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EFFECTS OF SEMI-RIGID GIRDER CONNECTIONS ON THE DYNAMIC RESPONSE OF STEEL STRUCTURES

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ABSTRACT

The nonlinear response of a ten story steel structure with semi-rigid girder connections is studied under conditions of dynamic loading. The dynamic loading used in this study is the north-south component of the may 18, 1940 El Centro, California earthquake. To deal with the complexity of the problem the structure is idealized by a series of equivalent masses, lumped at the floor levels and restrained by weightless members. The physical model used to represent individual members consists of a flexible central beam with springs attached at both ends. All connections have the capability of exhibiting bilinear hysteresis curves.

The analysis is accomplished within the general purpose computer program SAP 2000 V 10. Semi-rigid girder connections affect the properties of a structure in three ways: (a) by altering the relative girder to column stiffness, (b) by changing the strength or yield deformation characteristics, and (c) by decreasing the stiffness of the structure. The effects that these variables have on structural response are determined. The ground motion characteristics, intensity, and duration are also investigated.

Keywords: Semi-rigid, fixity factor, nonlinear response, damping, stiffness, ductility

1. INTRODUCTION

Hechtman and Johnston [1] in a progress report recommend that a dependable percentage of end restraint can be used in design for several types of semi-rigid connections. However, before high speed computers became available the analysis of structures with semi-rigid connections was difficult and time consuming. Then analyses of such structures for static loading became feasible through the use of matrix methods and high speed computer [2]. Little is known, however, as how semi-rigid connections affect the dynamic response of structures.

The object of this paper is a computer investigation of the nonlinear response of steel structures with semi-rigid connections subjected to seismic loading. In particular, the investigation is to consider the effects of semi-rigid connections on structural deflections and natural frequency by comparing the response of structures with connections of varying rigidity.

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To deal with the complexity of the problem, certain assumptions have been introduced. The structure is idealized by a series of equivalent masses, lumped at the floor levels and restrained by weightless members. Connections are simulated by inserting springs at both ends of each member. The spring stiffness is varied from zero, for a pin connection, to infinity, for a rigid connection, through the use of fixity factors. All connections have the capability of exhibiting bilinear hysteresis curves. The seismic loading used in this investigation is the north-south component of the may 18, 1940 El Centro, California earthquake accelerogram.

From classical analyses such as the elastic response spectrum it became obvious that inelastic action was taking place in structures for even moderate earthquake forces, and to build structures to exhibit only elastic deformation is not economically practical. Further studies in this area led to the conclusion that the maximum deflections based on elastic considerations can be quite different from the results based on nonlinear considerations. Therefore it became necessary to consider nonlinear effects for structures in order to prevent limits on the height of structures that can be built in zones of high earthquake forces. Rayleigh damping is supplied as a percent of critical damping of either the mass or the stiffness matrix. Static loads are also considered. The physical model used to represent individual members consists of a flexible beam with springs attached at both ends. This model is particularly effective in calculating plastic deformation for either static or dynamic loading.

The analysis is accomplished by means of the general purpose computer program SAP 2000 V 10. Information on static tests of semi-rigid connections were obtained from reports by J. F. Baker [3], Pippard and Baker [4], Munse,Bell and Cheson [5], Batho [6], Hechtman and Johnston [1] and many others. A complete listing of publications concerning semi-rigid connections is given by Gere [8]. An extensive review of literature on research work in earthquake engineering is given by E. Rosenblueth [9].

2. METHOD OF ANALYSIS

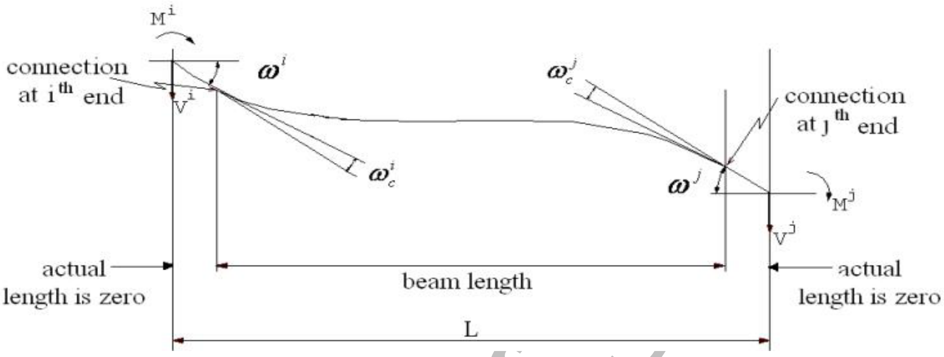
2.1 Individual member stiffness matrix

The stiffness of structural elements is assumed linear during all the intervals. Hence, the general relationship between member-end moments and member-end rotation apply whether the structure is elastic or partially elastic-partially plastic. The difference between the two states is the connection rigidities that are used in computing deformations. Neglecting axial deformations, the force-displacement relationship is shown in Figure 1. The force-displacement relationship is given in matrix form as follows:

$$\{q\} = [k]\ \{d\} \tag{1}$$

Where $\{q\}$ –member force vector
$\{d\}$ –member displacement vector
$[k]$ –member stiffness matrix

(a) Sign convention for member displacements. Directions as shown are considered positive



(b) Force and displacement quantities for semi-rigidly connected members

Figure 1. Member force and displacement quantities and sign convention.

Table 1. List of symbols used in Fig. 1 and associated developments.

M^i, M^j	Bending moment at i and j ends of a member.
V^i, V^j	Shear at i and j ends of a member.
η^i, η^j	Displacement at i and j ends of a member.
ω^i, ω^j	Rotation at i and j ends of a member.
ω_c^i, ω_c^j	Total rotation of the connection at i and j ends of a member.
$\omega_{ce}^i, \omega_{ce}^j$	Elastic rotation of the connection at i and j ends of a member.
$\omega_{cn}^i, \omega_{cn}^j$	Plastic rotation of the connection at i and j ends of a member.
K_c^i, K_c^j	Stiffness of the connection at i and j ends of a member.
$M_{y_0}^i, M_{y_0}^j$	Bending moment at i and j ends of a member at which plastic deformation begins
$\theta_{y_0}^i, \theta_{y_0}^j$	Rotation at i and j ends of a member at which plastic deformation begins

These matrices are of the form:

$$\{q\} = \begin{bmatrix} M^i \\ M^j \\ V^i \\ V^j \end{bmatrix} \{d\} = \begin{bmatrix} \omega^i \\ \omega^j \\ \eta^i \\ \eta^j \end{bmatrix} \tag{2}$$

$$[K] = \frac{2EI}{L} \begin{bmatrix} 2\gamma_{11} & \gamma_{12} & \frac{2\gamma_{11}+\gamma_{12}}{L} & -\frac{2\gamma_{11}+\gamma_{12}}{L} \\ \gamma_{12} & 2\gamma_{22} & \frac{2\gamma_{22}+\gamma_{12}}{L} & -\frac{2\gamma_{22}+\gamma_{12}}{L} \\ \frac{2\gamma_{11}+\gamma_{12}}{L} & \frac{2\gamma_{22}+\gamma_{12}}{L} & \frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} & -\frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} \\ -\frac{2\gamma_{11}+\gamma_{12}}{L} & -\frac{2\gamma_{22}+\gamma_{12}}{L} & -\frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} & \frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} \end{bmatrix} \tag{3}$$

Where E: modulus of elasticity of the member
I : moment of inertia of the member
L: length of the member

$$\gamma_{11} = \left(\frac{3\phi^i}{4-\phi^i\phi^j}\right), \gamma_{12} = \left(\frac{3\phi^i\phi^j}{4-\phi^i\phi^j}\right), \gamma_{22} = \left(\frac{3\phi^j}{4-\phi^i\phi^j}\right) \tag{4}$$

ϕ^i, ϕ^j are the connection fixity factors for the I and j ends of the member respectively.

The connection fixity factors are used to express the stiffness of the connection K_c as a function of the stiffness of the beam K_m in the form:

$$K_c = P\,K_m \tag{5}$$

Where

$$K_m = \frac{4EI}{L} \tag{6}$$

and

$$P = \frac{3}{4}\left(\frac{\phi}{1-\phi}\right) \tag{7}$$

The value of ϕ varies from zero for a pinned connection to 1.0 for a rigid connection. Using the Eq. (5), the connection stiffness can be related to the fixity factor used in the stiffness matrix. Using the above representation for semi-rigid connections yields results very similar to those obtained by Giberson [10]. Giberson introduces springs into the member as it commences plastic deformation. The springs, initially rigid, are capable of exhibiting a curvilinear hysteresis loop. The stiffness of the spring K_s used by Giberson is related to the beam stiffness K_m as in the form:

$$K_s = f\,K_m \quad \text{or} \quad f = P$$

The two methods will yield identical results for structural members with connections that are initially rigid and that exhibit the same hysteresis curves. The approach investigated herein leads itself to modification for use with curvilinear hysteresis loops. The values of the stiffness of the spring used in this investigation are taken from the work of Elnashai [13] and are given in Table 2.

