrix} 2a & 0 & 0 & a & 0 & 0 \\ 0 & 156b & 22Lb & 0 & 54b & -13Lb \\ 0 & 22Lb & 4L^2b & 0 & 13Lb & -3L^2b \\ a & 0 & 0 & 2a & 0 & 0 \\ 0 & 54b & 13Lb & 0 & 156b & -22Lb \\ 0 & -13Lb & -3L^2b & 0 & -22Lb & 4L^2b \end{bmatrix} \tag{4}$$

Where

$$a=\frac{\rho AL}{6} \quad and \quad b=\frac{\rho AL}{420}$$

The stiffness and mass matrices in global coordinate system are considered as:

$$[K] = [L]^t[k][L] \tag{5}$$

$$[M] = [L]^t[m][L] \tag{6}$$

in which L is a transformation matrix:

$$[L]=\begin{bmatrix} l & m & 0 & 0 & 0 & 0 \\ -m & l & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & l & m & 0 \\ 0 & 0 & 0 & -m & l & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Where $l = \cos\alpha$ and m = $\sin\alpha$, α being the angle between the element and the global axis X.

The equation which governs the dynamic behavior of an undamped structure is:

$$[M]\{\square\}+[K]\{x\}=0 \tag{8}$$

2.2. Damage formulation

Here, damage is considered as a reduction in stiffness which is incorporated into the equations by a reduction factor β. When damage occurs in an element, the stiffness matrix of the element is modified as:

$$k_{id} = \beta_i [k_i] \tag{9}$$

Here, the parameter β ranges from 0.2 to 1, introducing a maximum of 80 percent damage in each element.

The mass matrix [M] of the structure is assumed to be unchanged. The *j*th eigenvalue equation of the damaged structure will be derived by substitution of the structure's stiffness matrix by that of the damaged one:

$$[K_d]\{\phi_{jd\}}\} - \lambda_{jd}[M]\{\phi_{jd}\} = \{0\} \tag{10}$$

Where λ_{jd} and ϕ_{jd} are the *j*th natural frequency and the *j*th shape mode of the damaged structure, respectively.

2.3. Objective Function

The objective function of the optimization is considered as:

$$F(X) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(f_i^a - f_i^c)^2} \tag{11}$$

where, X is the solution vector representing the state of damage; *n* is the number of natural frequencies involved in the objective function; f_i^a and f_i^c are the *i*th actual (measured) and computed natural frequencies, respectively.

3. OPTIMIZATION ALGORITHM

Charged System Search (CSS) algorithm introduced by Kaveh and Talatahari [2] is improved and utilized here as the optimization algorithm. In this section the standard CSS is first present concisely. The improved CSS is then introduced.

3.1. Standard CSS

Charged System Search is a population based meta-heuristic algorithm proposed by Kaveh and Talatahari [1]. This algorithm is based on laws from electrostatics of physics and Newtonian mechanics.

The Coulomb and Gauss laws provide the magnitude of the electric field at a point inside and outside a charged insulating solid sphere, respectively, as follows [19]:

$$\mathrm{E}_{ij} = \begin{cases} \dfrac{k_e q_i}{a^3} r_{ij} & if\ r_{ij} < a \\ \dfrac{k_e q_i}{{r_{ij}}^2} & if\ r_{ij} \geq a \end{cases} \tag{12}$$

Where k_e is a constant known as the Coulomb constant; r_{ij} is the separation of the centre of sphere and the selected point; q_i is the magnitude of the charge; and a is the radius of the charged sphere. Using the principle of superposition, the resulting electric force due to N charged spheres is equal to [1]:

$$\mathrm{F}_j = k_{eq} \sum_{i=1}^{N} \left(\frac{q_i}{a^3} r_{ij} . i_1 + \frac{q_i}{{r_{ij}}^2} . i_2 \right) \frac{r_i - r_j}{\left\| r_i - r_j \right\|} \quad \begin{cases} i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a \\ i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} \geq a \end{cases} \tag{13}$$

Also, according to Newtonian mechanics, we have [19]:

$$\Delta r = r_{\text{new}} - r_{\text{old}} \tag{14}$$

$$\mathrm{v} = \frac{r_{new} - r_{old}}{\Delta t} \tag{15}$$

$$\mathrm{a} = \frac{v_{new} - v_{old}}{\Delta t} \tag{16}$$

Where r_{old} and r_{new} are the initial and final positions of the particle, respectively; v is the velocity of the particle; and a is the acceleration of the particle. Combining the above equations and using Newton's second law, the displacement of any object as a function of time is obtained as [19]:

$$r_{\text{new}} = \frac{1}{2} \frac{F}{M} . \Delta t^2 + \mathrm{v}_{\text{old}} + r_{old} \tag{17}$$

Inspired by the above electrostatic and Newtonian mechanics laws, the pseudo-code of the CSS algorithm is presented as follows [16]:

Level 1: Initialization

Step 1. Initialization. Initialize the parameters of the CSS algorithm. Initialize an array of charged particles (CPs) with random positions. The initial velocities of the CPs are taken as zero. Each CP has a charge of magnitude (q) defined considering the quality of its solution as:

$$q_i = \frac{fit(i) - fit_{worst}}{fit_{best} - fit_{worst}} \qquad i = 1,2,\ldots.N \tag{18}$$

Where fit_{best} and fit_{worst} are the best and the worst fitness of all the particles; *fit(i)* represents the fitness of agent *i*. The separation distance r_{ij} between two charged particles is defined as:

$$r_{ij} = \frac{\left\| X_i - X_j \right\|}{\left\| \frac{\left(X_i + X_j\right)}{2} - X_{best} \right\| + \varepsilon} \tag{19}$$

where X_i and X_j are the positions of the *i*th and *j*th CPs, respectively; X_{best} is the position of the best current CP; and ε is a small positive to avoid singularities.

Step 2. CP ranking. Evaluate the values of the fitness function for the CPs, compare with each other and sort them in increasing order.

Step 3. CM creation. Store the number of the first CPs equal to charged memory size (CMS) and their related values of the fitness functions in the charged memory (CM).

Level 2: Search

Step 1. Attracting force determination. Determine the probability of moving each CP toward the others considering the following probability function:

$$p_{ij} = \begin{cases} 1 & \frac{fit(i) - fit_{best}}{fit(j) - fit(i)} > rand \vee fit(i) > fit(j) \\ 0 & else \end{cases} \tag{20}$$

and calculate the attracting force vector for each CP as follows:

$$F_{ij} = q_j \sum_{i,i \neq j} \left(\frac{q_i}{a^3} r_{ij} \, . \, i_1 + \frac{q_i}{r_{ij}^{\ 2}} \, . \, i_2 \right) p_{ij} \left(X_i - X_j \right) \quad \begin{cases} j = 1,2,\ldots,N \\ i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a \\ i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} \geq a \end{cases} \tag{21}$$

Where F_j is the resultant force affecting the *j*th CP.

Step 2. Solution construction. Move each CP to the new position and find its velocity using the following equations:

$$X_{j,new} = rand_{j1} \, . \, k_a \, . \, \frac{F_j}{m_j} \, . \, \Delta t^2 + rand_{j2} \, . k_v \, . \, V_{j,old} \, . \, \Delta t + X_{j,old} \tag{22}$$

$$V_{\mathrm{j,new}} = \frac{X_{j,new} - X_{j,old}}{\Delta t} \tag{23}$$

Where $rand_{j1}$ and $rand_{j2}$ are two random numbers uniformly distributed in the range (1,0); m_j is the mass of the CPs, which is equal to q_j in this paper. The mass concept may be useful for developing a multi-objective CSS. Δt is the time step, and it is set to 1. k_a is the acceleration coefficient; k_v is the velocity coefficient to control the influence of the previous velocity. In this paper k_v and k_a are taken as:

$$k_a = c_1(1 + iter/iter_{\max}), k_v = c_2(1 - iter/iter_{\max}) \tag{24}$$

Where c_1 and c_2 are two constants to control the exploitation and exploration of the algorithm; *iter* is the iteration number and $iter_{\max}$ is the maximum number of iterations.

Step 3. CP position correction. If each CP exits from the allowable search space, correct its position using the HS-based handling as described by Kaveh and Talatahari [1].

Step 4. CP ranking. Evaluate and compare the values of the fitness function for the new CPs; and sort them in an increasing order.

Step 5. CM updating. If some new CP vectors are better than the worst ones in the CM, in terms of their objective function values, include the better vectors in the CM and exclude the worst ones from the CM.

Level 3: Controlling the terminating criterion

Repeat the search level steps until a terminating criterion is satisfied.

3.2. Improved CSS

In the standard CSS, each solution candidate is represented by a vector with a predefined dimension i.e. the number of variables is kept unchanged during the optimization process. In structural damage identification problems on the other hand, the number and location of the damaged problems is not previously known. However, it is observed that the number of damaged elements is usually far smaller than the total number of structural members. Hence, using the standard CSS would logically lead to one of the following solutions:

1. Taking the number of variables equal to the total number of structural members; this usually leads to unreasonably large search spaces and may cause convergence difficulties. Consequently, many structural members with small structural damage appear in the final solution which unfavorably affect the objective function, and thus, hinder the algorithm from finding the actual damaged members.

2. Solving the problem several times with different number of variables. This solution is apparently time-consuming. Moreover, in each of the runs, the algorithm may face the problem mentioned in case 1.

In order to address the abovementioned problem, the CSS algorithm is slightly modified and made capable of solving problems where the number of variables is not previously known. In other words, the number of variables (the number of damaged members) is treated as a variable and the algorithm tries to optimize its value.

