

ROBUST OPTIMIZATION FOR TMD WITH UNCERTAIN BOUNDED SYSTEM PARAMETERS AND STOCHASTIC EXCITATION

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Abstract

In this work a robust optimum design for mechanical parameters of linear Tuned Mass Damper devices is proposed. In this field, standard approaches are based on the implicit assumption that all system parameters are deterministically known quantities. When this hypothesis is removed, a robust optimum design criterion for Tuned Mass Damper should be developed, where robustness is obtained by finding solutions which are less sensitive to variation of system parameters, originated by the uncertainty.

In this study the load condition for the analysed system is represented by a stationary stochastic process which models the base acceleration, and here modelled by the Kanai-Tajimi stochastic process. The main system, which is equipped by a single Tuned Mass Damper, is described by a system with a single degree of freedom: system mass and stiffness are assumed to be affected by uncertainty, and then are represented by random-bounded variables. The ratio between the protected and unprotected main system covariance displacement is assumed as Objective Function, and then its mean and standard deviation are evaluated. Robust optimum design is formulated as a multi-objective optimization problem, in which both the first and the second statistical moments are minimized simultaneously, with different weights. In this way, optimal Pareto fronts are obtained: after that a sensitivity analysis is carried out in order to assess the variation of robust solution with respect to some parameters, and moreover in order to evaluate the differences with respect to conventional deterministic solution.

1. Introduction

It is well known that all problems in the field of structural engineering deal with uncertainty. Nevertheless, in order to reduce conceptual and computational difficulties, often standard methods have been developed in structural analysis, assuming that all quantities are

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deterministically known. Actually, the parameters involved in structural problems, such as load or mechanical and geometrical configuration, are only partially known quantities, or simply they have an intrinsic uncertain nature.

In structural dynamics a widely used simplified approach assumes that the loads represent the only source of uncertainty. For example, earthquake or wind actions can be suitably modelled as stochastic processes, and then the standard random vibration theory can be employed [1] for obtaining structural responses, assuming that all the other parameters are deterministically defined. This approach in structural design leads to “Conventional Stochastic Structural Optimization” (CSSO), where the objective functions and the constraints may concern system response processes or related quantities.

In the field of random vibrations the first definition of structural optimization problem was proposed by Nigam [2], leading to a standard nonlinear constrained problem, in which constraints were defined by probabilistic structural response quantities and the objective function (OF) by structural weight.

This paper concentrates the attention on a particular optimization problem that is the vibration control of systems subject to random dynamic loads. This is a very broad context, which embraces different branches of engineering, from the mechanical to the seismic one. In this latter field, the use of a stochastically defined OF, after the Nigam proposal, was adopted in many different circumstances, regarding passive vibration control (for example [3-8]). A CSSO was proposed in [9], where a stochastic reliability - based design criterion for linear structures subject to random vibrations was developed. Both OF and constraints were defined in a stochastic way. In detail, constraints impose a limit to the failure probability associated with the first threshold crossing of a structural response.

In above cited studies it is implicitly assumed that uncertainties in structural systems have negligible effects on system response, obtaining in this way an oversimplification in many real problems. However, it is reported that the uncertainty in system parameters may have equal or greater influence on response than uncertainty in excitation [10]. This aspect is of great relevance particularly in cases in which the solution is strongly influenced by the variation of system parameters, as in the case of structural optimization. Therefore, for a more realistic analysis, system parameters must be treated with a suitable description of uncertainty which affects nominal values.

For these reasons, real structures in mechanical or civil engineering are often described by means of random variables, owing to various factors of uncertainty in materials, measurement, manufacturing and installation. For the same reasons, safe domain and input process parameters have to be considered uncertain quantities. This means that CSSO may become unfeasible due to the scatter of structural behaviour. Therefore, it is reasonable to explore the effects of uncertainty on the design of structures subject to random vibrations. For this reason, in the last twenty years, various nondeterministic methods have been developed in order to deal with optimum design under uncertainty. These methods can be classified in two main branches, namely “reliability-based methods (RBMs)” and “robust design-based methods” (RDBMs).

The RBMs, based on the knowledge of probability distribution of the random parameters, estimate the probability distribution of the system response, and are mainly used for risk analysis, by computing the probability of system failure. However, the solution variation which

derives from uncertainty of system parameters is not minimized in reliability approaches [11], which are focused on rare events at the tail of the probability distribution [12].

The RDBMs optimize a performance index in term of mean value, and at the same time they minimize its variability which derives from environmental uncertainty, obtaining a solution less sensitive to parameters variation. This is achieved by optimizing the design variables in order to make the performance minimally sensitive to the different causes of variation. A RDBM solution is not able to give the best performance in an absolute sense but it gives a low sensitivity of solution.

In the field of vibration control, that is the area of this work, recently a robust design method intended for a vibration absorber of a system having an uncertain mass and stiffness, has been used in order to demonstrate the robust design approach in dynamics, based on a frequency approach [13]. The uncertainty of parameters is defined by the mean and the covariance, and it concerns with main system mass and damping. As local performance index, the maximum of the dimensionless displacement transfer function, over a limited frequency band, is used, and the robust optimization has been obtained by minimizing its deviation in mean and variance, that are obtained by a direct first order perturbation method based on a Taylor series expansion.

A reliability-robust optimization has been also been proposed in [20]. In this work, the failure probability, related to the crossing of system response over a given threshold, is adopted as conventional OF, in order to optimize the TMD parameters for a system subject to a white noise input. A multi-objective optimization is performed in order to obtain Pareto fronts. The mean and variance are evaluated by approximated asymptotic evaluation.

In this paper, a RDBM is applied for the optimization of a TMD, in the hypothesis that some main system parameters have an uncertain nature. More precisely, uncertain parameters are described as random variables, and are represented by means of bounded independent probability density functions. The PDF here used has a uniform law. The main system, described by a single degree of freedom model, is protected by a linear single TMD in order to reduce vibration level induced by base acceleration, here modelled by means of the stationary Kanai-Tajimi stochastic process.

The design vector collects the TMD frequency and damping ratio. The ratio between the root mean squares (RMSs) of displacement of the protected and unprotected main system is adopted as OF. The robust optimum solution is obtained by using a multi-objective OF instead of a single conventional one. The mean and standard deviation are estimated by a direct numerical integration in order to evaluate the OF. The Pareto front, in the space mean-RMS, is then obtained by using different weight coefficient values, in the range [0,1]. Finally, a comparison between conventional and robust optimum solutions is performed.

2. Structural Model and Motion Equations

Tuned Mass Damper (TMD) is one of the simplest and the most reliable passive device for vibration control in a wide range of applications. It consists in an additional mass connected to a main system by a spring and a damper.

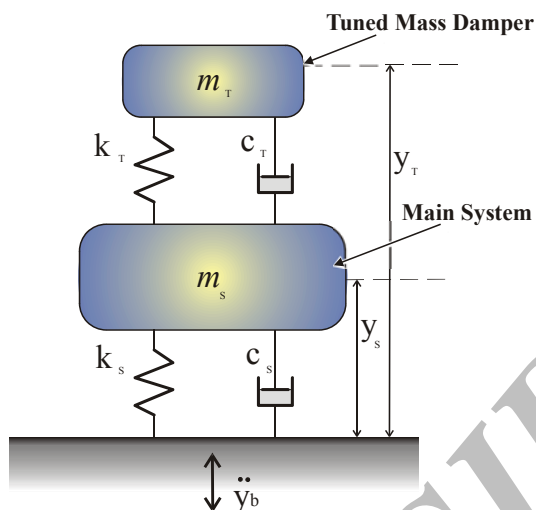


Figure 1. Linear model of TMD system

The mechanical model is represented in Figure 1: more precisely, the main system is described by a single mass m_s , linked with the base by a linear spring and a dashpot, whose mechanical characteristics are described by stiffness and damping parameters k_s and c_s , respectively. It is excited by a base acceleration $\ddot{y}_b(t)$, and it is connected with a secondary mass m_T by a spring and a dashpot, whose characteristics are k_T and c_T , respectively. y_T , \dot{y}_T and \ddot{y}_T denote the displacement, the velocity and the acceleration of the TMD with respect to the base, and y_s , \dot{y}_s and \ddot{y}_s are the displacement, the velocity and the acceleration of the main system with respect to the base.

