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A UNIFIED MODEL FOR RESOURCE-CONSTRAINED PROJECT SCHEDULING PROBLEM WITH UNCERTAIN ACTIVITY DURATIONS

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ABSTRACT

In this paper we present a unified (probabilistic/possibilistic) model for resource-constrained project scheduling problem (RCPSP) with uncertain activity durations and a concept of a heuristic approach connected to the theoretical model. It is shown that the uncertainty management can be built into any heuristic algorithm developed to solve RCPSP with deterministic activity durations. The essence and viability of our unified model are illustrated by fuzzy examples presented in the recent fuzzy RCPSP literature.

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KEY WORDS: project scheduling; stochastic scheduling; fuzzy scheduling; resource-constrained project; heuristic algorithm; simulation; central limit theorem; frequency histogram; density function; membership function

1. INTRODUCTION

In the resource-constrained project scheduling literature it is a usual assumption that each activity can be characterized by a deterministic duration. Naturally in our more or less always uncertain world this assumption is far from the reality. The solution is theoretically simple: to decrease or bridge the gap between the models and the reality we have to manage the uncertainty about the activity durations somehow in the resource-constrained project scheduling models (see e.g.: Herroelen et al. [1], Herroelen and Leus [2], and Ke and Liu [3].

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We have two ways: (1) when the probabilities of the different duration occurrences are known from the past and the future can be described from the past then a stochastic (probabilistic or density function oriented) approach may be useful; (2) when we are unable to forecast the future from the history, than, according to our best knowledge and experience we have to imagine and describe somehow the possible events connected to the uncertain activity durations and must to use a fuzzy (possibilistic or membership function oriented) approach.

Naturally, every approach has some drawbacks:

- In the stochastic approach, apart from requiring detailed information about probability distribution functions, also has a drawback that the computational expense of solving these models is very high. In the continuous (highly non-linear) case needs complicated multiple integration techniques to compute the convolutions and in the discrete representation of the uncertainty we have to manage an extremely large tree of the different scenarios (see e.g.: Pollack-Johnson and Liberatire [4], and Ballestín [5].
- In the fuzzy approach we have to cope with the non-smooth membership functions. Apart from the trivial case when the membership function is only a line similarly to the uniform distribution function; a membership function in generally is a non-smooth composite of linear segments. It may be triangular, trapezoidal or any other shape can be build up from line segments. It is well-known that there are several tricks in the operations research (OR) which can be used to replace this non-smooth problem with an equivalent linear one, but the real drawback is connected to the definition of the break-points. In the fuzzy approach, according to our knowledge and experience, we "have to see the future" in the form of a parameter pair (optimistic-pessimistic durations), a triplet (optimistic-most likely-pessimistic durations), a sextet (Wang [6]) and so on. A good summary of the fuzzy approach from managerial point of view can be found in Bonnai et al. [7].

In a recent article (Bhaskar et al. [8]) a heuristic method has been proposed for the resource-constrained project scheduling problem (RCPSP) with fuzzy activity durations. The apropos of this state-of-the-art work, we try to identify and illuminate a popular misconception about fuzzification of RCPSP. The main statement of their approach, similarly to the other fuzzy approaches (see, for example, Wang [6]) is simple: the project completion time can be represented by a "good" fuzzy number. This statement is naturally true: in a practically axiomatic fuzzy thinking and model building environment, using only fuzzy operators and rules, we get a fuzzy output from the fuzzy inputs.

According to our opinion, the presented theoretically correct, innovative and easy to understand approach has three drawbacks:

- In the presented approach it is remains an open question that from managerial point of view what is the real meaning of this "good" fuzzy number and how could be used in the managerial practice.
- There is no theoretical model behind the presented heuristic, which as a fix point could be used to investigate the quality of the developed heuristic in the function of its tunable parameters.
- The presented pure fuzzy approach is unable to manage the real problems which are usually mixed from deterministic, probabilistic and possibilistic elements.

The possibilistic (fuzzy) approach, traditionally, defines itself against the probabilistic approach, so in the "conservative" fuzzy community everything is prohibited which is connected to somehow to the probability theory. For example, the Central Limit Theorem (CLT) is in the taboo list of this community. We have to emphasize, CLT is a humanized description of a miracle of nature. When we fight against CLT, we fight against nature. The situation in the "liberal" fuzzy community is not much better, because they try to redefine somehow the probability theory within the fuzzy approach without using "forbidden" statistical terms. According to the robustness of CLT, the distribution function of the completion time of real-size projects remains nearly normal, which is a manager friendly, natural and usable result. The fuzification, as an abstraction is unable to modify the order of nature.