In the improved CSS each solution candidate is a (2n+1)-dimensional vector, n being the largest possible number of damaged elements. The first variable determines the number of damaged elements. The value of this variable is shown by n_d which may be different for different CPs. The next n_d entries in the solution vector represent the indices of the damaged members. This part of the solution vector is called the index part. And finally, the next n_d entries represent the percentage of damage in the damaged members. This part of the solution vector is called the percentage part. The rest of the variables $(2n+1 - (2n_d+1))$ will be filled by zeros to keep the dimensions of all the solution vectors equal.

In the next iterations, once the values of the first variable of all CPs (n_d) are established, all the redundant variables in the percentage part ($n - n_d$) should be set to zero. For example, if $n_d = 2$ for a CP and the largest possible number of damaged elements is assumed to be equal to 5, there may (or may not) be up to 3 redundant variables in the percentage part of this CP. Occurrence of these redundant variables is inevitable due to the movements of the Charged Particles within the search space. In fact, nothing guarantees the number of non-zero elements in the percentage part of a solution vector to be equal to n_d.

Different strategies may be used for choosing redundant variables. Here the redundant variables are assumed to be those having the smallest percentages. This assumption which is proved to be appropriate through experimental observations is a result of the fact that the algorithm is supposed to be viable of managing big damages.

All of the other aspects of the improved algorithm are the same as those of the standard CSS.

4. NUMERICAL EXAMPLES

Three numerical examples (a beam and two frames) with different damage cases are considered here to show the efficiency of the proposed improved algorithm. Comparisons of the results show that the modifications have improved the performance of the algorithm. In all cases a population of 50 CPs and a total number of 100 iterations is used. Each problem has been solved 20 times with each of the algorithms and the best results are reported here.

Three algorithms are used for solving the problems here:

1. Standard CSS with number of variables taken equal to the total number of members of the structure. (Standard CSS 1)
2. Standard CSS with number of variables taken 3 times the number of damaged members. (Standard CSS 2)
3. Improved CSS.

4.1. A Two-span beam

The two-span beam depicted in Figure 1 is considered as the first example. The finite element model of this structure is defined using 20 beam elements with a uniform section (W12x65). The modulus of elasticity and the material density are 207 GPa and 7780 kg/m^3, respectively. The first 10 natural frequencies of the structure are used to form the objective function. Two damage cases are considered for this structure:

Case 1: 50 percent damage in element 3 and 30 percent damage in element 8.

Case 2: 40 percent damage in element 2, 60 percent damage in element 8, 50 percent damage in element 11, and 40 percent damage in element 16.

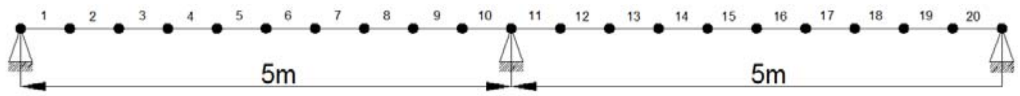

Figure 1. A two-span beam

Figures 2 and 3 represent the damage states found by the algorithms together with the actual damage states in Case 1 and Case 2, respectively.

Tables 1 and 2 represent the rounded percentages of damage found in the damaged members found by different algorithms for the beam structure in Case 1 and Case 2, respectively.

Figures 4 and 5 show the variations of the normalized objective function values versus the iteration number for the improved CSS algorithm in Case 1 and Case 2, respectively.

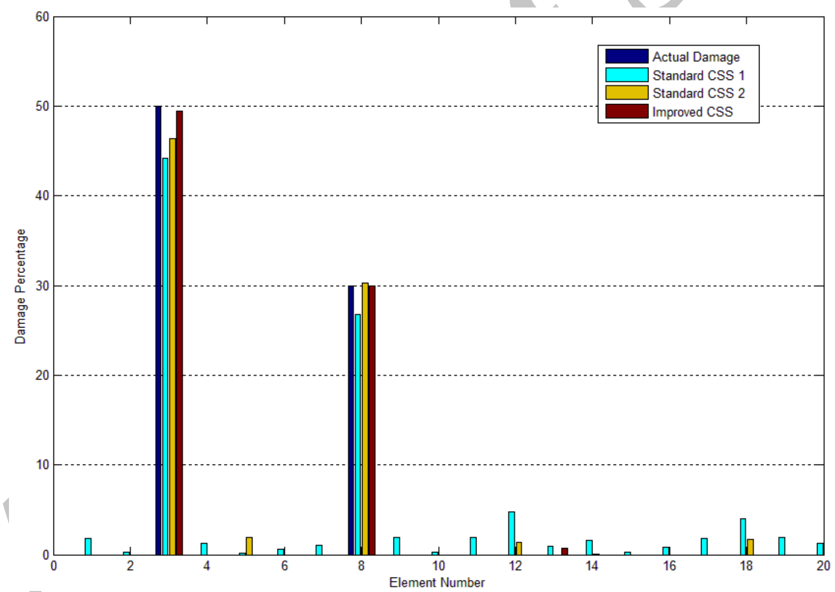

Figure 2. The damage states found by the algorithms together with the actual damage for the two-spam beam (Case 1)

Table 1. The rounded percentages of damage found in the damaged members by the different algorithms for the two-span beam (Case 1)

Element number	Actual damage	Standard CSS 1	Standard CSS 2	Improved CSS
3	50	44	46	49
8	30	27	30	30

It can be seen that the improved algorithm has attained the best approximation of damage among all. Moreover, as visible in Figure 2, the improved algorithm has detected only one redundant element while the others have detected several of them.