In this case, system response is determined by solving the dynamic equilibrium system equation:

$$\mathbf{M}\ddot{\bar{Y}}(t) + \mathbf{C}\dot{\bar{Y}}(t) + \mathbf{K}\bar{Y}(t) = \bar{r}\ddot{y}_b(t) \quad (1)$$

where $\bar{Y}(t)$, $\dot{\bar{Y}}(t)$ and $\ddot{\bar{Y}}(t)$, denote the displacement, velocity and acceleration vectors, \bar{r} is the drag vector; finally, \mathbf{M} , \mathbf{C} and \mathbf{K} are respectively the mass, damping and stiffness matrices of the combined system.

A widely adopted model in stationary case for $\ddot{y}_b(t)$ is obtained by filtering a white noise process, acting at the bed rock, through a linear filter which represents the surface ground. This is the well known Kanai-Tajimi stochastic process [20] which is able to characterise input frequency content for a wide range of practical situations.

In the stationary case base acceleration $\ddot{y}_b(t)$ is:

$$\ddot{y}_b(t) = \ddot{x}_f(t) + w(t) = -\left(2\xi_g\omega_g\dot{x}_f(t) + \omega_g^2x_f(t)\right) \quad (2)$$

$$\ddot{x}_f(t) + 2\xi_g \omega_g \dot{x}_f(t) + \omega_g^2 x_f(t) = -w(t) \quad (3)$$

In Eqs. (2) and (3) $x_f(t)$ is the response of the filter representing the ground, ξ_g and ω_g are, respectively, the damping ratio and the frequency of this filter, $w(t)$ being the white noise process, representing the excitation at the bed rock.

Introducing the space state vector:

$$Z = (y_T \quad y_S \quad x_f \quad \dot{y}_T \quad \dot{y}_S \quad \dot{x}_f)^T \quad (4)$$

the stochastic response is completely known by means of the covariance matrix R_{ZZ} , obtained by solving the Lyapunov equation:

$$AR_{ZZ} + R_{ZZ}A^T + B = 0 \quad (5)$$

where the state A is:

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\omega_T^2 & +\omega_T^2 & +\omega_f^2 & -2\xi_T \omega_T & +2\xi_T \omega_T & +2\xi_f \omega_f \\ +\mu \omega_T^2 & -(\mu \omega_T^2 + \omega_S^2) & +\omega_f^2 & +\mu 2\xi_T \omega_T & -(\mu 2\xi_T \omega_T + 2\xi_S \omega_S) & +2\xi_f \omega_f \\ 0 & 0 & -\omega_f^2 & 0 & 0 & -2\xi_f \omega_f \end{pmatrix} \quad (6)$$

and where the 6 order matrix B has all null elements, except the last on the main diagonal: $[B]_{6,6} = 2\pi S_0$, where S_0 is the intensity of the white noise.

System mechanical parameters in matrix (6) are:

$$\omega_T = \sqrt{\frac{k_T}{m_T}} \text{ - TMD frequency, } \omega_S = \sqrt{\frac{k_S}{m_S}} \text{ - main system, frequency, } \xi_T = \frac{c_T}{2\sqrt{m_T k_T}} \text{ -}$$

TMD damping ratio, $\xi_S = \frac{c_S}{2\sqrt{m_S k_S}}$ - main system damping ratio, $\mu = \frac{m_T}{m_S}$ - mass ratio.

Previous formulation deals with the assumption that all system parameters have a deterministic nature, but this is usually if referred to many real cases.

3. Conventional Stochastic Structural Optimization of TMD

Generally speaking, the structural optimization problem can be formulated as the selection

of a set of decision variables (that are the parameters of the design which characterize structural configuration), collected in the so called design vector (DV) \bar{b} , over a possible admissible domain Ω_b . The optimum DV must be able to minimize a given Objective Function (OF) and to satisfy, in general, some constraint conditions. Both constraints and OF must be defined over a given time interval, as the problem regards dynamic system response.

The optimization problem so defined has been stated first by Nigam [2] and then transformed into a standard nonlinear programming one. It could be stated as:

$$\text{Find } \bar{b} \in \Omega_b \quad (7)$$

$$\text{Which Minimizes } OF(\bar{b}, t) \quad (8)$$

$$\text{Subject to } P_f(\bar{b}, t) \leq \bar{P}_f \quad (9)$$

The OF could be defined either by a standard deterministic way (e.g., the total structural weight or the volume of the elements) or in a stochastic one. In the latter case, response statistics could be used, as covariance or spectral moments of variables of interest (e.g., displacement, acceleration or structural stress in relevant elements). The reliability constraint imposes that possible DVs must guarantee a failure probability $P_f(\bar{b}, t)$ smaller than a given maximum acceptable one, defined by \bar{P}_f .

In this study the unconstrained optimization of TMD mechanical parameters, which are collected in the two dimensional design vector $\bar{b} = (\omega_T, \xi_T)$, will be performed assuming that the mass ratio μ is a given quantity. The criterion selected for the optimization is the minimization of the dimensionless peak of the displacement of the protected system with respect to the unprotected one. The OF is defined in dimensionless way as the ratio between σ_{Y_s} and the $\sigma_{Y_s}^0$, which are the root mean squares of displacement of the protected and unprotected system, respectively.

This function represents a direct stochastic index of vibration protection efficiency, which will be more effective as its value decreases from one. At the same time, a value close to one will indicate a practically negligible efficiency of the vibration control strategy.

The conventional optimum design is performed assuming that all parameters involved in the problem are deterministic. Therefore, the unconstrained problem can be formulated as follows:

$$\text{Find } \bar{b} = (\omega_T, \xi_T) \in \Omega_b \quad (10)$$

$$\text{Which Minimizes } \frac{\sigma_{Y_s}(\bar{b})}{\sigma_{Y_s}^0} \quad (11)$$

The results of this conventional optimization can be observed in Figure 2, where the optimum solution is represented in the space $\rho_T = \frac{\omega_T}{\omega_S}$ (that is the ratio of the TMD frequency on the system one), and ξ_T . Input is characterised by $\xi_f = 0.4$ and $\omega_f = 25(\text{rad/sec})$. The system has a frequency $\omega_s = 45$ (rad/sec) and a damping ratio $\xi_s = 5\%$. The mass ratio is assumed $\mu = 0.1$.

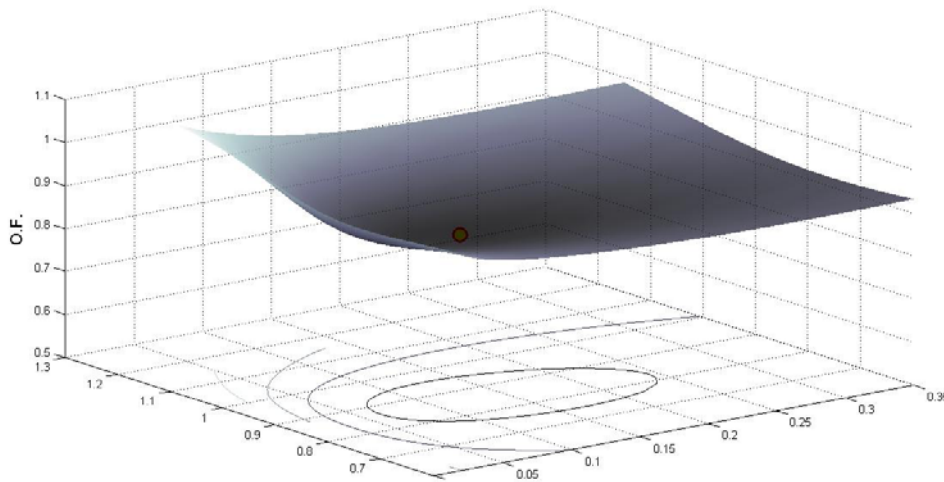


Figure 2. Conventional optimum solution in the space ρ_T, ξ_T .

