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FORMEX FORMULATION OF FREEFORM STRUCTURAL SURFACES

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ABSTRACT

Structures for covering large public spaces are more and more designed with innovative shapes involving combinations and/or modified forms of classical simple shapes such as plane, cylinder, sphere, ... etc. These innovative shapes are termed "freeform" and are created through the interaction between practical considerations for the structures and the artistic sense of the designers. Formex algebra and its programming language Formian can be of help in generating the geometry of freeform structures. In particular, there are two formex concepts that are of value in this context. These are the concepts of "pellevation" and "novation". The purpose of the present paper is to discuss the manner in which pellevation and novation can be used for freeform generation. In the paper, to begin with, the concepts of pellevation and novation are explained in terms of some simple examples. The process of freeform generation is then illustrated using a number of examples.

Keywords: Freeform, configuration processing, formex algebra, formian

1. PELLEVATIONAL EFFECTS

Consider the plane configuration shown in Figure 1. This configuration may be transformed into a dome using a "pellevation function". The dome is shown in Figure 2. The formulation shown under the grid in Figure 1 is a "formex formulation" for the grid. To follow the general ideas in the present paper, it is not necessary for the reader to be an expert in "formex algebra". However, to take full advantage of these ideas for the practical creation and processing of configurations, one needs to have a reasonable understanding of formex algebra and its programming language "Formian". Basic information about formex algebra and Formian is found in Refs. [1-3].

A formex formulation for the dome of Figure 2 is shown under the dome. In this formulation:

- "capel" is the name of a pellevation function that creates a dome-like effect.
- The first parameter inside the parentheses following the function name, that is, integer 1,

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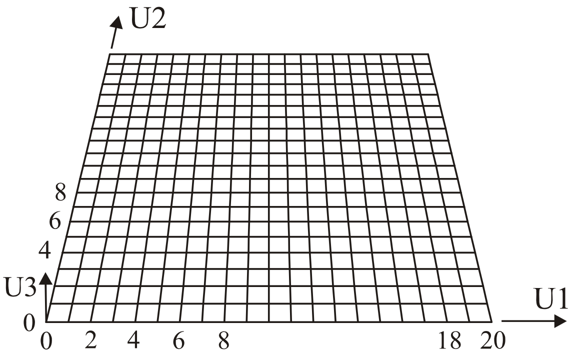
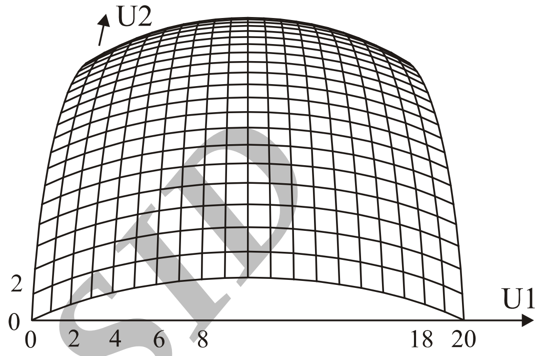
specifies a particular mode of the capel function that creates a "spherical cap" effect.

- The second and third parameters specify the U1 and U2 coordinates of the centre of the spherical cap.
- The fourth parameter (that is, 10√2), specifies the diameter (span) of the spherical cap.
- The last parameter (that is, 6), specifies the height (rise) of the spherical cap.



Grid=rinid(20,21,1,1)|[0,0,0; 1,0,0]#
rinid(21,20,1,1)|[0,0,0; 0,1,0]

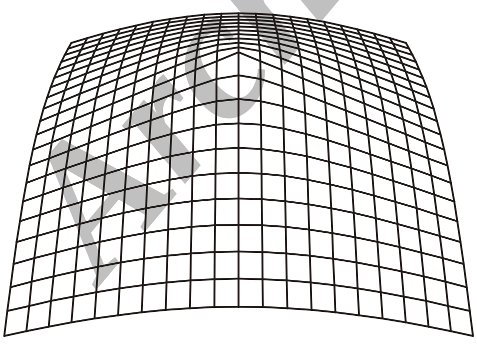
Figure 1. A plane grid in U1-U2-U3 coordinate system