Table 2. Values of K_s

Fixity Factor	K_s **(kNm/rad)**
0.8	7.6E3
0.6	5.5E3
0.4	3.8E3

2.2 Ductility factor

Failure of a member is closely associated with the nonlinear displacement that takes place during plastic deformation. A ductility factor, defined such that it becomes a measure of this nonlinear yielding, is defined as:

$$\mu_n = \begin{cases} M/M_0 & M \le M_y \\ 1 + \dfrac{\omega_{cn}}{\theta y_0} & M > M_y \end{cases} \tag{8}$$

Where μ_n is the ductility factor which defines nonlinear or permanent rotational deformation of a connection.

Because no elastic deformation of the connections takes place for structures with rigid connections, the definition given in Eq. (8) also defines the total connection deformation. This is not the case when semi-rigid connections are considered. To determine the total rotational deformation that a connection undergoes, a second definition of ductility factor is given and calculated in this investigation as:

$$\mu_t = 1 + \frac{\omega_c}{\theta y_0} \tag{9}$$

Where μ_t is a ductility factor which defines the total rotational deformation of a connection.

For members with rigid connections μ_t equals μ_n for $M \geq M_y$

The expression for initial yield rotation θy_0 is given as

$$\theta y_0 = \frac{My_0 L}{6EI} \frac{4 - \phi_e^i \phi_e^j}{\phi_e^i \left(2 + \phi_e^j\right)} \tag{10}$$

Hence, the ductility factors are calculated in terms of connection moment rather than connection rotations. This is first done for μ_n as follows:

-Determine the nonlinear connection deformation ω_{cn}, with $M > M_y$,by means of the following equation:

$$\omega_{cn} = \omega_c - \omega_{ce} = \frac{M - M_y}{P_n K_m} - \frac{M - M_y}{P_e K_m} \tag{11}$$

This is rewritten in terms of fixity factors as:

$$\omega_{cn} = \frac{4}{3}\left(\frac{\phi_e^i - \phi_n^j}{\phi_e^i . \phi_n^i}\right)\frac{M - M_y}{K_m} \tag{12}$$

The ductility factor μ_n can then calculated from Eq. (8) as

$$\mu_n = \begin{cases} M / My_0 & M \leq M_y \\ 1 + 2\left(\dfrac{2 + \phi_e^j}{4 - \phi_e^i \phi_e^j}\right)\left(\dfrac{\phi_e^i - \phi_n^i}{\phi_n^i}\right)\left(\dfrac{M - M_y}{My_0}\right) & M > M_y \end{cases} \tag{13}$$

The ductility factor μ_t given in Eq. (9) is next determined as follows:

-Evaluate the total connection deformation ω as

$$\omega_c = \begin{cases} \dfrac{M}{P_e K_m} & M \leq M_y \\ \dfrac{M_y}{P_e K_m} + \dfrac{M - M_y}{P_n K_m} & M > M_y \end{cases} \tag{14}$$

The ductility factor μ_t given in Eq. (9) is then calculated in terms of fixity factors as:

$$\mu_t = \begin{cases} 1+2\left(1-\phi_e^i\right)\left(\dfrac{2+\phi_e^j}{4-\phi_e^i\phi_e^j}\right)\dfrac{M}{My_0} & M \le M_y \\ 1+2\left(\dfrac{2+\phi_e^j}{4-\phi_e^i\phi_e^j}\right)\left[\left(1-\phi_e^i\right)+\dfrac{\phi_e^i}{\phi_e^j}\left(1-\phi_n^i\right)\left(\dfrac{M-M_y}{My_0}\right)\right] & M > M_y \end{cases} \tag{15}$$

Therefore, two ductility factors are calculated μ_n and μ_t . With the ductility thus defined both the nonlinear deformation and the total connection deformation can be determined.