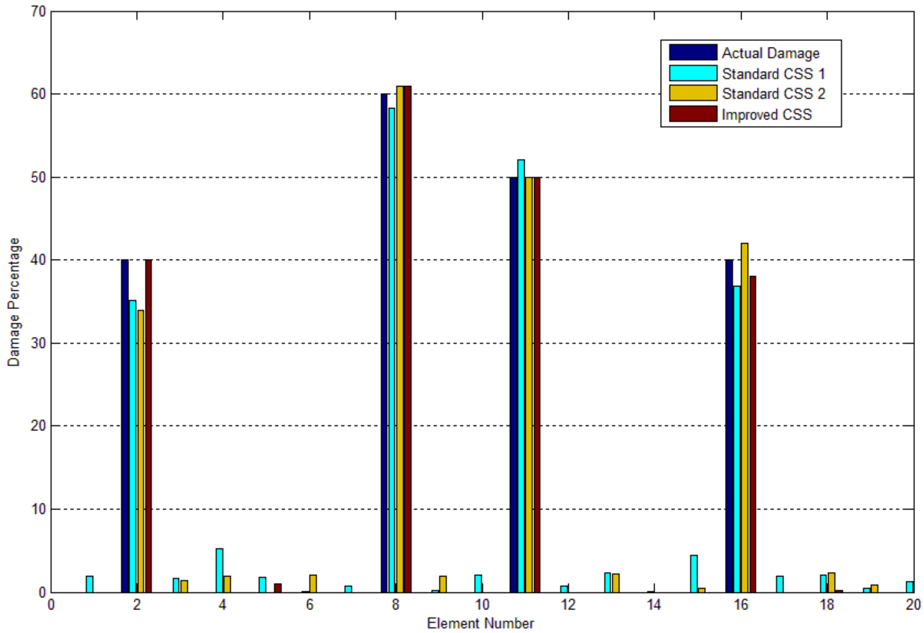

Figure 3. The damage states found by the algorithms together with the actual damage for the two-spam beam (Case 2)

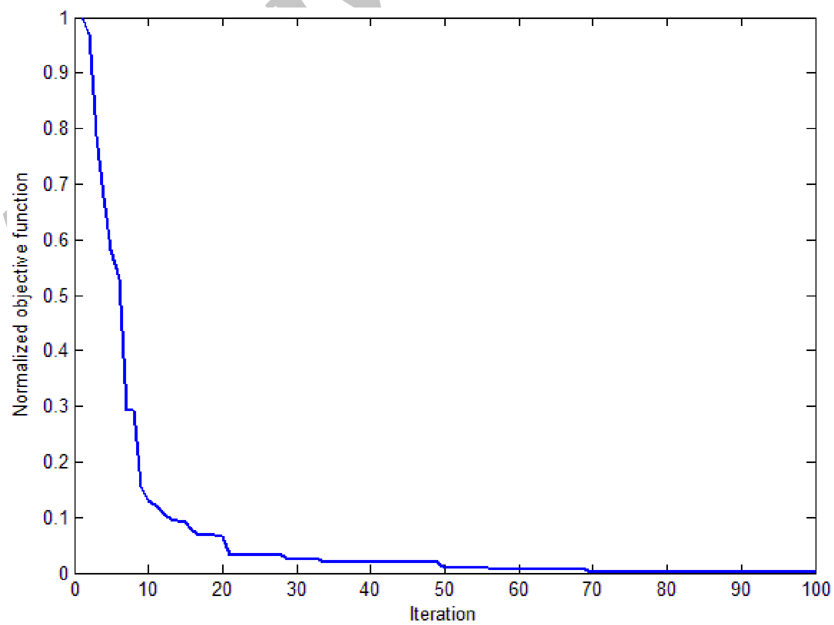

Figure 4. The variations of the normalized objective function values versus the iteration number by the improved CSS algorithm for the two-span beam (Case 1)