4. Robust Design Based Optimization with Uncertain Bounded System Parameters

The probability theory is the most used approach for modelling the uncertainty involved in structural parameters. It is based on the assumption that uncertain parameters are random variables, and are characterised by a standard joint probability density distribution (PDF). This approach is intuitive, even if it offers analytical and numerical difficulties and presents some serious conceptual limitations, due to the impossibility of completely characterize random parameters, and also due to the complex statistical analysis involved. Nevertheless, the assumption of a given PDF cannot be justified for common situations, in which detailed statistical input data are not available. Moreover, a complete characterization of random variables requires knowledge of all infinite joint moments. This is an extremely difficulty and, sometimes, practically inapplicable procedure. A common simplification consists in assuming that all variables have independent normal or lognormal distributions, due to the application of the central limit theorem. This simplification is valid only for an asymptotic and well-defined random variables relation.

Alternative “non-probabilistic” approaches, usually called “possibilistic”, have also been

developed. These methods include the “convex modelling”, the “fuzzy set theory” and, finally, the “interval analysis” [17]. This last method assumes that the only knowledge of uncertain parameters is a finite hyper rectangle where they lie, and that its vertices are the only available information. A different and intermediate approach is based on the probabilistic description of uncertain parameters by using finite bounded distributions. In this method it is assumed that all the involved random parameters in structural problem are bounded in nature [18]. This is a realistic consideration, also because in many cases engineers can define a suitable and realistic variation range for all involved parameters. The bound limits can be assigned through experience and/or physical evidences. By means of this approach the uncertain parameters are defined on a finite hyper-rectangle only, as by in the interval analysis. Anyway, in any internal point of the hyper-rectangle the finite bounded distribution gives a finite probability and, therefore, a more qualified level of information. This simple approach can be associated with the uniform probability distribution in absence of any experimental or analytical information about mean and variance of the bounded variables. This means that every possible value has the same probability of happening. The expression for this affirmation is $1/\Delta$, where Δ is the variation range. The probability distribution could be more sophisticated and accurate in the sense of entropic information, as the *beta distribution*, in case of additional statistical information as for example mean and variance data.

For the design of structures with random parameters and subject to random dynamic loads, a possible approach is to define the optimum condition as the mean value that corresponds to deterministic optimum solution. However, the optimum solution obtained by minimizing the expected value of the objective function may be still quite sensitive to the fluctuation of the stochastic parameters, then causing scatter of the performance. Thus a more robust design concept has to be adopted to overcome this limitation. A solution that could be defined robust is that which characterises completely the OF as a random variable, so by knowledge of its probability density function. This way offers many difficulties and only in few cases it could be found in an analytical form. In order to overcome this limitation, the complete OF statistical description is replaced by the knowledge of its first two statistic moments, the mean value and the variance. Denoting with $\bar{\mathcal{G}}$ the random vector which contains the uncertain parameters, and with $p_{\bar{\mathcal{G}}}(\bar{\mathcal{G}})$ its joint probability density function, the first two statistical moments are

$$\mu_{OF}(\bar{b}) = \int_{\Omega_{\bar{\mathcal{G}}}} OF(\bar{b} | \bar{\mathcal{G}}) p_{\bar{\mathcal{G}}}(\bar{\mathcal{G}}) d\bar{\mathcal{G}} \quad (12)$$

$$\sigma_{OF}^2(\bar{b}) = \int_{\Omega_{\bar{\mathcal{G}}}} (OF(\bar{b} | \bar{\mathcal{G}}))^2 p_{\bar{\mathcal{G}}}(\bar{\mathcal{G}}) d\bar{\mathcal{G}} - \mu_{OF}^2(\bar{b}) \quad (13)$$

where $\Omega_{\bar{\mathcal{G}}}$ is the domain of the uncertain system parameters vector, and $OF(\bar{b} | \bar{\mathcal{G}})$ is the value of the OF evaluated under the assumption that parameters affected by uncertainty have

the value of $\bar{\mathcal{G}}$.

A possible method for developing a robust optimum design is to minimize the dispersion of the OF by means of a multi-criteria measure of the goal performance. By adopting this formulation, the proposed problem becomes a vectorial minimization, in which the two conflicting criteria are the mean and the RMD of the OF. The optimization problem, therefore, can be formulated as follows:

$$\text{Find } \bar{b} \in \Omega_b \quad (14)$$

$$\text{Which Minimizes } \{ \mu_{OF}(\bar{b}), \sigma_{OF}(\bar{b}) \} \quad (15)$$

Figure 3 shows that in the point where μ_{OF} reaches its minimum values, σ_{OF} has a not negligible value. This outcome indicates the final performance scatter due to uncertainty and therefore, a trade-off between them must be made.

In order to obtain a Pareto optimum set, a widely used method consists in replacing the vector of objective functions with a scalar function: in this work this is obtained by the conventional weighted sum method. The objective function for the robust optimization becomes:

$$OF_{rob}(\bar{b}) = \beta \mu_{OF}(\bar{b}) + (1 - \beta) \sigma_{OF}(\bar{b}) \quad (16)$$

where the weighting factor $\beta \in [0, 1]$ is the weight of each objective.

It must be noted that integrals (6) and (7) usually present serious analytical and numerical difficulties, especially when it is assumed that probability distributions has tails. For this reason, many approximate methods with different levels of difficulty and accuracy have been proposed. The perturbation method is computationally the least expansive and it needs only knowledge of the first two statistical moments, instead of complete joint PDF of uncertainty parameters. Moreover, it is reasonably applicable only for a limited number of cases and for relatively small levels of uncertainty [19].

Alternative approaches are based on expanding the conditional response moments for given values of the uncertain system parameters in terms of a series of orthogonal functions, but they are usually computationally expensive. Finally, asymptotic expansion approximations have been proposed in order to solve approximately the involved integrals [21-23].

In this work in order to overcome numerical and analytical difficulties in solving integrals (13) and (14), the uncertainty about the parameters is analysed by means of a simplified probabilistic model. It is assumed that these parameters are uncorrelated, uniformly distributed stochastic parameters with a given band amplitude, centred on nominal mean value:

$$p_{\theta_i}(\theta_i^0, \Delta_{\theta_i}) = \begin{cases} \frac{1}{\Delta_{\theta_i}} & \text{if } \theta_i \in \left[\theta_i^0 - \frac{\Delta_{\theta_i}}{2}, \theta_i^0 + \frac{\Delta_{\theta_i}}{2} \right] \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

where θ_i^0 is the nominal mean value of each uncertain parameter.