In this paper, we will show that the nature is working totally independently from our abstraction used to describe our uncertanty about the future. In our approach, for example, a triangular membership function is nothing else that, a simplified triangular density function as an acknowledgement of our limited knowledge.

When we want to add a practical scheduling method to the project managers we have to destroy the borders between the probabilistic and possibilistic approaches and have to define a "unified" approach to decrease the gap between scientific beliefs and reality. This simple means, that we have to accept, that idenpedently from the mixed partly probabilistic and partly possibilistic input the output will be a probabilistic variable which follows the CLT.

In this paper we present a new unified (probabilistic/possibilistic) model and a conception of a heuristic connected to the unified model for RCPSP with uncertain activity durations. In Section 2 we present a unified theoretical model. In Section 3 we describe the conception of the uncertainty management according to the theoretical model. The essence and viability of our unified model will be illustrated by fuzzy examples in Section 4. Finally, Section 5 draws conclusions from this study.

2. A THEORETICAL MODEL

In this section we describe our theoretical model for RCPSP with uncertain activity durations. The approach produces “robust” schedules which are immune against uncertainties in the activity durations. The optimality criterion is defined as a linear combination (weighted sum) of resource-feasible makespans connected to the key terms of the applied uncertainty formulation. Theoretically the optimal robust schedule searching process is formulated as a multi-objective mixed integer linear programming problem (MOMILP) where the number of objectives corresponds to the number of key terms (parameters) of uncertainty formulation.

In this paper, we replaced the MOMILP with a MILP by scalarization. The resulting MILP can be solved directly in the case of small-scale projects within reasonable time. The proposed model is based on the so-called “forbidden set” concept. The output of the model is the set of the optimal conflict repairing relations. Obviously, the solution of the problem depends on the choice of the weights for the objective functions.

In order to model uncertain activity durations in projects, we consider the following resource-constrained project-scheduling problem: A single project consists of N real activities $i \in \{1, 2, ..., N\}$.

In this paper, without loss of generality, we assume that each activity duration can be described by three parameters: $\{D_{i1}, D_{i2}, D_{i2}\}$ $i \in \{1, 2, ..., N\}$ where triplet $\{D_{i1}, D_{i2}, D_{i2}\}$ may define a triangular membership function in the possibilistic approach, or a density function from beta distribution in the probabilistic approach. We have to note, that in the probabilistic approach the triplet is estimated from a sample using standard statistical tools, assuming that the future can be described from the past, but in the possibilistic approach it is only an abstraction which describe the future according to knowledge of the project managers.

The fuzzy community, under the spell of the challenging but manageable nature of the membership function (it is non-smooth composite of linear segments) tries to recreate everything from the beginning. For example, "normalization" is a "coded message" that the triangle is not a density function, and the horizontal line corresponding to "α-cut" is a theoretically questionable replacement of the two vertical lines, which define the confidence interval in the probabilistic approach. Changing the position of α we change our risk-taking habit, but, at the same time, we omit/add duration segments with totally different left/right tail probability (Figure 1).

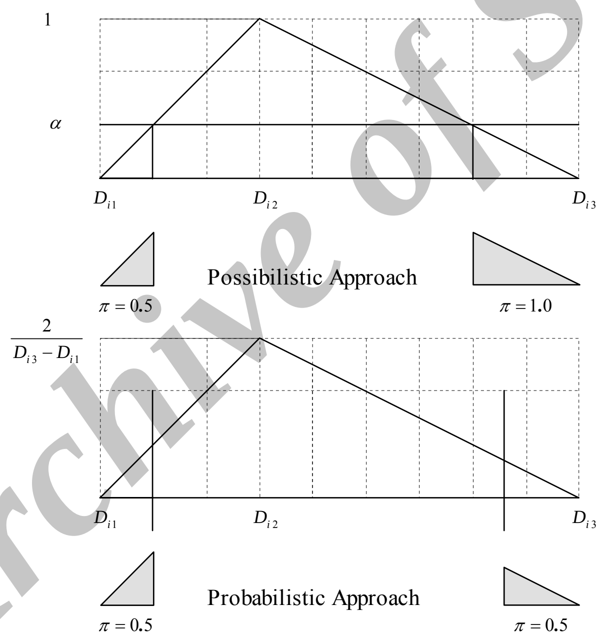

Figure 1. Possibilistic and probabilistic approaches

Our opinion about the uncertainty management in project scheduling is very simple: we have to replace the triangular membership function with the equivalent triangular density function, have to let the CLP to work. Formalisms which in the uncertainty dimension try to redefine statistical terms without statistical terms are meaningless and misleading.