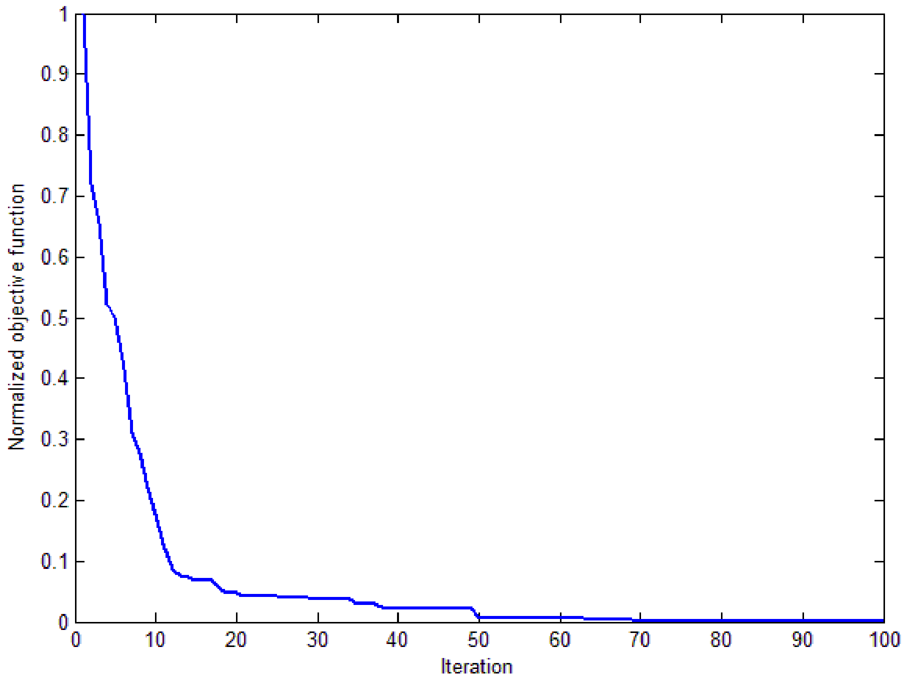

Figure 5. The variations of the normalized objective function values versus the iteration number by the improved CSS algorithm for the two-span beam (Case 2)

Table 2. The rounded percentages of damage found in the damaged members by the different algorithms for the two-span Beam (Case 2)

Element number	Actual damage	Standard CSS 1	Standard CSS 2	Improved CSS
2	40	35	34	40
8	60	58	61	61
11	50	52	50	50
16	40	37	42	38

In both cases it can easily be seen that the improved CSS outperformed the Standard CSSs. As mentioned before, the Standard CSS 1 finds many redundant damaged members with small damage percentages.

4.2. A Two-span three-story frame

The geometry and element numbering of a two-span three-story frame is depicted in Figure 6. The beams and columns are modeled using 3 and 2 finite elements, respectively. The sections used for the beams and columns are (W12x65) and (W14x120), respectively. The modulus of elasticity is 207 GPa and the material density is 7780 kg/m^3 like the previous example. The first 15 natural frequencies of the structure are used to form the objective function. Two damage cases are considered for this structure:

Case 1: 40 percent damage in element 3, 55 percent damage in element 21, and 50 percent damage in element 35.

Case 2: 25 percent damage in element 5, 30 percent damage in element 7, 40 percent damage in element 16, 60 percent damage in element 24, 50 percent damage in element 25, and 60 percent damage in element 34.

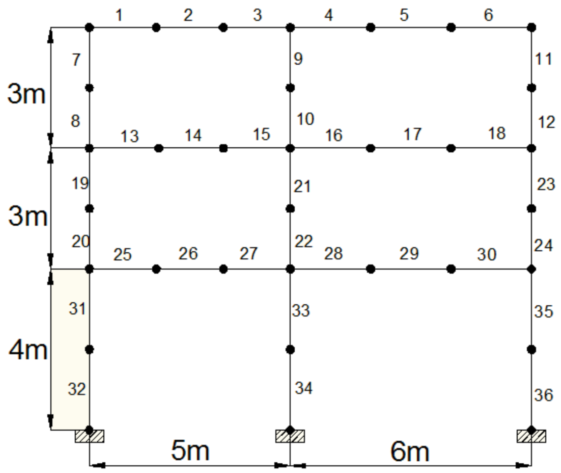

Figure 6. A two-span three-story frame.

Figures 7 and 8 represent the damage states found by the algorithms together with the actual damage states in Case 1 and Case 2, respectively.

Tables 3 and 4 represent the rounded percentages of damage found in the damaged members by the different algorithms for the frame structure in Case 1 and Case 2, respectively.

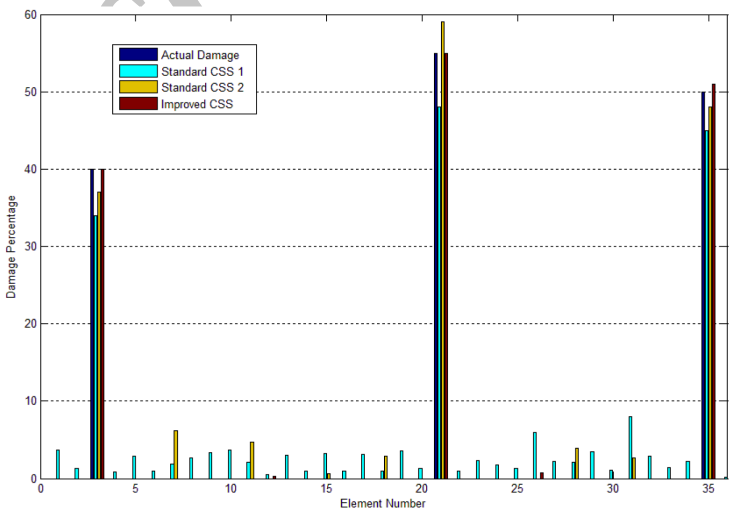

Figure 7. The damage states found by the algorithms together with the actual damage state for the frame structure (Case 1)