The OF mean and covariance can thus be directly evaluated numerically for each DV value by solving the two integrals:

$$\mu_{OF}(\bar{b}) = \int_{\theta_1^L}^{\theta_1^U} \int_{\theta_2^L}^{\theta_2^U} \dots \int_{\theta_{n_\theta}^L}^{\theta_{n_\theta}^U} \left(\frac{\sigma_{Y_s}(\bar{b}, \bar{\theta})}{\sigma_{Y_s}^0(\bar{\theta})} \right) p_{\theta_1}(\theta_1) p_{\theta_2}(\theta_2) \dots p_{\theta_{n_\theta}}(\theta_{n_\theta}) d\theta_1 d\theta_2 \dots d\theta_{n_\theta} \quad (18)$$

$$\sigma_{OF}^2(\bar{b}) = \mu_{OF}(\bar{b}) = \int_{\theta_1^L}^{\theta_1^U} \int_{\theta_2^L}^{\theta_2^U} \dots \int_{\theta_{n_\theta}^L}^{\theta_{n_\theta}^U} \left(\frac{\sigma_{Y_s}(\bar{b}, \bar{\theta})}{\sigma_{Y_s}^0(\bar{\theta})} \right)^2 p_{\theta_1}(\theta_1) p_{\theta_2}(\theta_2) \dots p_{\theta_{n_\theta}}(\theta_{n_\theta}) d\theta_1 d\theta_2 \dots d\theta_{n_\theta} - \mu_{OF}^2(\bar{b}) \quad (19)$$

where, n_θ is the number of uncertain parameters. Moreover, the upper and lower interval limits of each uncertainty parameter are:

$$\theta_i^L = \theta_i^0 - \frac{\Delta_{\theta_i}}{2} \quad (20)$$

$$\theta_i^U = \theta_i^0 + \frac{\Delta_{\theta_i}}{2} \quad (21)$$

5. Numerical Examples

In this work as case of study, it is considered that the parameters afflicted by uncertainty are the frequency ω_s of the main system and the mass ratio μ previous defined. These are collected in the uncertain vector $\bar{\mathcal{G}}$:

$$\bar{\mathcal{G}} = \begin{pmatrix} \omega_s \\ \mu \end{pmatrix} \quad (22)$$

Concerning the system frequency, it is well known that this is often difficult to predict accurately. The actual value is, in fact, usually determined by full-scale measurements after that the structure has been constructed, and it may vary with the time. For this reason, in many real applications the natural frequency of the TMD has been designed as tuneable on site [15].

Moreover, one should consider that also the mass of the main system may be afflicted by significant variation during the service life, especially in presence of time variable masses, as for example in civil buildings or bridges. For this reason, the mass ratio has been assumed as an uncertain parameter in this study. No uncertainty about the main system damping has been assumed, even if in real situations one should also consider that there are some uncertainties about the dissipation during the dynamic motion of the system. Nevertheless, different authors [16], [6] have observed that this parameter has a very limited influence on optimal TMD parameters.

Figures 3 show the mean and the RMS of OF obtained by using Eqs. (12) and (13) in the space of variables $\rho_T = \frac{\omega_r}{\omega_s^0}$ and ξ_T . The characteristics of the analysed system are given in Table 1.

Table 1. Characteristics of the analysed system

Deterministic parameters	Symbol	Deterministic value
Input filter damping ratio	ξ_f	0.4
Input filter frequency	ω_f	62.83 (rad/sec) and 20.04 (rad/sec)
Input White Noise intensity	S_0	1000 (cm ² /sec ³)
Main system damping ratio	ξ_s	5%
Nondeterministic parameters		
	Symbol	
Main system frequency nominal value	ω_s^0	45 (rad/sec)
Main system frequency band amplitude	Δ_{ω_s}	8,706 (rad/sec)
Mass ratio nominal value	μ^0	0,05
Mass damping ratio band amplitude	Δ_{μ}	0,017

Figures 3(a) and 3(b) correspond, respectively; to two different values of parameter $\psi_\omega = \frac{\omega_s^0}{\omega_f}$ that is the ratio between the nominal value of the main system frequency and the predominant frequency of the motion.

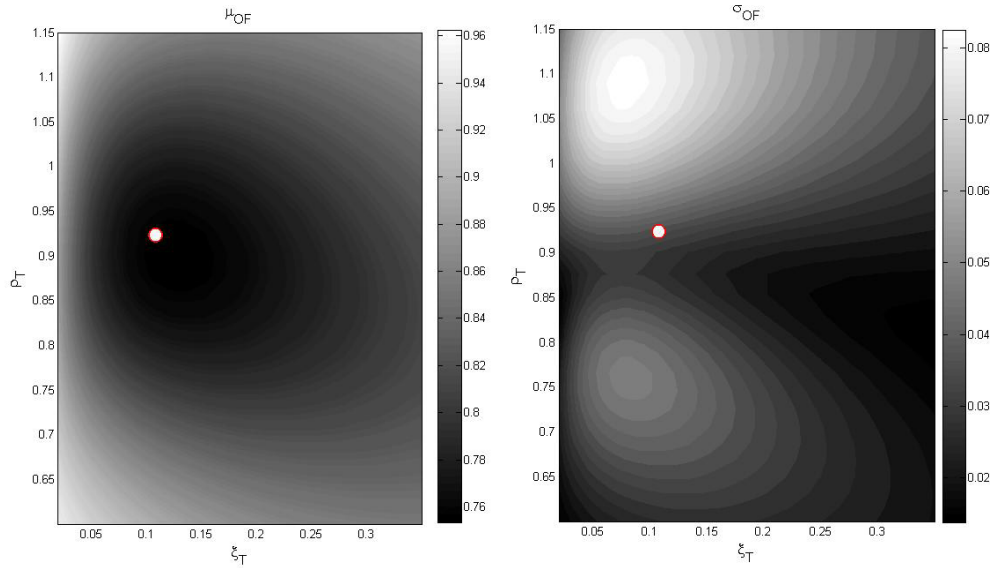


Figure 3(a). μ_{OF} and σ_{OF} in space of design variables ρ_T and ξ_T . $\psi_\omega = 0.71$

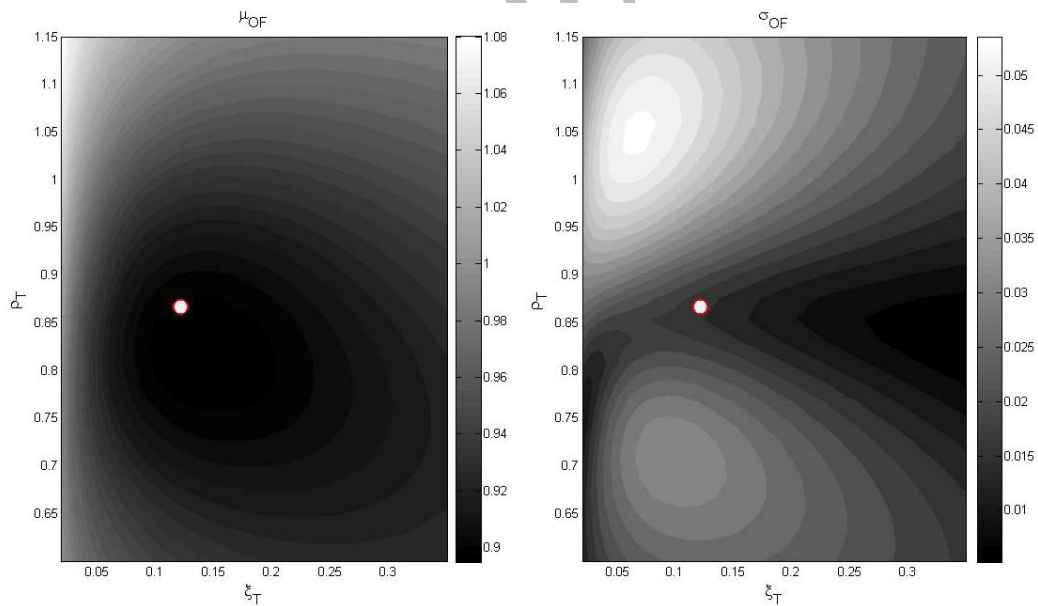


Figure 3 (b). μ_{OF} and σ_{OF} in the space of design variables ρ_T and ξ_T . $\psi_\omega = 2.2$

First of all, one must notice that the conventional solution, obtained by assuming that all involved parameters are deterministic (this solution is the white point in Figures 3(a) and 3(b)) does not correspond exactly to the minimum of the mean value of the OF. At the

contrary, the value of RMS of the OF is greater if compared to those relative to the minimum local point of μ_{OF} .

In the next part of this section the results of the sensitivity analysis carried out with regard the proposed TMD robust design method are shown. More precisely, the investigation concerns the uncertainty level of main system frequency and mass ratio, the weight factor and the frequency content of excitation. The parameters are listed in Table 1, where the nominal values of the main system frequency and mass ratio are reported together with the band amplitude in the case of uncertain parameters.

