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PERFORMANCE OF MAXWELL DAMPER BASED MULTIPLE TUNED MASS DAMPERS

C. Li*
Department of Civil Engineering, Shanghai University, No. 149 Yan Chang Road, Shanghai 200072, P.R. China

Abstract

In practical applications, it is difficult to link dashpot absolutely rigidly between the structure and the mass blocks of the multiple tuned mass dampers (MTMD). In order to cope with this practical issue, Maxwell damper based multiple tuned mass dampers (referred to as the MD-MTMD) have been presented for attenuating the response of structures excited by the ground acceleration. By resorting to the formulated transfer functions of the MD-MTMD structure system, the dynamic magnification factors (DMF) are then defined of the MD-MTMD structure system. The criterion for the optimum searching can thus be selected as the minimization of the minimum values of the maximum DMF (min. min. max. DMF). Employing this criterion, the effects of the normalized relaxation time constant (NRTC) are investigated on the optimum parameters and effectiveness of the MD-MTMD. Likewise, the effects of the RTC on the stroke of the MD-MTMD are estimated in terms of maximizing the dynamic magnification factors (DMF) of each MD-TMD in the MD-MTMD. The numerical results have indicated that the MD-MTMD is a feasible solution for the practical issue mentioned above of the traditional MTMD.

Keywords: Damping; multiple tuned mass dampers; Maxwell damper based multiple tuned mass dampers; dynamic magnification factors; normalized relaxation time constant; optimality

1. Introduction

It is quite well known that the tuned mass damper (TMD) may provide a cheaper and convenient solution for vibration suppression of structures. But, a single TMD is mainly used for passive control of narrowband vibrations. Employing more than one TMD, with different dynamic characteristics, is capable of improving the effectiveness and robustness of a single TMD, consequently able to suppress broad-band vibrations. The studies by Xu and Igusa [1] and Igusa and Xu [2] demonstrated that the multiple tuned mass dampers (MTMD), whose natural frequencies are distributed over a small range in the neighborhood of the natural frequency of a single-degree-of-freedom (SDOF) structure, may provide better

* E-mail address of the corresponding author: li-chunxiang@vip.sina.com (C. Li)

effectiveness and higher robustness with respect to a single TMD with equal total mass ratio. Likewise the MTMD has been comprehensively investigated by several researchers such as Yamaguchi and Harnpornchai [3], Abe and Fujino [4], Kareem and Kline [5], Jangid [6], Li [7,8], Park and Reed [9], Gu et al. [10], Chen and Wu [11], Hoang and Warnitchai [12], Yau and Yang [13], and Wang and Lin [14]. The MTMD is shown to be more effective and robust under excitation frequencies distributed over a wider band in comparison to a single TMD.

It is important to point out, however, that in practical applications, it is difficult to link dashpot absolutely rigidly between the structure and the mass blocks of the MTMD. As regards this issue, the recent study by Li and Qu [15] has demonstrated the influences of the stiffness ratio on the control performance of the AMTMD (including the MTMD) in the design. With reference to this recent study, the Maxwell damper based multiple tuned mass dampers (referred to as the MD-MTMD) have been further proposed in the present paper. The numerical results have shown that the proposed MD-MTMD is a feasible solution for this practical issue of the traditional MTMD.

2. Transfer Functions of the MD-MTMD Structure System

The MD-MTMD is taken into account here to control the specific vibration mode of a structure under the ground acceleration. The structure is modeled as an SDOF system using the mode-reduced method, characterized by the mode-generalized stiffness k_s, mode-generalized damping coefficient c_s, and mode-generalized mass m_s, respectively. The parameters of the jth MD-TMD in the MD-MTMD include the mass m_{Tj}, damping coefficient c_{Tj}, and stiffness k_{Tj} (k_{tj}). When the relative displacements of both the structure (y_s) and the jth MD-TMD in the MD-MTMD (y_{Tj}) with respect to their supports are introduced, as shown in Figure 1, the equations of motion of the MD-MTMD structure system under the ground acceleration [$\ddot{x}_g(t)$] can be formulated as follows:

$$m_s\left[\ddot{x}_g(t)+\ddot{y}_s\right]+c_s\dot{y}_s+k_s y_s=\sum_{j=1}^{n}\left[k_{Tj}y_{Tj}+p_j(t)\right] \tag{1}$$

$$m_{Tj}\left[\ddot{x}_g(t)+\ddot{y}_s+\ddot{y}_{Tj}\right]+k_{Tj}y_{Tj}+p_j(t)=0 \quad (j=1,2,\cdots,n) \tag{2}$$