2.3 Damping

For this investigation damping was assumed to be Rayleigh damping which is composed of both stiffness proportional viscous damping and mass proportional viscous damping as given by the following equation:

$$[C]=\alpha[M]+\beta[K] \tag{16}$$

Where [C] - The damping matrix.
[M]- The mass matrix.
[K] - The system stiffness matrix.
α - a scalar quantity that indicates the fraction of mass used for damping.
β - a scalar quantity that indicates the fraction of stiffness used for damping.

The fraction of mass and stiffness conventionally used in damping is determined as some percent of critical damping in the fundamental mode. This percent of critical damping has been related to α and β by O'Kelly [12] as follows:

$$\xi_n^m = \frac{\alpha}{2\omega_n} \qquad n = 1,2,\ldots,N \tag{17}$$

and

$$\xi_n^s = \frac{\beta\omega_n}{2} \tag{18}$$

Where ξ_n^m is the percent of mass proportional critical damping in the nth mode,
ξ_n^s is the percent of stiffness proportional damping in the nth mode,
ω_n is the nth circular frequency.

For the fundamental mode with n = 1, α and β are

$$\alpha = 2\omega_1\xi_1^m \tag{19}$$

$$\beta = \frac{2\xi_1^s}{\omega_1} \tag{20}$$

Thus the damping matrix takes the following form:

$$[C] = 2\omega_1 \xi_1^m [M] + \frac{2\xi_1^s}{\omega_1}[K] \tag{21}$$

2.3 Earthquake accelerogram

The accelerogram used in this investigation is the north-south component of the may 18, 1940 El Centro earthquake. This earthquake is believed representative of strong earthquake in the western part of the United States and its accelerogram is the strongest yet recorded. Studies by Clough and Benuska [11] indicate that structural response depends primarily on the peak acceleration impulse in the ground motion and that continuing motions of smaller amplitude have only a small effect on the maximum response. Therefore the duration of the earthquake used in this analysis was primarily limited to the first ten seconds of the El Centro earthquake.

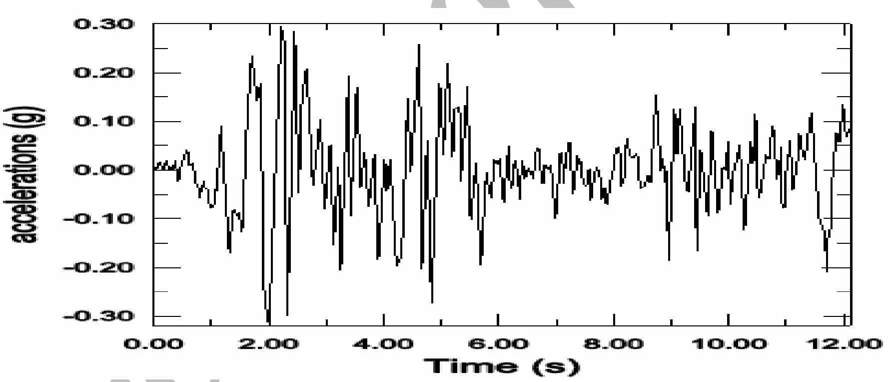


Figure 2. El Centro accelerogram (north-south component)

3. RESULTS AND DISCUSSIONS

3.1 Example used in the analysis

In order to evaluate the influence of semi-rigid connections on the dynamic response of structures, a ten story steel structure was designed in accordance with the European code EC 3[12]. The building was designed for vertical gravity loads plus static lateral forces. From the resulting internal member forces, relative member properties were obtained as shown in Figure 3.

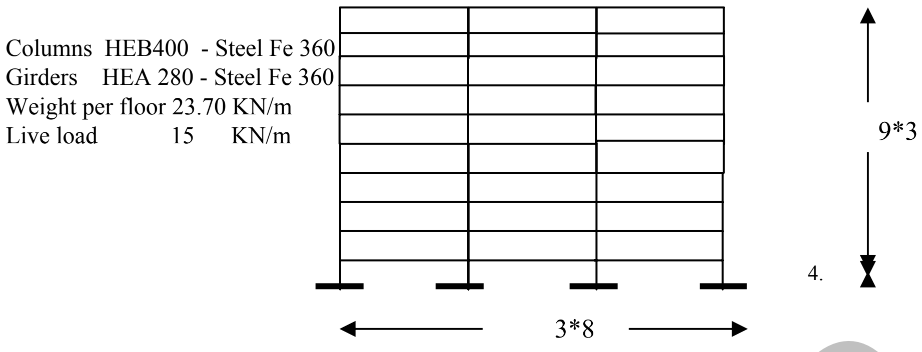


Figure 3. Building properties

3.2 Effect of semi-rigid girder connections on the fundamental period.

The effect of girder connection rigidity on the fundamental period was determined by varying the girder connection fixity factor from 1.0 to 0.4 and calculating the resulting period. The results are shown in Table 3. The range of typical girder connection fixity factors is also noted. It is seen that within this range of fixity factors the fundamental period can increase by as much as 70 percent of that of rigid connections.

