# SENSITIVITY OF THE OPTIMUM DESIGN OF REINFORCED CONCRETE FLAT SLAB BUILDINGS TO THE UNIT COST COMPONENTS AND CHARACTERISTIC MATERIAL STRENGTHS

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## Abstract

In this paper the influence of the unit cost of steel, concrete and formwork and the characteristic strengths of steel and concrete on the optimum design of reinforced concrete flat slab buildings is investigated. Size optimization of flat slab buildings according to the British Code of Practice is carried out. The objective function is the total cost of the building including the cost of floors, columns and foundations. The total cost of the building includes the cost of material and labour for concrete, reinforcement and formwork. Excavation cost is also considered in the cost of foundations the optimization process is handled in two levels. In the first level the optimum cross-sectional dimensions of reinforced concrete elements is determined using a hybrid algorithm based on genetic algorithm . In the second level an exhaustive search is applied to seek the optimum size and number of steel bars for each individual type of structural elements. A practical example is given to demonstrate the achieved cost saving and sensitivity of the optimum design to unit cost items and the characteristic strengths of steel and concrete.

**Keywords**: Structural optimization; genetic algorithm; hybrid optimization algorithm; flat slab buildings; sensitivity of the optimum design

## 1. Introduction

A reinforced concrete (RC) flat slab building is a kind of building in which floors are directly supported by columns without the use of intermediary beams. To increase punching shear resistance of flat slabs, columns may be flared to form a column head or column capital or the slab may be thickened around columns as a drop panel or both (Figure 1). Flat slab systems are popular for use in office and residential buildings, hospitals, schools and hotels. They are quick and easy to formwork and build. The architectural finish can be directly applied to the underside of the slab. Absence of beams allows lower story heights

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and as a result, cost saving in vertical cladding, partition walls, mechanical systems, plumbing and a large number of other items of construction especially for medium and high-rise buildings. They provide flexibility for partition location and allow passing and fixing services easily. Windows can be extended up to the underside of ceiling. The absence of sharp corners gives greater fire resistance and less danger of concrete spalling and exposing the reinforcement. Moreover, where the total height of a building is restricted, using a flat slab will result in more stories accommodated within the set height [1-3].

Design optimization of isolated reinforced or pre-stressed concrete slabs has been widely studied [4-8]. Skelton [9] carried out optimum design of RC slab and beam structures according to the British Standard (CP 114). Adib [10] studied minimum weight design of a RC flat plate floor including column weights at the top and bottom of the floor to the American Standard (ACI 318-71). This paper presents cost optimization of RC flat slab buildings according to the British Code of Practice for design and construction of reinforced concrete structures, BS 8110, [11]. The objective function is the total cost of the building including the cost of material and labour for concrete, reinforcement, formwork of floors, columns and foundations and also cost of foundation excavation. Sensitivity of the optimum design to unit cost of materials and the characteristic strengths of steel and concrete is studied in a main part of this paper.
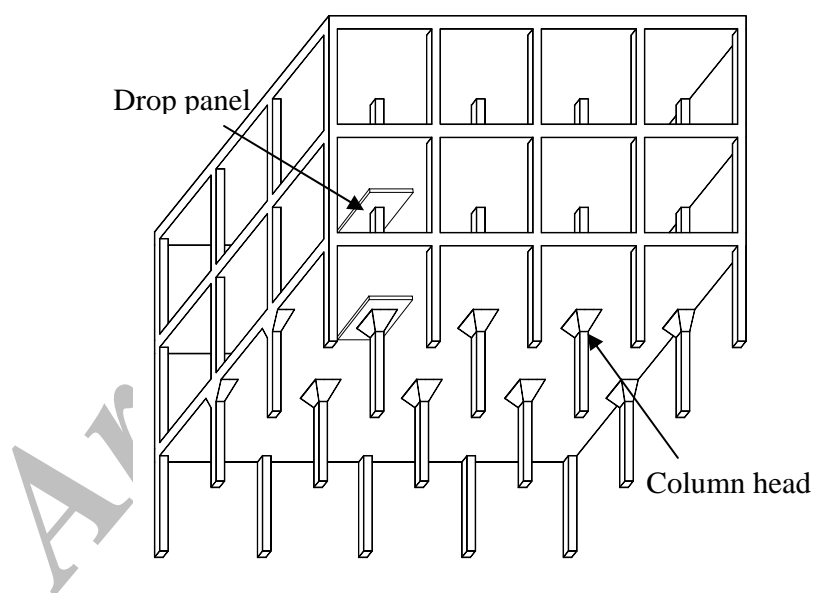