The activities are interrelated by precedence constraints: Precedence constraints force an activity not to be started before all its predecessors are finished. These are given by network-relations $i \rightarrow j$, where $i \rightarrow j$ means that activity j cannot start before activity i is completed. Furthermore, activity $i = 0\,(i = N + 1)$ is defined to be the unique dummy source (sink). Let $\boldsymbol{NR}$ be the set of the network relations.

Let R denote the number of renewable resources required for carrying out the project. Each resource $r \in \{1,...,R\}$ has a constant per period availability R_r. In order to be processed, each real activity $i \in \{1, 2,..., N\}$ requires $R_{ir} \geq 0$ units of resource $r \in \{1, ..., R\}$ over its duration.

A schedule is network-feasible if satisfies the predecessor-successor relations:

$$S_i + D_i \leq S_j \text{, for each } i \rightarrow j \in \mathrm{NR} \tag{1}$$

Let $\Re$ denote the set of network-feasible schedules. For a network feasible schedule $\mathrm{S} \subset \Re$, let $A_t = \{i \mid S_i \leq t < S_i + D_i\}$, $t \in \{1,...,T\}$ denote the set of active (working) activities in period t and let

$$U_{t\,\mathrm{r}} = \sum_{i \in A_t} r_{i\,\mathrm{r}}\,,\ t \in \{1,...,T\},\ r \in \{1,...,R\} \tag{2}$$

be the amount of resource r used in period t, where T is an upper bound of the resource-feasible makespan.

A network-feasible schedule $\mathrm{S} \subset \Re$ is resource-feasible if satisfies the resource constraints:

$$U_{t\,\mathrm{r}} \leq R_r\,,\ t \in \{1,...,T\},\ r \in \{1,...,R\} \tag{3}$$

Let $\overline{\Re} \subseteq \Re$ denote the set of resource-feasible schedules.

The presented unified MILP formulation is based on the forbidden set concept. In MILP model the total number of zero-one variables is $|\boldsymbol{RR}|$, where $\boldsymbol{RR}$ denotes the set of the possible resource-conflict repairing relations and the formulation is based on the well-known "big-M" formulation. The presented MILP model is a modified and simplified version of the original forbidden set oriented model developed by Alvarez-Valdés and Tamarit [9] for the deterministic case.

A forbidden activity set is identified such that: (1) all activities in the set may be executed concurrently, (2) the usage of some resource by these activities exceeds the resource availability, and (3) the set does not contain another forbidden set as a proper subset. A resource conflict can be repaired explicitly by inserting a network feasible precedence relation between two forbidden set members, which will guarantee that not all members of the forbidden set can be executed concurrently. We note, that an inserted explicit conflict repairing relation (as its side effect) may be able to repair one or more other conflicts implicitly, at the same time.

Let $T = \sum_{i=1}^{N} D_{i3}$, which is an "extremely weak" resource-feasible upper bound and fix the position of the unique dummy sink in period $T+1$. Naturally, this "weak" upper bound can be replaced by any "stronger" one.

Let $D_i = D_{i1}, i \in \{1, 2,..., N\}$, and let F denote the number of forbidden sets and let $\boldsymbol{RR}_f$ denote the set of explicit repairing relations for forbidden set $\boldsymbol{F}_f$, $f \in \{1,2,...,F\}$ according to the "optimistic" durations and resource-feasible upper-bound T.

Let

$$\boldsymbol{RR} = \left\{ \bigcup_f \boldsymbol{RR}_f \;\middle|\; f \in \{1,2,...,F\} \right\} \tag{4}$$

denote the set of all the possible repairing relations. In the forbidden set oriented model, a resource-feasible schedule is represented by the set of the inserted resource conflict repairing relations (Alvarez-Valdés and Tamarit [9]). According to the implicit resource constraint handling, in this model the resource-feasibility is not affected by the feasible activity shifts (movements).

In the time oriented model, a resource-feasible schedule is represented by the activity starting times. In this model, according to the explicit resource constraint handling, an activity movement may be able to destroy the resource-feasibility.

