ASIAN JOURNAL OF CIVIL ENGINEERING (BUILDING AND HOUSING) VOL. 8, NO. 3 (2007)
PAGES 267-282

EFFECTS OF CREEP AND SHRINKAGE ON THE DEFLECTION OF RCC TWO WAY FLAT PLATES

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ABSTRACT

BS 8110-1997, ACI 318-2000 and IS 456-2000 Codes provide methods for calculating the short-term deflection, deflection due to Creep effects and Shrinkage effects for beams and one-way slabs. Though the codes are giving the provision for two-way slabs, as span to depth ratio for deflection control, much variation exists among them. Deflection calculation with creep and shrinkage effects is not included for two way slabs. Hence an attempt is made in this paper, to determine the total deflection including creep and shrinkage effects by suitably modifying the available formula for beams as per Equivalent frame method. A programme is developed in MATLAB to determine the exact deflection for two-way beamless slabs including short-term effects, Creep effects and Shrinkage effects. A numerical example is solved for flat plates by varying the parameters such as total thickness, characteristic compressive strength of concrete, clear cover of reinforcement, creep coefficient and the disparities among BS 8110-1997, ACI 318-2000, and IS 456-2000 are highlighted.

Keywords: stiffness, creep, shrinkage, deflection, flat plates

1. INTRODUCTION

RCC floor systems are the main horizontal structural members in multistorey buildings, which provide space for the occupants. The design of the members should not only satisfy the strength criteria, but also the serviceability requirements (Park and Gamble 2000, p.515) should be satisfied.

Codes suggest that the minimum thickness to be calculated based on span to depth ratio. In this paper, the available beam formula as per the three codes (i) BS 8110-1997 (ii) IS 456-2000 and (iii) ACI 318- 2000 are modified considering the slab as wide beams. A program in MATLAB is developed and compared for the determination of deflection of RCC floor system particularly with reference to flat plates including creep and shrinkage effects. The following are the span to depth ratios to be adopted for the members as per the relevant codes.

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- **BS 8110-1997**

The following span to depth ratio should be considered while assuming the thickness of the member, for beams up to 10m spans. For slabs also, the same span to depth ratio has to be used with suitable modification factor that based on the tension and compression reinforcement.

Cantilever	7
Simply supported	20
Continuous	26

If the drop width is greater than one-third of the span, the above values can be applied directly and for others, a modification factor 0.9 to be used.

- **IS 456-2000**

The control of deflection states that for beams and one-way slabs, the vertical deflection limits may generally be assumed to be satisfied if span to effective depth ratio of the member does not exceed the allowable values given below:

Cantilever	7
Simply supported	20
Continuous	26

For spans above 10m, the above values may be multiplied by 10/span in metres, except for cantilever, for which the deflection has to be calculated. Also the above values have to be multiplied by suitable modification factor based on the area and stress of steel for tension reinforcement, area of compression reinforcement.

Two-way slabs with shorter spans up to 3.5m with mild steel reinforcement and for loading class up to 3 kN/m^2, the span to overall depth ratios given below are generally assumed to satisfy vertical deflection limits.

➢ Simply supported slabs	35
➢ Continuous slabs	40

The above value should be multiplied by the factor 0.8 for Fe415 grade steel. For slabs without drops, the multiplication factor is 0.9.But the clause does not specify span to depth ratio for the spans greater than 3.5m and loading class above 3 kN/m^2.

- **ACI 318-2000**

Table 1 is to be adopted, for calculating the thickness of flat plate.

Table 1. Span to Depth Ratio for Flat plate (Without Drop Panels) (ACI 318-2000)

Yield Strength (N/mm^2)	**Exterior Panels**		**Interior Panels**
	Without Edge Beams	**With Edge Beams**	
300	$l_n/33$	$l_n/36$	$l_n/36$
420	$l_n/30$	$l_n/33$	$l_n/33$
520	$l_n/28$	$l_n/31$	$l_n/31$

But this value is based on yield strength of steel and not on span or loading class.

2. LITERATURE REVIEW

Hwang and Chang (1996, p.160) proposed two procedures for the control of deflection for two-way slabs and in the first procedure, considered minimum thickness requirement and in the second procedure with more flexibility in determining the mid-panel deflection. These were compared with the available experimental data and ACI 318-89. In their companion paper, Chang and Hwang (1996, p.150) proposed an algebraic equation for slabs subjected to uniform gravity loads based on plate theory and then calibrated using the results of finite element analysis. Numerical examples were given based on their proposed equation.

Nayak and Menon (2004, p.19) made experimental study on six one-way slab specimens on short-term deflection excluding long-term effects. Also they compared the existing provisions given in the codes such as IS 456- 2000, BS 8110- 1997, ACI 318- 2002, Eurocode 2 and highlighted the difference among them. Finally they proposed an improved procedure. Das (2004, p.144) reviewed the design approach for plain and reinforced concrete slabs based on IS 456- 2000 and provided charts to find out the effective depth satisfying both bending and serviceability conditions for beams and slabs.