Dome=capel(1,10,10,20√2,6)|Grid

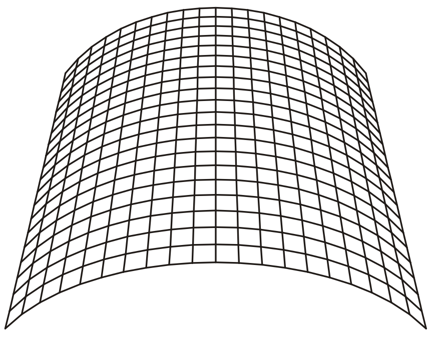
Figure 2. A dome obtained by applying a spherical cap pellevation

The effects created by the pellevation functions are not limited to spherical caps. Examples of some other effects that can be created by various pellevation functions are shown in Figures 3, 4, 5 and 6. The pellevational effects in Figures 4 to 6 are cylindrical, where the type of cylinder, plan coordinates of two points of its axis, its span and its rise are to be specified.



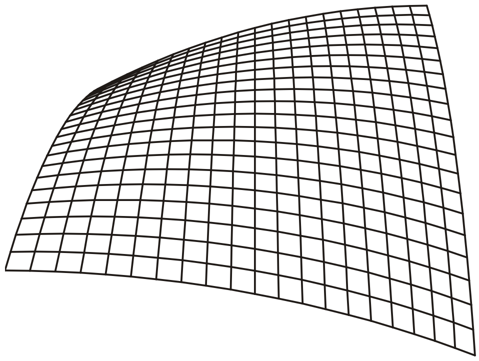
Dome=capel(4,10,10,20√2,7)|Grid

Figure 3. A dome obtained by a conical cap pellevation



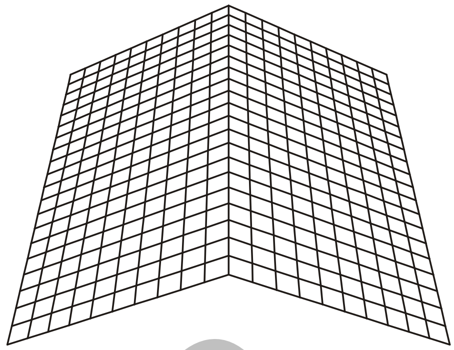
Barrel=bapel(1,10,0,10,20,20,5)|Grid

Figure 4. A barrel vault obtained by a cylindrical pellevation



Barrel=bapel(3,0,0,20,20,20√2,6)|Grid

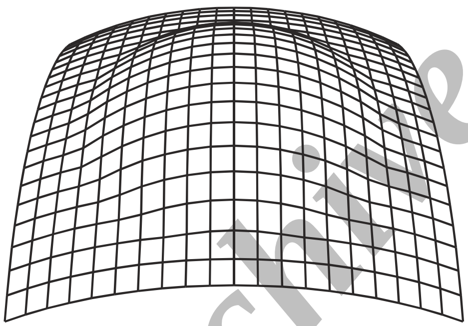
Figure 5. A barrel vault obtained by a parabolic cylindrical pellevation along the diagonal of the grid of Figure 1



Barrel=bapel(4,10,0,10,20,20,5)|Grid

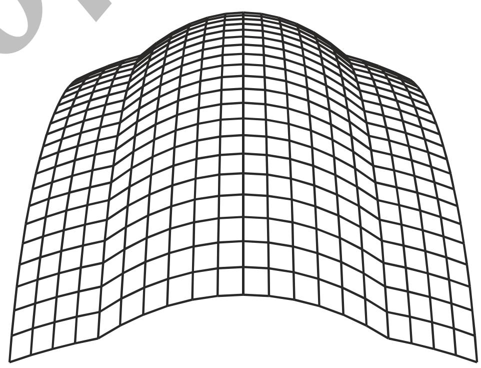
Figure 6. A barrel vault obtained by a wedge shaped pellevationsl effect

Pellevational effects may be combined (by adding or subtracting) to create many different shapes. Examples of combinations of pellevational effects are shown in Figures 7, 8, 9, and 10.



