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VARIATIONAL MODELLING OF REINFORCED CONCRETE BEAMS

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ABSTRACT

This paper presents improved analytical method for predicting deflection of reinforced concrete beams. This approach considers stiffness of steel and concrete separately. A number of diverse analytical models have been proposed by various researchers aimed at accurately predicting deflection of reinforced concrete bent elements. In this paper energy model is formed and varied. Deflection is determined from the potential energy model. To establish the validity of the proposed model correlation of analytical solution of existing models are conducted. Variation model solutions generally show very good agreement with results obtained other analytical methods.

Keywords: reinforced concrete, stiffness, deflection, potential energy, analytical model

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1. INTRODUCTION

Numerous proposals have been made in predicting deflection of reinforced concrete beams which takes into account tension stiffening, level of loading and percentage of reinforcement [2, 4].The variation method is used effectively to approximate solutions of the differential equations with accuracy sufficient for engineering calculations. The majority of the problems connected to research in stress–strain condition of structures are reduced, as a rule, to the solution of one or several differential equations of equilibrium of an element corresponding to one or several unknowns.

The exact solution of such equations does not represent difficulties in some elementary cases. When solving real problems frequently it is necessary to handle such volume of computing works, that does not coincide with the exact solution and in many cases the exact solution of a problem in general is impossible, as boundary conditions simply are not expressed in analytical form. Therefore as a rule when solving practical problems it is advisable to use variation methods. Variation method is illustrated by various scientists: Ritz, Treffez, Vlasov and others [1].

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2. SINGLE SPAN SIMPLY SUPPORTED BEAM

In solving problems using variation method the following assumptions are taken [1]:

1) Strain in cross section of an element are distributed on height linearly, the hypothesis of plane sections (Bernoulli) is applicable.
2) Tensile stress is taken by steel
3) Concrete does not work in the tensile zone
4) In the compressed zone, concrete is elastic, and dependences between stress and strain are linear.

In this study a simply supported plane beam is considered. The analytical diagram is taken as follows: concrete works only in compression and steel works only in tension, take the stress diagram in the compressed zone triangular (Figure.1)

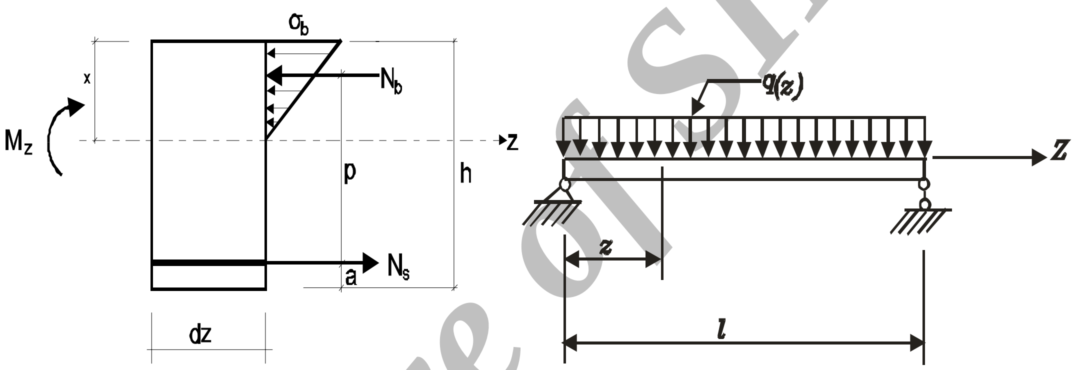

Figure 1. A beam and the stress distribution

Energy function is written as

$$Э = \frac{1}{2}\int_0^l E_b A_b \left(u_b'\right)^2 dz + \frac{1}{2}\int_0^l E_s A_s \left(u_s'\right)^2 dz - \int_0^l q(z)\cdot y(z)\,dz \qquad (1)$$

Energy function in term forces is expressed as

$$Э = \frac{1}{2}\int_0^l \frac{N_b^2}{E_b A_b} dz + \frac{1}{2}\int_0^l \frac{N_s^2}{E_s A_s} dz - \int_0^l q(z)\cdot y(z)\,dz \qquad (2)$$

Taking into consideration the hypothesis of plane sections (Figure.2)
From the equation Σz=0 we shall express them in one value

$$\frac{u_s}{h-x-a} = \frac{u_b}{\frac{2}{3}x} \;;\; u_b \frac{\frac{2}{3}}{h-x-a} u_s \;;\text{ Let }\; u_b = x_0 u_s \,,$$