It is very important to note, that after inserting an appropriate conflict repairing set, the "immunised" schedule will invariant to the duration change. In other words, the schedule will be resource-feasible on the set of the possible (and allowed) activity durations because we immunised it according to the optimistic (shorter) durations:

$$D_i \in \{D_{i1}, D_{i,2}, D_{i3}\}, i \in \{1, 2,..., N\} \tag{5}$$

Let S_{ip} denote the starting time of activity i, where $i \in \{0, 1, 2, ..., N+1\}$ and $p \in \{1, 2, 3\}$. By definition, in the optimistic, most likely, and pessimistic schedules the durations are optimistic, most likely, and pessimistic durations:

$$D_i = D_{ip},\ i \in \{1, 2, ..., N\},\ p \in \{1,2,3\} \tag{6}$$

Defining the binary decision variables:

$$Y_{ij} = \begin{cases} 1 & \text{if } i \to j \text{ inserted} \\ 0 & \text{otherwise} \end{cases},\ i \to j \in \boldsymbol{RR} \tag{7}$$

the following MILP model arises:

$$\sum_{p=1}^{3} W_p * S_{N+1\,p} \to \boldsymbol{min} \tag{8}$$

$$\sum_{i \to j \in \boldsymbol{RR}_f} Y_{ij} \geq 1, \quad f \in \{1,...,F\} \tag{9}$$

$$S_{ip} + D_{ip} \le S_{jp} + \left(\overline{S}_i - \underline{S}_j + D_{ip} \right) * \left(1 - Y_{ij} \right),\ i \to j \in \boldsymbol{RR},\ p \in \{1, 2, 3\} \tag{10}$$

$$S_{ip} + D_{ip} \le S_{jp},\ i \to j \in \boldsymbol{NR},\ p \in \{1, 2, 3\} \tag{11}$$

The objective function (8) minimizes the linear combination of the resource-constrained makespans, where the weights characterize risk-taking habit of the project manager (for example: "best pessimistic" may be a good scheduling policy, when the project manager is a risk-avoider).

Constraint set (9) assures the resource feasibility (we have to repair each resource conflict explicitly or implicitly, therefore from each conflict repairing set we have to choose at least one element).

Constraint sets (10) take into consideration the precedence relations between activities in the function of the inserted repairing relations.

Constraint sets (11) take into consideration the original precedence (network) relations between activities.

In the "big-M" formulation $\underline{S}_i \left(\overline{S}_i \right)$ define the earliest (latest) starting time of activity i, in the optimistic schedule according to upper-bound T.

We have to note again, that the optimal solution is a function of $W_p,\ p \in \{1, 2, 3\}$ weights. According to the model construction in the optimal schedule every possible activity movement is resource feasible, and schedule is “robust” because it is invariant to the variability of activity durations. In other words, a non-critical activity movement (a non-critical delay) or a longer (but possible) activity duration is unable to destroy the resource feasibility of the schedule.

3. A HEURISTIC ALGORITHM

In this section, shortly describe the conception of a heuristic algorithm connected to the presented theoretical model, Without loss of generality, we assume that we have a deterministic list scheduling algorithm width forward-backward improvement (FBI) to produce resource-feasible schedules in an arbitrary meta-heuristic frame. According to the essence of the algorithm, we generate the resource-feasible schedules by taking the selected activities one by one in the given activity order and scheduling them at the earliest (latest) feasible start time using the optimistic activity durations. After that, using FBI we try to improve the quality of the generated schedule.

Then the algorithm, inn the forward-backward list scheduling process, inserts a precedence relation between an already scheduled activity and the currently scheduled activity whenever they are connected without lag, than we get a schedule without "visible" resource-conflicts in which, according to applied "thumb rule", the number of "hidden" conflicts is drastically decreased.

The importance of the "thumb rule" may be explained by the fact, that in this way we are able to resolve resource conflicts, without explicit forbidden set computation. After that, the

algorithm is able (in exactly one step) to repair all of the hidden (invisible) conflicts, inserting always the “best” conflict repairing relation for each forbidden set.
In this context “best” means a relation between two forbidden set members for which the lag is maximal. Naturally, the algorithm memorizes the best schedule found so far by computing the durations of the schedule according to the key point durations. In the search process, according to the "from optimistic to pessimistic" strategy, the algorithm resolves the visible (hidden) resource conflicts using the optimistic durations, after that replaces the optimistic durations with the most likely, and pessimistic ones. The algorithm exploits the fact, that we cannot destroy the resource-feasibility, replacing the optimistic durations in a conflict free optimistic solution with longer durations.