A Maxwell damper consists of a spring and a dashpot in series as shown in Figure 1. The damper force [$p_j(t)$] and velocity [$\dot{y}_{Tj}(t)$] in the Maxwell damper can be related by the following first-order differential equation**:**

$$p_j(t)+\eta_j\dot{p}_j(t)=c_{Tj}\dot{y}_{Tj}(t) \quad (j=1,2,\cdots,n) \tag{3}$$

Let one now introduce the following notation**:**

$$\omega_s^2 = \frac{k_s}{m_s};\ \xi_s = \frac{c_s}{2m_s\omega_s};\ \omega_{Tj}^2 = \frac{k_{Tj}}{m_{Tj}};\xi_{Tj} = \frac{c_{Tj}}{2m_{Tj}\omega_{Tj}}$$

$$\mu_{Tj} = \frac{m_{Tj}}{m_s};\ \ \mu_T = \sum_{j=1}^{n}\mu_{Tj};\ \ \omega_T = \sum_{j=1}^{n}\frac{\omega_{Tj}}{n};\ \ f = \frac{\omega_T}{\omega_s}$$

$$\beta = \frac{\omega_{Tn} - \omega_{T1}}{\omega_T};\ \ r_{Tj} = \frac{\omega_{Tj}}{\omega_s};\ \ \xi_T = \sum_{j=1}^{n}\frac{\xi_{Tj}}{n};\ \lambda = \frac{\omega}{\omega_s}$$

$$\tau_j = \omega_s^2\eta_j = \omega_s^2\,\frac{c_{Tj}}{k_{tj}}$$

where η_j is the relaxation time constant (RTC) of the MD-MTMD and τ_j the normalized relaxation time constant (NRTC) of the MD-MTMD. Evidently, when the NRTC is set equal to zero, the MD-MTMD will degenerate into the traditional MTMD.

Letting $\ddot{x}_g(t) = \ddot{X}_g e^{-i\omega t}$, in which $\ddot{X}_g$ represents the amplitude of the ground acceleration, $y_s = \left[TF_{y_s}(-i\omega)\right]e^{-i\omega t}$, and $y_{Tj} = \left[TF_{y_{Tj}}(-i\omega)\right]e^{-i\omega t}$ in Eqs**.** (1)-(3), the transfer functions [$TF_{y_s}(-i\omega)$ and $TF_{y_{Tj}}(-i\omega)$] of the MD-MTMD structure system can be derived, which have the following form**:**

$$TF_{y_s}(-i\lambda) = \omega_s^2\left[\frac{TF_{y_s}(-i\omega)}{\ddot{X}_g}\right] = -\left[\frac{T_{r3}(\lambda) + iT_{r4}(\lambda)}{T_{r1}(\lambda) + iT_{r2}(\lambda)}\right] \tag{4}$$

$$TF_{y_{Tj}}(-i\lambda) = \omega_s^2\left[\frac{TF_{y_{Tj}}(-i\omega)}{\ddot{X}_g}\right] = -\frac{1}{R_1 + iR_2}\left[1 + \lambda^2\,\frac{T_{r3}(\lambda) + iT_{r4}(\lambda)}{T_{r1}(\lambda) + iT_{r2}(\lambda)}\right] \tag{5}$$

in which in the present study, λ is set within the range from 0.4 to 3.4; the intermediate variables in Eqs. (4) and (5) can be represented explicitly as follows**:**

$$T_{r1}(\lambda) = 1 - \lambda^2 - \lambda^2\sum_{j=1}^{n}\left[\frac{\mu_{Tj}(E_A^*R_1 + E_BR_2)}{R_1^2 + R_2^2}\right]$$

$$T_{r2}(\lambda) = -2\xi_s\lambda - \lambda^2\sum_{j=1}^{n}\left[\frac{\mu_{Tj}(E_BR_1 - E_A^*R_2)}{R_1^2 + R_2^2}\right]$$

$$T_{r3}(\lambda) = 1 + \sum_{j=1}^{n}\left[\frac{\mu_{Tj}(E_A^*R_1 + E_BR_2)}{R_1^2 + R_2^2}\right]$$

$$T_{r4}(\lambda) = \sum_{j=1}^{n}\left[\frac{\mu_{Tj}(E_BR_1 - E_A^*R_2)}{R_1^2 + R_2^2}\right]$$

$$E_A = \frac{2\xi_{Tj} r_{Tj} \tau_j \lambda^2}{1+\tau_j^2 \lambda^2};\ E_A^* = E_A + r_{Tj}^2;\ E_B = \frac{-2\xi_{Tj} r_{Tj} \lambda}{1+\tau_j^2\lambda^2}$$

$$R_1 = r_{Tj}^2 - \lambda^2 + E_A;\ R_2 = E_B$$

$$r_{Tj} = f\left[1+\frac{j\beta}{n-1}-\frac{(n+1)\beta}{2(n-1)}\right];\ \xi_T = \frac{\xi_{Tj} f}{r_{Tj}}$$