Dome=capel(3,10,9,15,2)|
capel(1,10,10,20√2,5)|Grid

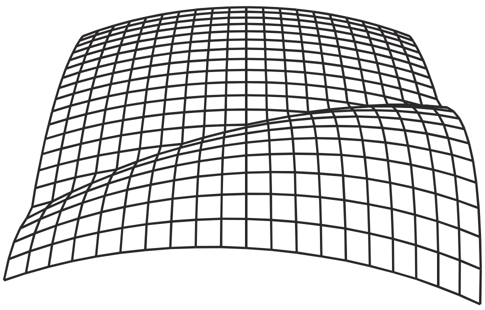
Figure 7. A combination of spherical and paraboloidal cap pellevations



Dome=bapel(3,10,0,10,20,12,2)|
capel(1,10,10,20√2,5|Grid

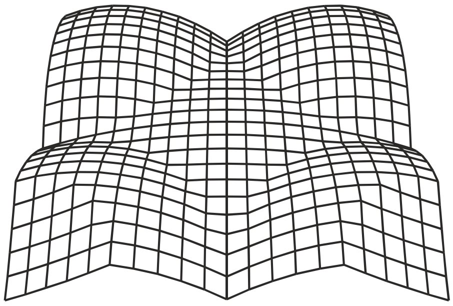
Figure 8. A combination of spherical and parabolic cylindrical pellevations

The configuration which is subjected to pellevation need not be a simple plane configuration. It may be a configuration of any form. For example, Figure 11 shows a geodesic dome on an ellipsoidal surface. Figure 12 shows the dome of Figure 11 with a spherical pellevational effect imposed on the central part of it. Also, Figure 13 shows the dome of Figure 11 with a cylindrical pellevational effect imposed on it. Further details regarding pellevation functions are found in Refs. [4-6].



Dome=bapel(3,5,0,20,5,12,3)|
capel(1,10,10,20√2,4|Grid

Figure 9. A combination of spherical and parabolic cylindrical pellevations



Dome=capel(3,0,0,20,4)|capel(3,0,20,20,4)|
capel(3,20,0,20,4)|capel(3,20,20,20,4)|
capel(-3,0,0,10,2)|capel(-3,20,0,10,2)|
capel(1,10,10,20√2,4)|Grid

Figure 10. A combination of one spherical and six paraboloidal cap pellevations (two of which are subtractive)

E=pol(1,5,'[1-5]',50,[0,0; 18,0]|
genid(9,9,2,√3,1,-1)|
rosad(1,√3/3,120)|[0,0; 2,0]
Geodome=dil(3,1/3)|
trac(1,[0,0,0,50],4,-15)|E

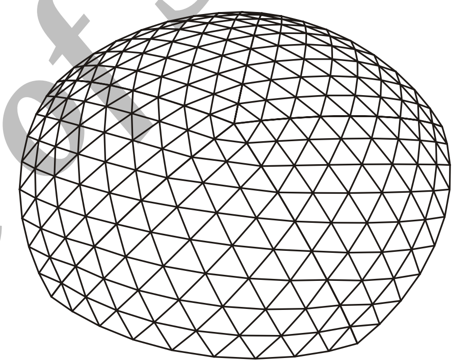
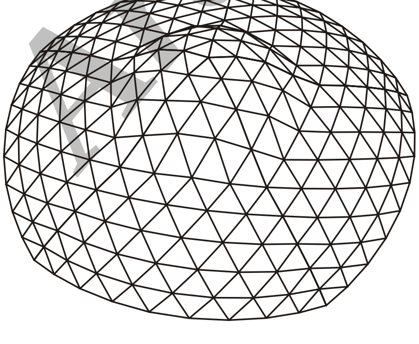
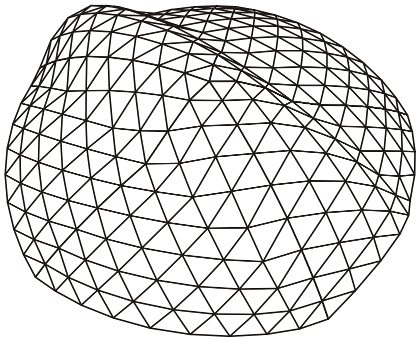


