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OPTIMAL DESIGIN OF STRUCTURES WITH FREQUENCY CONSTRAINTS USING WAVELET BACK PROPAGATION NEURAL

E. Salajegheh*, S. Gholizadeh, P. Torkzadeh
Department of Civil Engineering, University of Kerman, Kerman, Iran

ABSTRACT

A combination of improved genetic algorithm and neural networks is proposed to find the optimal weight of structures subject to multiple natural frequency constraints. The structural optimization is carried out by an evolutionary algorithm employing the discrete design variables. To reduce the computational time of the optimization process, the natural frequencies of structures are evaluated by using properly trained back propagation (BP) and wavelet back propagation (WBP) neural networks. The numerical results reveal the robustness and high performance of the suggested methods for the structural optimization with frequency constraints. It is found that the best results are obtained using WBP network.

Keywords: Natural frequency, optimal design, genetic algorithm, wavelet, back propagation, neural network

1. INTRODUCTION

In the recent years, much progress has been made in the field of optimum design of structures subject to stress and displacement constraints. They have mostly employed the conventional and traditional methods for the constraints approximation and optimization [1-4]. These methods usually employ derivative calculations.

In this study, an efficient method is presented to find the optimal design of structures subject to multiple natural frequency constraints utilizing an evolutionary algorithm. The weight of structures is considered as the objective function. The design variables are cross sectional areas of the structural elements. The optimization algorithm is carried out by virtual sub population (VSP) method [5]. In this method, all the necessary mathematical models of the natural evolution operations are implemented on a small initial population to access the optimum solution on iterative basis.

The stochastic nature of the evolutionary algorithms makes the convergence of the process slow. Furthermore, evaluating the natural frequencies of structures by the finite element method can impose vast computation during the optimization process. To reduce

* Email-address of the corresponding author: eysasala@mail.uk.ac.ir

computer efforts, the natural frequencies of structures are evaluated by using properly trained back propagation (BP) and wavelet back propagation (WBP) neural networks [6, 7]. Wavelet neural network is presently the most popular mapping of neural networks. In the WBP, the activation function of hidden layer neurons is substituted with a wavelet function. In the civil and structural engineering cases, various types of WBP have been mostly utilized [7-9].

Illustrative examples are solved to assess the robustness and efficiency of the suggested methods. The numerical results demonstrate the computational advantages of the proposed methods.

2. BACK PROPAGATION NEURAL NETWORKS

Back Propagation was created by generalizing the Widrow-Hoff learning rule to multiple layer networks and nonlinear differentiable transfer functions. Input vectors and the corresponding target vectors are used for training a network until it can approximate a function, associate the input vectors with the specific output vectors. Networks with a sigmoid layer and a linear output layer are capable of approximating any function with a finite number of discontinuities.

Standard back propagation employs a gradient descent algorithm, as the Widrow-Hoff learning rule, in which the network weights are moved along the negative of gradient of the performance function. The term of back propagation refers to the manner in which the gradient is computed for nonlinear multilayer networks. There are a number of variations on the basic algorithm based on the other standard optimization techniques, such as conjugate gradient and Newton methods. In this study, the scaled conjugate gradient (SCG) algorithm that was developed by Moller is employed [10]. The basic back propagation algorithm adjusts the weights in the steepest descent direction. This is the direction in which the performance function is decreasing most rapidly.

3. FUNDAMENTALS OF WAVELET THEORY

Wavelet theory is the outcome of multi-disciplinary endeavours that brought together mathematicians, physicists and engineers. This relationship creates a flow of ideas that goes well beyond the construction of new bases or transforms. The term of wavelet means a little wave. A function $h \in L^2(R)$ (the set of all square integrable or a finite energy function) is called a wavelet if it has zero average on $(-\infty,+\infty)$ [11]:

$$\int_{-\infty}^{+\infty} h(t)dt = 0 \tag{1}$$

This little wave must have at least a minimum oscillation and a fast decay to zero in both the positive and negative directions of its amplitude. These three properties are the Grossmann-Morlet admissibility conditions of a function that is required for the wavelet transform. The wavelet transform is an operation which transforms a function by integrating it with modified versions of some kernel functions. The kernel function is called the mother wavelet and the modified version is its daughter wavelet. A function $h \in L^2(R)$ is admissible if:

$$c_h = \int_{-\infty}^{+\infty} \frac{|H(\omega)|^2}{|\omega|} d\omega < \infty \tag{2}$$

where H(ω) is the Fourier transform of h(t). The constant c_h is the admissibility constant of the function h(t). For a given h(t), the condition $c_h < \infty$ holds only if $H(0) = 0$. The wavelet transform of a function $h \in L^2(R)$ with respect to a given admissible mother wavelet h(t) is defined as:

$$W_f(a,b) = \int_{-\infty}^{+\infty} f(t) h^*_{a,b}(t) dt \tag{3}$$

where * denotes the complex conjugate. However, most wavelets are real valued.