Figures 9 and 10 show the variations of the normalized objective function values versus the iteration number by the improved CSS algorithm for two-span three-story frame in Case 1 and Case 2, respectively.

Table 3. The rounded percentages of damage found in the damaged members by the different algorithms for the two-span three-story frame (Case 1)

Element number	Actual damage	Standard CSS 1	Standard CSS 2	Improved CSS
3	40	34	37	40
21	55	48	59	55
35	50	45	48	51

According to Table 3 the improved CSS has found a better approximation of the damage state comparing to the standard versions. Both of the standard versions have found several undamaged members as damaged ones. This is probably because there is very little chance for the algorithms to set the value of an existing variable equal to zero.

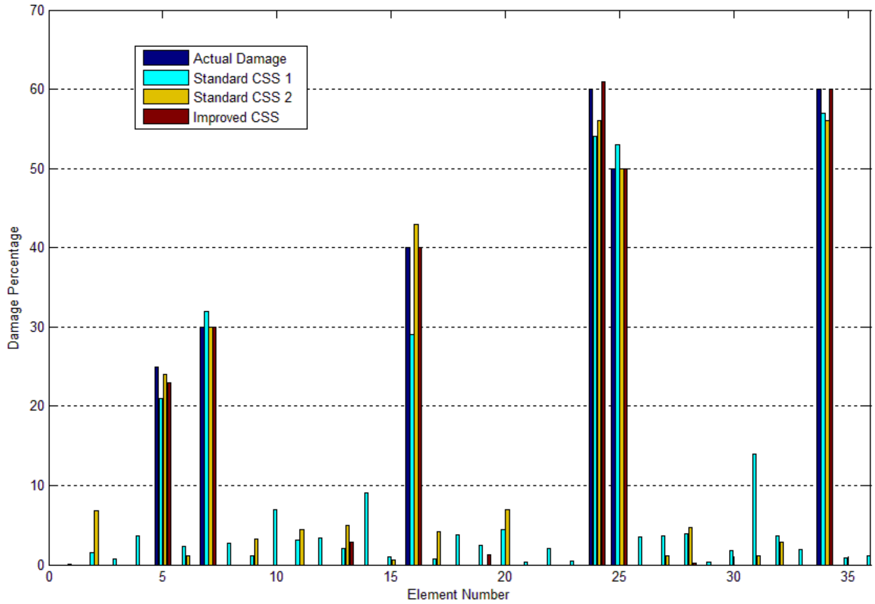

Figure 8. The damage states found by the algorithms together with the actual damage state for the frame structure (Case 2)

Again, it can be seen that the performance of the improved algorithm is far better than the performances of the standard CSSs in both cases. Here, the increase of the number of the members of the structure has worsened the performance of the standard CSS. The improved CSS detects the damaged members correctly in both cases (with only three extra members with little damages). The percentages are also rationally close to the actual values.

Table 4. The rounded percentages of damage found in the damaged members by the different algorithms for the two-span three-story frame (Case 2)

Element number	Actual damage	Standard CSS 1	Standard CSS 2	Improved CSS
5	25	21	24	23
7	30	32	30	30
16	40	29	43	40
24	60	54	56	61
25	50	53	50	50
34	60	57	56	60

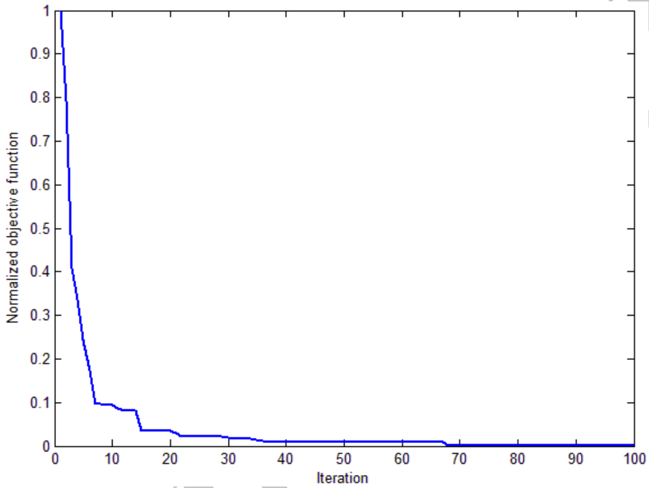

Figure 9. The variations of the normalized objective function values versus the iteration number by the improved CSS algorithm for the two-span three-story frame (Case 1)