After the "best conflict repairing combination" searching phase, the makespan distribution function is generated by simulation. In the simulation phase we have to replace the membership functions with the appropriate density functions (for example: we replace triangular membership functions with triangular density functions and use a triangular random number generator to get duration instances).

4. EXAMPLES

To solve the presented problem to optimality the callable version of Cplex 12.2 was used. The computational results were obtained by running the algorithm on a 1.8 GHz Pentium IV IBM PC under Microsoft Windows XP® operating system. The conception of Section 3 was inserted to the "Sounds of Silence" harmony search metaheuristic frame (Csébfalvi et al. [10], Csébfalvi et al. [11], Csébfalvi [12], Csébfalvi et al. [13], and Danka [14]) and its improved version (SoS-ProPos) will be presented in a forthcoming paper.

Example 1

The first fuzzy example was borrowed from Bhaskar et al. [8]. The project is shown in Table 1 and Figure 2. The project has only one renewable resource type. The "weak" upper-bound is T=397.

Table 1. A fuzzy RCPSP

Activity	**Duration triplet**	**Resource requirement**
1	{42, 50, 61}	8
2	{36, 40, 42}	17
3	{35, 50, 79}	12
4	{39, 50, 59}	3
5	{16, 25, 30}	13
6	{43, 51, 57}	17
7	{52, 58, 69}	16
Resource availability		30

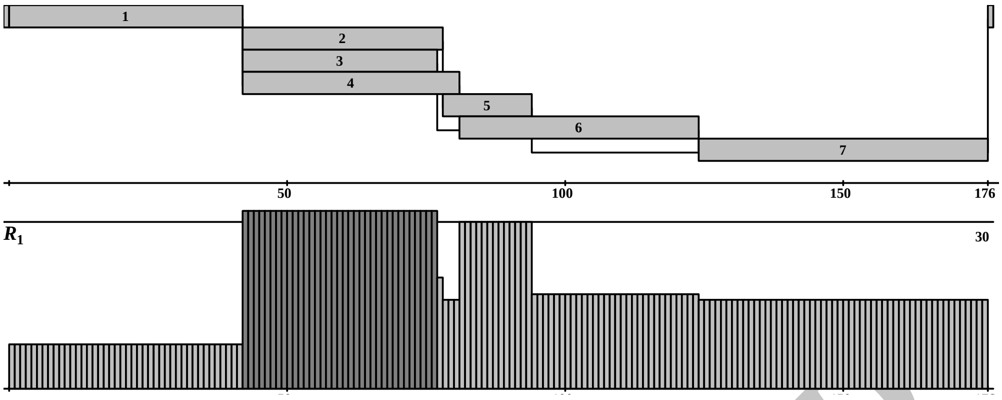

Figure 2. A fuzzy RCPSP with optimistic durations

Using the (Bhaskar et al. [8]) fuzzy RCPSP algorithm based on a "distance base ranking of fuzzy numbers" method, the authors presented the following "good" triangular fuzzy makespan: $\{212, 249, 278\}$ as an acceptable heuristic solution. The project is extremely small and one can easily prove by explicit enumeration, that this result is incorrect because it is better than the Pareto front. According to the $T = 397$ setting and using the optimistic duration estimations, the problem has only two forbidden sets (see Table 2).

The implicit enumeration tree is presented in Figure 3. In this figure apart from the root node, every node describes a MILP problem without or with extra equality constraints according to the logic of the Pareto front computation. In Figure 3 the expandable nodes are represented by light grey squares, the leaves by grey squares, and the element of the Pareto front by dark grey squares. The first child of the root node is the "optimistic-optimal" solution without extra constraints. Its first child node is the "most-likely-optimal" solution on the set of "optimistic-optimal" solutions with one $S_{81} = 208$ extra constraint. Easy to see, that from managerial point of view the valuation of the two non-dominated solutions may be totally different depending on the risk-taker or risk-avoider habit of the project manager. For example, the "best-pessimistic-solution" may be preferred in a fundamentally risk-averse managerial strategy.