Concerning the level of uncertainty, three different uncertain configurations will be considered: these are defined by imposing that the mean value (equal to the nominal one in Table 1) and RMS are the only known parameters, and that the PDF has a uniform distribution, as stated before. By using this assumption, and considering that the distribution amplitude is $\Delta = \sigma\sqrt{12}$, one obtains the three configurations listed in Table 2, where

$$k_{\mu} = \frac{\sigma_{\mu}}{\mu^o} \tag{23}$$

$$k_{\omega_s} = \frac{\sigma_{\omega_s}}{\omega_s^o} \tag{24}$$

Table 2. Different levels of uncertainty of main system frequency and mass ratio

Parameters	k_{μ}	k_{ω_s}	$\Delta\mu$	$\Delta\omega_s$
Configuration a	0,050	0,050	0,009	7,379
Configuration b	0,075	0,075	0,013	11,67
Configuration c	0,100	0,100	0,017	15,58

Optimum solution has been investigated for different input filter frequencies. Specifically, the numerical analyses have been carried out with regard to two different input filter configurations, defined by means of the parameter $\psi_{\omega} = \frac{\omega_s^0}{\omega_f}$, assuming that the damping ξ_f is equal to 0.4.

In Table 3 the deterministic solution obtained by means of the conventional approach is shown, where it is assumed that the nominal values of uncertain parameters are representative of a deterministic condition, which is unaffected by uncertainty.

The optimum solution is expressed in terms of optimum value of design variables: optimum TMD damping ratio TDM ξ_T^{opt} , and optimum frequency ratio ρ_T^{opt} , which define

the optimum design vector \bar{b}^{opt} . In the following, the symbols $\xi_{T\ conv}^{opt}$ and $\rho_{T\ conv}^{opt} = \left(\frac{\omega_{T\ conv}^{opt}}{\omega_S^0} \right)$, collected in \bar{b}_{conv}^{opt} , will indicate the optimum solution in terms of design vector, in the case of deterministic conventional solution.

Table 3. Deterministic optimum solutions for two different soil configurations

ω_S^0 (rad/sec)	ω_f (rad/sec)	$\psi_\omega = \frac{\omega_s'}{\omega_f}$	$OF = \left(\frac{\sigma_{\omega_s}}{\sigma_{\omega_s^0}} \right)$	$\left(\xi_{T\ conv}^{opt} \right)$	$\rho_{T\ conv}^{opt} = \left(\frac{\omega_{T\ conv}^{opt}}{\omega_S^0} \right)$
45.0	62.8	0.71	0.7199	0.1079	0.9238
45.0	20.04	2.20	0.8811	0.1217	0.8667

Table 3 shows that in the hypothesis of conventional optimization a reduction of about 28% and 12% takes place for the covariance response. The different level of TMD efficiency depends on the parameter $\psi_\omega = \frac{\omega_s'}{\omega_f}$, which indicates the resonance condition of the system motion with respect to the ground one. Obviously, the maximum reduction takes place for the resonant system, which corresponds to $\psi_\omega = \frac{\omega_s'}{\omega_f} \approx 1$.

A comparative analysis on optimum solution has been carried out with the aim to show differences that take place if system parameters are considered as uncertain and not as deterministic quantities.

In Figure 4 the optimized OF is plotted as the parameter ψ_ω varies. More precisely, the Figure on the left shows three curves which represent, respectively, the conventional solution, which corresponds to the optimum solution \bar{b}_{conv}^{opt} (continuous line), the mean value of the OF evaluate in \bar{b}_{conv}^{opt} by considering the uncertainty in the parameters (dash-dot), and the mean value of the OF obtained in the case of complete robust optimization (dashed line).

Moreover, on the right of the same Figure there are shown the standard deviation $\sigma(OF)$ in case of robust optimization (dashed line) and the standard deviation $\sigma(OF)$ evaluated in \bar{b}_{conv}^{opt} by considering the uncertainty in the parameters (dash-dot).

From Figure 4 one can deduce that the deterministic solution in terms of OF, obtained by means of a conventional optimization, is underestimated if compared with its mean value evaluated by Eq. (5), obtained by adopting \bar{b}_{conv}^{opt} and then carrying out the optimization by considering the uncertainty in system parameters. Moreover, it is evident that if the uncertainty which affects the parameters of the system is not considered, the TMD

performance is overestimated. One should point out that a robust solution (the weight coefficient has been assumed in the example equal to 0.5) furnishes in mean a performance just little bit lower than that obtained by considering \bar{b}_{conv}^{opt} . On the contrary it is evident by analyzing $\sigma(OF)$ that the robust solution shows a scatter lower than those obtained by considering \bar{b}_{conv}^{opt} , that means the conventional optimization gives a solution less sensitive to variability of uncertain parameter.

Finally, with regard to the sensitivity analysis carried out with respect to ψ_ω , one can observe from Figure 4, as previously mentioned, that the higher performance is attained when $\psi_\omega = \frac{\omega_s}{\omega_f} \approx 1$. Moreover discrepancies between conventional and robust solution grow as this parameter increases.

Also $\sigma(OF)$ can be strongly influenced by ψ_ω . More precisely, when ψ_ω is smaller than 1, the values of $\sigma(OF)$ are not much different each other. Instead, if this parameter becomes greater than the unit, the difference between the robust solution and that obtained by assuming \bar{b}_{conv}^{opt} is more emphasized. Anyway, the results here obtained indicate that a major dispersion in TMD performance evaluation takes place if $\psi_\omega < 1$ when a conventional optimization is adopted rather than a robust one.

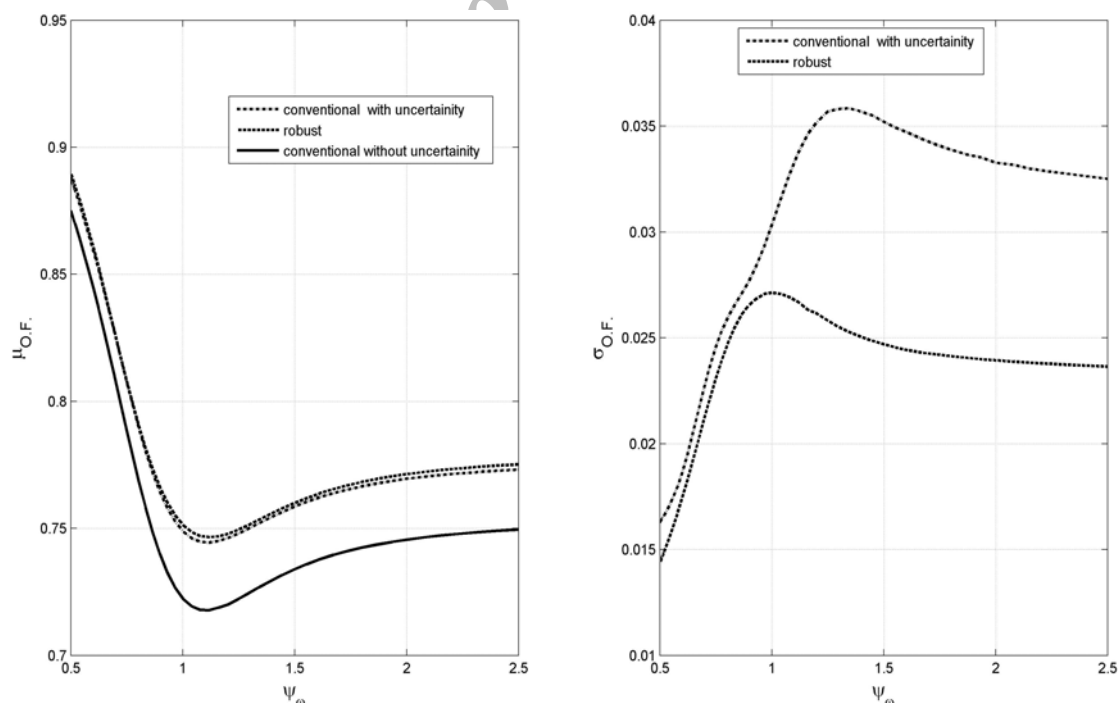


Figure 4. Mean value and variance of OF evaluated in case of conventional and robust optimization. It is assumed $\beta=0.5$