Sets of wavelets are employed for approximation of a signal and the goal is to find a set of daughter wavelets constructed by dilated and translated original wavelets or mother wavelets that best represent the signal. The daughter wavelets are generated from a single mother wavelet h(t) by dilation and translation factors as follows:

$$h_{a,b}(t) = \frac{1}{\sqrt{a}} h(\frac{t-b}{a}) \tag{4}$$

where a > 0 is the dilation factor and b is the translation factor. The constant term of $1/a^{1/2}$ is for energy normalization which keeps the energy of the daughter wavelet equal to the energy of the original mother wavelet [11].

4. WAVELET NEURAL NETWORKS

Wavelet neural networks (WNN) employing wavelets as the activation functions have been recently researched as an alternative approach to the neural networks with sigmoidal activation functions. The combination of wavelet theory and neural networks has lead to the development of WNNs. WNNs are feed forward neural networks using wavelets as activation functions. In WNNs, both the position and the dilation of the wavelets are optimized besides the weights. Wavenet is another term to describe WNN. Originally, wavenets refer to neural networks using wavelets. In wavenets, the position and dilation of the wavelets are fixed and the weights are optimized [11].

5. WAVELET BACK PROPAGATION NEURAL NETWORKS

BP network is now the most popular mapping of neural network. But it has few problems such as trapping into the local minima and slow convergence. Wavelets are powerful signal analysis tools. They can approximately realize the time-frequency analysis using a mother wavelet. The mother wavelet has a square window in the time-frequency space. The size of the window can be freely variable by two parameters. Thus, wavelets can identify the localization of unknown signals at any level. Activation function of hidden layer neurons in BP network is a sigmoidal function shown in Figure 1-a. To design wavelet back propagation (WBP) network we substitute hidden layer sigmoidal activation function of BP network with POLYWOG1 wavelet [12]:

$$h_{POLYWOG\ 1}(t) = \sqrt{e}\,(t)\, e^{(-(t)^2)/2} \tag{5}$$

Diagram of POLYWOG1 with a = 1 and b = 0 is shown in Figure 1-b. Typical topology of WBP is also shown in Figure 2.

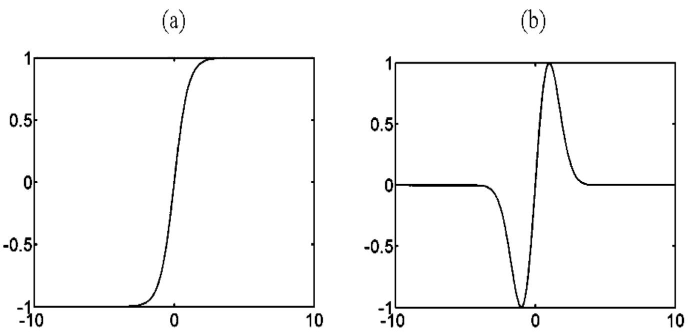

Figure 1. (a) Sigmoidal function, (b) POLYWOG1 wavelet

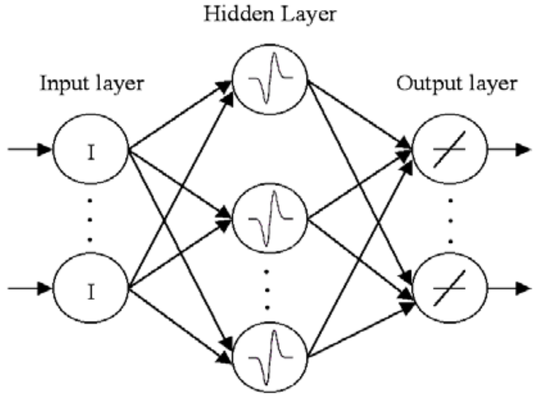

Figure 2. Typical topology of WBP

In this WBP, the position and dilation of the wavelets as the activation function of hidden layer neurons are fixed and the weights of network are optimized using SCG algorithm. In this study, we obtain good results considering b = 0 and a = 2.25.

