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EXPERIMENTAL INVESTIGATION OF LOW TO HIGH-STRENGTH STEEL FIBER REINFORCED LIGHTWEIGHT CONCRETE UNDER PURE TORSION

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ABSTRACT

Results of an experimental investigation carried out to assess the effect of various concrete strength levels on behavior of steel fiber reinforced lightweight concrete (SFRLWC) under pure torsion are reported. The variables of the testing program were: compressive strength of concrete, volume of steel fibers, and the aspect ratio of steel fibers. The concrete strengths investigated are 9, 12, 30 and 61 MPa. Fiber content ranges from zero to 3.0 percent by volume of matrix. Addition of approximately two percent of fibers led to maximum torsional strength and toughness enhancement of SFRLWC; however, the improvement in higher strength concrete was more pronounced than that in lower strength concrete. In general, the torsion strength gained was higher for high strength level specimens with higher percentages of fiber volume and larger fiber aspect ratio. A proposed torsional strength formulation provided good agreement with the test results.

Keywords: fiber reinforced concrete, high-strength concrete, lightweight concrete, torsion strength

1. INTRODUCTION

The production and use of lightweight aggregate concrete has received considerable attention for structural purposes during the last two decades. Lightweight concrete (LWC) with compressive strengths up to 50 MPa can be made readily with high-quality lightweight aggregates. Malhotra[1] and Zhang and Gjorv[2] have demonstrated that strengths of 70 MPa and higher and a density of less than 2000 kg/m³ are also achievable when silica fume and a superplasticizer are used in the mix. These studies on the material properties of high-strength lightweight concrete (HSLWC) have rapidly advanced material development.

In certain applications such as bridge decks and parking garages, the self-weight of structural components represents a large portion of the total load. By reducing the self-

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weight, considerable savings could be attained, not only in materials but also in construction costs.

With the rapid development of concrete technology in recent years, higher-strength concrete can be produced much more easily than before. Since 1980, several investigations on mechanical properties of lightweight high-strength concrete have been reported [1-7]. Investigation of the torsional behavior of LWC, however, has not received adequate attention. The cracking and ultimate torsional strengths of LWC and HSLWC can determine the maximum torsion permitted in concrete section not requiring torsional reinforcements according to ACI 318-02 [8].

Fiber reinforced concrete has been successfully used for a variety of applications over the last 35 years [9-12]. Recently, investigators have studied some of the mechanical properties of lightweight concrete reinforced with fibers [13-14] .

Increase in strength of concrete is associated with increase in material compressive and tensile strengths and elastic modulus. However, concretes with higher strengths are more brittle than normal strength concrete, representing a significant limitation for their wide-range application in innovative structural design [15-17]. The addition of steel fibers to HSLWC provides a substantial increase in splitting tensile strength and modulus of elasticity (up to 160 and 80 percent, respectively). The compressive, flexural and shear strengths improve up to 20, 90 and 80 percent respectively. Also, fibers improve the ductility under all the four modes of loading [14].

Experimental research is required to understand the effect of concrete density and strength on torsional behavior of concrete reinforced with fibers. The objective of this research study is to provide information on torsional strength of SFRLWC utilizing lightweight concrete with different levels of strength, various fiber volumes, and aspect ratios.

2. RESEARCH SIGNIFICANCE

With the increased use of high-strength lightweight concrete in various structures, where torsion can be crucial in design, it seems necessary for any code to determine the ultimate torsion strength of plain concrete. The toughness of concrete including that of high-strength concrete is anticipated to improve for torsion when reinforced with randomly distributed steel fibers. Experimental testing of the behavior of high strength concrete beams subjected to torsion is limited. This study reports the behavior of plain and steel fiber reinforced lightweight concrete with low, normal and high strength under pure torsion.

3. EXPERIMENTAL PROGRAM

3.1 Materials and fabrication of test specimens

Figure 1 shows the test set-up. The specimens had dimensions of 700×200×100mm (27.56×7.87×3.94 in). Three parameters were investigated: (1) concrete strength, four levels of strength representing low strength, 9 MPa (1.1 ksi) and 12 MPa (1.74 ksi), normal strength, 30 MPa (4.35 ksi), and high strength, 61 MPa (8.84 ksi); (2) steel fiber to matrix ratio, six ratios including 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0 percent and zero percent for control; and (3) length of steel fiber, 25 mm (1.0 in.), 32 mm (1.25 in.) and 50 mm (2.0 in.) long fibers representing aspect ratios of 42, 47 and 57, respectively. Information regarding fiber reinforced torsion specimens and specimens for compressive strength determination are shown in Table 1. In this table the concrete strengths are identified as: low strength lightweight concrete (LC1) and (LC2), normal strength lightweight concrete (NC), and high-strength lightweight concrete (HC). The designation of the specimen reflects its major properties. For example in LC2-0.5-25, LC2 is for low strength (2), 0.5 is the percent of fiber and 25 is the length of the fiber. The experimental program included two specimens for each type of torsion specimen used, and a minimum of two 6″×12″ cylindrical specimens for determination of concrete strength. In total seventy two (thirty six pairs) rectangular specimens and 84 cylindrical specimens were tested.

