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METHODOLOGICAL STUDY OF THE ULTIME LIMIT SECTION IN REINFORCED CONCRETE UNDER BIAXIAL BENDING AND AXIAL COMPRESSION

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ABSTRACT

The complexity of the geometrical shape in reinforced concrete amplified the difficulties of shearing resistance in the boundaries limits state in particular for a section, which is submitted to the eccentrited biaxial loading (biaxial force plus bending).

The difficulties in this study results in the determination of the ultimate forces N_u , M_{ux} and M_{uy} and the relationship between them. These difficulties are essentially du to the geometrical shape, the steel disposition and the law behaviour of the concrete and steel. The main objectif of this paper is to present a methodological study based on the integration numerical method that would determine the equations of the interaction curves fitting for the determination of the steel sections and the verification of the shearing resistance.

1. INTRODUCTION

In this case of simple loading such as bending and compression, the value of shearing is less difficult because it depends on one parameter; $\mathbf{M_u}'$ (ultimate limit moment) for the simple bending and $\mathbf{N_u}'$ (ultimate limit force) for the simple compression.

For the shearing resultats, we have to verify the following condition:

$\mathbf{M} < \mathbf{M_u}'$ for simple bending.
$\mathbf{N} < \mathbf{N_u}'$ for simple compression.

where **M** and **N** are forces du to external loading.

Whereas in axial force plus bending, the problem become more difficult because it depend on two parameters ($\mathbf{N_u}$ and $\mathbf{M_u}$) in this case of the axial force plus bending and on third parameters ($\mathbf{N_u}$, $\mathbf{M_{ux}}$ and $\mathbf{M_{uy}}$) in the case of the biaxial force plus bending.

The axial force plus bending parameters aren't independent, therefore:

$\mathbf{N_u} = \mathbf{f_1}(\mathbf{M_u})$ for axial force plus bending.

$\mathbf{N_u} = \mathbf{f_2}(\mathbf{M_{ux}}, \mathbf{M_{uy}})$ for bi axial force plus bending.

The function $\mathbf{f_1}$, Figure 1, define the interaction curves.

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Their graphical performance is flat and the function $\mathbf{f_2}$ (Figure 1) defines the interaction surfaces and their graphical representation is space.

To verify the shearing resistance under axial force plus bending (eccentricity), you must insure that at each time:

a) In the case of axial force plus bending, the coordonnâtes point (**N**, **M**) must be inside the delimited surface by the interaction curve defined by $\mathbf{f_1}$.

b) In the case of biaxial loading plus bending, the coordonnates point ($\mathbf{N}$, $\mathbf{M_x}$, $\mathbf{M_y}$) must be inside the defined volume by the interaction surface whuchis $\mathbf{f_2}$.

Where:

N is the normal compression load provoked by external loading**.**

$\mathbf{M_x}$ is the moment over the principal axis **xx** provoked by external loading.

$\mathbf{M_y}$ is the moment over the principal axis **yy** provoked by external loading.

The problem to be solved is to find functions $\mathbf{f_1}$ and $\mathbf{f_2}$ which depend on some factors such as, geometrical shape of sections, the mechanical characteristics of materials (the behaviour diagram of concrete and steel) and the position of the stroke steel. Those factors make these equations very complicated.

Although these difficulties exist, the only solution which could be employed are the graphical ones.

The problem is more difficult for biaxial loading plus bending because the graphical representation is spaced, which wouldn't allow their use over a plan.

To solve this problem, we must find firstly a relationship between $\mathbf{M_u = f_3 (M_{ux},M_{uy})}$ and therefore establish a relationship $\mathbf{N_u = f_4 (M_u)}$ and this is to reduce the spaced problem to the plan problem which makes the graphical method's useful.

Many authors such as Pannel [1], Bressler [2], Ramamarthy and Khan [3], Mallikajuna and Mahdevappa [4], Wolfgang [5] and Cerniak [6] have looked to this problem for particular sections defined and by differents approachs.