$$h_{\text{POLYWOG }1}(t) = \sqrt{e}\,(\frac{t}{2.25})\, e^{(-(\frac{t}{2.25})^2)/2} \tag{6}$$

6. MAIN STEPS OF OPTIMIZATION

The main steps for the structural optimization with multiple frequency constraints by VSP method using BP and WBP neural networks are summarized as follows:

a) Selecting some parent vectors from the design variables space.
b) Evaluating the natural frequencies of the structure employing trained BP and WBP networks.
c) Evaluating the objective function.
d) Checking the constraints for feasibility of parent vectors.
e) Generating offspring vectors using crossover and mutation operators.
f) Employing the trained BP and WBP networks for predicting the natural frequencies of the offspring population.
g) Evaluating the objective function.
h) Checking the constraints; if satisfied continue, else change the vector and go to step (f).
i) Checking convergence; if satisfied stop, else go to step (f).
j) Selecting the majority parent vectors from the previous solution and some random design variables as a VSP.
k) Repeating steps (c) to (k) until the proper solution is met.

As the size of populations in VSP method is small, the method converges rapidly. It can be observed that the modal analysis of structures is not necessary during the optimization process.

7. NUMERICAL RESULTS

In this study, two structures are selected as numerical examples to be optimized. These structures are:

1) 72-bar aluminium space truss
2) 200-bar steel double layer grid

Example 1 is taken from Ref. [1] and the other example is arbitrary chosen. The optimization is carried out by VSP using the following frequency analysis methods:

(a) Exact analysis
(b) BP neural network
(c) WBP neural network

The computational time is measured in CPU time used by a Pentium IV 2.0 GHz. Also, the errors between the exact and approximate frequencies are calculated using the following equation:

$$\text{error} = \left| \frac{f_{ap} - f_{ex}}{f_{ex}} \right| \times 100 \tag{7}$$

where f_{ap} and f_{ex} represent the approximate and exact frequencies, respectively.
The specifications of VSP method are displayed in Table 1.

Table 1. Specifications of VSP method

Population size	**30**
Crossover method	One, two and three points crossover
Crossover rate	0.9
Mutation rate	0.001
Maximum generation	30

The input space consists of cross sectional areas of the structural elements, and the corresponding natural frequencies of them are considered as the target space components. Also, for training and testing the neural networks, MATLAB [13] is utilized and the modal analysis of structures is performed using ANSYS [14].

7.1 Example 1: 72-bar aluminium space truss

The 72-bar aluminium space truss is shown in Figure 3. Cross sectional areas of the elements are selected from the available sections in Table 2.

Table 2. Available sections for Example 1

No.	**Area (cm^2)**	**No.**	**Area (cm^2)**
1	0.64	9	8.20
2	3.40	10	8.30
3	3.50	11	12.50
4	3.60	12	12.80
5	7.80	13	13.00
6	7.90	14	17.00
7	8.00	15	17.20
8	8.10	16	17.50

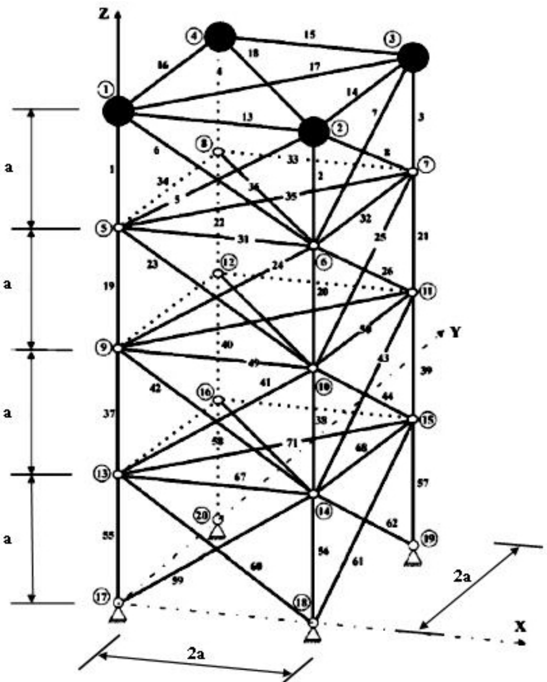

Figure 3. 72-bar aluminium space truss

Modulus of elasticity and density are equal to 6.86×10^9 kg/m^2 and 2770 kg/m^3, respectively. The mass of 2270 kg is lumped at nodes 1 to 4. The multiple natural frequency constraints are considered as: $F_1 \geq 4$ Hz, $F_3 \geq 6$ Hz.