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where

$$x_0 = \frac{2x}{3(h-x-a)}.$$

If triangular diagram is taken then,

$$N_b = \frac{1}{2}bx\sigma_b \text{ or } \sigma_n = 0,5\sigma_b,$$

It is necessary to change the position of the reaction force on the other side

$$tg\varphi = \frac{u_s}{h-x-a} \cong z', \text{ hence } u_s = (h-x-a)\cdot z',$$

When x-const $u_s' = (h-x-a)\cdot z''$

Let $h_0 = (h-x-a)$ and assume x-const, then the energy function can be written as

$$Э = \frac{1}{2}\int_0^l E_b A_b (x_0 h_0 z'')^2 dz + \frac{1}{2}\int_0^l E_s A_s (h_0 z'')^2 dz - \int_0^l q(z)\cdot y(z)dz \tag{3}$$

Since all characteristics taken are constant, including q=const, then the first two members of (3) can be combined

$$Э = \frac{1}{2}(E_b A_b x_0^2 + E_s A_s)h_0^2 \int_0^b (z'')^2 dz - q\int_0^l y(z)dz \tag{4}$$

For simply supported beam, a polynomial function (5) is taken which satisfies the following boundary conditions:

$$w_{(z)} = a_1 z(l-z) + a_2 z^2 (l-z)^2 \tag{5}$$

$$x=0;\ w=0;\ M=0;$$
$$x=l;\quad w=0;\quad M=0.$$

For convenience the following of (5) calculations are carried out

$$w_{(z)} = a_1 z(zl - z^2) + a_2(z^2 l^2 - 2z^3 l + z^4)\ w'_{(z)} = a_1(l-2z) + a_2(2zl^2 - 6z^2 l + 4z^3)$$

$$w''_{(z)} = a_1(-2) + a_2(2l^2 - 12zl + 12z^2) \tag{6}$$

Square equation (6)

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$$\left[w''_{(z)}\right]^2 = 4a_1^2 - 4a_1a_2\left(2l^2 - 12lx + 12x^2\right) + a_2^2\left(4l^4 + 144l^2x^2 + 144x^4 - 48l^3x + 48l^2x^2 - 288lx^3\right) \quad (7)$$

Integrals are calculate separately

$$W = \frac{1}{2}\left(E_bA_bx_0^2 + E_sA_s\right)h_0^2 \cdot \int_0^l \Big[4a_1^2 - 4a_1a_2\left(2l^2 - 12lx + 12x^2\right)$$
$$+a_2^2\left(4l^4 + 192l^2x^2 + 144x^4 - 48l^3x - 288lx^3\right)\Big]dz = \frac{1}{2}\left(E_bA_bx_0^2 + E_sA_s\right)h_0^2 \quad (8)$$
$$\left[4a_1^2l - 4a_1a_2\left(2l^3 - 6l^3 + 4l^3\right) + a_2^2\left(4l^5 + 64l^5 + \frac{144}{5}l^5 - 24l^5 - 72l^5\right)\right]$$
$$= \left(E_bA_bx_0^2 + E_sA_s\right)h_0^2\left(2a_1^2l + \frac{4}{10}a_2^2l^5\right)$$

$$U = q\int_0^l\left[a_1\left(zl - z^2\right) + a_2\left(z^2l^2 - 2z^3l + z^4\right)\right]dz = q\left[a_1\left(\frac{l^3}{2} - \frac{l^3}{3}\right) + a_2\left(\frac{l^5}{3} - \frac{1}{2}l^5 + \frac{l^5}{5}\right)\right] = q\left(a_1\frac{l^3}{6} + a_2\frac{l^5}{30}\right) \quad (9)$$

Energy function is expressed as

$$Э = \left(E_bA_bx_0^2 + E_sA_s\right)h_0^2\left(2a_1^2l + \frac{2}{5}a_2^2l^5\right) - q\left(a_1\frac{l^3}{6} + a_2\frac{l^5}{30}\right); \quad (10)$$

Coefficients a_i are determined from varying energy function (10)

$$\frac{\partial э}{\partial a_1} = \left(E_bA_bx_0^2 + E_sA_s\right)\cdot h_0^2 \cdot 4a_1l - q\frac{l^3}{6} = 0; \quad a_1 = \frac{ql^2}{24h_0^2\left(E_bA_bx_0^2 + E_sA_s\right)};$$
$$\frac{\partial э}{\partial a_2} = \left(E_bA_bx_0^2 + E_sA_s\right)\cdot h_0^2 \cdot \frac{4}{5}a_2l - q\frac{l^5}{30} = 0; \quad a_2 = \frac{q}{24h_0^2\left(E_bA_bx_0^2 + E_sA_s\right)}$$

Deflection function takes the following form

$$y_{(x)} = \frac{q}{24h_0^2\left(E_bA_bx_0 + E_sA_s\right)}\left[l^2z\left(l - z\right) + z^2\left(l - z\right)^2\right] \quad (11)$$