The studies were made mainly on one-way slab with or without the long-term effects. Hwang and Chang (1996, p.160) studied and suggested equation for the deflection of two-way slab; it was based on plate theory. Though Branson (1977, p.293) approach uses the Equivalent Frame method for deflection of the two-way slab, for long-term effects thrice the value of dead load deflection is added with the short-term deflection. In the codes also, provisions are not available to calculate the deflection due to creep and shrinkage effects for two-way slabs. The formulae available in the codes are only for beams. Hence in this paper Equivalent Frame method is used to calculate the total deflection of two-way slab for long-term effects by suitably modifying the available beam formula.

3. METHODS

The following two methods are adopted by all the three (BS 8110-1997, ACI 318-2000 and IS 456-2000) codes for two-way beamless slabs:

- Direct design method
- Equivalent Frame method

The direct design method is applicable only to certain flat slab systems which satisfies the

following conditions:

- There should be minimum of three continuous spans in each direction.
- The successive span lengths in each direction shall not differ by more than one-third of the longer span.
- Panels shall be rectangular with the ratio of longer span to the shorter span not greater than two.

- Columns shall not be offset or staggered more than 10% of the span in the direction of the offset.
- The design live load shall not exceed three times the design dead load.

4. MODELING OF SLAB

The slab is modeled by using the Equivalent frame method. The Equivalent frame method is a more comprehensive method that can be used for slab system of unequal span and does not meet the limitation of direct design method. This method consists of approximating the actual three dimensional slab-and-column system to two-dimensional frames on column centerline in both, longitudinal and transverse directions. Thus each planer frame consists of a row of columns with connecting slab-beam strips at each floor having width equal to the distance between the panel centerline. In this method a system of wide beams or equivalent frames is used to determine the mid panel deflection (Figure 1).

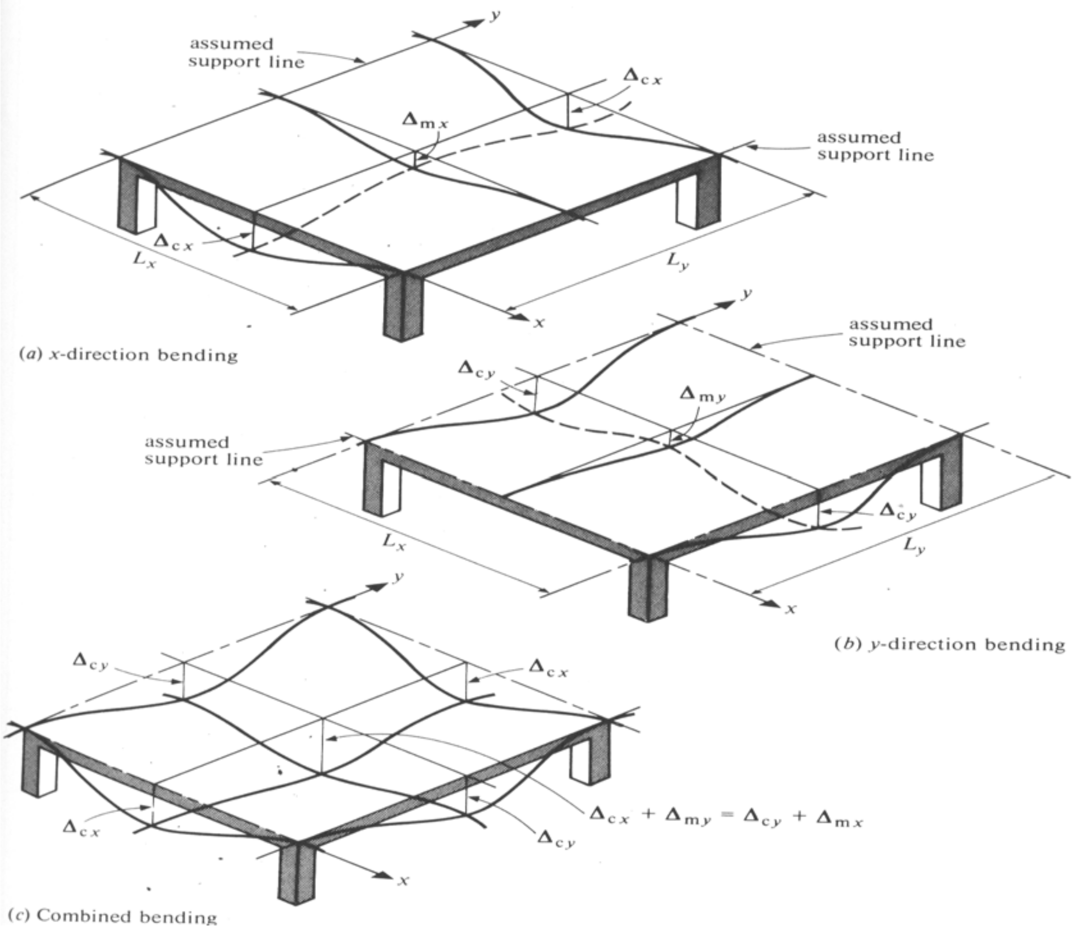

Figure 1. Midpanel deflection of two-way slab (Branson 1977, p. 289)

Hence in this paper for two-way beamless slabs, the deflection of mid panel for shrinkage and creep is calculated by using the Equivalent frame method. Also the percentage contribution of creep and shrinkage in the total deflection is calculated. For deflection due to shrinkage and creep, the relevant provisions given in the respective codes for beams are suitably modified and used. A program in MATLAB is developed and based on the parametric study the influence of parameters is identified. Parameters considered are the total thickness of slab, clear cover, grade of concrete. The following are the basic equations, which are used for calculating deflection.