Figure 1. A typical flat slab building

## 2. Statement of the Problem

Cost optimization of a RC flat slab building, as shown in Figure 2, with a typical rectangular plan of $n_x$ and $n_y$ spans of lengths $l_x$ and $l_y$ in $x$ and $y$ directions respectively is carried out. The building can have many storeys of arbitrary heights.

### 3. Design Variables

Figure 2 illustrates design variables in a typical floor of a flat slab building. The thickness of the floor slab, $t_i$, and the number and size of reinforcement in different positions over the floor slab are design variables for a typical floor slab.
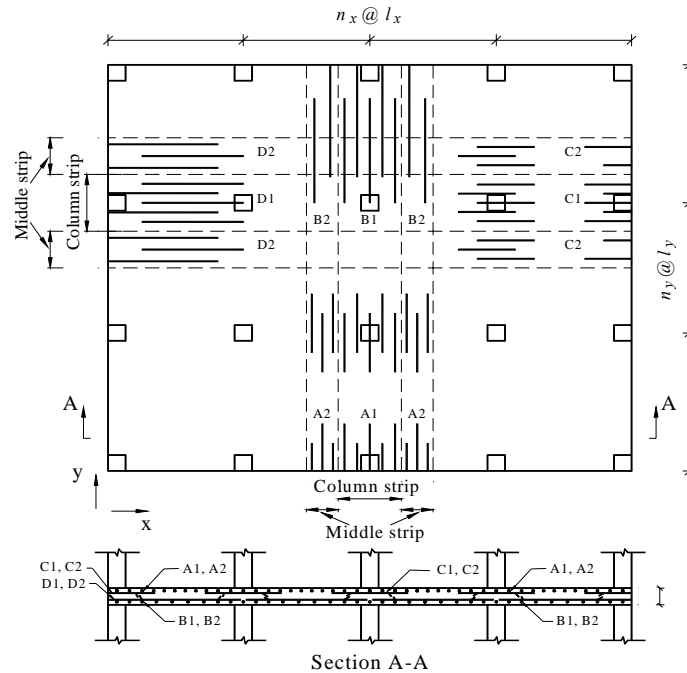


Figure 2. Illustration of design variables in a typical floor slab

Figure 3 shows a typical layout of shear reinforcement around a column-slab connection. The size of reinforcing bars, $\phi$, for the connection and the number of reinforcement in each layer, $N_1$ $N_2$ etc. are considered as design variables. The number of required layers of shear reinforcement for each column-slab connection depends on the magnitude of punching stresses around column. The use of column head can also be considered as an optional feature for increasing punching strength of slabs (Figure 4). The cross-sectional dimensions of a column head in contact with the floor slab are the design variables for the column head. A typical reinforcement detailing is considered for column heads. Therefore, the number and size of column head steel reinforcement are not considered as design variables. Four types of column-slab connections have been considered in each floor. These are a corner connection, two edge connections for columns located in longitudinal and transversal sides of the building and an intermediate connection.

Figure 5 shows design variables for a column. To simplify the problem, it is assumed that all columns have rectangular cross-section, in a cross-section of column all reinforcement have same diameter and they are symmetrically concentrated in four corners of the section.