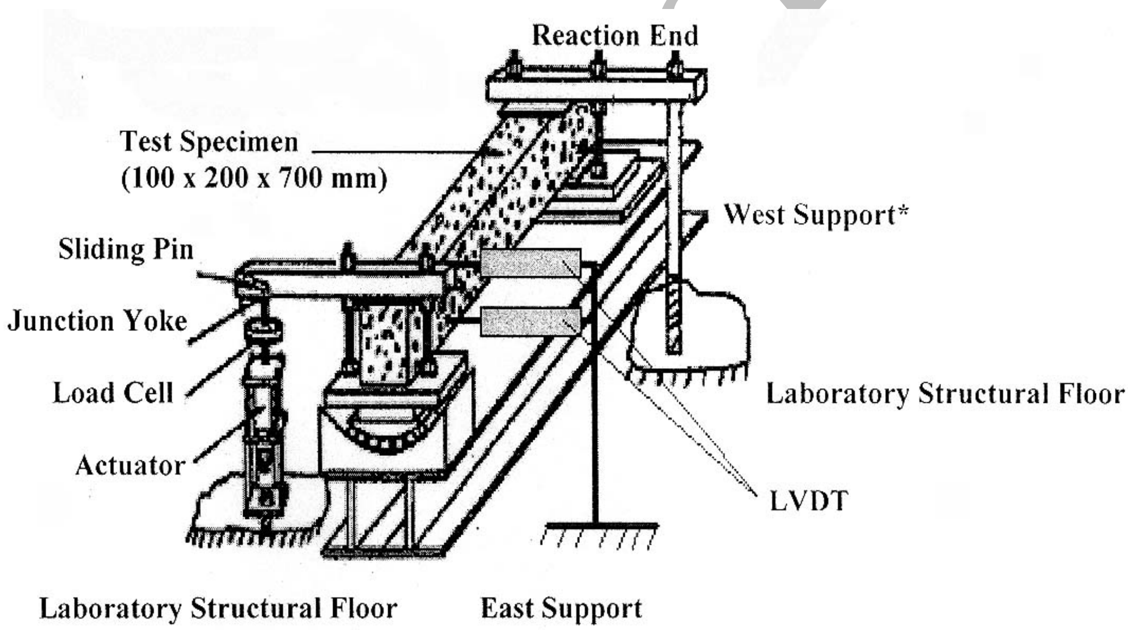


Figure 1. Test setup for pure torsion

Table 2 presents the four concrete mix proportions used in the testing program. Cement used was Type I Portland cement conforming to ASTM C150. Coarse lightweight aggregate with a maximum size of 19 mm (0.75 in.) and a relative density of 1.54 was used in a saturated surface dry (SSD) condition for HC and NC design mixes. Coarse aggregate (LECA) with a maximum size of 25 mm (1.0 in.) and a relative density of 1.1 was used in a SSD condition for LC1 and LC2 design mixes. Fine aggregate had a fineness modulus of 3.06 and a relative density of 2.6.

Table 1. Experimental program for torsion and control specimens

compressive strength, MPa	Concrete type and target	Specimen identification	Unit weight, (kg/m³)	Fiber by volume, percent	Aspect ratio	Number 6″×8″ of control specimens for compressive strength
		LC1	1767	0.0	---	4
		LC1-0.5-32	1793	0.5	47	2
Low	1, 9	LC1-1.0-32	1819	1.0	47	2
		LC1-1.5-32	1845	1.5	47	2
		LC1-2.0-32	1871	2.0	47	2
		LC1-3.0-32	1923	3.0	47	2
		LC2	1901	0.0	---	4
		LC2-0.5-25	1919	0.5	42	2
		LC2-0.5-32	1919	0.5	47	2
		LC2-1.0-25	1949	1.0	42	2
		LC2-1.0-32	1949	1.0	47	2
Low	2, 12	LC2-1.5-25	1978	1.5	42	2
		LC2-1.5-32	1978	1.5	47	2
		LC2-2.0-25	2007	2.0	42	2
		LC2-2.0-32	2007	2.0	47	2
		LC2-3.0-25	2066	3.0	42	2
		LC2-3.0-32	2066	3.0	47	2
		NC	1743	0.0	---	6
		NC-0.5-32	1759	0.5	47	2
Normal	30	NC-1.0-32	1777	1.0	47	2
		NC-1.5-32	1795	1.5	47	2
		NC-2.0-32	1812	2.0	47	2
		NC-3.0-32	1846	3.0	47	2
		HC	1986	0.0	---	6
		HC-0.5-25	2008	0.5	42	2
		HC-0.5-32	2008	0.5	47	2
		HC-0.5-50	2020	0.5	57	2
		HC-1.0-25	2030	1.0	42	2
		HC-1.0-32	2030	1.0	47	2
High	61	HC-1.0-50	2042	1.0	57	2
		HC-1.5-25	2052	1.5	42	2
		HC-1.5-32	2052	1.5	47	2
		HC-2.0-25	2074	2.0	42	2
		HC-2.0-32	2074	2.0	47	2
		HC-3.0-25	2117	3.0	42	2
		HC-3.0-32	2117	3.0	47	2