Table 2. Forbidden sets and repairing relations

Forbidden Sets		**Explicit Repairs**	**Implicit Repairs**
1	{2,6}	2→6 6→2	2→3 2→4
2	{2,3,4}	2→3 3→2 2→4 4→2 3→4 4→3	6→2

The problem has two non-dominated (Pareto-optimal) solutions: $\{208, 249, 308\}$ and $\{212, 249, 288\}$, which illustrate the fact, that a good optimistic schedule not necessarily will be a good pessimistic one and vice versa. The "good" solution $\{212, 249, 278\}$ from (Bhaskar et al. [8]) is better than a non-dominated solution $\{212, 249, 288\}$, which is impossible. This error reveals the fact, that without a clear theoretical model which can be used as an etalon it is very hard to investigate the affectivity or control the results of a given approach.

When we apply the model of Section 2 to the presented fuzzy problem with unit weights, we get $\{212, 249, 288\}$ as optimal solution within 0.05 sec. In this case, the optimal resource conflict repairing relations are: $2 \rightarrow 6$ and $4 \rightarrow 2$. The optimistic optimal solution is presented in Figure 4.

After the "best conflict repairing combination" searching phase which is motivated by the managerial style, the makespan distribution function is generated by simulation. In the simulation phase we replaced the membership functions with density functions (in this case we replaced the triangular membership functions with a triangular density functions and used a triangular random number generator to get duration instances).

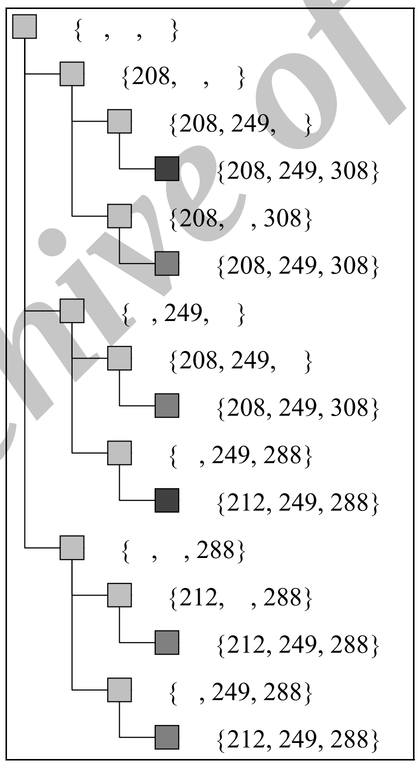

Figure 3. Explicit enumeration tree

We have to mention it, that simulation is a cheap operation, so the sample size may be large enough. In the presented example we set the sample size to ten thousand.

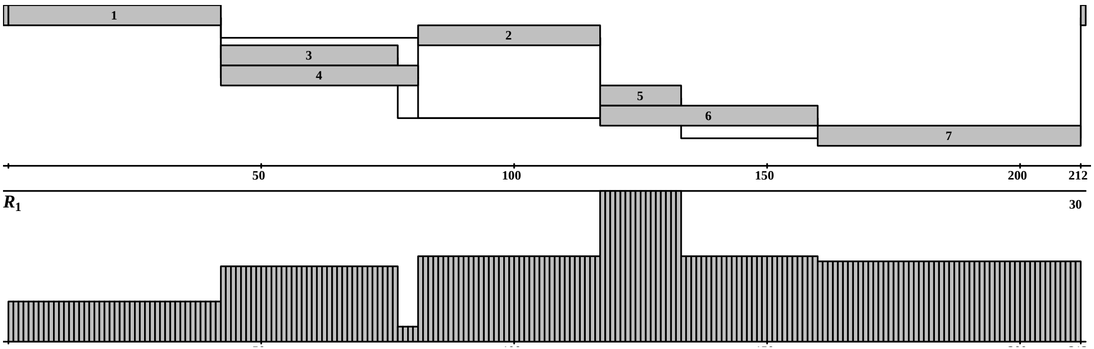

Figure 4. Optimal solution with optimistic durations

Using the Kolmogorov-Smirnov test, we cannot reject a null hypothesis that the sample comes from a normal distribution with the following parameters:

$$\pi = 0.158,\ \mu = 258,\ \sigma = 10.0 \tag{12}$$

where $\pi = 0.158$ is the probability of the largest difference (in absolute value) between the observed and theoretical distribution functions when the null hypothesis is true with mean $\mu = 256.4$ and standard deviation $\sigma = 10.0$. The histogram in Figure 5 reviles the fact, that the nature knows nothing about the fuzzification and does its best according to the CLP.