Since it has been assumed that lateral loads are resisted by shear walls or other systems capable of withstanding lateral forces there is no considerable shear force in the column section, therefore the size and space between the links are calculated according to the Code recommendations to prevent outward buckling of the longitudinal bars and providing ductility of columns. Four different typical columns are considered in each storey, which are corner column, two edge columns in longitudinal and transversal sides of building and one intermediate column.
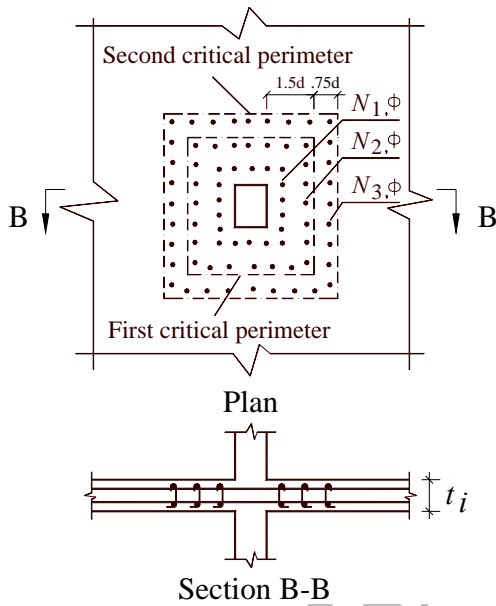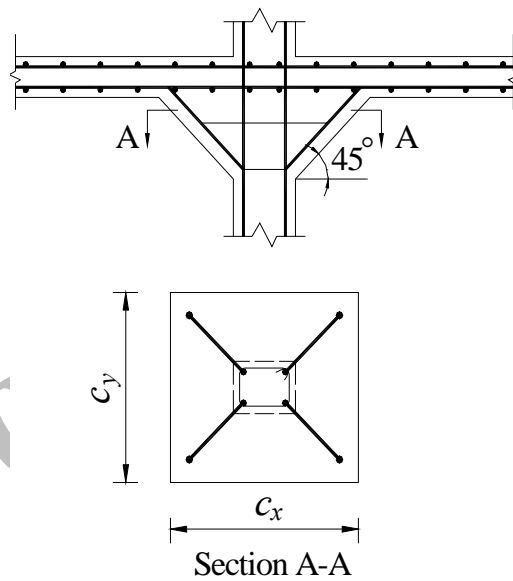


Figure 3. Shear reinforcement around columns
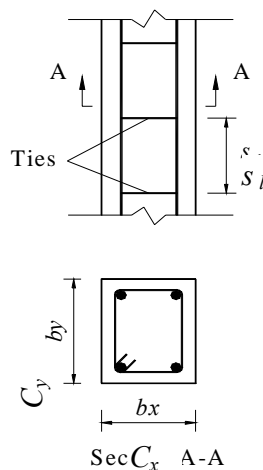


Figure 4. A typical column head



Figure 5. Details of a typical column

## 4. Objective Function and Design Constraints

The objective function is the cost of labour and material for concrete, reinforcement and formwork for, $n_f$, floors, $n_c$ typical columns and foundations and the cost of the foundation excavation for a quarter of the building as follows:

$$C = \sum_{i=1}^{n_f} C_i(x_f) + \sum_{j=1}^{n_c} C_j(x_c) + C_f(x) \qquad (1)$$

subject to:

$$G_i(x_f, x_c) \leq 1 \qquad i = 1, 2, \ldots, n_g, \qquad (2)$$

$$x_j^l \leq x_j \leq x_j^u \qquad j = 1, 2, \ldots, n_s, \qquad (3)$$

$$x = (x_1, x_2, \ldots, x_n) = (x_f, x_c). \qquad (4)$$

The first term in the Eq. (1) represents the sum of the cost of floors, the second term is the sum of the cost of all typical columns and $C_f$ is the total cost of the foundation for a quarter of the building. In the Eq. (2) $G_i$ is the $i$-th behavioural constraint function which is resulted from a design provision. Eq. (3) shows a side constraint on the $j$-th design variable, $x_j$. In these equations $n_g$ and $n_s$ are the number of behavioural and side constraints, respectively. The foundation cost is approximately calculated by assuming that, the type of all foundations is pad and they are identical. The cost of shear reinforcement around columns has been also included in the cost of floors. The vector of design variables includes two components, which are $x_f$ for design variables of flat plate floors and $x_c$ for design variables of columns. Design constraints are introduced according to BS 8110 [11] code requirements. These design constraints includes strength, serviceability, stability and ductility requirements. Also, architectural and practical considerations are applied to the problem. The possibility of decreasing the column dimensions from lower floors to upper floors but not in the opposite direction and identical slope of 45° for all lateral sides of the column capitals are two of these practical consideration.

In order to handle the constraints, the problem has been converted to an unconstrained problem using the exterior penalty function method. Here the penalized objective function, $\tilde{C}(x)$, is defined by adding a penalty for each constraint violation to the objective function, $C(x)$, as follows:

$$\tilde{C}(x) = C(x) + r \sum_{i=1}^{m} \Phi_i(x) \qquad (5)$$

where $r$ is the penalty multiplier, $m$ is the number of constraints and $\Phi_i$ is the $i$-th penalty function which can be expressed in a general form as follows:

$$\Phi_i(\mathbf{x}) = [\max((G_i(x) - 1), 0)]^n \qquad (6)$$

where $n$ is the power of penalty function and $G_i(x)$ is the value of the $i$-th constraint. A Comprehensive discussion on the appropriate values of penalty multipliers and the mathematical form of the penalty function has been presented in reference [12].

## 5. Genetic Algorithms

GAs are numerical optimization techniques inspired by the natural evolution laws [13]. A GA starts searching design space with a population of designs which are created over the design space at random. In the basic GA, every individual of population is described by a binary string formed by 0s and 1s. GA uses three main operators: selection (reproduction), crossover (recombination) and mutation to direct the density of the population of designs towards the optimum point.

In the selection process some individuals of a population are selected by some randomized method as parents to create the next generation. The fitter individuals have a greater chance to be selected.

Crossover allows the characteristics of the designs to be altered, depending on the crossover probability, $Pc$, for creation of a better generation of designs. In this process different digits of binary strings of each parent are transferred to their children (new designs produced by the crossover operation).

Mutation is an occasional random alteration of the value of some digits in binary code strings of some randomly selected individuals. The mutation operation changes each bit of string from 0 to 1 or vice versa depending on the mutation probability, $Pm$. Mutation can be considered as a factor preventing from premature convergence.

## 6. Design Optimization Procedure

The design optimization procedure is handled in two levels. In the first level the section dimensions of the columns and the thickness of slabs are found. In this level a hybrid optimization algorithm based on a GA is employed. The algorithm includes two stages. In the first stage a modified GA is initially used for a global search to find the optimum or a near-optimum solution for the cross-sectional dimensions of RC elements. In the second stage this solution is considered as a base point for a local search by a discretized form of Hooke and Jeeves method [14].

Two modifications have been applied in a basic GA. The first modification is that GA starts by a larger size of randomly created individuals (designs) over the design search space [15] and then in second generation a certain number of the best designs are selected to carry on the rest of the GA process. The second modification limits the number of copies of each group of designs with the same fitness to one. In this manner the population size decreases dynamically during the GA process, but it is not permitted the population size becomes less

than a predefined minimum allowable population size [8].

In the second level, using an exhaustive search method [16], the optimum amount of reinforcement (the number and diameter of steel bars) for each group of members with given dimensions and the column capital dimensions are determined.

A full discussion about the optimization algorithm employed for design optimization of concrete flat slab buildings has been presented in reference [17]. In this research a specific computer program for analysis and design optimization of flat slab buildings based on above mentioned procedure has developed. To find more information and details about the program refer to [18].