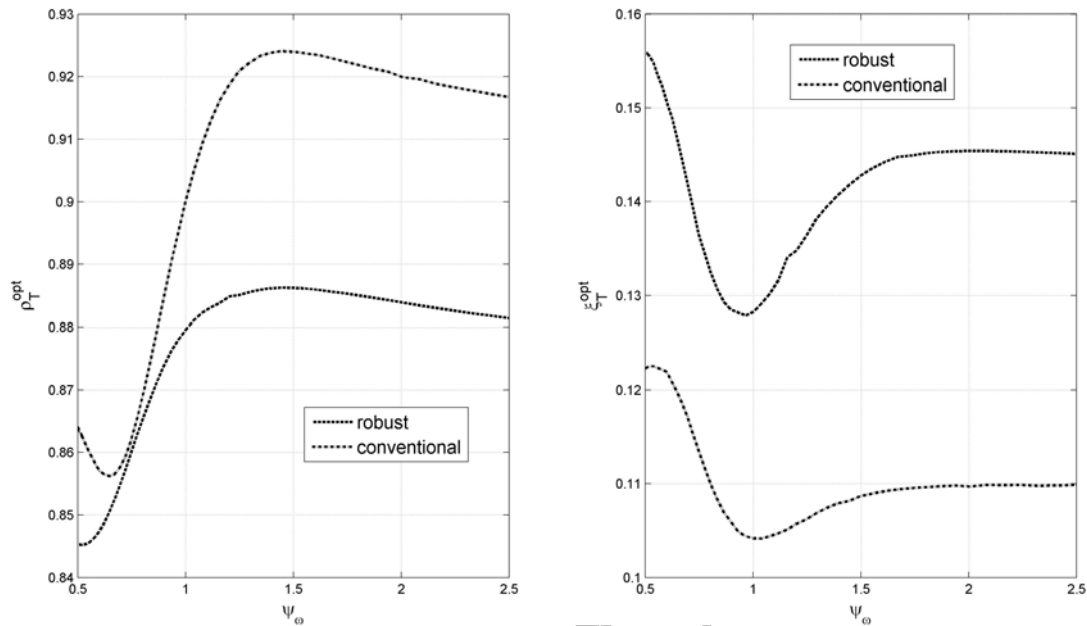


Figure 5. Optimum solution in terms of design variables in case of conventional and robust optimization. It is assumed $\beta = 0.5$

Concerning optimum design variables, from Figure 5 one can deduce that robust and conventional solutions give dissimilar results. More precisely, for what concerns the design variable $\rho_T^{opt} = \left(\frac{\omega_T^{opt}}{\omega_S^0} \right)$, the robust solution overestimates this parameter if compared with the robust one (moreover, as previous shown, the performance is larger). The difference between the two solutions grows up as the parameter ψ_ω increases and then remains almost constant.

With regard the optimum TMD damping ratio $\xi_T^{opt}_{conv}$, from Figure 5 one can deduce that the conventional optimum values are underestimated with respect to the robust solution. Also for this parameter the difference between robust and conventional becomes larger as ψ_ω increases.

In Figures 6(a) and 6(b) the results of the multi-objective robust design of TMD is shown for three uncertain configurations (Table 2) and for two different values of ψ_ω . More precisely, optimum Pareto fronts are obtained for $\psi_\omega = 2.2$ (in Figure 6(a)) and for $\psi_\omega = 0.71$ (Figure 6(b)). The weight coefficient has been assumed $\beta = 0.5$. Different lines correspond to various uncertainty levels.

The conventional optimum solution, that is a single OF evaluated without considering any uncertainty of system parameters is also represented by the single dash-dot vertical line. Moreover, the points represent the mean and the RMS of the OF, obtained by adopting the

conventional solution \bar{b}_{conv}^{opt} and then taking into account the uncertainty of system parameters.

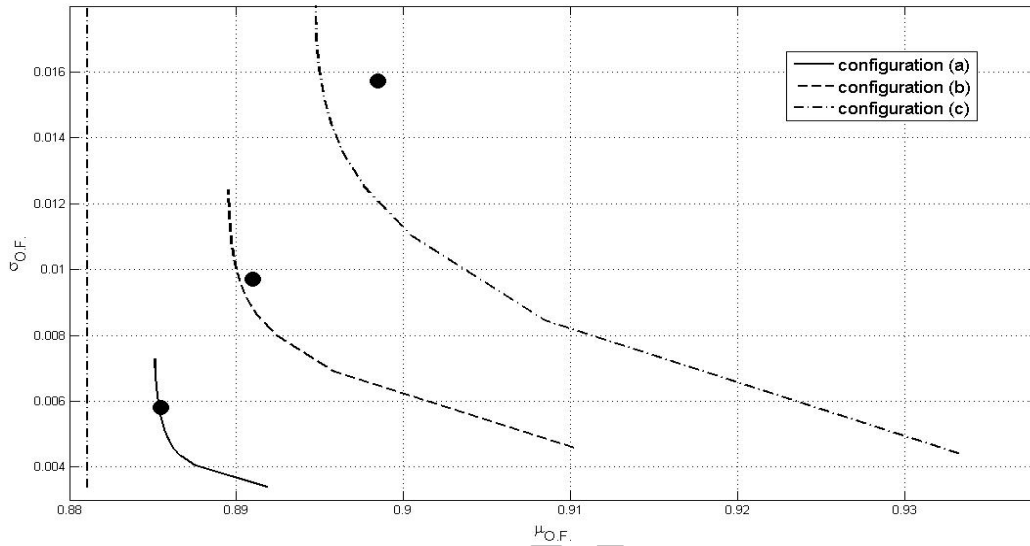


Figure 6(a). Pareto set evaluated for different uncertain configurations given in Table 2 ($\psi_\omega = 2.2$)

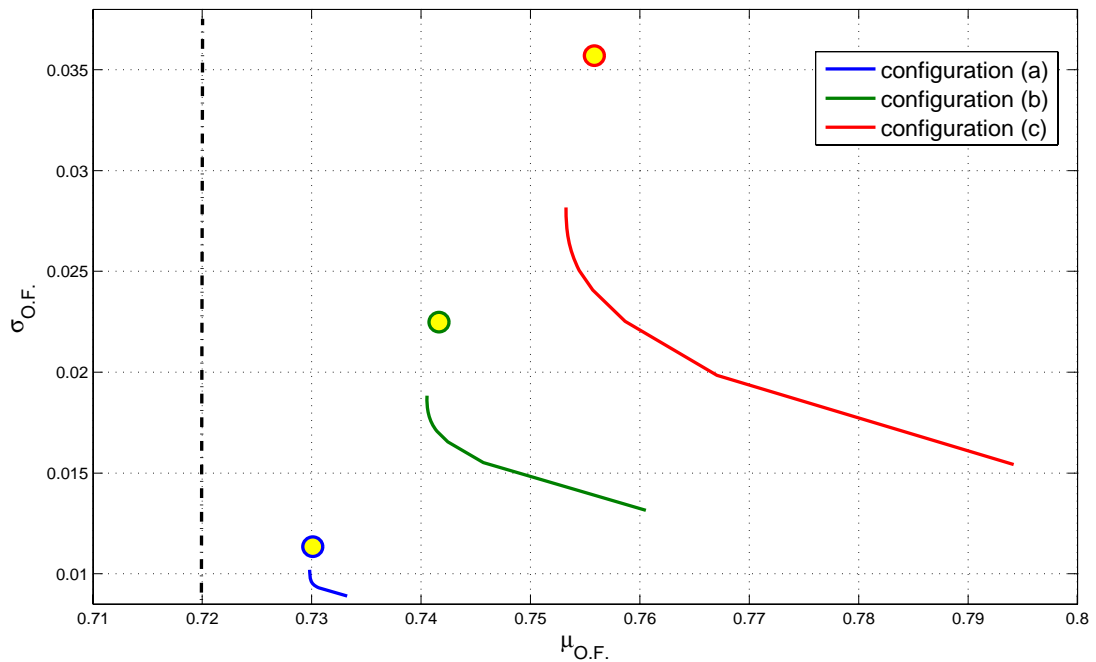


Figure 6(b). Pareto set evaluated for different uncertain configurations given in Table 2 $\psi_\omega = 0.71$