Table 3. Periods of vibration (T in seconds)

Girder connection fixity factors		**Rigid**	**Semi-rigid**		
		1.0	**0.8**	**0.6**	**0.4**
Periods of vibration	T1	3.4516	5.6378	6.1659	6.8059
	T2	1.0745	1.6108	1.7213	1.8554
	T3	0.5743	0.7618	0.7911	0.8229
	T4	0.3593	0.4328	0.4423	0.4520
	T5	0.2449	0.2767	0.2843	0.2840
	T6	0.1920	0.1935	0.1943	0.1959
	T7	0.1873	0.1927	0.1936	0.1938
	T8	0.1780	0.1921	0.1926	0.1930
	T9	0.1373	0.1435	0.1443	0.1450
	T10	0.1371	0.1383	0.1384	0.1385

3.3 Effect of semi-rigid girder connections on nonlinear response

Semi-rigid connections influence the dynamic response in the following three ways. By decreasing the girder connection fixity factors (a) the relative stiffness of the girders are reduced, (b) the strength or yield moments of the girder connections are reduced, and (c) the overall stiffness of the structure is reduced or the period of vibration is increased.

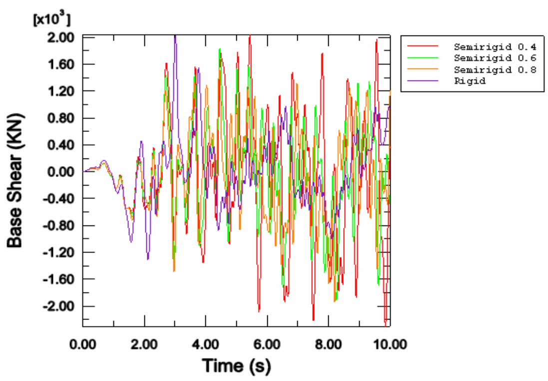


Figure 4. Linear base shear

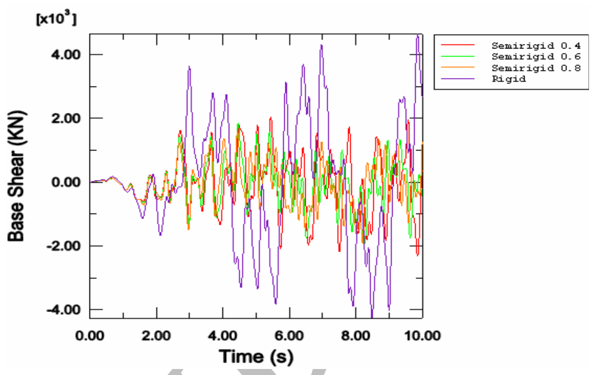


Figure 5. Nonlinear base shear

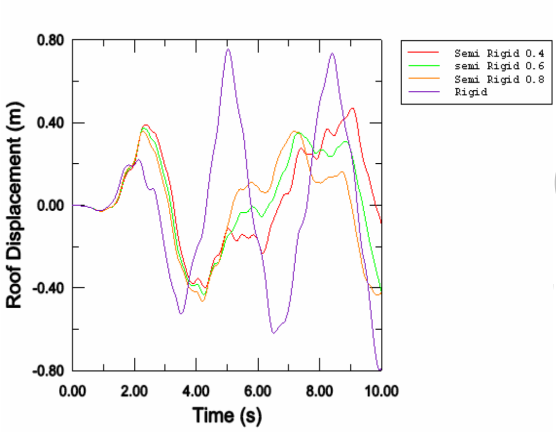


Figure 6. Linear roof displacement

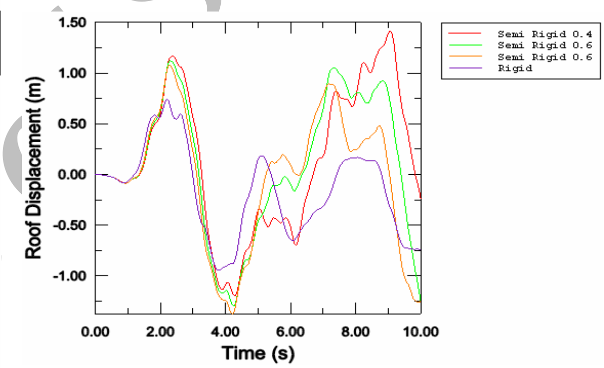


Figure 7. Nonlinear roof displacement

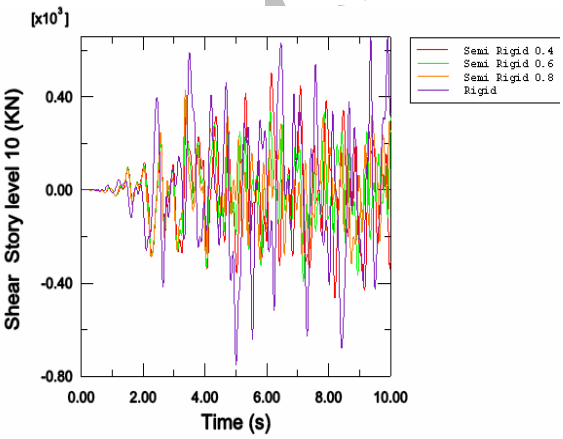


Figure 8. Linear shear story level 10

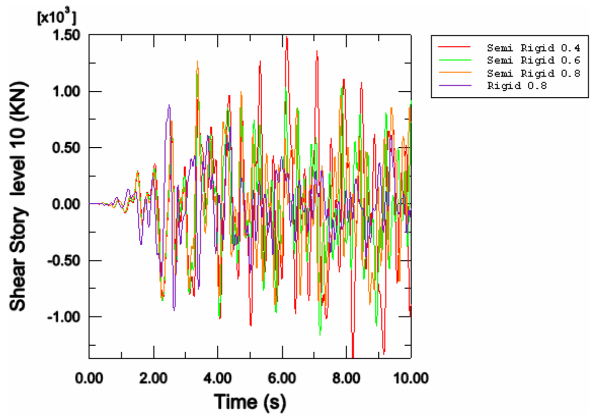


Figure 9. Nonlinear shear story level 10

3.4 Influence of intensity of ground motion

The ground motion record used in this investigation was the 1940 El Centro,California earthquake accelerogram. To obtain the influence of earthquake intensity, the earthquake acceleration was multiplied by a scale factor SF. Thus a modified earthquake accelerogram of intensity S was defined. The results of using the el Centro earthquake of intensity 1.0, 2.0 and 3.0 are shown in Figure 4 through 15.