For simplicity and practical demands, the truss elements are divided into 16 groups based on the cross-sectional areas, as shown in Table 3.

Table 3. Groups of the 72-bar aluminium truss elements

Group	Elements	Group	Elements
1	1-4	9	37-40
2	5-12	10	41-48
3	13-16	11	49-52
4	17-18	12	53-54
5	19-22	13	55-58
6	23-30	14	59-66
7	31-34	15	67-70
8	35-36	16	71-72

7.1.1 Neural networks training and testing

For this example, information of training and testing the neural networks is summarized in Table 4. Also, the networks errors due to frequencies approximation in the testing mode are displayed in Figures 4 and 5. The number of samples in training and testing modes is 120 and 80, respectively.

Table 4. Information on training and testing the neural networks for Example 1

Network	Training time (sec)	Mean of errors (%)	
		F_1	F_3
BP	40.0	9.968	6.057
WBP	32.0	3.304	4.410

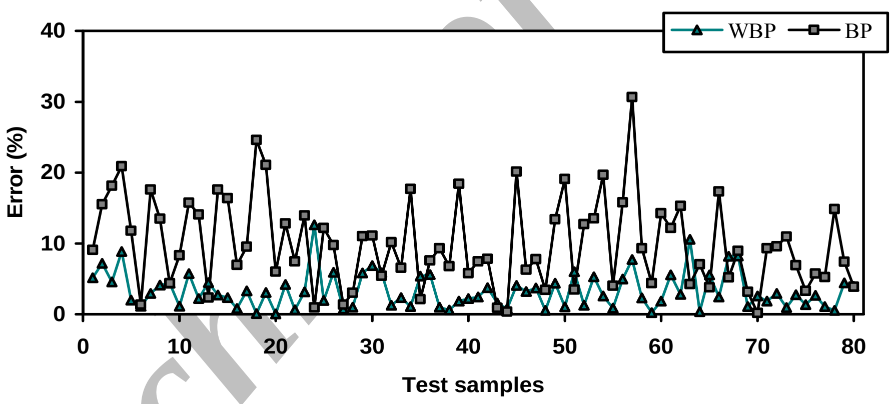

Figure 4. Errors of the first approximate frequency

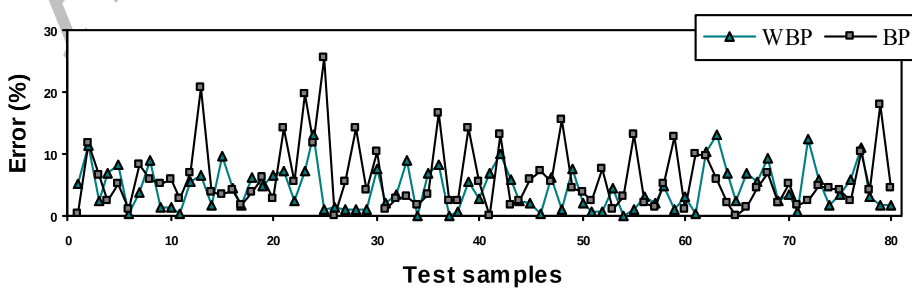

Figure 5. Errors of the third approximate frequency

The optimum solutions obtained by the various methods are shown in Table 5.

Table 5. Optimum designs of the 72-bar aluminium truss

Variable No.	**Optimum design (cm^2)**		
	Exact	**BP**	**WBP**
1	3.400	13.000	3.500
2	7.900	8.100	7.900
3	0.645	0.645	0.645
4	0.645	0.645	0.645
5	7.800	3.400	13.000
6	8.300	7.800	7.900
7	0.645	0.645	0.645
8	0.645	0.645	0.645
9	12.500	12.800	12.500
10	7.800	7.800	7.900
11	0.645	0.645	0.645
12	0.645	0.645	0.645
13	17.200	17.000	13.000
14	8.200	8.000	7.800
15	0.645	0.645	0.645
16	0.645	0.645	0.645
Weight(kg)	326.99	332.28	323.67
Generations	202	138	110
time (sec)	132.0	39. 0	31.0

It is observed that all the solutions are proper and the best solution is attained using WBP neural network.