From Figures 6(a) and 6(b), one can deduce that the conventionally optimized OF overestimates TMD performance with respect to real situation, in which uncertainty of system parameters is considered. This outcome is evident by observing that all Pareto fronts are on the right of the vertical line which indicates the conventional OF value. Moreover, the distance between this line and the Pareto front points measures the difference between real (with uncertainty) and purely ideal (without uncertainty) solution. This distance obviously tends to grow up with the increase of the level of the uncertainty.

Furthermore, one can observe that Pareto fronts become wider as the uncertainty level increases; moreover, as the level of the uncertainty grows up, the same performance in mean value is obtained with a greater RMS.

In addition, one can observe that the variation of the OF mean value and RMS is narrow in the configuration (a), while this variation becomes wider in the case of configuration (b) and, especially in (c). Points which correspond to the conventional solutions are always dominated in relation to the corresponding Pareto's fronts obtained through a robust method. This means that it is possible by means of robust solution to obtain better performance without increase the scatter of solution. Greater levels of uncertain structural parameters lead to greater dominance of the solutions. Particularly, in the case of configuration (a), the points corresponding to the conventional solution lie in the front (Figure 6(a)), while in case of (b) and (c) they clearly lie inside the feasible domain.

Regarding the analysis developed in this work, one can deduce that the conventional solution is not different from that obtained by means of a multi-objective robust approach, only in the case of low levels of uncertainty. In this condition the Pareto front is narrow and the conventional solution is localized on the same front. Moreover, the distance between the conventional solution and each point of Pareto front is moderate. On the contrary, in case of high levels of uncertainty, the conventional solution does not represent the real TMD efficiency and the solution lies more inside the feasible domain.

In Figures 7(a) and 7(b) ($\psi_\omega = 2.2$ and $\psi_\omega = 0.71$, respectively) the optimized μ_{OF} and σ_{OF} (continuous lines) are plotted for different values of weight factor β . In these Figures the conventional solutions are also represented (dashed lines). One can immediately point out that by increasing the weighting factor value, the robust optimal solution tends to reduce the OF mean value, but at the same time it increases its RMS. This result means that a more robust optimal solution is characterised by a general reduction of structural performance, expressed by the OF mean value. This is, on the other hand more stable and less sensitive to uncertainty sources. This variation in consideration is expressed by a general reduction of OF RMS. These variations in OF first and second statistical moments are more evident for greater uncertainties (configuration b and c), while they tend to be negligible for low levels of uncertainty (configuration a).

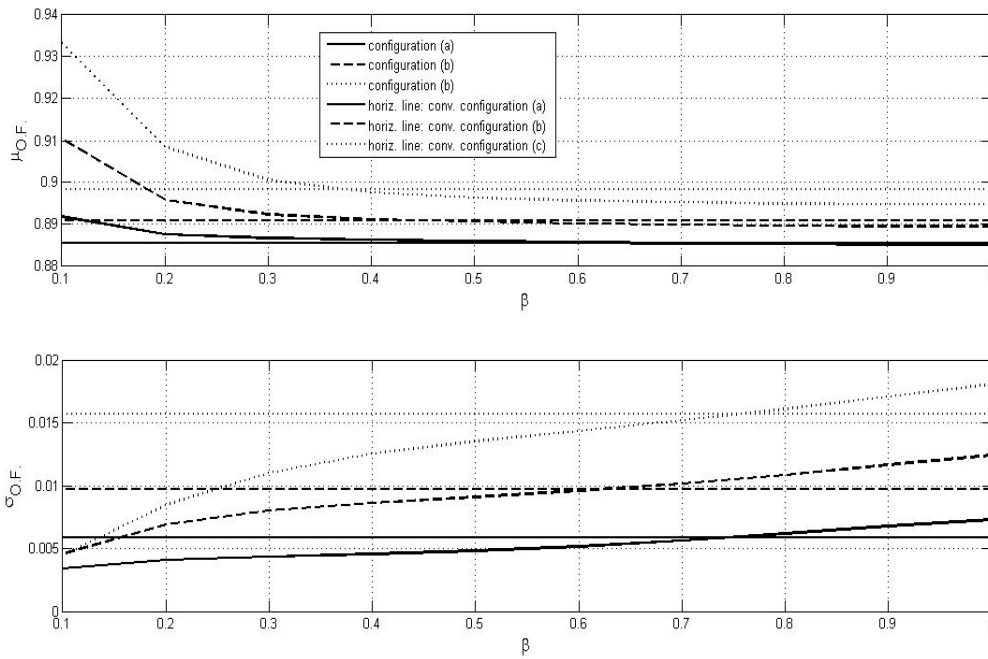


Figure 7(a). Variation in optimum OF mean and RMS for different weight values β . It is assumed $\psi_{\omega} = 2.2$

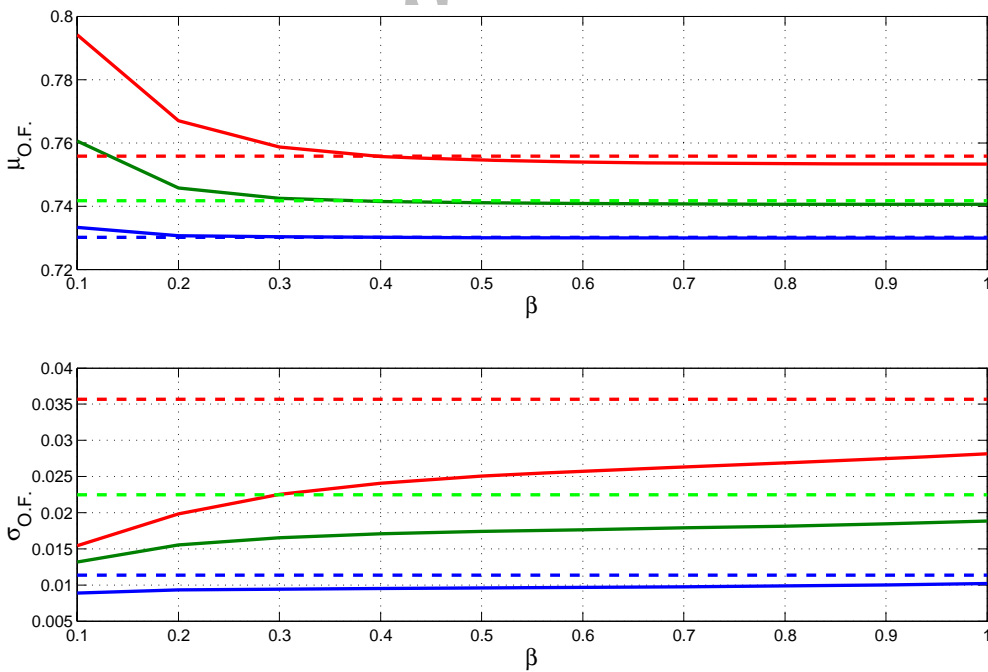


Figure 7(b). Variation in optimum OF mean and RMS for different weight values β . It is assumed $\psi_{\omega} = 0.71$

Finally, elements of the optimum robust design vector \bar{b}_{rob}^{opt} are plotted in Figures 8(a) ($\psi_\omega = 2.2$) and 8(b) ($\psi_\omega = 0.71$), where also the conventional solution is shown. Concerning the optimum TMD frequency ratio ρ_T^{opt} , one can deduce that this parameter shows two different trends as β varies in relation with the value assumed by ψ_ω . More precisely if $\psi_\omega \leq 1$ (Figure 8(a)) when β increases the optimum TMD frequency first grows up, reaching a maximum value, then it decreases. If $\psi_\omega > 1$ (Figure 8(b)), ρ_T^{opt} decreases by the increase of weight factor β . Moreover, one can notice that as the uncertainty level increases from configuration a) to configuration c) the optimum TMD frequency ratio decreases. Furthermore, it can be larger or smaller with respect to the conventional configuration in relation to the value assumed by β , for $\psi_\omega = 2.2$, whereas for $\psi_\omega = 0.71$ the conventional solution gives always values greater than the robust ones.