## 7. Sensitivity of the Optimum Design to Unit Costs of Materials and Material Strengths

The influence of the unit cost of steel, concrete and formwork and the characteristic strength of steel and concrete on the optimum design of a RC flat slab building is investigated by a numerical example. This design example has been chosen from a report on the comparative costs of concrete and steel framed office buildings [19] that has been recommended to be a benchmark for future studies. The conventional design of this example has been carried out by a team of professional engineers [19]. The building includes three identical storeys, each of 3.95 m height. A typical plan of the building is as shown in Figure 6. The live load on intermediate floors is 5.0 KN/m$^2$ and on the roof is 1.5 KN/m$^2$. Dead loads are self-weight and the imposed dead load of 1.5 KN/m$^2$. In this study the average of the unit prices for each material presented in the aforementioned report, is used in the design optimization of the building. On this basis the unit prices of materials and labours for concrete, shear and main reinforcement and formwork have been considered as $u_c$=53.5£/m$^3$, $u_f$=0.4 £/kg and $u_f$=18.5£/m$^2$, respectively. The average unit cost of foundation excavation is composed of different items, therefore, an average unit cost equal to 18.5 £/m$^3$ is considered for foundation excavation which also includes cost of disposal and backfill of soil. The characteristic strengths of main and shear reinforcement and concrete are $f_y = 460$ N/mm$^2$, $f_y= 250$ N/mm$^2$, $f_{cu} = 35$ N/mm$^2$, respectively. Cover of steel bars of the floors is 25 mm and of the columns is 40 mm. Minimum and maximum bar diameter for main reinforcement of floors and columns and shear reinforcement are 10, 25 and 10, 32 and 6, 12 mm, respectively. The allowable bearing pressure of soil is 200 kN/m$^2$. There is no strap between pad foundations.

Figure 7 shows the comparison of the total cost components of concrete, reinforcement and formwork of the structure obtained from conventional and optimum designs. The breakdown of costs of the floors and columns is also shown in this Figure. The total cost of the superstructure according to the conventional design is 55.46 £/m$^2$, and according to the optimum design is 42.57 £/m$^2$. As a result, design optimization of the structure has been led to 23.3% saving. According to this Figure the cost of floors and columns is about 89% and 11% of the total cost for the conventional design and 91% and 9% of the total cost for the optimum design, respectively. Those results show that the cost of floors is the major part of the structural cost and emphasises the importance of the optimization of floors in the flat