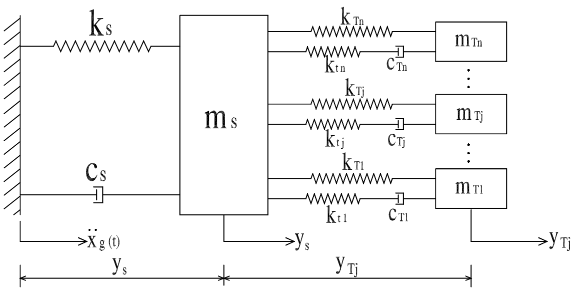

Figure 1. Model of the structure with the Maxwell damper based multiple tuned mass dampers (referred to as the MD-MTMD, non-traditional MTMD) under the ground acceleration

3. Evaluation Criteria of the MD-MTMD

The MD-MTMD is manufactured through keeping the stiffness and damping coefficient constant but varying the mass (i.e., $k_{T1} = k_{T2} = \cdots = k_{Tn} = k_T$; $k_{t1} = k_{t2} = \cdots = k_{tn} = k_t$; $c_{T1} = c_{T2} = \cdots = c_{Tn} = c_T$; and $m_{T1} \neq m_{T2} \neq \cdots \neq m_{Tn}$). For the sake of simplicity, but without any loss of generality, it is further assumed that the RTC of the MD-MTMD maintains constant (i.e., $\eta_1 = \eta_2 = \cdots = \eta_n = \eta$), then implying that the NRTC of the MD-MTMD holds constant (i.e., $\tau_1 = \tau_2 = \cdots = \tau_n = \tau$). Here the dynamic magnification factors (DMF) of the MD-MTMD structure system are explicitly defined as

$$DMF = \left|\left[TF_{y_s}(-i\lambda)\right]\right| \tag{6}$$

$$DMF_j = \left|\left[TF_{y_{Tj}}(-i\lambda)\right]\right| \tag{7}$$

The evaluation on the effectiveness, optimum frequency spacing, used for reflecting the robustness, optimum tuning frequency ratio, and optimum average damping ratio of the MD-MTMD can then be carried out through the minimization of the minimum values of the maximum dynamic magnification factors (DMF) of the structure with the MD-MTMD, which has the following form**:**

$$R = min.min.max.DMF \tag{8}$$

The estimation on the MD-MTMD stroke can be simultaneously performed in terms of the maximum dynamic magnification factors (DMF_j) of each MD-TMD in the MD-MTMD using the obtained optimum parameters of the MD-MTMD based on Eq**.** (8), which has the following form**:**

$$R_j = max.DMF_j \tag{9}$$

4. Performance of the MD-MTMD

The minimization of the maximum values of the dynamic magnification factors (DMF) of the structure with the MD-MTMD (i.e., $min.min.max.DMF$) is used as the criterion for searching the optimum parameters of the MD-MTMD. Then, it can be said that the MD-MTMD which gives the value of the $min.min.max.DMF$ is the optimum MD-MTMD, with which the structure achieves the minimum vibration level. It is also indicated that the optimum MD-MTMD which renders the smaller value of the $min.min.max.DMF$ is a more effective MD-MTMD. In the present study, a structural damping ratio $\xi_s = 0.02$ is taken into account. Examination has been made on the performance of the proposed MD-MTMD based on the obtained optimum parameters and effectiveness. Taking advantage of the criterion of the optimization procedure selected and based on the analytic results, the variations in the optimum average damping ratio, optimum tuning frequency ratio, optimum frequency spacing ratio and the corresponding $min.min.max.DMF$ versus the normalized relaxation time constant (NRTC) are studied considering the different values of both the total number (n) and total mass ratio (μ_T).

Figure 2 depicts the relation curves between the optimum average damping ratio and the normalized relaxation time constant (NRTC) for the MD-MTMD. It is seen from Figure 2 that the optimum average damping ratio of the MD-MTMD increases with an increase in the total mass ratio while decreases with an increase in the total number. More importantly, with reference to the traditional MTMD, namely the MD-MTMD with NRTC=0, the MD-MTMD generally has relatively larger optimum average damping ratio. Likewise, it is seen from Figure 2 that the optimum average damping ratio of the MD-MTMD generally increases with the increase of the NRTC.