Figure 11. A geodesic dome on an cllipsoidal surface



Dome=capel(1,0,0,40,10)|Geodome

Figure 12. A geodesic dome with a central spherical cap



Dome=bapel(3,0,0,0,10,30,9)|Geodome

Figure 13. A geodesic dome with a cylindrical rib

2. NOVATIONAL EFFECTS

Again, consider the plane grid of Figure 1. Suppose that it is required to change the coordinates of a node of the grid. For instance, consider the node whose U1-U2-U3 coordinates are 7,7,0 and let it be required to change these coordinates to 7,7,5. This operation can be done by a formex function called "novation". The result is shown in Figure 14. In the formex formulation given in Figure 14:

- "nov" is an abbreviation for "novation".
- The first parameter (that is, 1) indicates the mode of operation.
- The second parameter gives the coordinates of the node to be moved and the third parameter gives the coordinates of the point to which the node is to be moved.

Mode 1 of the novation function is for moving one or more specified nodes of a configuration to specified positions.

Now, consider the situation illustrated in Figure 15. In this case, mode 2 of the novation function is used. In this mode, the effect of the novation function is to apply the specified movements of the nodes, and also move all the remaining nodes of the configuration in a manner which is conformable with the specified nodal movements. In the formulation of Figure 15, "Corner" represents the four corner nodes of the grid. The third parameter of the novation function (that is, Corner # [7,7,0]) represents five nodes,

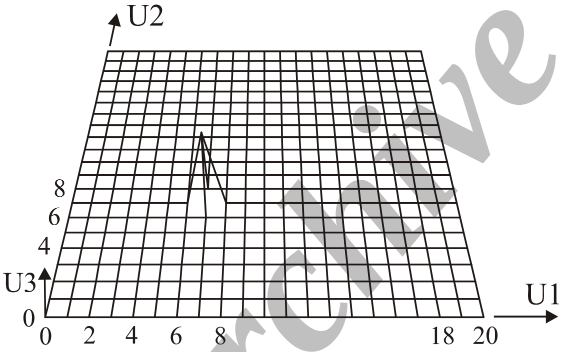


Figure 14. Moving a node of the grid using a novation function

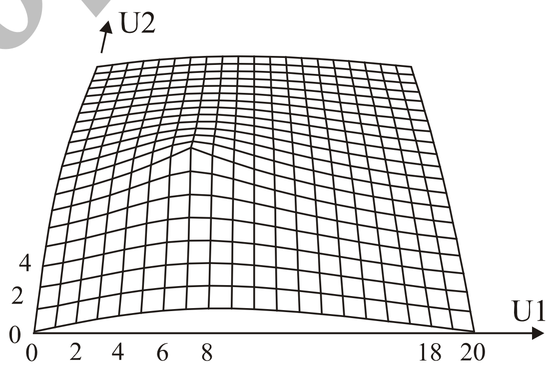


Figure 15. Moving a node of the grid with all the remaining nodes (except the corner nodes

namely, the four corner nodes and the node with coordinates 7,7,0. The last parameter of the novation function (that is, Corner # [7,7,5]), again represents five nodes, namely, the corner nodes and the new position of the node 7,7,0. The effect is that the corner nodes remain unmoved (since their specified movements is zero), the node 7,7,0 will move to the point 7,7,5 and all the remaining nodes will move in harmony with the specified movements. However, to determine what may be regarded as "moving in harmony", one needs a "rule". Such a rule for mode 2 of the novation function is in accordance with a formula that is referred to as the "exponential decay conformity rule" or ED conformity rule, see Ref. [7]. A graphical representation of the general effect of this rule is shown in Figure 16. In general, for any conformity rule, the further away a node from a specified movement is, the smaller the conformity movement of the node will be. In Figure 16, the speed with which the curves decay is controlled by a parameter C, where C varies from 0 to ∞ and the larger it is, the more rapid the decay will be. To illustrate the effect of the decay control parameter C, the arrangement of Figure 15 that corresponds to C=1 is shown again in Figures 17 and 18 for C=0.5 and C=2, respectively. The control parameter C appears as the second parameter in a novation function (In the case of mode 1 novation, this control parameter has no effect and its presence is optional).