To illustrate the essence of the proposed manager-friendly and absolutely correct "biased-free" approach, imagine the last conversation between the project manager (PM: who is an apostle of fuzzification and has a magic fuzzy makespan triangle) and the top manager (TM: who is thinking always in odds and hates the "academic" explanations) as the end of a sad tale:
TM: what is the chance that the project will be finished within 270 days?
PM: hm! … who knows? … the membership function …
TM: what? …

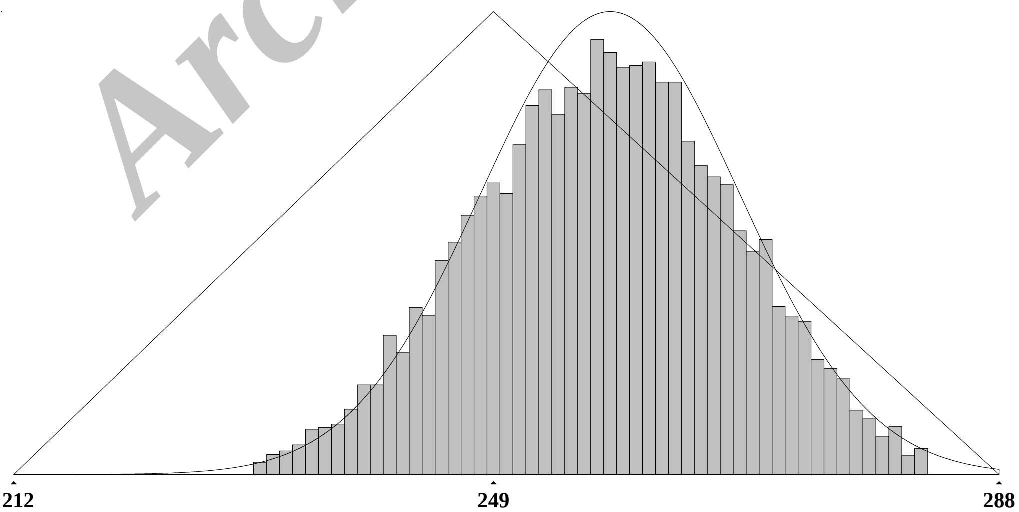

Figure 5. Makespan estimation by simulation: $\{\mu = 258, \sigma = 10\}$

After firing, the PM start for find a job in a deterministic world and in this wonderful world he lives happily until his death.

Naturally, forgetting the fuzzification and using the estimated makespan distribution function, the correct and easy answer would be the following: boss, we have a good chance (with more than probability 0.5) that we will be able to finish the project within 270 days.

Example 2

The objective of Wang [6] research was to develop a robust scheduling methodology based on fuzzy set theory for uncertain product development projects. The imprecise temporal parameters involved in the project were represented by fuzzy sets. A measure of schedule robustness based on qualitative possibility theory (Dubois and Prade, [15]) was proposed to guide the search process to determine the robust schedule; i.e., the schedule with the best worst-case performance. A genetic algorithm approach was developed for solving the problem with acceptable performance. An example of electronic product development project was used to illustrate the concept developed. For the demonstration purpose, the author simplified the entire project with hundreds of activities into 51 activities. The resource types were aggregated into four types ($R = 4$): systems engineers (R_1), software engineers (R_2), hardware engineers (R_3), and supporting engineers (R_4). It was assumed that $\varepsilon = 0$ and $\alpha = 0.5$ for all fuzzy temporal parameters and according to the applied notation the "index of optimism" was set to 0.5 in the example. Each real activity duration was described by a sextet $D_i = \{ D_{i1}, D_{i,2}, \dots, D_{i6} \}$, $i \in \{ 1, 2, \dots, 51 \}$ according to Figure 6.

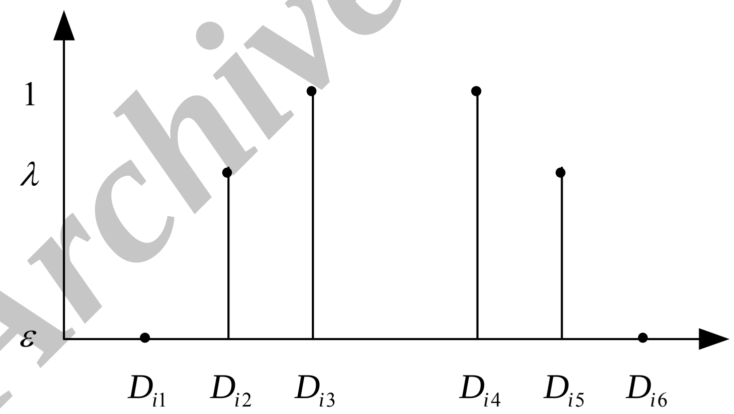

Figure 6. Fuzzy duration representation by a sextet

Our first critical comment is connected to the applied "precise" uncertainty representation: it is very hard to imagine (it is a nonsense) that a group of experts after some discussion would be able to agree with six parameters as assumed by Wang [6] in a product development project where the membership functions "imitate" density functions.