Basic mid-span deflection of the panel, (Eq. 1) assumed as fixed at both ends given by

$$\delta = \frac{wl^4}{384 E_c I} \tag{1}$$

This has to be proportioned to separate deflection (Branson 1977, p.293) of the column strip and middle strip, such that;

- The midspans deflection of Flat plate (Eq. 2) is determined by the formula

$$(\delta) = \delta_{cx} + \delta_{my} = \delta_{mx} + \delta_{cy} \tag{2}$$

Where δ_{cx}, δ_{cy} = deflection of column strip along x and y direction respectively.
δ_{mx}, δ_{my}= deflection of middle strip along x and y direction respectively.

5. INSTANTANEOUS DEFLECTION

Short term or immediate deflection is due to permanent load and live load under service conditions. The BS 8110-1997, IS 456-2000 and ACI 318-2000 codes uses the same formula by considering the curvature and support conditions. The Young's modulus of concrete and the moment of inertia are to be used in uncracked stage.

$$\delta = \frac{wl^4}{384EI} \tag{3}$$

6. DEFLECTION DUE TO SHRINKAGE

Concrete shrinks when it dries and hardens. The shrinkage deflection (Eqs. 4,6) is due to reduction in the moisture content. Shrinkage deflection depends on the shrinkage curvature. In the Codes, shrinkage strains are given and the deflection is based on this strain and support condition.

- **BS 8110-1997**

$$Curvature3 = \frac{\varepsilon_{cs} * \alpha_e * \delta_s}{I_x} \tag{4}$$

ε_{cs}=free shrinkage strain=$300*10^{-6}$ (in the absence of experimental evidence)

- **IS 456-2000**

$$\delta_{cs} = k_3 * \psi_{cs} * l^2 \tag{5}$$

$$\psi_{cs} = \frac{k_4 * \varepsilon_{cs}}{D} \tag{6}$$

Where k_3, k_4 is a constant based on support conditions and percentage of reinforcement respectively.

ε_{cs} - Ultimate shrinkage strain=0.003 (in the absence of test data)

7. DEFLECTION DUE TO CREEP

Creep deflection is the deformation due to sustained load. The strain due to creep is much larger than the elastic strain. When the load is removed, the strain due to creep will not be recovered completely. Creep may be expressed by a creep coefficient or by value of strain per unit of concrete stress. In this paper, deflection due to creep (Eqs. 7,8) is calculated by creep coefficient method.

- **BS 8110-1997**

$$Curvature4 = \frac{M_{r2}}{E_{eff} * I_x} \tag{7}$$

where E_{eff} = modified young’s modulus of concrete

θ=3.1 (creep coefficient for 28 days age at loading)
=1.6 (creep coefficient for one year age at loading)

- **IS 456-2000**

$$\delta_{i,cc(perm)} = \frac{wl^4}{384 E_{ce} I_{eff}} \tag{8}$$

where E_{ce}= modified young's modulus of concrete=$\frac{E_c}{1+\theta}$

θ=1.6 (creep coefficient for 28 days age at loading)

=1.1 (creep coefficient for one year age at loading)

The young's modulus of slab is calculated for the cracked stage.

8. TOTAL DEFLECTION

8.1 BS 8110-1997

The total deflection is calculated (Macjinley and Choo 1990) based on the equation (Eq. 12). The Curvature1 and curvature2 are the curvatures due to permanent loads and temporary loads, which gives the effect due to instantaneous effects. The curvature3 gives the value for shrinkage effects and the curvature4 gives the value for creep effects. The total curvature is calculated using the Equation (Eq. 10).

$$\text{Curvature}=\text{Curvature1}-\text{Curvature2}+\text{Curvature3}+\text{Curvature4} \tag{9}$$

$$\text{Deflection}= \text{constant}*\text{curvature}*l^2 \tag{10}$$

8.2 IS 456-2000

The following equation (Eq. 11) is used for total deflection including short-term effects and long-term effects. The creep and shrinkage effects are accounted in the formulae by modifying the young's modulus and moment of inertia in the cracked stage.

$$\delta_T = \delta_{i,cc(perm)} + \delta_{cs} + \delta_{i(temp)} \tag{11}$$

8.3 ACI 318-2000

The total deflection is calculated using the following formula, (Eq. 12) which is calculating the deflection of two-way slab for the creep and shrinkage effect as three times the deflection due to dead load[5] and deflection due to live load.