Figure 3 presents the variation of the optimum frequency spacing ratio, used for measuring the robustness, of the MD-MTMD with respect to the normalized relaxation time constant (NRTC). Note that the optimum frequency spacing ratio of the MD-MTMD increases with an increase in both the total number as well as the total mass ratio, indicating that the robustness of the MD-MTMD becomes better. Likewise, Figure 3 clearly demonstrates that the optimum frequency spacing ratio of the MD-MTMD practically maintains constant with an increase in the NRTC. Therefore, the robustness of the MD-MTMD is basically equal to that of the traditional MTMD.

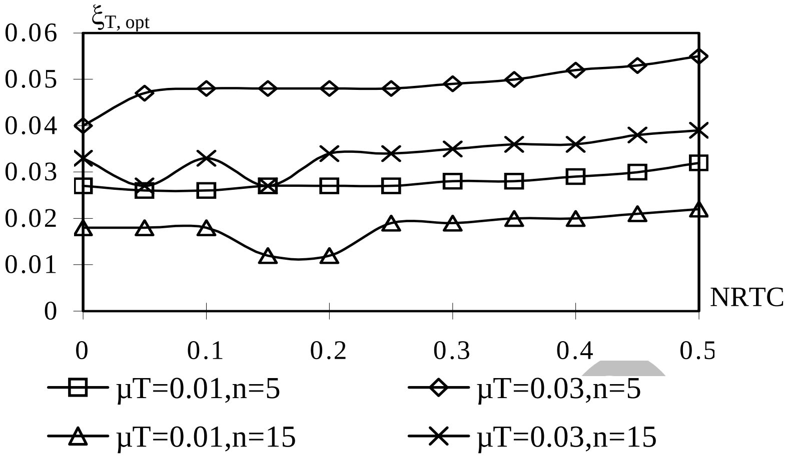

Figure 2. Variation of the optimum average damping ratio of the MD-MTMD, non-traditional MTMD [traditional MTMD (NRTC=0)] for vibration control of structures due to the ground acceleration with respect to the normalized relaxation time constant

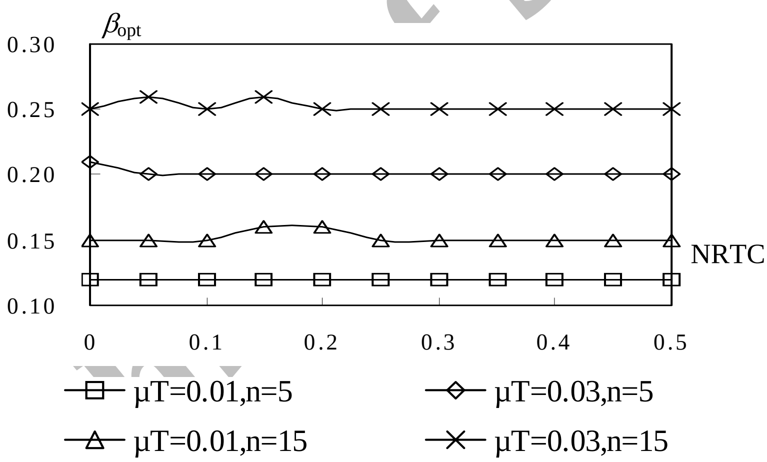

Figure 3. Variation of the optimum frequency spacing ratio of the MD-MTMD, non-traditional MTMD [traditional MTMD (NRTC=0)] for vibration control of structures due to the ground acceleration with respect to the normalized relaxation time constant

Figure 4 demonstrates how the optimum tuning frequency ratio of the MD-MTMD varies with the normalized relaxation time constant (NRTC). Note that the optimum tuning frequency ratio of the MD-MTMD decreases with an increase in both the total number, and while increases with the increase of total mass ratio. With reference to the traditional MTMD, the MD-MTMD possesses the smaller optimum tuning frequency ratio. Note also that, there exists a changing trend that the optimum tuning frequency ratio decreases with the increase of the NRTC.