Table 2. Concrete mix proportions

Materials	Mix HC (61 MPa)	Mix NC (30 MPa)	Quantity Mix LC1 (12 MPa)	Mix LC2 (9 MPa)
Type I cement, kg/m³	530	410	310	234
Fine aggregate, kg/m³	640	280	1050	940
Coarse aggregate[1], kg/m³	600	870	339	450
Water, kg/m³	154	138	186	140
Superplasticizer/cement, by weight	2.2 percent	2.2 percent	---	---
Silica Fume, kg/m³	45	30	---	---
Water/cementitious ratio	0.27	0.31	0.6	0.6
Fresh concrete density, kg/m³	1980	1739	1890	1762

[1]Coarse aggregate used in Mix HC & Mix NC is Stalite and in Mix LC1 & LC2 is LECA
1kg/m³ = 0.0624 lb/ft³

The fiber concrete mixes had slumps of 60 to 120 mm (2.36 to 4.72 in) for the various fiber contents. The specimens were cast with the wide face 700 × 200 (27.55 × 7.87 in) placed horizontally. Also, control cylinders of 152.4 × 304.8 mm (6 ×12 in.) were cast for plain and steel fiber reinforced lightweight concrete. All the specimens were compacted using a table vibrator at a frequency of 5.5 cycle/sec for two minutes immediately after the placement of concrete. All specimens were cured for 24 hours in the mold under a polyethylene sheet and then stored in the laboratory environment (temperature of 23°C ±1°C and relative humidity of about 100%) until expected testing time.

Four mix proportions were used for achieving the low, normal and high levels of concrete strength with respect to the desired unit weight for lightweight concrete. Curing time was the basis for obtaining the anticipated concrete strengths. When the anticipated stength was reached, a set of two torsion specimens with various steel fiber contents and aspect ratios along with their corresponding control specimens for compressive strength were tested.

3.2 Loading setup and measurements

Details of the test setup are shown in Figure 1. One 20 kN hydraulic actuator was used to apply the load near the east support. The load had a 525 mm lever arm from the centroidal axis of the specimen, giving the test rig a 10.5 kN-m torque capacity. A 50 kN tension load cell was used to measure the applied load. The actuator had a stroke length of 152 mm providing a minimum of 13 degree twist capacity of the specimen. A reaction arm was used near the west support to balance the applied load by attaching the arm to the laboratory strong floor. The reaction rod had a 525 mm eccentricity from the centroidal axis of the specimen as well. After cracking, the specimen elongates longitudinally. In order to avoid any longitudinal restraint and subsequent compression, the specimen was allowed to slide and elongate freely. This was achieved by supporting the west end of the specimen on rollers. The twist angle of the specimen due to the applied torque was measured by two sensitive linear variable differential transformers (LVDTs).

4. RESULTS AND DISCUSSIONS

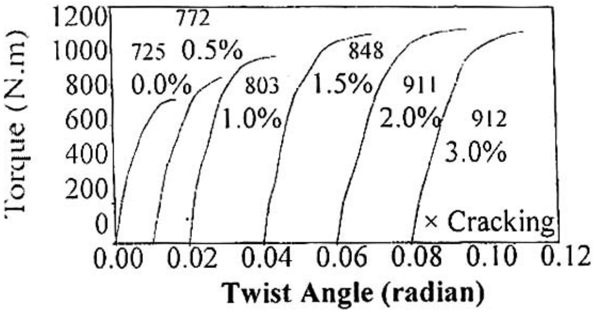
4.1 Strength and rotation behavior

The test results of the thirty six pairs of specimens are summarized in Table 3. The ultimate torsion strength ($T_{u,\exp}$) and the cracking torsion strength (T_{cr}) are reported as average of each pair of specimens tested. The percent increase in ultimate strength of SFRLWC specimens with respect to plain specimens due to the presence of the fibers are also presented in Table 3.

Table 3. Test results for LC1, LC2, NC, and HC concrete specimens

Type	JC-25	JC-32	JS-50
Length, mm	25	32	50
Width, mm	0.80	1.00	2.00
Thickness, mm	0.35	0.37	0.30
Cross-sectional area, mm²	0.28	0.37	0.60
Equivalent diameter,mm	0.60	0.69	0.87
Aspect ratio	42	47	57
Tensile strength, Mpa	650	650	650

The torsion-rotation behavior (i.e., torque versus twist angle curves) of the steel fiber reinforced lightweight concrete for LC1-32, LC2-25, LC2-32, NC-32, HC-25, HC-32 and HC-50 specimens are shown in Figure 2. For the specimens reinforced with steel fibers, the behavior up to first cracking was almost similar to that of the plain specimens. After cracking, the SFRLWC specimens withstood the cracking load for a small rotation. Then the fibers in the concrete matrix contributed significantly, resulting in a strength increase of up to 84.0 percent to the plain specimen depending on concrete strength, fiber content and fiber aspect ratio (Table 3). Between the stages of cracking and maximum torsion strength, the SFRLWC specimens demonstrated a reduced stiffness and a considerable rotation in the concrete prior to failure. The rotation of the fiber reinforced specimens at ultimate conditions was about twice that at cracking. The failure occurred at the ultimate torsion strength, and afterwards the specimens did not show a softening response for the torque-twist angle curve.