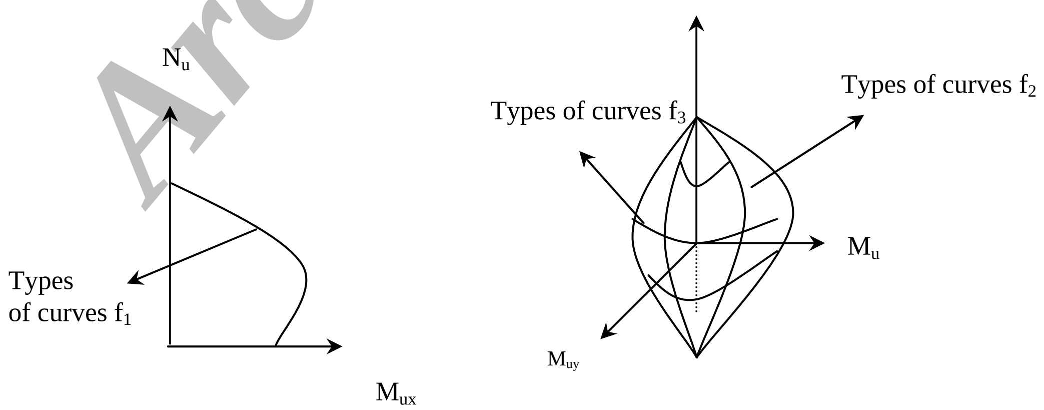


Figure 1. Interaction surfaces and curves

2. ASSUMPTIONS

1. Material behavior as shown in Figure 2.
2. We consider a good grip or adherence between steel and the concrete.
3. The tension concrete is neglected.
4. The straight section remains straight even after deformation.
5. The section has to be taken short which does not allow distortion.

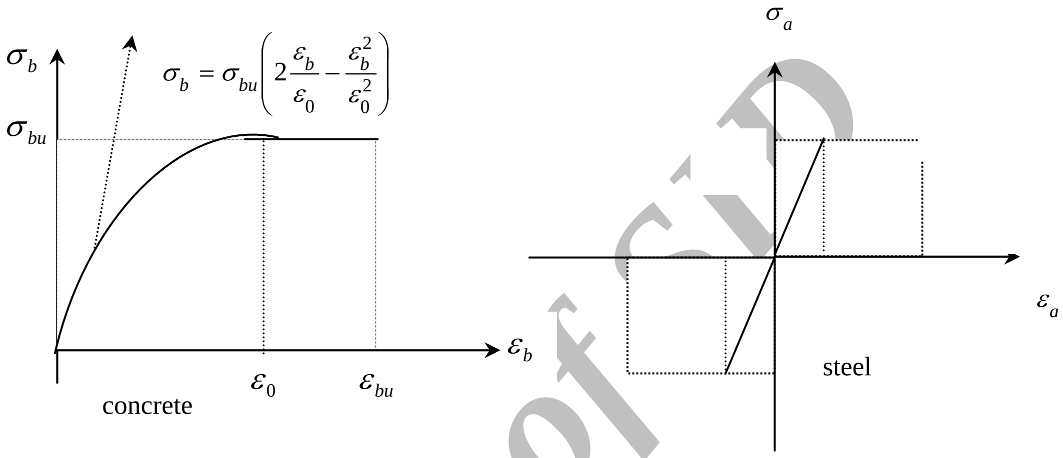


Figure 2. Behaviour's law of the material

3. METHODOLOGICAL STUDY

3.1 Analysis procedure:
In order to determine the outline curve f_2, we must change the orientation of the neutral axis on (from 0 to 360°) (Figure 3) and for each orientation of the angle we must do a translation of the neutral axis (from one interval of $0{,}1h_0$ to $2{,}4h_0$).

For each translation we can determine N_u, M_{ux}, M_{uy} which really represented a point in the curve f_2.

The efforts N_u, M_{ux}, M_{uy} inside the reinforced concrete are determined in function of the position of the elementary section of concrete ds_i and of the steel A_i (from the neutral axis and the principal central axis).

$$N_u = \int_s \sigma_{bi}.dsi \quad + \sum A_i.\sigma_{ai}$$

$$M_{ux} = \int_s \sigma_{bi}.y_{bi}.dsi \quad + \sum A_i.\sigma_{ai}.y_{ai}$$

$$M_{uy} = \int_s \sigma_{bi}.x_{bi}.dsi \quad + \sum A_i.\sigma_{ai}.x_{ai}$$

To determine the effort, a numerical program based over numerical integration methods is essential and needed.

Once the obtained efforts are known, we do an analysis to determine a relationship of type f_3 which could be independent of the orientation of the angle of neutral axis and of the steel.