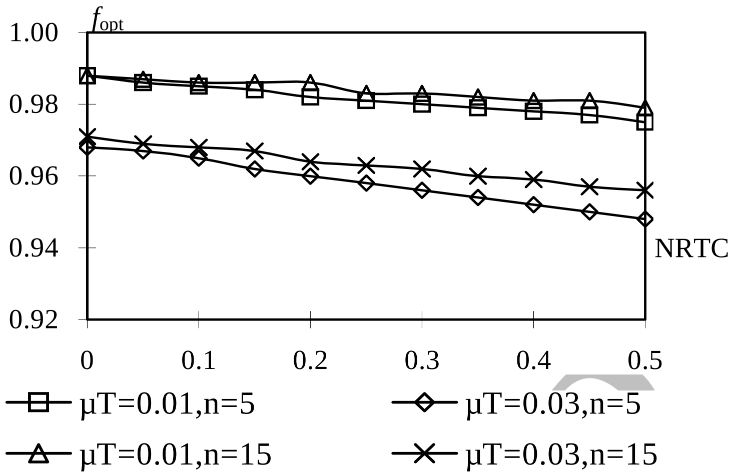

Figure 4. Variation of the optimum tuning frequency ratio of the MD-MTMD, non-traditional MTMD [traditional MTMD (NRTC=0)] for vibration control of structures due to the ground acceleration with respect to the normalized relaxation time constant

Figure 5 plots the interrelation between the *min.min.max.DMF* of the MD-MTMD and the normalized relaxation time constant (NRTC). From the Figure 5, it is apparent that the effectiveness of the MD-MTMD increases with the increase of both the total number and the total mass ratio. Notwithstanding this, the effectiveness of the MD-MTMD cannot be significantly enhanced by resorting to larger total number. Also, it is important to note that the MD-MTMD yields the same control effectiveness as the traditional MTMD, irrespective of the NRTC.

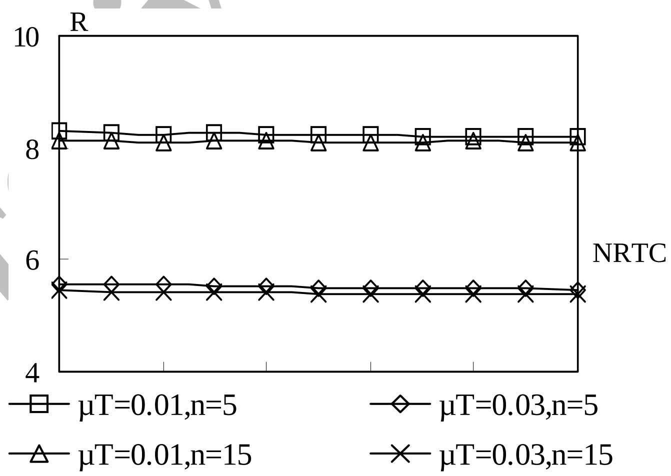

Figure 5. Variation of the *min.min.max.DMF* (used for measuring the effectiveness) of the MD-MTMD, non-traditional MTMD [traditional MTMD (NRTC=0)] for vibration control of structures due to the ground acceleration with respect to the normalized relaxation time constant

Table 1 offers the variation of the $max.DMF_j$, used for estimating the stroke, of each MD-TMD in the MD-MTMD using the obtained optimum parameters with reference to the normalized relaxation time constant (NRTC) with the total number and total mass ratio equal to 5 and 0.01, respectively. The numerical results listed in the Table 1 indicate that the MD-MTMD achieves approximately the same stroke as the traditional MTMD, regardless of the NRTC.

The foregoing investigations suggest that the MD-MTMD basically provides the same robustness, effectiveness, and stroke as the traditional MTMD. Hence, the proposed MD-MTMD is a feasible solution for the practical issue, that in practical applications, it is difficult to link dashpot absolutely rigidly between the structure and the mass blocks of the MTMD.

Table 1. Variation of the $max.DMF_j$ (used for measuring the stroke) of each MD-TMD in the MD-MTMD, non-traditional MTMD [traditional MTMD (NRTC=0)] for vibration control of structures due to the ground acceleration with reference to the normalized relaxation time constant (NRTC) in the case of μ_T=0.01 and $n = 5$.

NRTC	MD-TMD1	MD-TMD2	MD-TMD3	MD-TMD4	MD-TMD5
0.0	172.61	162.28	154.98	143.61	134.48
0.1	178.71	168.27	161.06	149.62	141.35
0.2	175.55	166.49	160.03	149.15	141.19
0.3	177.09	168.16	161.80	150.79	142.62
0.4	180.60	171.75	165.57	154.53	146.69
0.5	174.94	167.78	162.58	152.36	144.88

5. Conclusions

It is demonstrated from the preceding elucidation that (1) the optimum average damping ratio of the MD-MTMD generally increases with an increase in the normalized relaxation time constant (NRTC); (2) the optimum tuning frequency ratio of the MD-MTMD decreases with an increase in the NRTC; (3) the MD-MTMD basically render the same robustness, effectiveness, and stroke as the traditional MTMD, regardless of the NRTC. Consequently, the proposed MD-MTMD is a feasible solution for the practical issue, that in practical applications, it is difficult to link dashpot absolutely rigidly between the structure and the mass blocks of the MTMD.

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