$$\delta_{tot} = 3*\delta_{DL} + \delta_{LL} \tag{12}$$

By using the three codes, all the values required are assumed suitably and a numerical example is solved for the parameters given at Appendix II, to calculate the total deflection of flat plate by varying the parameters like total thickness, characteristic compressive strength of concrete, clear cover to reinforcement, creep coefficient by using the Equations given in the three codes BS 8110-1997 (Eq. 9), IS 456-2000 (Eq. 12) and ACI 318-2000 (Branson approach 1977, p.293)

The total deflection is calculated for the square interior panel of size 6m span and for live load of 3KN/mm^2 by considering the following parameters as the variables for all the three codes (BS 8110-1997, IS 456-2000 and ACI 318-2000) and the Figures 2 to 4 are plotted.

- Total thickness (minimum 125mm)
- Concrete strength is assumed as 20 N/mm^2.
- Clear cover as 20mm, 25mm and 30mm.
- Creep coefficient for 28 days age at loading

3.1- BS 8110-1997

1.6- IS 456-2000

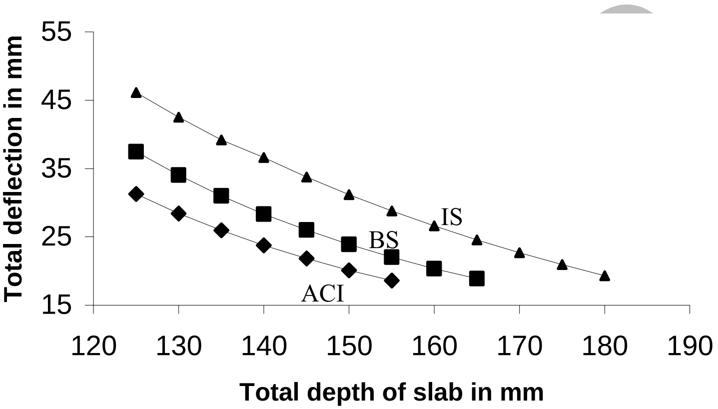

Figure 2. Total Deflection in mm fck=20 N/sq.mm and cc=20mm

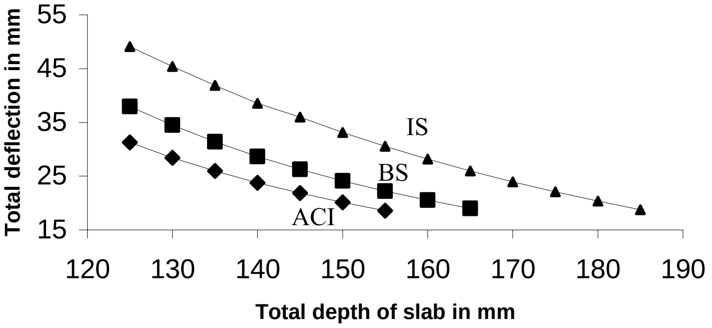

Figure 3. Toatl Deflection in mm fck=20 N/sq.mm and cc=25mm

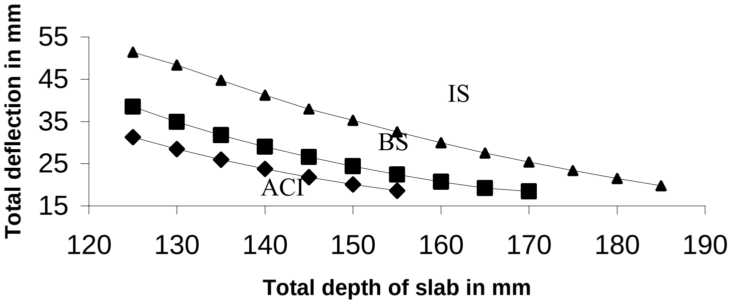

Figure 4. Total Deflection in mm fck=20 N/sq. mm and cc=30mm

As per BS 8110-1997, the curvature is calculated for the long-term deflection based on the neutral axis depth and hence the clear cover has its role in deflection calculation. The increase in the clear cover by 5mm increases the deflection of member by 0.6%. Here also the increase in total thickness by 5mm reduces the deflection of the member to minimum 7.3%. The total thickness required as per the calculation is 165mm.

Since the Branson approach (1977) is used for ACI 318 –2000, the deflection is the same for all the clear covers. But the increase in total thickness by 5mm has significant impact in reducing the deflection of the member to minimum 7.5%. The total thickness required as per the calculation is 155mm.

It is observed that, the deflection based on IS 456-2000, is very much conservative, because the total thickness required for the same span is 180mm when compared to 155mm for ACI 318-2000 and 165mm for BS 8110-1997. Since the formula uses the neutral axis depth for calculating the long-term deflection, the increase in clear cover by 5mm has an impact in increasing the total deflection by minimum 7.5%. The increase in total thickness reduces the total deflection by minimum 6%.