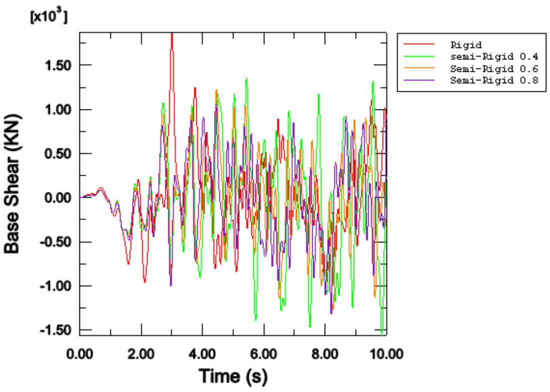


Figure 10. Base shear SF2

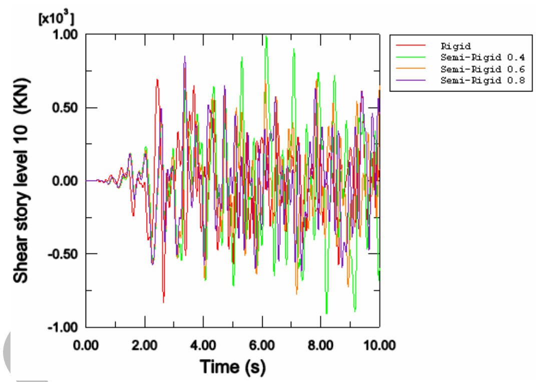


Figure 11. Shear story level 10 SF2

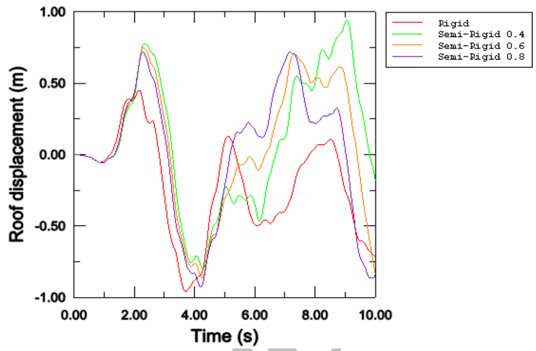


Figure 12. Roof displacement SF2

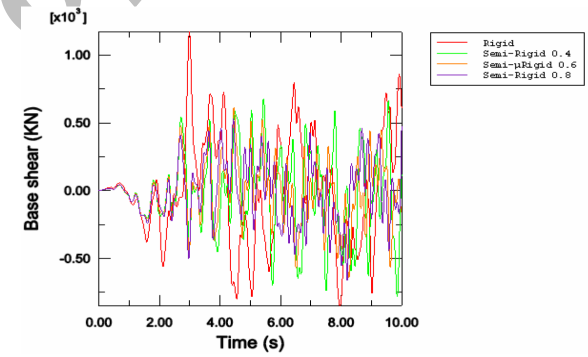


Figure 13. Base shear SF1

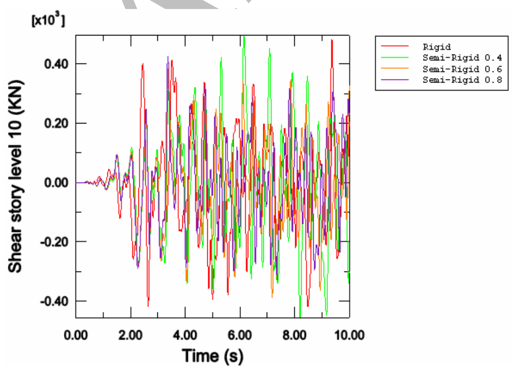


Figure 14. Shear story level 10 SF1

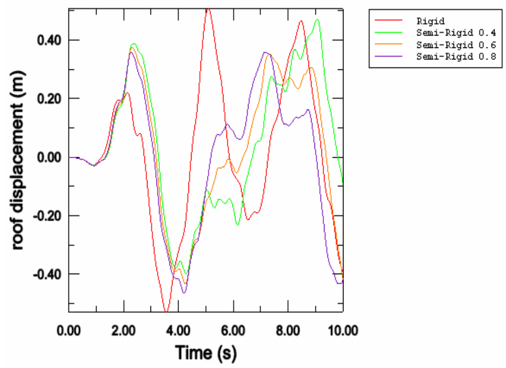


Figure 15. Roof displacement SF1

From Figures. 6-12 and 15 it is observed that the resulting ratio of maximum lateral floor deflection (roof displacement) is between 1.5 and 1.8. Likewise Figure 5, 10, and 13 show that the ratio of the maximum story shear (base shear for an earthquake scale of 1.0 to 3.0 varies form 1.0 to 1.35. Figures 9, 11 and 14 show that the ratio of the tenth story shear varies from 1.0 to 2.0.

From studying the results it is evident that this building is designed to adequately withstand the El Centro earthquake with a scale factor of 1.0. For this excitation only slight plastic rotation occurs in the girder connections. This is true for the structure with rigid semi-rigid connections.

Since this study was intended primarily as an investigation of the non linear response of structure under dynamic loading; it was concluded that a scale factor of 3.0 would be used in all of the tests to insure that the response would include nonlinear deformations.

3.5 Influence of duration of ground motion

To determine the influence of the duration of the ground motion, the maximum response values at the end of each second of earthquake time were given by the SAP program.