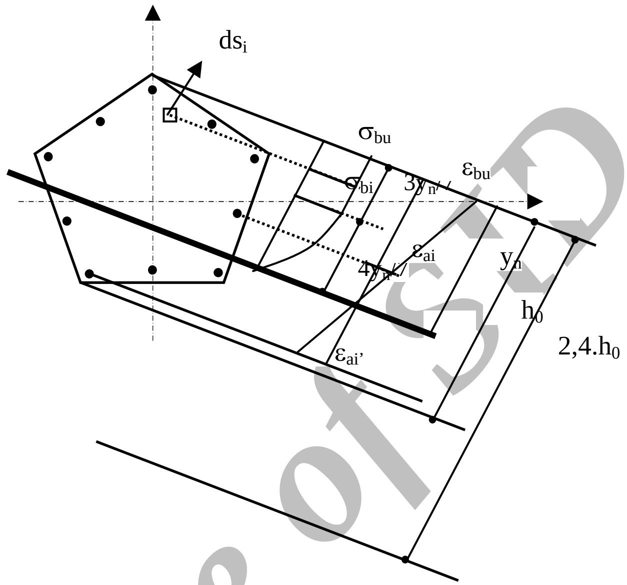


Figure 3. Analysis curve

4. POLYGONAL SECTIONS CASES

4.1 Concrete only

4.1.1 Geometrical parameters

We take the geometrical parameters in function of «h» to consider the sections adimensionel.

Let us take N a number of polygonal sides.

- Angle β and α

$$\beta = \pi \frac{N-2}{2.N} \qquad \alpha = \frac{\pi}{N}$$

- Width of the polygonal side:
- for N even $a.h = h.\sin\alpha$
- for N uneven or oddnumber $a.h = 2\frac{\sin\alpha}{1+\cos\alpha}.h$

reduced height h_G (from the peak to gravity center of the reduced section of polygonal):

$$h_G.h = \frac{a.h}{2.\sin\alpha}$$

4.1.2 basis elements

The all polygonal section are constitued of a (2xN) triangles represented on triangles, Figure 4:

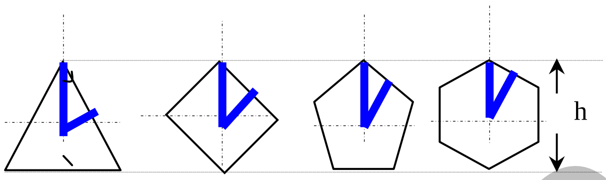


Figure 4. Polygonal sections

The basic triangle is divided in many elementary sections ds_i (Figure 5)

$n =$ number of elementary section

$I_{\max} =$ number of line

$J_{\max} =$ number of column

since $I_{\max} = J_{\max}$

$$n = \frac{I_{\max}^2 + I_{\max}}{2}$$

let us take $b.h$ and $v.h$ respectively the basis and the height of the triangle.

The dimensions of the elementary section will be then:

the basis $g.h = \dfrac{b.h}{J_{\max}}$

the height $d.h = \dfrac{z.h}{I_{\max}}$

the elementary section surface $a_e.h^2 = g.d.h^2 = \dfrac{b.z}{I_{\max}.J_{\max}}.h^2$

Remarks:

$$b.h = \frac{a.h}{2}$$
$$z.h = h_G.h$$

In general when

$$J \neq J_{\max} \rightarrow \quad x_i = (J-1)d + \frac{d}{2}$$
$$I \neq I_{\max} \rightarrow \quad y_i = (I-1)g + \frac{g}{2}$$

$$J = J_{max} \rightarrow \quad x_i = (J-1)d + \frac{d}{3}$$
$$I = I_{max} \rightarrow \quad y_i = (I-1)g + \frac{g}{3}$$

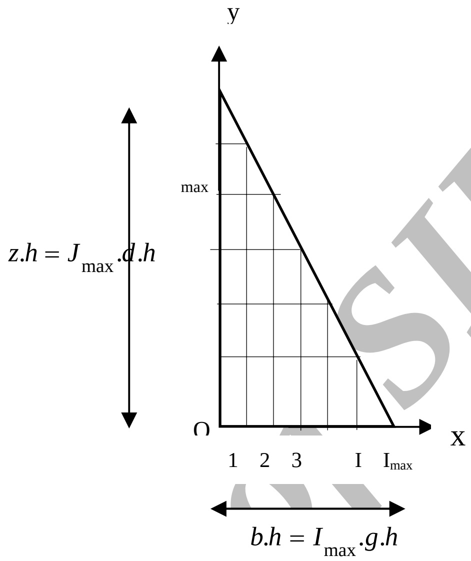


Figure 5. Elementary sections

If OX and OY are the principal central axis of the total section, θ the rotated angle of the axis ox and oy from the OX , OY and X_0 ,Y_0 the coordonates of the point o from OX, OY, therefore:

$$X_i = X_0 + x_i \cos\theta + y_i \sin\theta$$
$$Y_i = Y_0 - x_i \sin\theta + y_i \cos\theta$$

with

$$\theta = \alpha + \pi \qquad X_0 = \frac{\sin 2\alpha}{2}.h_G$$
$$Y_0 = h_G.\cos^2\alpha$$

Starting from the calculation program essentially based over the numerical integration methods, we determine:

$$N_b = \sum ds_i.\sigma_{bi}$$
$$M_{bux} = \sum ds_i.\sigma_{bi}.Y_i$$
$$M_{buy} = \sum ds_i.\sigma_{bi}.X_i$$

The reduced forces (for the adimensional section) will follow this form:

$$\nu_b = \frac{N_{bu}}{\sigma_{bu}.h^2} \quad ,$$
$$\mu_{bx} = \frac{M_{bx}}{\sigma_{bu}.h^3} \quad ,$$
$$\mu_{bx} = \frac{M_{by}}{\sigma_{bu}.h^3}$$

the steel framework

The efforts N_{ai} and M_{ani} inside each steel framework are calculated in function of the imposed displacement by the concrete and the distance behind the neutral axis (Figure 5) considering the law for the behaviour of the steel.

The efforts in the steel framework section are calculated in the following manner:

$$N_a = n\sum N_{ai} \quad M_{an} = \sum N_{ai}.e_{ani}.h$$
$$\varepsilon_{ai} = \frac{\varepsilon_{bu}}{k.h} e_{ani}.h \qquad \frac{\varepsilon_{ai}}{\varepsilon_{au}} = \frac{E_a.\varepsilon_{bu}}{k.\varepsilon_{au}} e_{ani} = \psi.e_{ani}$$

- if $\psi.e_{ani} \geq 1 \rightarrow \frac{\sigma_{ai}}{\sigma_{au}} = 1$ (plastic compression domain)
- if $-1 < \psi.e_{ani} < 1 \rightarrow \frac{\sigma_{ai}}{\sigma_{au}} = \psi.e_{ani}$ (elastic compression or tensile domain)
- if $\psi.e_{ani} \leq 1 \rightarrow \frac{\sigma_{ai}}{\sigma_{au}} = -1$ (plastic tensile domain)

Hence

- $p = \frac{n_t.(A.h^2)}{A_b} \rightarrow$ (steel percentage)
- $m = \frac{\sigma_{au}}{\sigma_{bu}}$ (equivalent coefficient)
- "$p.m$" is called mechanical percentage
- $a_0 = \frac{A_b}{n_t.h^2}$ (remind constant)

$$N_{ai} = (A.h^2.\sigma_{ai}).\frac{n_t.A_b.\sigma_{au}}{n_t.A_b.\sigma_{au}}\frac{\sigma_{bu}.h^2}{\sigma_{bu}.h^2} =$$
$$a_0.pm.\frac{\sigma_{ai}}{\sigma_{au}}(\sigma_{bu}.h^2) = a_0.pm..\Omega_i.(\sigma_{bu}.h^2)$$

- N_a and N_{ai} are respectively the total effort in the framework and the effort in the framework i;
- M_{an} is the provoked moment by the overall steel compared to the neutral axis.
- e_{ani}, e_{axi} and e_{ayi} are the eccentricities respectively compared to the axis nn, xx and yy;
- A and σ_{ai} are respectively the framework steel section and the effort in the framework steel i.
- n_t and n_b are respectively the total number of steel and the steel number by side of the hexagonal section.