With regard the optimum TMD damping ratio, the robust solution supplies always a solution larger than those obtained by conventional optimization. Moreover, values decrease with the increase of the weight factor β . Uncertainty configuration can have an important effect: the greater is the uncertainty level, the greater is the required optimal damping for TMD device. This trend is the same in both frequency ratio ψ_ω configurations here considered.

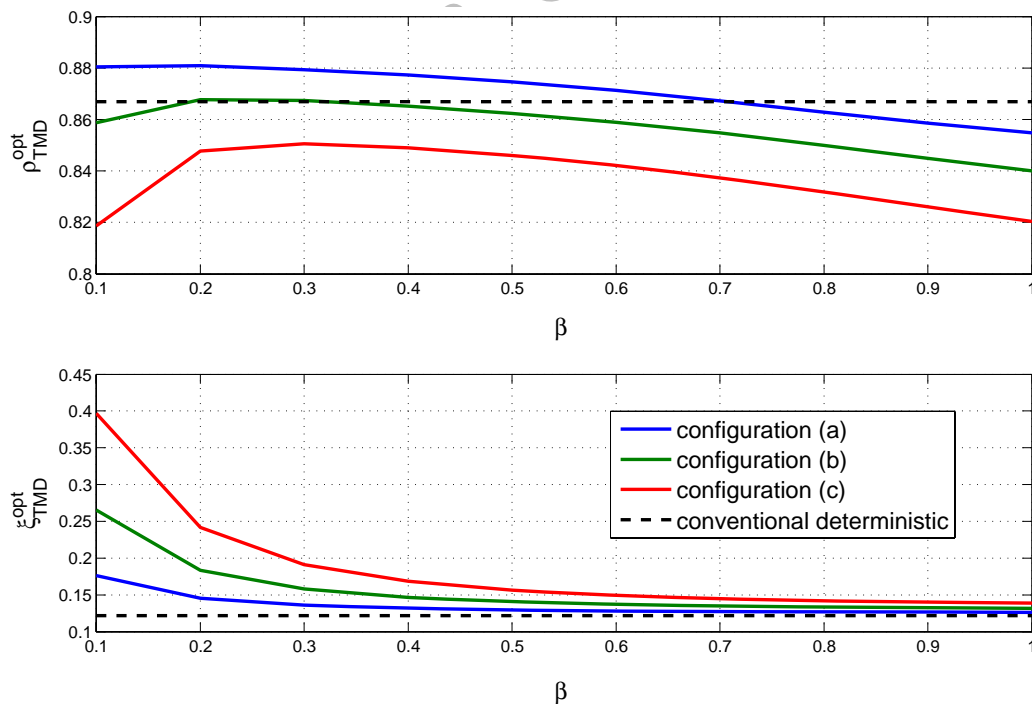


Figure 8(a). Sensitivity of optimum design vector elements with respect to β ; $\psi_\omega = 2.2$

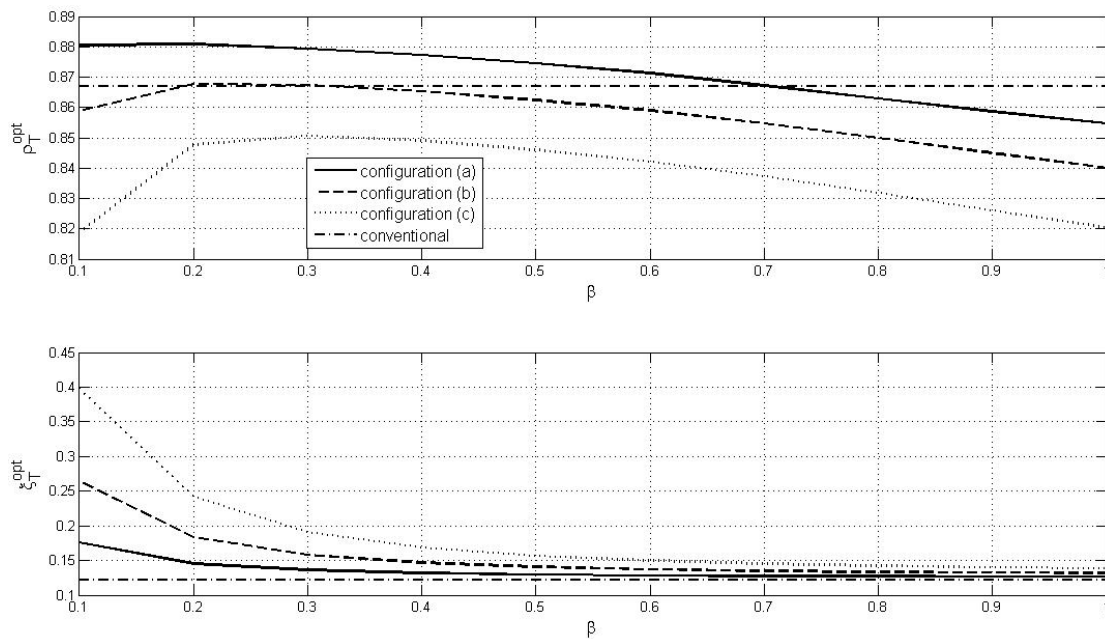


Figure 8(b). Sensitivity of optimum design vector elements with respect to β ; $\psi_{\omega} = 0.71$

Conclusions

A robust optimal design criterion for a single TMD device intended for suppressing the vibration level in dynamic systems is here proposed. Robustness is obtained by finding solutions insensitive to variation of system parameters due to uncertainty. The dynamic input is represented by a random base acceleration modelled by a stationary filtered stochastic process. The main system is described by a single degree of freedom; its mass and stiffness are assumed to be affected by uncertainty so that the main system frequency and the TMD mass ratio are represented by random uniform bounded stochastic variables. The performance index is assumed to be the factor of reduction of system displacement. In order to obtain a robust optimum design, mean and standard deviation of Objective Function are then evaluated. Robust optimization is formulated as a multi objective optimization problem, in which both the mean and the standard deviation of the deterministic OF are minimized. It is assumed that uncertain parameters are described by two independent random variables with uniform distributions, so that only the distribution extremes are needed for a complete statistical characterisation.

Some interesting conclusions can be drawn with regard to the results obtained. First, results attained have showed an improvement in the evaluation of performance of TMD, and a limitation of OF real values dispersion, especially if compared with standard conventional solutions. In fact, robust solutions are able to control and to reduce the final OF dispersion, by limiting the standard deviation. Results have pointed out that the deterministic solution in terms of OF, obtained by means of a conventional optimization, is underestimated if

compared with its mean value obtained by adopting \bar{b}_{conv}^{opt} (conventional solution) and then carrying out the optimization by considering the uncertainty in system parameters. Moreover, it is evident that if the uncertainty which affects the parameters of the system is not considered, the TMD performance is overestimated.

The proposed method can also be suitably used when more accurate information about uncertain parameters are known, for instance by using different probability distributions such as *beta*. Finally, the number of uncertain sources can be incremented to take into account more parameters, without a serious computational cost increment.

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