The parameters considered for calculating the total deflection to plot the Figures 5, 6 and 7 are the same that are used to plot the Figures 2 to 4, except that the concrete strength is assumed as 25 N/mm^2. The total deflection is calculated by the same method as for the Figures 2 to 4 and the Figure 5, 6 and 7 are plotted and as follows:

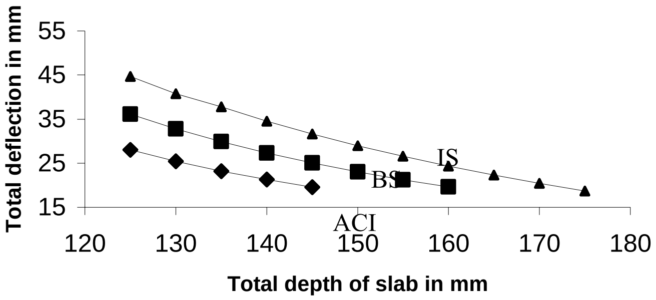

Figure 5. Total Deflection in mm fck=25 N/sq. mm and cc=20mm

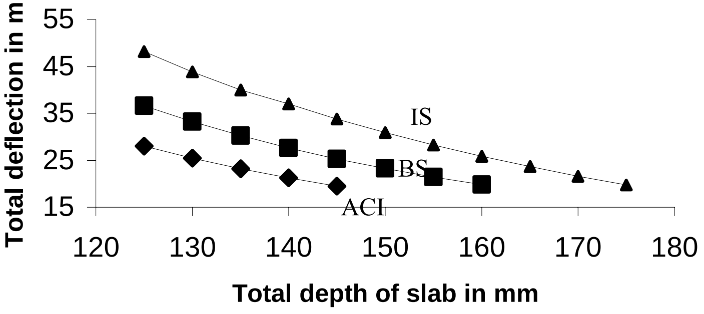

Figure 6. Total Deflectin in mm fck=25 N/sq. mm and cc=25mm

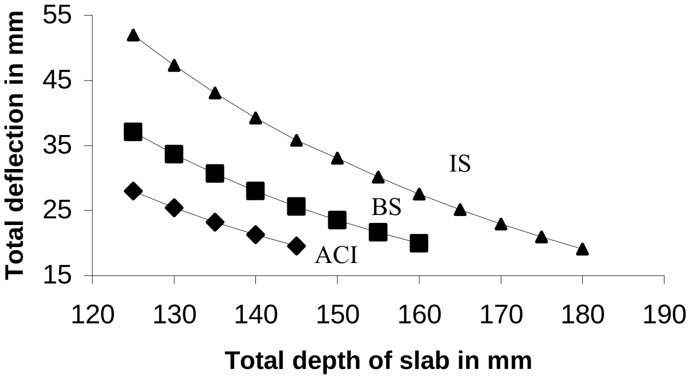

Figure 7. Total Deflection in mm fck=25N/sq. mm and cc=30mm

It is observed that the deflection decreases, if the strength of concrete is higher. For the same span, by taking the other parameters constant, only with increase in the strength of concrete, reduces the total deflection resulting in requirement of total thickness of 165mm for BS 8110-1997, 145 mm for ACI 318-2000 and 175 mm for IS 456-2000 as against 165 mm for BS 8110-1997, 180 mm for IS 456-2000 and 155 mm for ACI 318-2000.