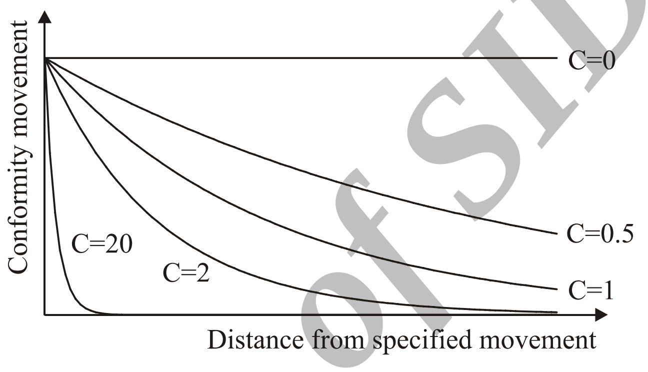
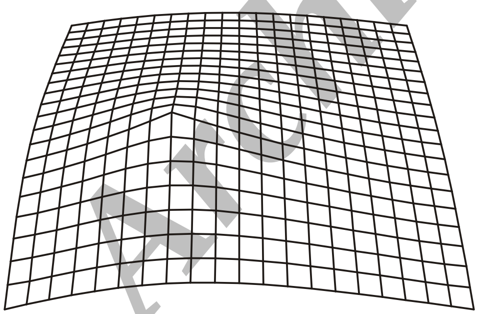
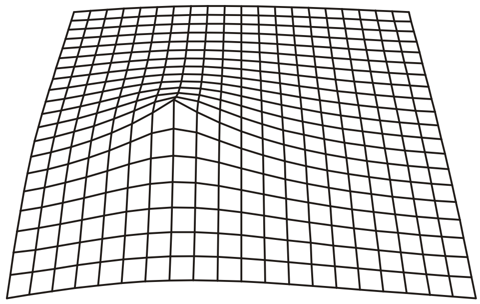


Figure 16. A graphical representation of the Exponential Decay (ED) conformity rule for some values of the decay control parameter C. Mode 2 of the novation function uses the ED conformity rule.



Corner=lamid(10,10)|[0,0,0]
Form=nov(2,0.5,Corner#[7,7,0],
Corner#[7,7,5])|Grid

Figure 17. An arrangement similar to Figure 15 but with C=0.5



Corner=lamid(10,10)|[0,0,0]
Form=nov(2,2,Corner#[7,7,0],
Corner#[7,7,5])|Grid

Figure 18. An arrangement similar to Figure 15 but with C=2

It should be mentioned that, in applying a novation function, when there are more than one specified movement then the conformity movements are to be determined by the simultaneous consideration of the effects of all the specified movements [7].

Novation function has a third mode corresponding to a conformity rule that is referred to as the "cubic decay conformity rule" or CD conformity rule. A graphical representation of this conformity rule is illustrated in Figure 19. The effects are shown for some values of the decay control parameter C. The range of the values of C for CD conformity rule is from 0 to ∞. However, as it may be seen from Figure 19, the curves corresponding to values larger than 2 are very close to one another. The application of novation mode 3 to the problem of Figure 15 will give rise to the configuration shown in Figure 20. It is seen that the formex formulation in Figure 20 is identical to that in Figure 15 except for the mode which is 3 rather than 2. However, it may be seen that the effects of the two modes are quite different. The shape created by mode 3 has a dome-like form, whereas the shape created by mode 2 resembles a fabric structure in tension. Another example of the application of novation mode 3 is shown in Figure 21.