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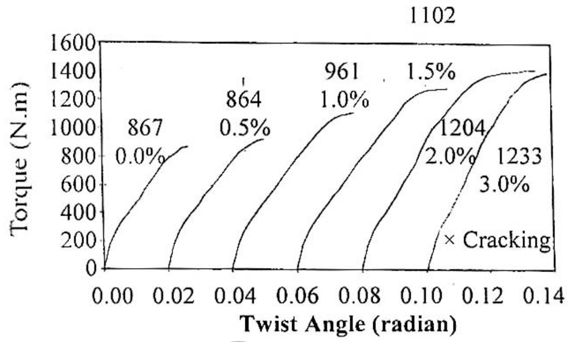

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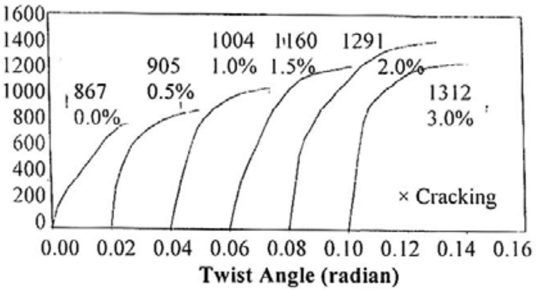

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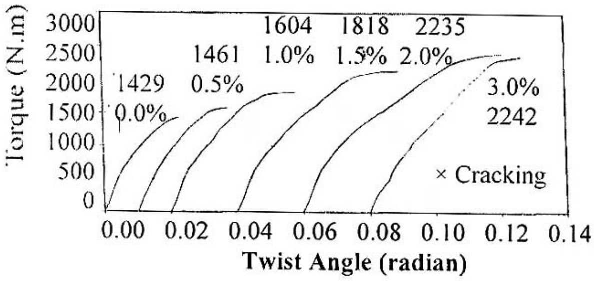

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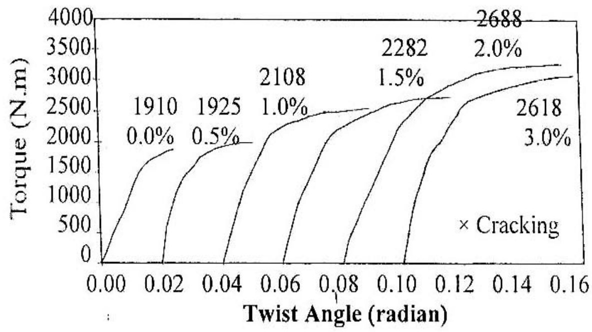

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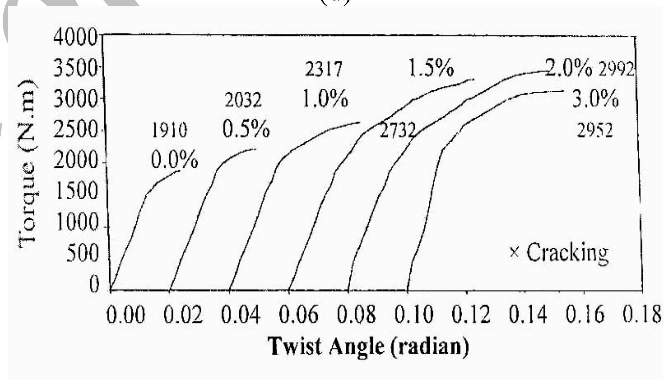

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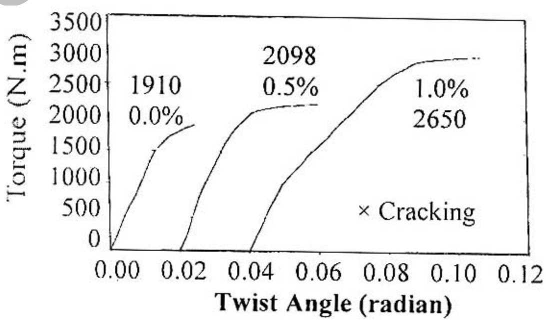


(g)

Figures 2(a), 2(b), 2(c), 2(d), 2(e), 2(f) and 2(g)- Influence of Steel Fiber Content on Torque-Twist Angle Response of LC1-32, LC2-25, LC2-32, NC-32, HC-25, HC-32 and HC-50 specimens, respectively

4.2 Torsional Strength of Plain Lightweight Concrete (Cracking Strength)

The ultimate strength of plain specimens is almost equal to their cracking strength. The cracking torsional moment under pure torsion for normal weight concrete, T_{cr}, is given in ACI 318-02 by the following equation,

$$T_{cr} = 1/3\sqrt{f'_c}\left[\frac{A^2_{cp}}{P_{cp}}\right] \quad (1)$$

in which the units are MPa and mm. A comparison between the test results for plain LWC under pure torsion with different concrete strength, and that of ACI (Eqs.1) is shown in Figure 3. It is observed that the ACI equation is conservative and may not require adjustment for LWC under pure torsion.