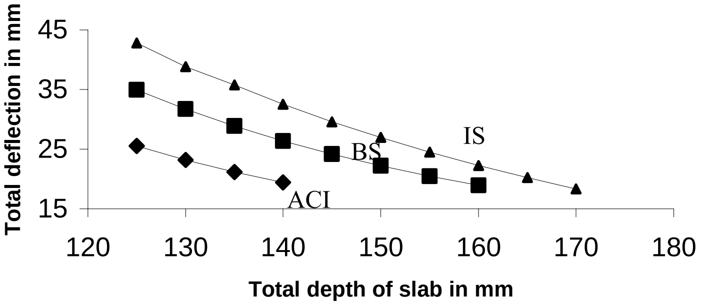

Figure 8. Total Deflection in mm fck=30 N/sq. mm and cc=20mm

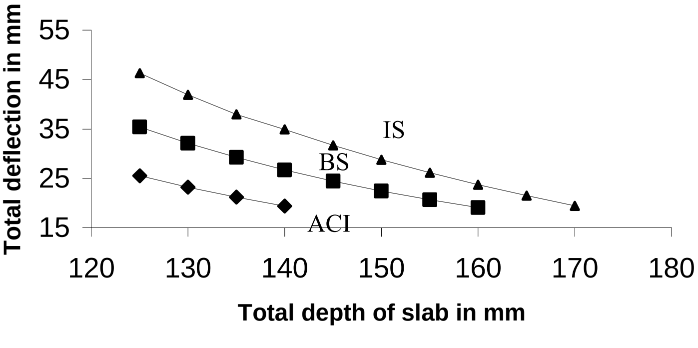

Figure 9. Total Deflection in mm fck=30 N/sq. mm and cc=25 mm

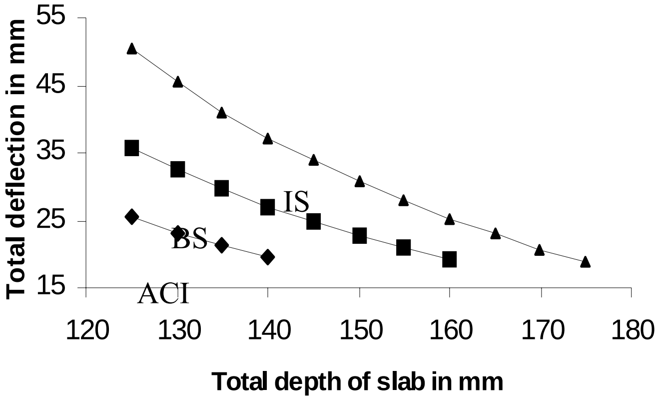

Figure 10. Total Deflection in mm fck=30 N/sq.mm and cc=30mm

Similarly the Figures 8 to 10 are plotted for the same parameters as that of Figure; 5 to 7 except that the concrete strength is assumed as 30 N/mm^2. When compared to the other two grades, this grade gives much reduction in deflection and hence the total thickness required is 160mm for BS 8110-1997, 170mm for IS 456-2000 and 140mm for ACI 318-2000.

Similarly the Figures 11 to 19 are plotted in the same above procedure (Figures 2 to 10) with all the same parameters except the creep coefficient value. The 28 days creep coefficient value is used for the Figures 2 to 10 and one-year creep coefficient value is used for the following Figures 11 to 19.

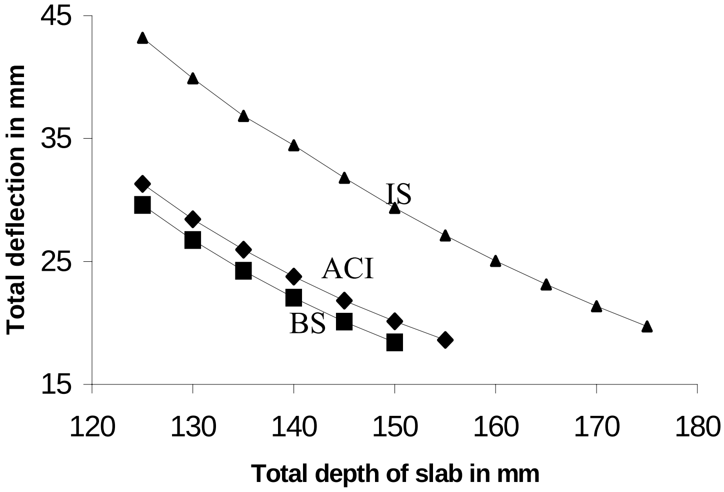

Figure 11. Total Deflection in mm fck=20 N/sq.mm and cc=20mm

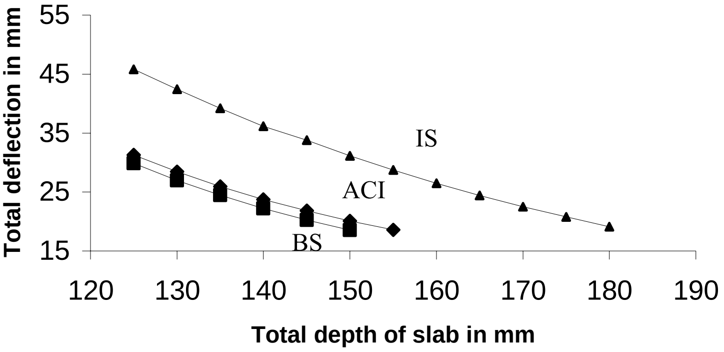

Figure 12. Total Deflection in mm fck=20 N/sq.mm and cc=25mm

It is observed that when compared to the deflection of flat plate for the 28 days age at loading which are plotted in Figures 2 to 4 and the deflection of the flat plate for the one year age at loading (Figures 11 to 13) are different for the three codes.