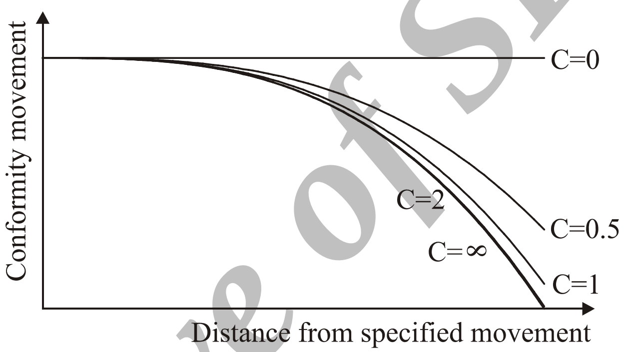


Figure 19. A graphical representation of the Cubic Decay (CD) conformity rule for some values of the decay control parameter C. Mode 3 of the novation function uses the CD conformity rule

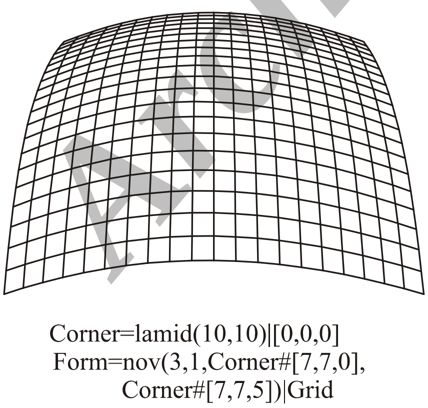


Figure 20. Moving a node of the grid with all the remaining nodes (except the corner nodes) moving in accordance with the CD conformity rule with C=1

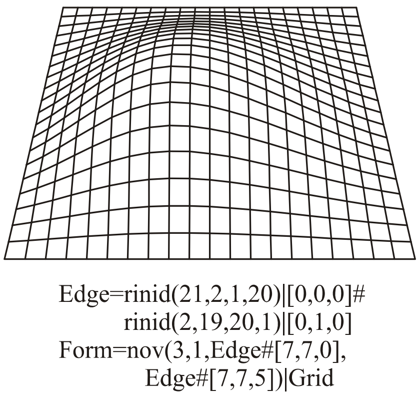


Figure 21. The arrangement in this figure is similar to that of Figure 20, except that all the edge nodes (rather than corner nodes) have specified zero movement

3. FREEFORM CONFIGURATIONS

In this section a number of examples are employed to illustrate the use of the concepts of pellevation and novation in generating freeform geometries. However, it should be appreciated that the range of possibilities for freeform shapes is unlimited. Therefore, the examples can only be indicative of the general approach and cannot cover all the possibilities.

The freeform in Figure 22 is obtained by novating a plane grid configuration. In the formex formulation of Figure 22:

- The variable Grid represents the plane grid that is subjected to novation.
- The variable Corner represents the four corner points of the grid.
- The variable Base represents the initial positions of four internal points of the grid.
- The variable Target represents the points to which the initial points are to be moved.
- The variable Form represents the novated configuration shown in Figure 22, where, mode 3 novation function is used and where the points represented by Corner#Base are specified to be moved to the points represented by Corner#Target.

```
Grid=rinid(24,25,1,1)|[0,0,0; 1,0,0]#
        rinid(25,24,1,1)|[0,0,0; 0,1,0]
Corner=lamid(12,12)|[0,0,0]
Base={[14,2,0], [8,18,0], [20,16,0], [4,22,0]}
Target={[14,2,5], [8,18,0], [20,16,3], [4,22,2]}
Form=nov(3,1,Corner#Base,
         Corner#Target)|Grid
```