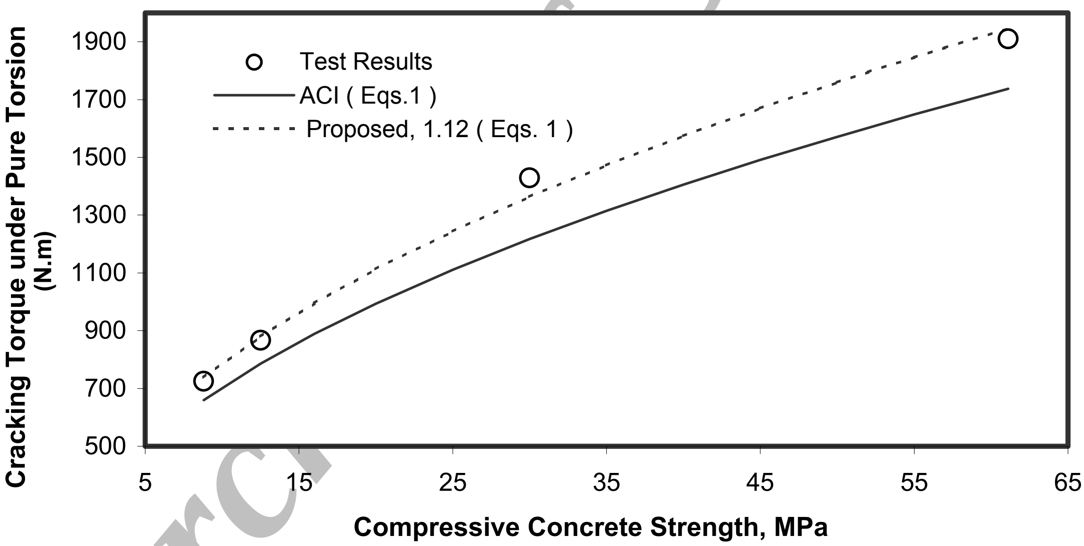


Figure 3. Comparison between the test results for plain LWC under pure torsion and ACI (Eqs. 1) for normal concrete

4.3 Influence of concrete strength, v_f and l/d of fibers

All the SFRLWC specimens provided higher ultimate torsion strength than that of their respective plain concrete (Table 3). In general, the specimens with higher uniaxial compressive strength (f'_{cf}) and higher fiber volume showed higher percent increase in ultimate torsion strength for a specific fiber aspect ratio. The specimens with higher steel fiber volume (up to 2.0 percent) resulted in larger values for ultimate torsion strength regardless of concrete strength and fiber aspect ratio. For instance, in NC specimens with fiber aspect ratio of 47, the percent increase in torsion strength was 10, 27, 49 and 74 corresponding to 0.5, 1.0, 1.5 and 2.0 percent fiber volume, respectively. The maximum percent increase in torsion strength due to fiber addition belongs to the HC-2.0-32 specimen with 84 percent.