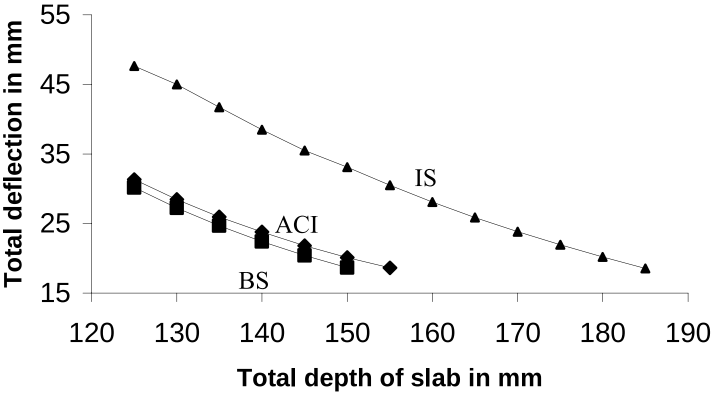

Figure 13. Total Deflection in mm fck=20 N/sq.mm and cc=30mm

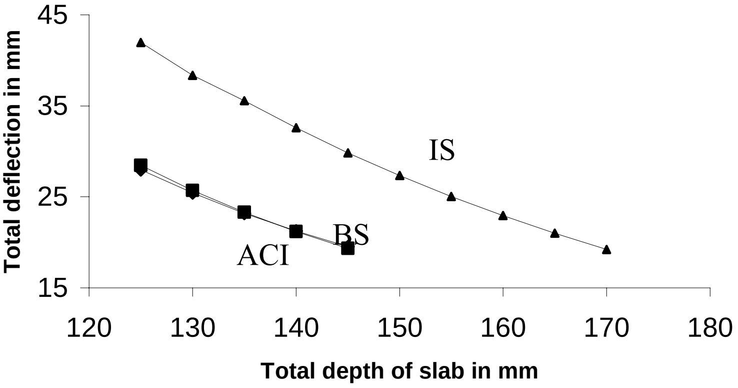

Figure 14. Total Deflection in mm fck=25 N/sq.mm and cc=20mm

In BS 8110-1997, the deflection at the one-year age at loading is 25% less when compared to the deflection at the 28 days age at loading. Hence the maximum deflection occurs at the 28 days loading at concrete. The total thickness required is observed to be 150mm for one year loading as against 165mm for 28 days age at loading of concrete.

For the ACI 318-2000, since Branson approach is used, the creep at different age at loading does not have any effect and hence the deflection is the same in both the cases as plotted in Figures 2 to 4 and Figures 11 to 13.

As per IS 456-2000, the deflection at one year loading is 6% less compared to 28 days loading. The total thickness required is observed to be 5mm less in case of clear cover for 25mm and 30mm.

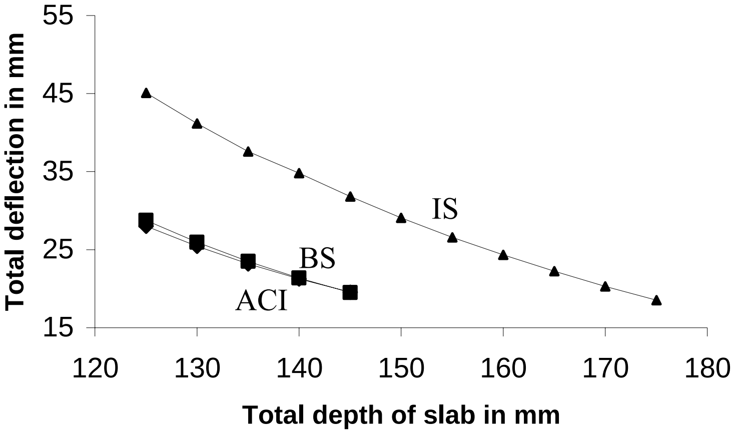

Figure 15. Total Deflection in mm fck=25 N/sq.mm and cc=25mm

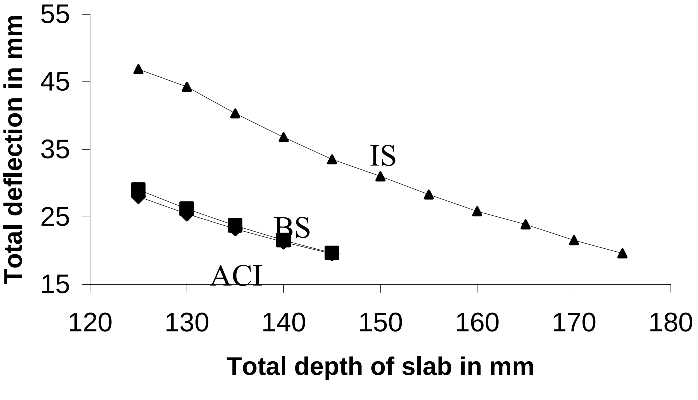

Figure 16. Total Deflection in mm fck=25 N/sq.mm and cc=30mm

The same difference is observed for the Figures 14 to 16 with Figures 5 to 7 and Figures 17 to 19 and Figures 8 to 10.