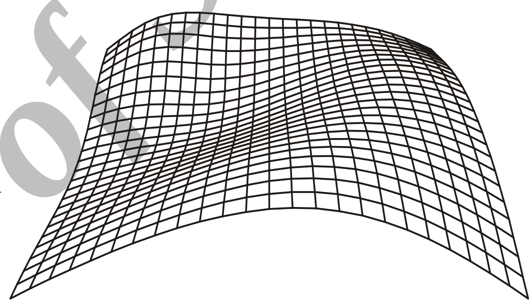
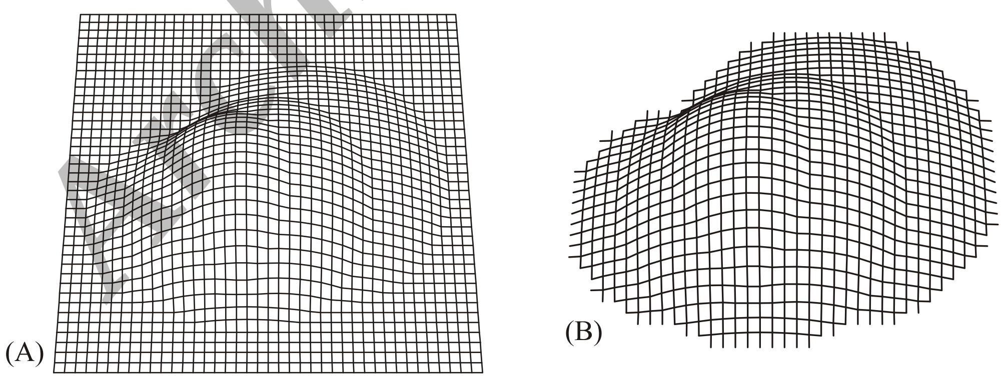


Figure 22. A freeform obtained by subjecting a plane grid to novation mode 3




```
Grid=rinid(40,41,1,1)|[0,0,0; 1,0,0]# rinid(41,40,1,1)|[0,0,0; 0,1,0]
Forma=capel(3,12,16,20,4)|capel(3,18,16,24,5)|capel(3,24,20,28,6)|Grid
Formb=rel(U(1,3)>0 || U(2,3)>0)|Forma
```

Figure 23. (A) A configuration obtained by subjecting a plane grid to three successive pellevations (B) A freeform obtained by removing the elements in the plane ot the initial grid of Figure 23A

Figure 23A shows a configuration obtained by subjecting a plane grid to three successive parabolic cap pellevations. Also, Figure 23B shows a freeform obtained by removing the elements that remain in the initial plane of the grid in Figure 23A. In the formex formulation of Figure 23:

- The variable Grid represents the original plane grid.
- The variable Forma represents the configuration of Figure 23A.
- The variable Formb represents the freeform of Figure 23B. The removal of the elements is achieved using a relection function, see Section 2.A.10 of Ref. [2].

Figure 24A shows a configuration obtained by subjecting a diamatic dome to a novation of mode 2. In the formex formulation of Figure 24:

- The variable Dome represents a regular diamatic dome as described in Section 2.5 of Ref. [2]. The dome has 6 sectors and is of frequency 8.
- The variable Edge represents the edge nodes of the dome along 4 sectors of the dome opposite the part of the dome that has been novated.
- The variable Sb specifies the coordinates of the edge node that has been novated
- The variable Form represents the freeform of Figure24A.

The configuration shown in Figure 24B shows a side view of the freeform dome of Figure 24B.