For the optimum structures, the accuracy of approximate frequencies predicted by BP and WBP networks are displayed in Table 6.

Table 6. Errors of approximate frequencies for optimum designs (%)

Frequency No.	**BP**	**WBP**
1	5.33	1.81
3	7.28	5.27

As displayed in Table 6, accuracy of the approximate frequencies by WBP network is better than BP network. In comparison with the results presented in Ref. [1], it is found that combination of the VSP and neural networks creates a powerful tool for the structural optimization subject to multiple frequency constraints.

7.2 Example 4: 200-bar double layer grid

A 200-bar and 10×10 m double layer grid with a height of 0.5 m is considered. The top, bottom and diagonal layers of the double layer grid are shown in Figure 6. The structure is supported on the corner nods of the bottom layer. Cross sectional areas of the elements are selected from the available sections in Table 7. Due to the practical demands, the structural elements are grouped into 8 different types as shown in Table 8. Modulus of elasticity and density are equal to 2.1×10^{10} kg/m^2 and 7850 kg/m^3, respectively. The mass of 19620 kg is lumped at each free node of the top layer. To group the elements and to select the frequency constraints, the double layer grid structure is optimized subject to the gravity loads and its natural frequencies are considered as constraints:

$$F_1 \geq 3.5 \text{ Hz},\ \ F_3 \geq 5 \text{ Hz},\ \ F_5 \geq 7 \text{ Hz},\ \ F_7 \geq 9 \text{ Hz}$$

Table 7. Available sections for Example 2

No.	Area (cm^2)
1	1.213
2	2.540
3	3.733
4	4.534
5	5.229
6	6.669
7	10.670
8	11.810

Table 8. Groups of the 200-bar double layer grid elements

Group	Elements
1	1-24, 61-64
2	65-92
3	93-100
4	25-48
5	49-60
6	101-148
7	149-196
8	197-200