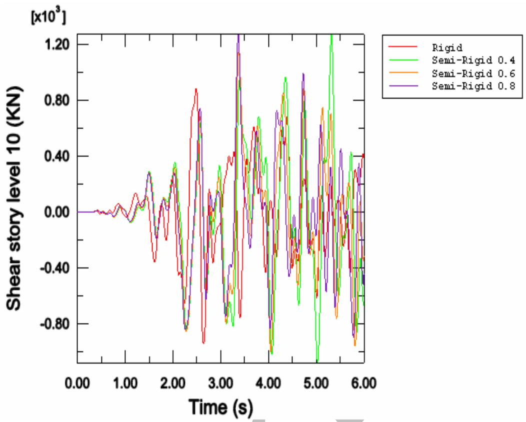


Figure 16. Shear story level 10 time duration 6 s

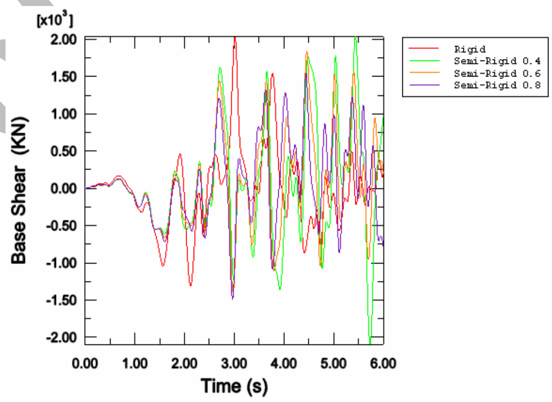


Figure 17. Base shear time duration 6 s

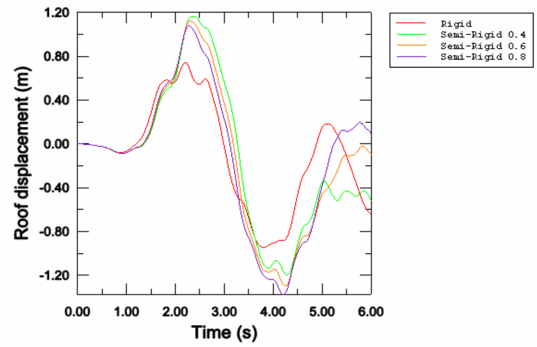


Figure 18. Roof displacement, time duration 6 s

The results at each second interval up to a maximum of 6 seconds are show in Figure 16, 17, and 18. Three separate tests, each carried out to a total of 6 seconds and considering both rigid and semi-rigid connections, produced results similar to those shown in Figures 6, 8 and 10. It was found that the time at which the maximum responses occurred decreased as the period of vibration increased. Clough and Benuska [11] found that the maximum structural response depends primarily on the peak acceleration impulse in the ground motion and it is not affected strongly by continuing motions of smaller amplitudes. Generally, this conclusion was borne out in this investigation. Thus it was concluded that the analyses would be limited to an earthquake of ten seconds.

3.6 Influence of girder connection fixity factor on structural response

The results show that the maximum lateral floor deflection and the maximum story shear are changed only slightly by using semi-rigid girder connections. Increasing the girder connection flexibility altered the stiffness relationship between girders and columns, with the girder becoming relatively more flexible. Therefore little variation would be expected in the maximum lateral deflection.

4. SUMMARY OF RESULTS AND CONCLUSIONS

Semi-rigid girder connections can affect the nonlinear response of a structure in the following three ways: (a) by reducing the relative stiffness of girders to columns with decreasing fixity factors, (b) by reducing the girder connection strength, or yield moments, (c) by reducing the overall stiffness of the structure or increasing the fundamental period of vibration. These three were isolated and investigated the independently. The results are summarized in Table 4. In column (1) the effect of reducing the relative stiffness of the girder to columns is shown. This is done by reducing the girder connection fixity factors while holding all other variables constant. In column (2) the effect of the reducing the strength of the girder connections is shown. This is accomplished by reducing the girder connection yield moments while holding all other variables constant. Column (3) shows the effect of reducing the overall stiffness of the structure or increasing the period of vibration while holding all other variables constant. Because of large axial forces in the lower columns of tall multistory buildings and the unknown requirements needed to prevent column instability, it is generally concluded that column ductility requirements must be minimized. It is of interest to note that reducing the girder connection yield moments will result in smaller maximum responses except for the girder nonlinear ductility factors. Thus, reducing the yield moments in semi-rigid girder connections may be an effective method of minimizing the ductility requirements of the columns of controlling the point of plastic deformation, and of partially controlling the maximum lateral deflections.

These results indicate that semi-rigid girder connections will have a significant effect on the structural response obtained, and that through a proper choice of the strength and stiffness properties of the connections, these responses may be altered to produce a beneficial effect on the structure.

Table 4. Summary of the effects of semi-rigid connections on structural responses

	Decreasing girder connection fixity factors (1)	**Decreasing girder connection yield moments (2)**	**Increasing period of vibration (3)**
Maximum lateral deflection	No change	Decrease	Increase
Maximum story shear	Slight increase	Decrease	Mixed
Maximum girder nonlinear ductility	Decrease	Increase	Decrease

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