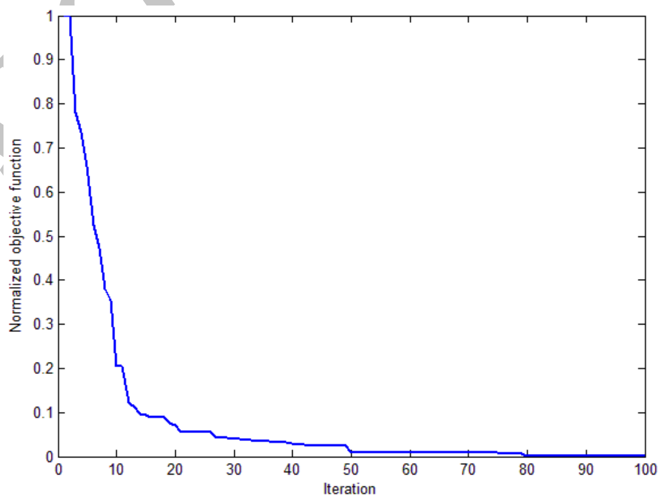

Figure 10. The variations of the normalized objective function values versus the iteration number by the improved CSS algorithm for the two-span three-story frame (Case 2)

4.3. A four-span five-story frame

A four-span five-story frame as depicted in Figure 11 is considered as the third example. The beams and columns are modeled using 3 and 2 finite elements, respectively. The sections used for the beams and columns are (W12×87) and (W14×145), respectively. The modulus of elasticity is 207 GPa and the material density is 7780 kg/m^3 like the previous examples. The first 15 natural frequencies of the structure are used to form the objective function. Two damage cases are considered for this structure:

Case 1: 40 percent damage in element 25, 55 percent damage in element 51, 50 percent damage in element 60, and 65 percent of damage in element 109.

Case 2: 25 percent damage in element 5, 40 percent damage in element 19, 35 percent damage in element 48, 65 percent damage in element 73, 50 percent damage in element 89, and 45 percent damage in element 104.

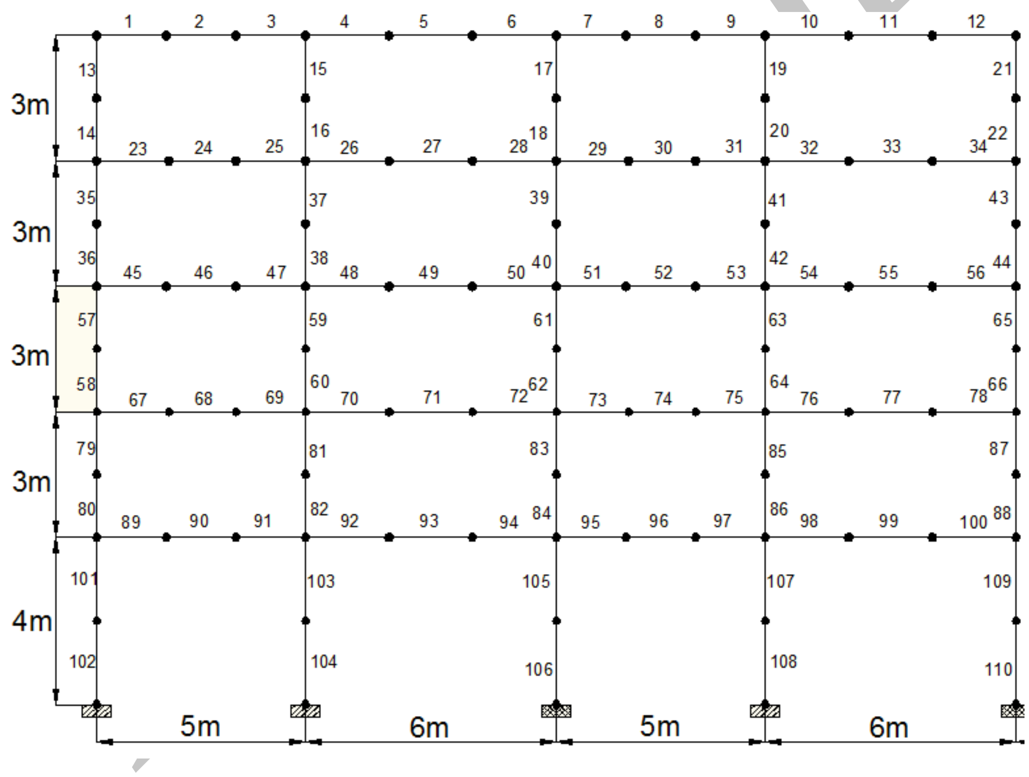

Figure 11. A four-span five-story frame

Tables 5 and 6 represent the percentages of damage found in the damaged members by different algorithms for the four-span five-story frame structure in Case 1 and Case 2, respectively. Figures 12 and 13 show the variations of the normalized objective function values versus the iteration number by the improved CSS algorithm in Case 1 and Case 2, respectively.