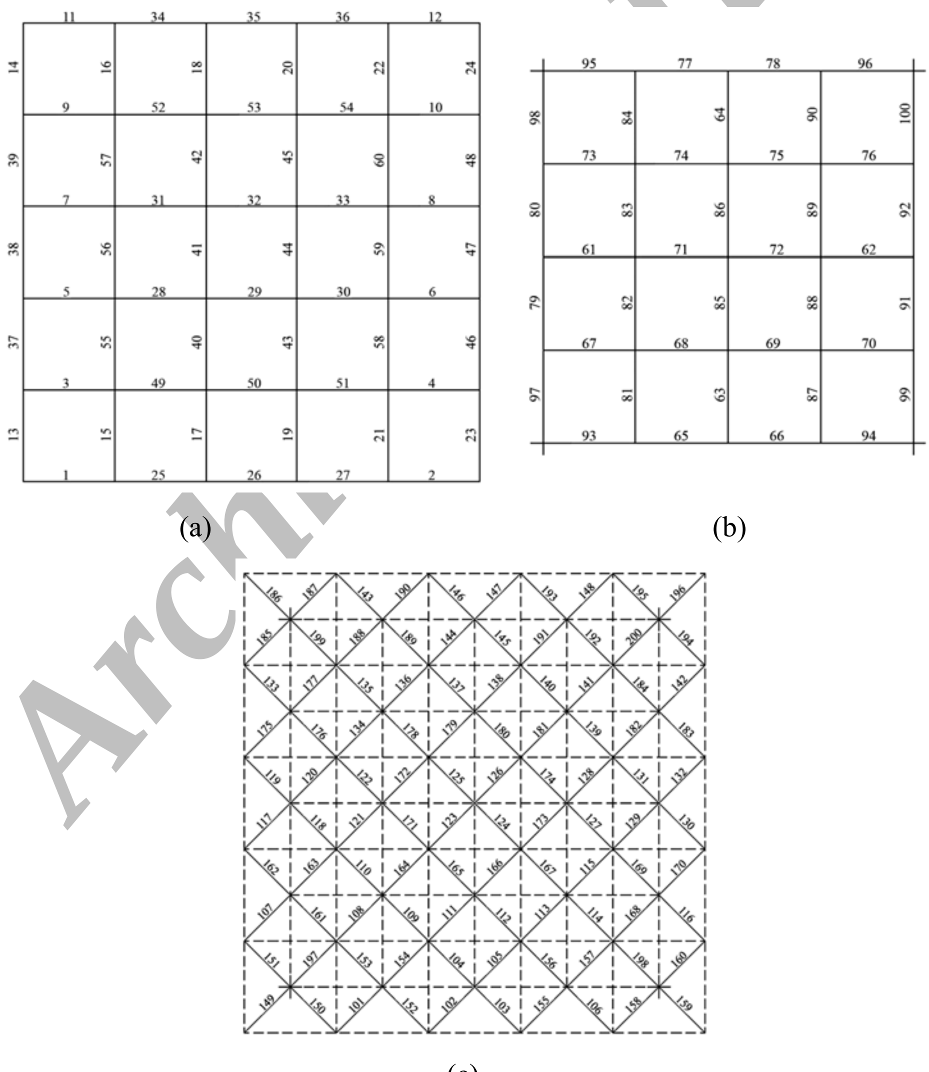

Figure 6. 200-bar double layer grid: (a) top layer, (b) bottom layer, (c) diagonal layer

7.2.1 Neural networks training and testing

For this example, the information of training and testing the neural networks is summarized in Table 9. Also, the neural networks errors due to frequencies approximation are displayed in Figures 7-10. The number of samples in training and testing modes is 120 and 80, respectively.

Table 9. Information on training and testing the neural networks for Example 2

Network	Training time (sec)	Mean of errors (%)			
		F_1	F_3	F_5	F_7
BP	30.0	5.87	7.07	6.77	9.79
WBP	20.0	2.93	3.21	2.92	3.29