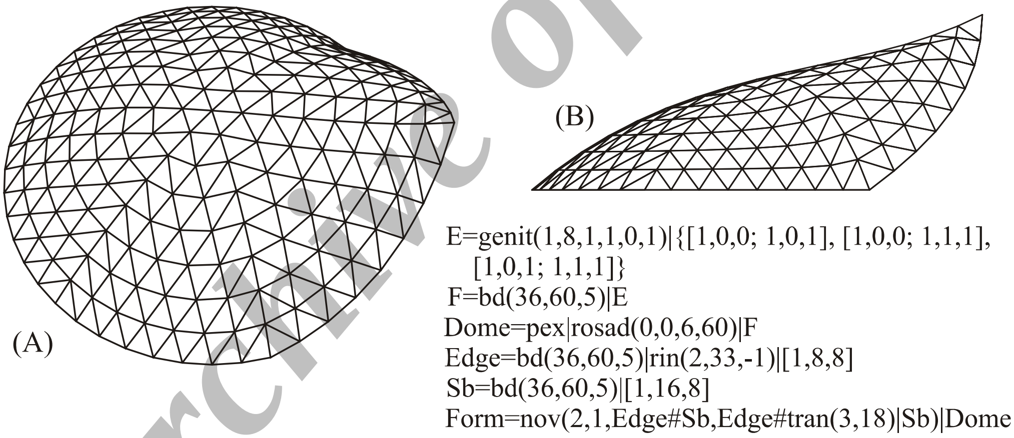
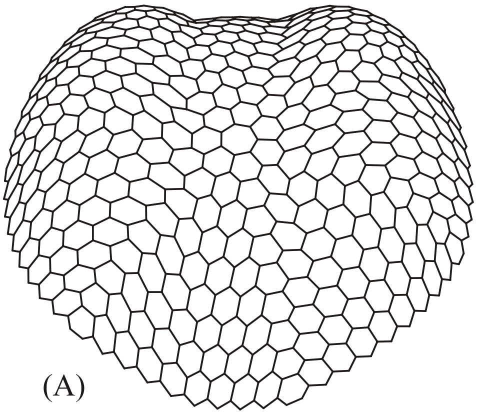
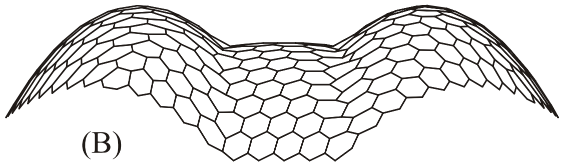


Figure 24. (A) Perspective view of a freeform dome obtained by novating a diamatic dome (B) Side view of the freeform dome of Figure 24A

The freeform dome of Figure 25A is obtained by applying two parabolic cylindrical pellevations on a diamatic dome with a honeycomb pattern. A side view of the freeform dome of Figure 25A is shown in Figure 25B. In the formex formulation of Figure 24, the formex variable Dome represents the initial diamatic dome. The formulation is carried out as explained in Section 2.5 of Ref. [2].



(A)



(B)

```
E=genit(1,9,3,3,0,1)|{[1,2,4; 1,1,2], [1,2,4; 1,1,5],
    [1,2,4; 1,4,5]}#rin(3,10,3)|[1,1,2; 1,-1,2]
F=bd(36,360/7,1.6)|E
Dome=rosad(0,0,7,360/7)|F
Form=verad(0,0,-45)|bapel(3,18,0,18,18,25,7)|
    bapel(3,-18,0,-18,18,25,7)|Dome
```

Figure 25. (A) Perspective view of a freeform dome obtained by pellevating a diamatic dome (B) Side view of the freeform dome of Figure 25A

The freeform configuration of Figure 26 is obtained by applying two parabolic cylindrical pellevations to a compound hypar consisting of four simple hypars. These hypars are connected together along their edges, with the combination being convex. In the formex formulation of Figure 26, the compound hypar is represented by the formex variable Hypar. The formulation has been carried out in the manner described in Section 3.2 of Ref. [2].

```
E=rinid(16,17,1,1)|[-8,-8,0; -7,-8,0]#
    rinid(17,16,1,1)|[-8,-8,0; -8,-7,0]
F=bhp(5/4,5/4,1,0.06)|E
Hypar=pex|rosad(10,10)|F
Form=bapel(3,0,0,0,20,20,10)|
        bapel(3,20,0,20,20,20,10)|Hypar
```

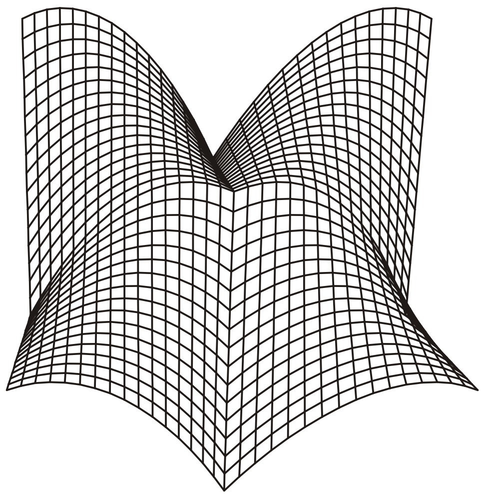


Figure 26. A freeform configuration obtained from pellevation of a compound hypar

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