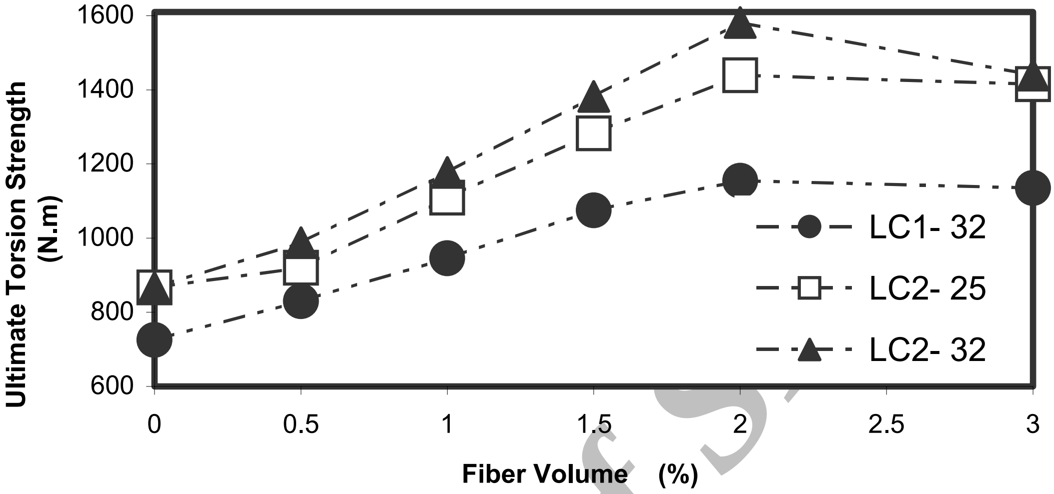
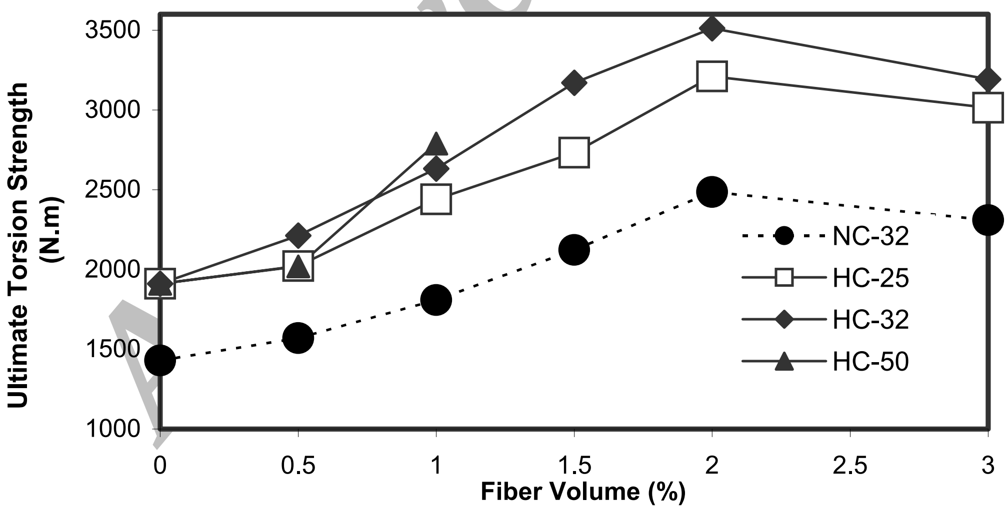


Figure 4(a). Representation of increase in torsion strength of low strength concrete (LC1 and LC2) for various fiber volume and fiber aspect ratio



(b). Representation of increase in torsion strength of normal and high strength concrete (NC and HC) for various fiber volume and fiber aspect ratio

Figure 4(a) and Figure 4(b) shows schematic representation of increase in torsion strength of LC1, LC2, NC and HC for various fiber volume and fiber aspect ratio. There was always an increase in the ultimate torsion strength due to increase of compressive strength of concrete. This increase was attributed to higher bond developed at the interface of fiber and concrete matrix for higher strength concrete. The increase in cracking strength of SFRLWC was higher for high-strength concrete containing larger volume of fibers (up to two percent) and with higher fiber aspect ratio. The rotation at cracking and ultimate torsion strength (failure) is larger for high-strength concrete (for example, rotation increase of 150 and 94 percent obtained for HC-2.0-32 compared to LC1-2.0-32 at cracking and failure, respectively). However, steel fiber demonstrates to be more effective in HC than in lower strength concrete, producing higher ultimate torsion strength as well as deformability characteristics at loads higher than cracking of the specimens. This is attributed to the improved bond characteristics between the fibers and the matrix with the higher strength concrete. Generally, the influence of concrete strength on rate of increase in ultimate torsion strength increased for specimens with higher percentage of fiber volume and aspect ratio.

The presence of high volume of steel fibers resulted generally in higher cracking torsion strength for various levels of concrete strength. Higher fiber volume in addition to providing higher strength contributed to slightly higher deformability of the materials. The torsion-rotation curve generally resulted in higher rotation due to the larger fiber aspect ratio; however, the increase was higher for higher strength concrete. Experimental results indicated that the concrete strength, fiber volume, and fiber aspect ratio have minor influence on torsion stiffness of the specimens up to cracking load. The SFRLWC specimens with the larger fiber aspect ratio experienced higher ultimate torsion strength for LC1, LC2, NC and HC concrete specimens.