We call

ν_{ai} the reduced effort in the framework i by mechanical percentage;

μ_{ani}, μ_{axi} and μ_{ayi} are the reduced moment by the mechanical percentage inside the framework i compared respectively to the axis nn, YY and XX; hence:

$$\nu_{ai} = p.m.\frac{N_{ai}}{\sigma_{bu}.h^2} \qquad \rightarrow \nu_{ai} = p.m.a_0.\Omega_i$$
$$\mu_{ani} = p.m.\frac{N_{ai}.(e_{ani}.h)}{\sigma_{bu}.h^3} \qquad \rightarrow \mu_{ani} = p.m.a_0.\Omega_i.e_{ani}$$
$$\mu_{axi} = p.m.\frac{N_{ai}.(e_{ayi}.h)}{\sigma_{bu}.h^3} \qquad \rightarrow \mu_{axi} = p.m.a_0.\Omega_i.e_{ayi}$$
$$\mu_{ayi} = p.m.\frac{N_{ai}.(e_{axi}.h)}{\sigma_{bu}.h^3} \qquad \rightarrow \mu_{ayi} = p.m.a_0.\Omega_i.e_{axi}$$

4.3 The effort in the reinforced concrete:

In the calculation program that we have done and realised in our laboratory of the University of Constantine, the reduced effort in the reinforced concrete are determined in function of the mechanical percentage *pm*, the number of steel franework by arete n_{br} also the wrapper *d* des armatures (*pm*, n_{br}, *d*) which we permitted to vary the equivalent coefficient (quality of steel and concrete) and the percentage steel also the disposition and the wrapper of steel . The reduced efforts inside the reinforced concrete are:

$$\nu = \nu_b + a_0 p.m \sum \Omega_i$$
$$\mu_x = \mu_{bx} + a_0 p.m \sum \Omega_i.e_{ayi}$$
$$\mu_y = \mu_{by} + a_0 p.m \sum \Omega_i.e_{axi}$$

4.4 Case of the hexagonal sections:

The program elaboration has permitted to determine the following relationship f_3 for the hexagonal sections. The results are down on the following curves, Figures 6, 7 and 8.

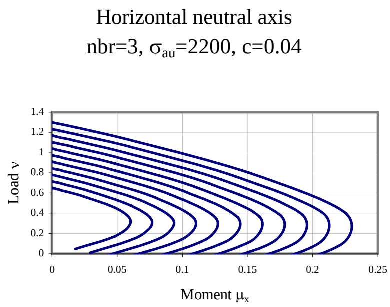


Figure 6. Type of results for neutral horizontal axis (Interaction curve $\nu = f_1(\mu_x)$)

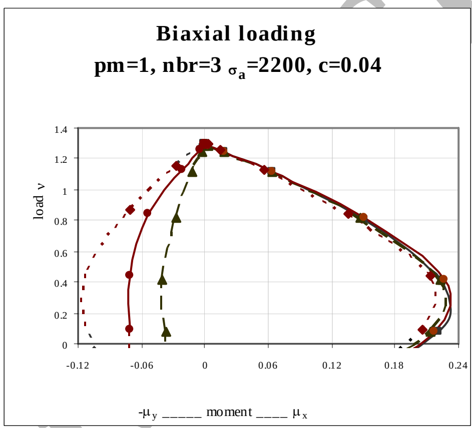
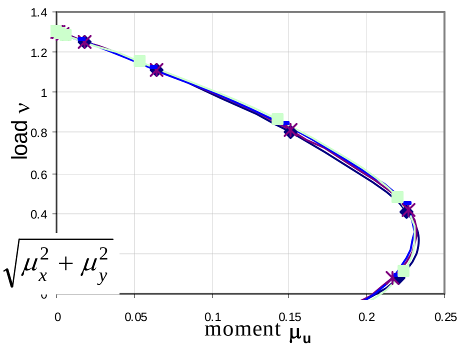


Figure 7. Type of results for neutral oblic axis (Interaction Curve $\nu = f_2(\mu_x, -\mu_y)$)



$$\mu_u = f_3(\mu_x, \mu_y) = \sqrt{\mu_x^2 + \mu_y^2}$$

Figure 8. Type of curve $\nu = f_4(\mu_u)$

5. CONCLUSION

This method based on numerical integration method's has shown that the calculated shearing resistance of the hexagonal section at the biaxial eccentrited compression would be reduced to the calculation of the shearing rersistance of the iniaxial eccentrited compression. This is shown in Figure 8 it is clearly shown that curves f_1 and f_4 are identical:

$$\nu = f_1\left(\mu_x\right) = f\left(\mu_u\right)$$

This methodological approach has for task and tarjet to verify the shearing resistance and to determine the bearing capacity of the considered section. Therefore the determination of the following f_1 relationship is necessarily. It is possible to enable this method to many types of sections.

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