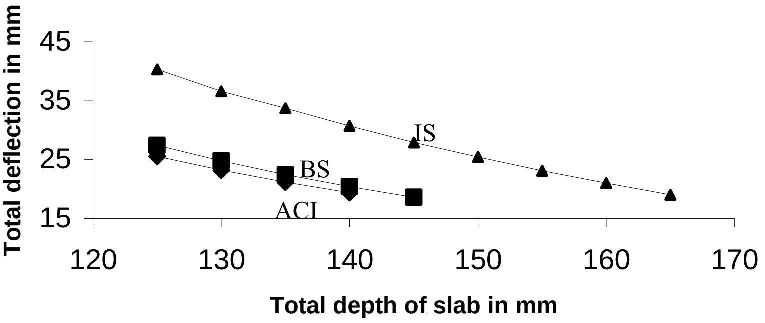

Figure 17. Total Deflection in mm fck=30 N/sq.mm and cc=20mm

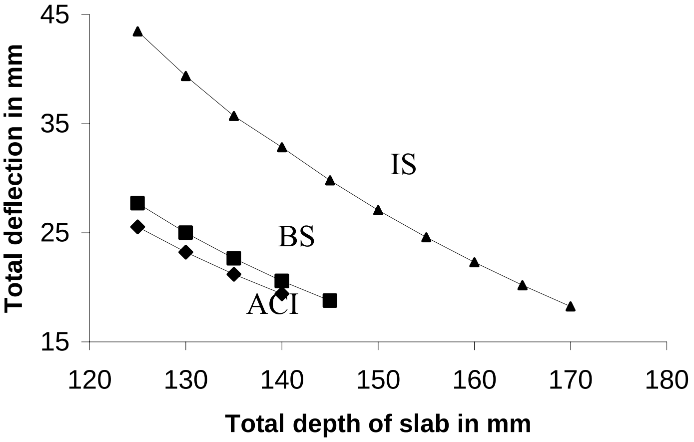

Figure 18. Total Deflection in mm fck=30 N/sq.mm and cc=25mm

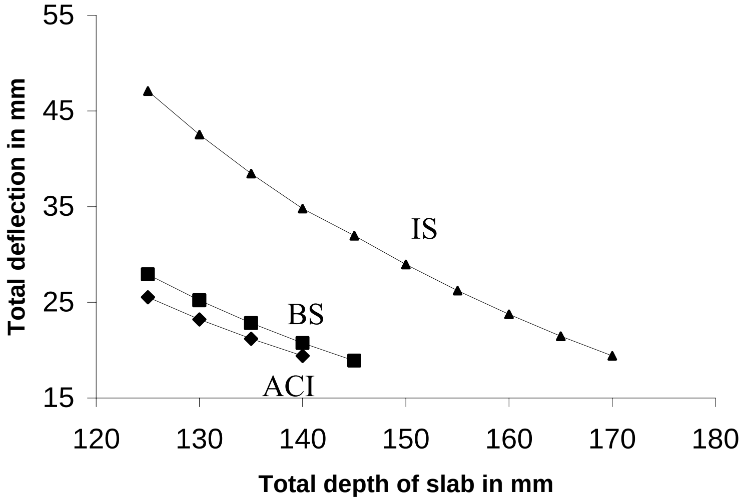

Figure 19. Total Deflection in mm fck=30 N/sq.mm and cc=30mm

From the Figures, it is observed that higher the strength of concrete leads to the reduction in the deflection of the slab. The increase in clear cover by 5mm increases the deflection. Also it is observed that the maximum deflection occurs within the 28 days age at loading as per BS and IS standards. The total thickness has much influence in reducing the deflection of the member.

9. CONCLUSIONS

The procedure given in the codes of BS 8110-1997, ACI 318-2000, and IS 456-2000 has the variation exist for the deflection calculation for beams and slabs. Also the above codes give the provision to calculate the long-term deflection only for beams. In beamless floors such as flat plates, the time dependent deflection is more and hence, to have adequate safety against serviceability for the deflection calculation, it is essential to include the shrinkage effects and creep effects. Hence in this paper RCC slab is modeled by using the Equivalent Frame approach and an attempt is made to determine the exact deflection of flat plate by considering the short-term effects, creep effects and shrinkage effects by suitably modifying the available provisions for beams.

A numerical example is solved to determine the total deflection by varying the parameters like total thickness, characteristic compressive strength of concrete, clear cover of reinforcement and creep coefficient. From the numerical example, it is observed that the above have significant influence in the reduction of deflection. This paper reveals the difference in the codal provisions for determining the deflection and hence it is essential to have a common model among the codes, by including all the parameters.

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APPENDIX I

Example

The Flat Plate of an Apartment building with the following data has demonstrated to calculate deflection by the exact method by using three code methods.

Span	longitudinal 6.0m
	transverse 6.0m
Concrete	Grade 20,25 and 30 (cube)
Reinforcement	f_y=415 N/mm^2
	Diameter =10mm
	Clear Cover=20mm, 25mm, 30mm
	Young's Modulus=200000 N/mm^2
Loading	Imposed = 3 KN/mm^2
	Floor finish=1.5 KN/mm^2
Creep coefficient	θ=1.6 &1.1 for IS 456-2000 method

θ=3.1 &1.6 for BS 8110-1997 method

The total deflection is determined using the above data. The calculations are not shown here.

Notations

E = Young's modulus

$E_c = 5700\sqrt{f_{ck}}$

x = depth of neutral axis;

$f_{cr} = 0.7*\sqrt{f_{ck}}$ Modulus of rupture of concrete;

I_{gr} = Moment of inertia of gross section;

Y_t =distance from centroidal axis of gross-section to extreme fibre in tension;

M = Maximum moment under service loads;

ε_{cs} =free shrinkage strain=$300*10^{-6}$;

$\alpha_e = E_s/E_{eff}$; E_s=Young's modulus of steel;