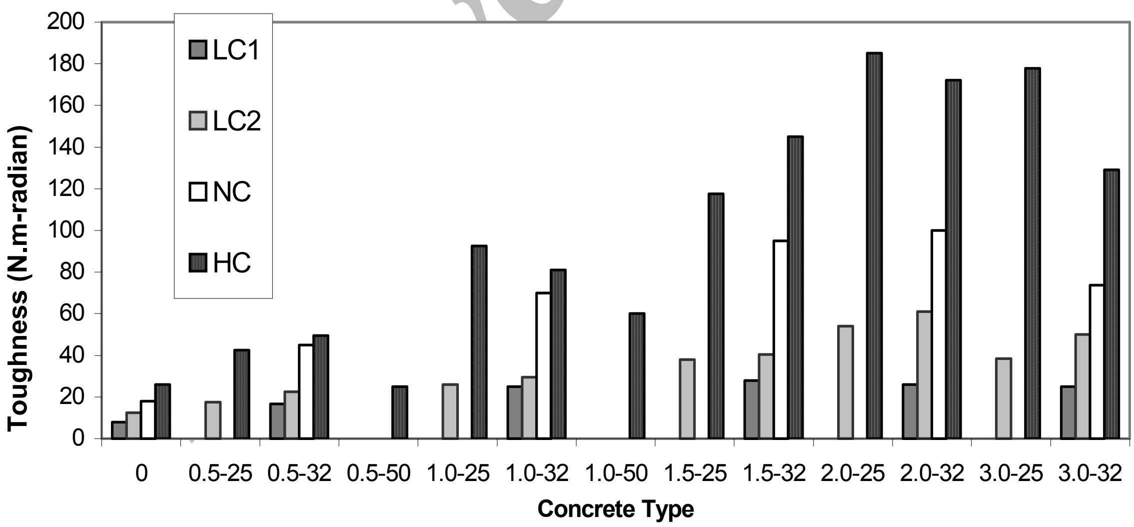


Figure 5. Toughness (area under the torque-twist angle response curve) values for various concrete strengths

4.4 Toughness

To evaluate the relative toughnesses associated with each type of specimen, the area under torsion-rotation curve up to failure was calculated and presented in Figure 5 as a bar graph comparing the toughness for various concrete strength specimens. The addition of fibers resulted in an improvement of toughness for all cases. The figure generally indicates that higher fiber volume and larger aspect ratio also contributed to the toughness increase. The specimens with higher strength (HC) manifested significant increase in toughness compared to that of lower strength (LC1 and LC2) specimens. Improvements of up to 5.4, 7.0, 5.5 and 7.1 times for LC1, LC2, NC and HC specimens, respectively, were obtained over their respective plain concrete specimens. Increase of concrete strength to 5.94 times (from LC1 to HC strength) resulted in a more ductile behavior possessing 5.61 times increase for toughness of specimen having 2.0 percent volume fiber with aspect ratio of 47.

4.5 Mode of Failure

The observed failure mode of the concrete specimens without fiber was very brittle, and increase of concrete strength caused further brittleness and violent failure for plain concrete. These specimens lost their integrity, breaking into two pieces. Figure 6 shows typical failure pattern of plain and steel fiber reinforced specimens. At torque above 85 percent of ultimate torque, a fine crack at an angle of about 45° developed with respect to longitudinal axis of the test specimen. The LC1 and LC2 specimens showed predominantly bond failure at the aggregate-matrix interfaces, whereas in NC and HC specimens, the torsional shear cracks passed through 20 to 35 percent of the aggregates along its path.

(a)

(b)

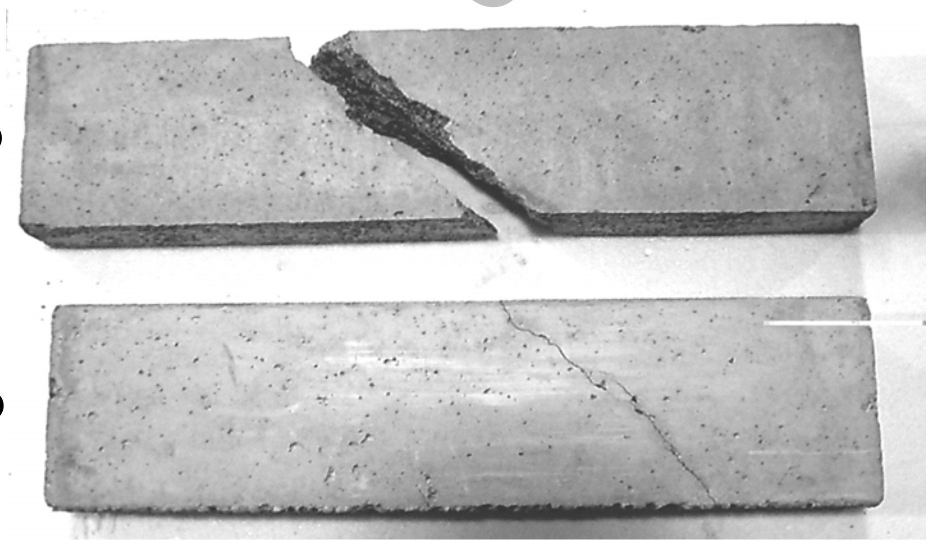


Figure 6. Failure pattern of (a) plain LWC specimens, (b) steel fiber reinforced LWC specimens

Specimens reinforced with fibers developed a fine crack at 80 to 90 percent of ultimate torsion strength and at angle similar to plain concrete specimens. At higher loads, sound of fibers being pulled out or ruptured was heard. The failure of SFRLWC specimens was ductile and ductility was further increased in specimens with higher fiber content and fiber aspect ratio. Even with considerably large crack width, the fibers were able to preserve the integrity of the specimen, maintaining it in one piece.