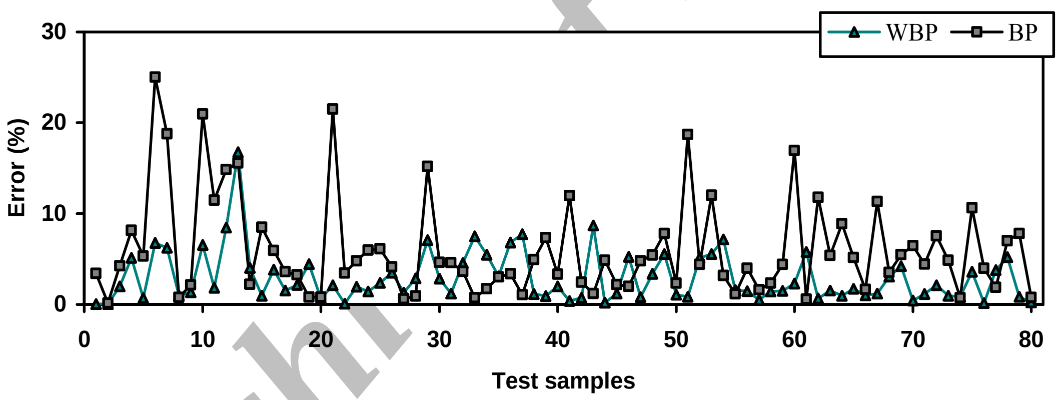

Figure 7. Errors of the first approximate frequency

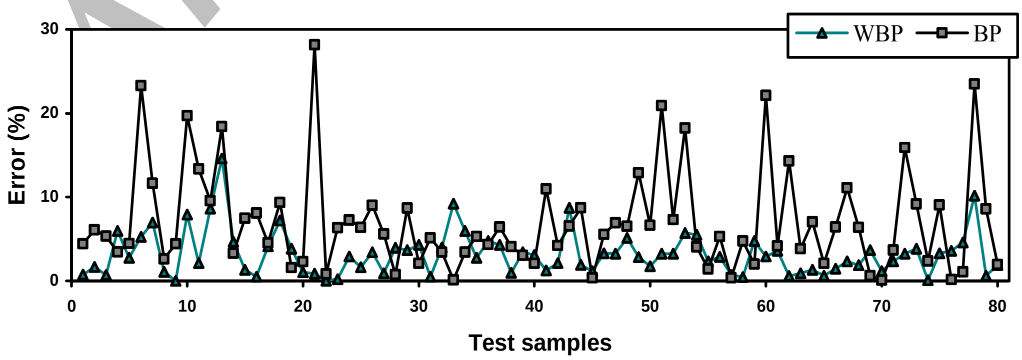

Figure 8. Errors of the third approximate frequency

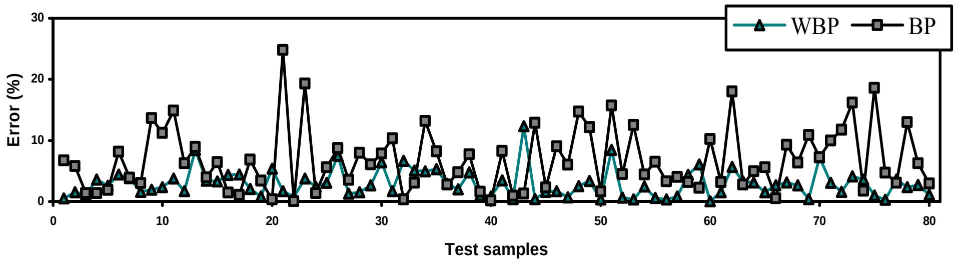

Figure 9. Errors of the fifth approximate frequency

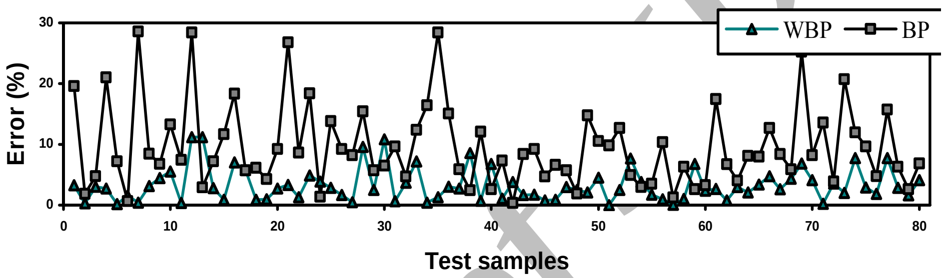

Figure 10. Errors of the seventh approximate frequency

The optimum solutions obtained by the various methods are shown in Table 10. As observed in this table, the best optimum design is obtained using WBP neural network.

Table 10. Optimum designs of the 200-bar double layer grid

Variable No.	Optimum design (cm^2)		
	Exact	BP	WBP
1	3.733	5.229	4.534
2	5.229	4.534	5.229
3	10.670	10.670	6.669
4	10.670	10.670	10.670
5	6.669	6.669	5.229
6	2.540	2.540	3.733
7	4.534	4.534	3.733
8	4.534	5.229	10.670
Weight(kg)	1476.9	1515.1	1485.7
Generations	196	184	154
time (sec)	315.0	11.0	8.0

The accuracy of approximate frequencies of the optimum structures predicted by BP and WBP neural networks are displayed in Table 11.

Table 11. Errors of approximate frequencies of optimum designs (%)

Frequency No.	BP	WBP
1	4.12	1.15
3	5.01	1.04
5	5.44	1.57
7	6.37	2.67

As displayed in Table 11, the accuracy of the approximate frequencies obtained by WBP network is superior.

6. CONCLUSIONS

A robust optimization procedure has been developed for the optimum design of structures with frequency constraints using discrete design variables. To achieve this, a combination of the evolutionary algorithm and neural networks has been utilized. The evolutionary algorithm employed is virtual sub population (VSP) method. The VSP method has eliminated the shortcomings of the standard genetic algorithm such as enormous computer efforts for structures with a great number of degrees of freedom. Implementing the structural optimization using the exact modal analysis is a time consuming procedure. To reduce the computational burden, the natural frequencies of structures have been evaluated by using properly trained BP and WBP neural networks. Also, to increase the performance generality of BP neural networks many BP neurons should be located in the input space. This can increase the computer efforts. To improve the performance generality by spending low computational burden, wavelet transform theory and concepts of neural network have been combined. Using these concepts, wavelet BP (WBP) neural network has been designed. The WBP hidden neurons activation function is a wavelet function with zero translation factors. To calculate the optimum value of dilation factor of WBP neurons, a simple procedure is implemented based on estimation of the performance generality of WBP neural network. The results of the neural networks test reveal the higher performance generality of WBP network comparing with BP network. Finally, numerical results indicate that the best solution is attained by employing WBP neural network.

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