Our second critical comment is connected to the presented solution: after 10 random runs, the obtained best fuzzy project duration was $\{ 175, 203, 231, 231, 257, 283 \}$ which is better than the unique exact optimal solution $\{ 176, 215, 231, 231, 271, 283 \}$. It can be proved by running our MILP model seven times: firstly we minimized the position of finishing milestone $D_{52\,p}$, $p \in \{1, 2, \ldots, 6\}$ separately, after that we minimized the weighted sum of the finishing milestones with unit weights.

The finishing milestone positions obtained by weighted sum minimization agree with the separately obtained minimized milestones. Therefore we have a unique exact optimum and a "better than the best "solution which is nonsense. The total solution time was 17245.047 sec, which means that this problem practically reached the size limit until a problem can be solved to optimality in reasonable time: $|\boldsymbol{RR}| = 182$. In the simulation phase, we replaced the hexagonal membership functions with hexagonal density functions and used a hexagonal random number generator to obtain duration instances. After the "best conflict repairing combination" searching phase, the makespan distribution function is generated by simulation. As we mentioned it before, the simulation is a cheap operation, so the sample size may be large enough. In the presented example the sample size was also ten thousand. Using the Kolmogorov-Smirnov test, we cannot reject a null hypothesis that the sample comes from a normal distribution with the following parameters: $\{\mu = 230, \sigma = 6\}$. In Figure 7 we show the simulation results and the "optimal hexagonal shape".

The difference between the "normal shape" and the "hexagonal shape" reveals the fact that the role of the makespan sextet is only in the "managerially best solution" selection. In other words, when we play with weights of the sextet in the objective function, we can get answers for several "what if" like questions. Each "deformed shape" moving backward as much as possible may describe a managerial strategy from a risk-avoider to risk-taker scale. We have to mention it that in the proposed unified approach similar statistical results can be generated for each real activity (starting-finishing time and duration distribution or any other statistics which can be useful in the planning phase from managerial point of view).

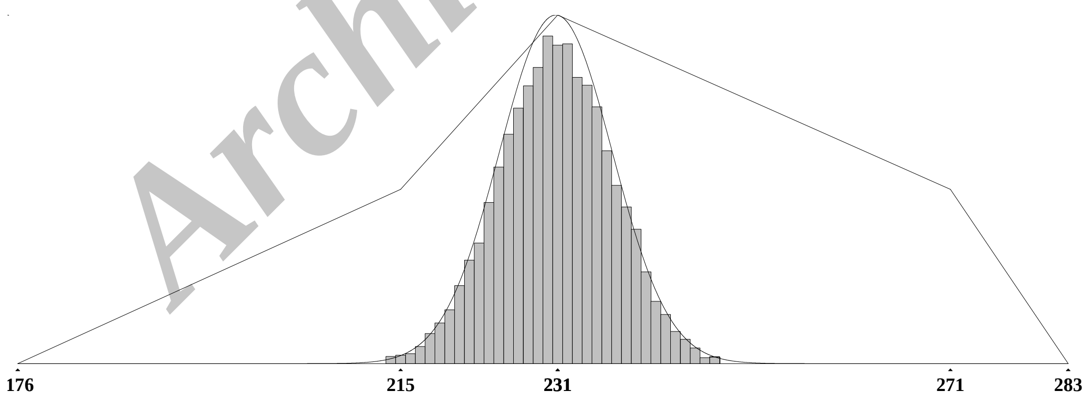

Figure 7. Makespan estimation by simulation: $\{\mu = 231, \sigma = 6\}$

5. CONCLUSION

In this paper, a new unified theoretical model and a concept of the corresponding heuristic approach to solve RCPSP with uncertain activity durations are presented. In the proposed heuristic approach, the uncertainty management is invariant to the applied heuristic frame; therefore it can be built into any other heuristic developed to solve RCPSPs. The essence and viability of our unified approach is illustrated by a fuzzy examples presented in the recent fuzzy RCPSP literature. For the unified (probabilistic/possibilistic) approach a problem-specific "Sounds-of-Silence" harmony search meta-heuristic version (SoS-ProPos) will be presented in a forthcoming paper.

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