Table 5. The rounded percentages of damage found in the damaged members by different algorithms for the four-span five-story frame (case 1)

Element number	Actual damage	Standard CSS 1	Standard CSS 2	Improved CSS
25	40	34	35	39
51	55	58	58	55
60	50	46	50	52
109	65	59	68	65

Table 6. The rounded percentages of damage found in the damaged members by the different algorithms for the four-span five-story frame (case 2)

Element number	Actual damage	Standard CSS 1	Standard CSS 2	Improved CSS
5	25	29	23	24
19	40	36	39	40
48	35	34	38	33
73	65	60	59	67
89	50	54	48	50
104	45	41	44	46

It is visible from Tables 5 and 6 that the improved CSS has reached better results in comparison to the standard forms. The percentages of damaged found by the improved CSS are reasonably close to the actual percentages. Both of the standard forms have introduced several undamaged members as damaged ones.

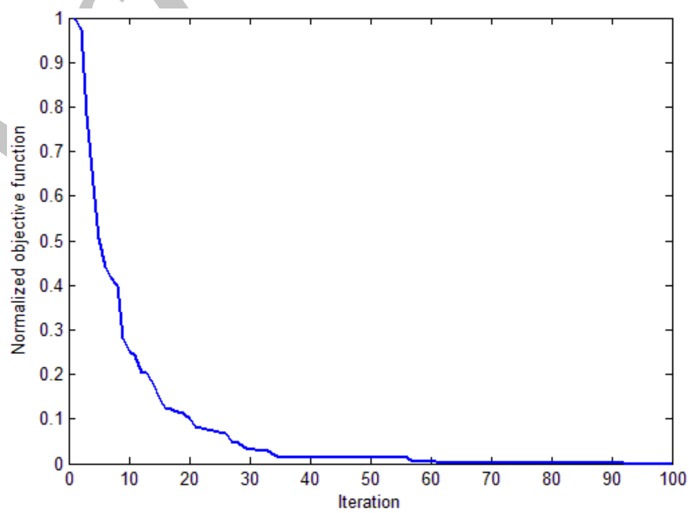

Figure 12. The variations of the normalized objective function values versus the iteration number by the improved CSS algorithm for the four-span five-story frame (case 1)

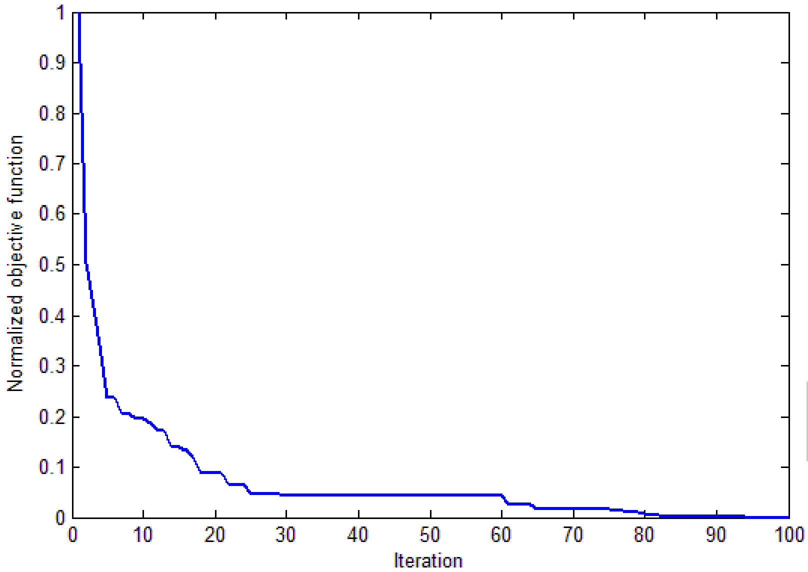

Figure 13. The variations of the normalized objective function values versus the iteration number by the improved CSS algorithm for the four-span five-story frame (case 2)

5. CONCLUDING REMARKS

Inverse problem of structural damage identification using changes in natural frequencies formulated as an optimization problem is considered in this paper. Since the number of damaged elements is not known before solving the problem, this is an optimization problem with unknown number of variables. The charged system search algorithm introduced by Kaveh and Talatahari [1] is improved and made capable of solving this class of problems.

Three numerical examples (a beam and two frames) are presented to show the functionality and the efficiency of the proposed improved algorithm. Comparisons of the results demonstrate that the improved algorithm performs better than the standard version. The difference between the performances of the algorithms becomes more visible as the size of the problem and the number of damaged elements increases.

Both of the standard versions tend to find several undamaged members as damaged ones and this, affecting the objective function, obstructs them from detecting the actual damaged structures. This is mainly because of the presence of the redundant variables. The improved algorithm solves this problem by defining the number of variables (damaged members) as a variable and changing it dynamically as the optimization process proceeds.

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