Observation of the failure surface of SFRLWC specimens reveals that the increase in fiber volume, aspect ratio, and especially concrete strength causes the failure plane to pass mainly through the aggregate. This is attributed to the delay of crack propagation and local confinement by fibers, for which reason the crack direction deviates from aggregate-matrix interface into the matrix and aggregate. In low and normal strength concrete specimens, the fibers bridging the cracks along the torsion plane were pulled out. However, for high strength concrete specimens, especially those with the larger fiber aspect ratios and higher fiber volume, some of the fibers bridging the crack along the torsion plane ruptured, indicating the improved bond provided by concrete of high strength level.

4.6 Torsion Strength Model

An empirical model is proposed for calculating the torsional strength of plain and steel fiber reinforced concrete. Ultimate torsion strength ($T_{u,pre}$) for various strengths of LWC, is expressed as a function of fiber aspect ratio (l/d) and fiber-volume percent (v_f) as follows,

$$T_{u,calc} = 0.22\left(\frac{0.85 \times 2\sqrt{f'_{cf}}}{3}\right)X^2Y\left(1 + m\left(v_f\right)^k\left(l/d\right)^n\right) \tag{2}$$

In which X and Y, respectively the width and the length of the cross-section are in mm, $T_{u,pre}$ is in N.mm, and f'_{cf} is in MPa. Based on the experimental results presented in Table 3 and regression analysis, the constants in Eqs. (2) are determined as, $m = 0.13$, $n = 1.5$ and $k = 1$. Hence the Eqs. (2) is reduced to,

$$T_{u,calc} = 0.125\sqrt{f'_{cf}}\,(X^2Y)\left(1 + 0.13v_f\left(l/d\right)^{1.5}\right) \tag{3}$$

The predicted ultimate torsion strengths using Eqs. (3) and percent difference with test results are given in Table 3. The percent difference in torsion strength falls within -12 and +10 percent, with a summation of the differences equal to -6.2 percent, which shows the model prediction is overall conservative.

5. SUMMARY AND CONCLUSIONS

In this paper, strength and ductility properties of plain and steel fiber reinforced low- to high-strength lightweight concrete under pure torsion are studied. The concrete strength levels included 9, 12, 30 and 61 MPA (1.30, 1.74, 4.35, and 8.84 ksi) representing low (LC1 and LC2), normal (NC), and high-strength concrete (HC), respectively. Based on the experimental results reported in this study, the following conclusions are drawn:

1. Fiber were more effective in increasing the torsional strength of high strength concrete

specimens compared to the strength of their respective unreinforced plain concrete specimens. For specimens with one percent fiber volume and aspect ratio of 47, the percent increase was 30, 36, 26, and 38 corresponding to LC1, LC2, NC and HC specimens, respectively. The maximum percent increase obtained belongs to the HC specimen with 84 percent for 2.0 percent of 32 mm fibers.

2. Fibers improved the twist angle and ductility characteristics of concrete. The twist angle at ultimate torsion strength (failure) was larger for concrete specimens with high strength level. Addition of one percent fiber with aspect ratio of 47 to high-strength concrete (HC) produced a relative toughness which was approximately 2.2 times than that for the plain high strength concrete specimen. The increase for two percent fiber volume in HC was about 5.7 times of that for plain HC. Comparable improvements in toughness were obtained in fiber reinforced concrete specimens with lower level of concrete strength. Higher fiber volume and especially larger fiber aspect ratio contributed to higher twist of the composite material. The toughness of HC specimen varied between 2 and 3 times of that for LC specimen.
3. For plain specimens, failure occurred in a very brittle manner with limited warning before collapse, and the SFRLWC specimens showed a relatively ductile type of failure. In both cases, a major continuous crack at 45° with respect to longitudinal axis caused final failure. In low and normal strength concrete specimens, the fibers bridging the crack were pulled out of the matrix; however, for high strength specimens, some of the fibers fractured indicating the improved bond provided by the concrete of high-strength level.
4. The empirical model predictions for the torsion strength of the tested specimens correlate well with the experimental results. The model includes the effect of concrete strength, fiber volume and aspect ratio.
5. Companion between the test result for plain lightweight concrete under pure torsion and that of ACI 318 for normal weight concrete (see Figure 3) shows that the ACI code underestimates the cracking torque for LWC by about 12 percent.

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LIST OF NOTATIONS

A_{cp} = area enclosed by outside perimeter of concrete cross-section

f_c' = plain concrete compressive strength

f'_{cf} = steel fiber reinforced compressive strength

l/d = aspect ratio of steel fibers

P_{cp} = perimeter of concrete cross-section

T_{cr} = cracking torsional moment of specimen

$T_{u,\exp}$ = experimental ultimate torsional moment of specimen

$T_{u,calc}$ = predicted ultimate torsional moment of specimen

v_f = volume of steel fibers

X = width of the specimens' cross-section

Y = length of the specimens' cross-section

σ_m = normal deviation defined as the square root of the average of square percent differences

ϕ_{cr} = curvature at cracking

ϕ_{ult} = curvature at ultimate