3. ANALYTICAL ANALYSIS BASED ON ALLOWED STRESSES [2]

The method is applied at elastic limit state taking into consideration of the assumptions stated above. As a result of the assumptions taken, triangular diagram of stress of concrete is considered. Cross-sectional area of tension steel A_S is replaces equivalent area of concrete αA_S and for compression steel from equilibrium of strain of concrete and steel

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$$\varepsilon_s = \frac{\sigma_s}{E_s} = \varepsilon_b = \frac{\sigma_b}{E_b}\text{; hence } \sigma_s = \alpha\sigma_b\text{, where } \alpha = \frac{E_s}{E_b}$$

Stress in concrete in a given uniform section

$$\sigma_b = \frac{Mx}{I_{red}}$$

Derived moment inertia of a given section is determined from equation (12)

$$I_{red} = \frac{bx^3}{3} + \alpha A_s\left(h_0 - x\right)^2 + \alpha A_s'\left(x - a'\right)^2 \quad (12)$$

Height of the compressed zone is determined from the equation of static moment (13) of the given section relative to the neutral axis and equated to zero

$$S_{red} = \frac{bx^2}{2} + \alpha A_s'\left(x - a'\right)^2 - \alpha A_s\left(h_0 - x\right)^2 = 0 \quad (13)$$

Deflection at the middle of the span

$$y_{\frac{l}{2}} = \frac{4ql^4}{384E_b I_{red}} \quad (14)$$

4. EXAMPLE

A simply supported beam with uniformly distributed load is considered as shown in (Figure 2)

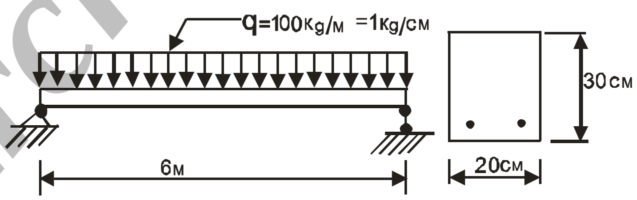

Figure 2. A simply supported beam

4.1 Based on allowed stresses

Concrete class– B25

$E_b = 306000 \text{kg/cm}^2$, Ref. [6]

Reinforcing steel– A-III

$E_s = 2.1 \times 10^6 \text{kg/cm}^2$, Ref. [5]

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$$\alpha = \frac{2.1 \cdot 10^6}{306000} = 6.863$$

Percentage of reinforcement μ0.08%

Concrete cover is taken as ***a=2.5***cm

$$A_b = b \cdot x = 20 \cdot 3 = 60cm^2$$

$$h_0 = h - x - a = 30 - 3 - 2.5 = 24.5cm$$

$$x_0 = \frac{2 \cdot x}{3h_0} = \frac{2 \cdot 3}{3 \cdot 24,5} = 0,0816$$

Determine ***x***,

$$\frac{20x^2}{2} - 6,863 \cdot 0,565(27,5 - x) = 0$$

$$10x^2 + 3,881x - 106,726 = 0$$

$$x^2 + 0,388x - 10,673 = 0$$

$$x = -\frac{0,388}{2} \pm \sqrt{(0,194)^2 + 10,673} = 3cm$$

$$I_{red} = \frac{20 \cdot 3^3}{3} + 6,863 \cdot 0,44(27,5 - 3)^2 = 1,999 \cdot 10^3 cm^4$$

Deflection at the middle of the span;

$$y_{\frac{l}{2}} = \frac{5 \cdot 1 \cdot 600^4}{384 \cdot 306000 \cdot 1,999 \cdot 10^3} = 2,767cm < 3cm$$, Ref. [2]

4.2 Based on the Variation model

Denominator,

$$24h_0^2\left(E_b A_b x_0 + E_s A_s\right) = 24 \cdot (24.5)^2 \left(306000 \cdot 60 \cdot 0,0816^2 + 2,1 \cdot 10^6 \cdot 0,44\right) = 1,507 \cdot 10^{10}$$

Deflection at the middle of the span when $z = \frac{l}{2}$

$$y_{\frac{l}{2}} = \frac{1}{1,507 \cdot 10^{10}}\left(600^2 \cdot 300^2 + \frac{600^2}{4}\left(\frac{600}{2}\right)^2\right) = 2,687cm < 3cm$$

Deviation

$$\frac{2,767 - 2,687}{2,767} \cdot 100 = 2,9\ \%$$

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5. CONCLUSIONS

Variation model solutions generally show very good agreement with results obtained by other analytical methods. The developed energy model is efficient in determining deflection at any point along the span of reinforced concrete beam. The model is currently being developed to take into account crack formation.

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