slab buildings, as concluded by other researchers [19,20]. It can be observed that the largest component of overall cost is the cost of formwork (39% and 51% for the conventional and optimum designs, respectively). The cost of concrete contributes 33% and 36% of the structural cost for the conventional and optimum designs, respectively. The smallest component is the cost of reinforcement being 28% and 13% for the conventional and optimum designs, respectively. The largest cost saving is achieved by reducing the cost of reinforcement (63% cost saving). Since the main part of the formwork cost relates to soffit of the floors which is identical for both of the conventional and optimum designs, the least cost saving is related to formwork (0.5% cost saving).
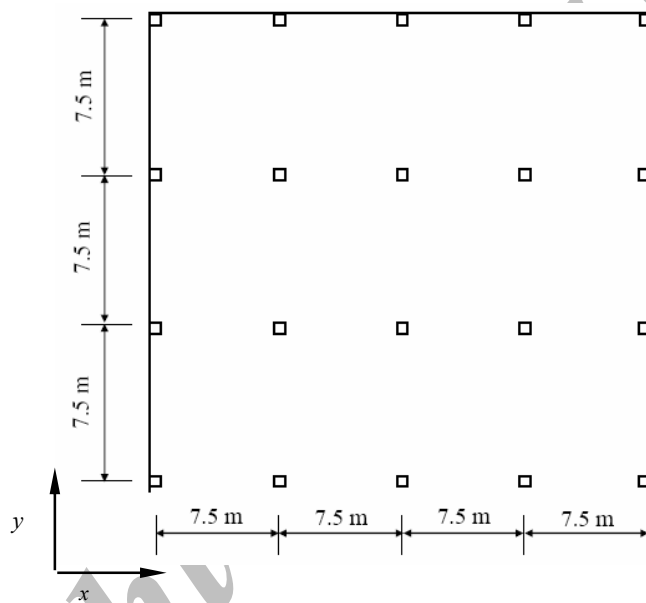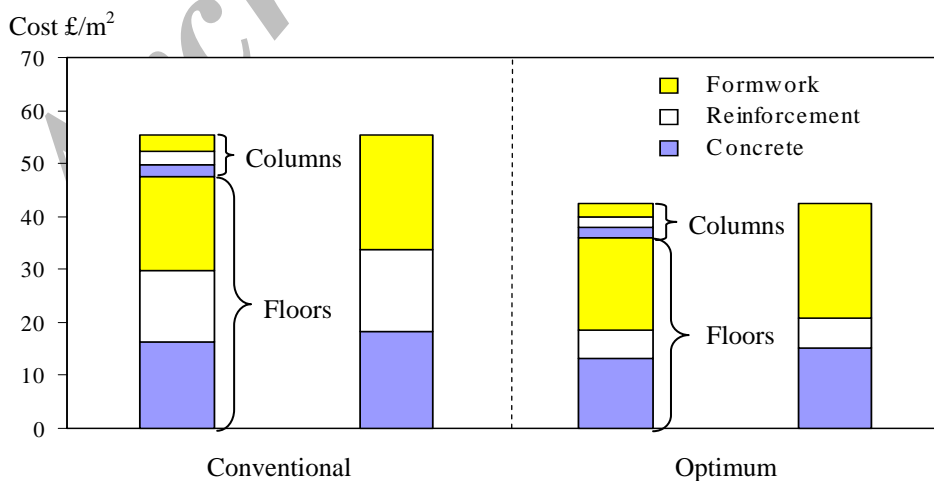


Figure 6. Plan of the building



Figure 7. Comparisons of cost items

Figure 8 shows the variations of the minimum structural cost against different concrete and reinforcement unit prices. Figure 8(a) has been obtained using a linear relationship characteristic strength of concrete and its price (Case 1). In Figure 8(b), the relationship between $u_c$ and $f_{cu}$ is non-linear (Case 2). In this Figure, the value of $u_c$ changes according to cement content of different concrete grades [20]. The base for variation of $u_c$ with respect to $f_{cu}$ in Figure 8(c) is the list of prices obtained from the Hanson Quarry Products Europe Limited (Case 3, a quotation). Table 1 shows the relationship between the characteristic strength of concrete and its unit price for Cases 2 and 3. As Figures 8(a) to c show with increasing $f_{cu}$ the minimum structural cost first decreases to a certain value and then starts to increase. The reason for this trend can be explained by looking at the variation of the amount of concrete and reinforcement with respect to $f_{cu}$ for the optimum structure. Figures 9 and 10 show the variation of concrete volume and reinforcement weight in the optimum structure per unit area of the building against $f_{cu}$, respectively. In these Figures the value of $u_r$ is 0.5 £/kg and the relationship between $f_{cu}$ and $u_c$ is as presented as Table 1 (Case 2). As Figure 9 indicates the concrete volume first follows a decreasing trend and then it remains almost constant. Figure 10 shows that the reinforcement weight is almost constant with the variation of $f_{cu}$. It can be concluded that when the amount of saving on structural costs due to reduction of materials consumption is balanced with the amount of increasing the structural cost due to using an expensive concrete, the optimum $f_{cu}$ is achieved. Consequently, the gradient of the curves in Figures 8(a) to c becomes zero when $f_{cu}$ approaches its optimum value. The optimum characteristic strength of concrete for a given building can be affected by the way of changing the concrete price with respect to its strength. For the Hanson Company prices, the optimum $f_{cu}$ is about 35 N/mm$^2$ while in the other two cases it is about 25 N/mm$^2$.
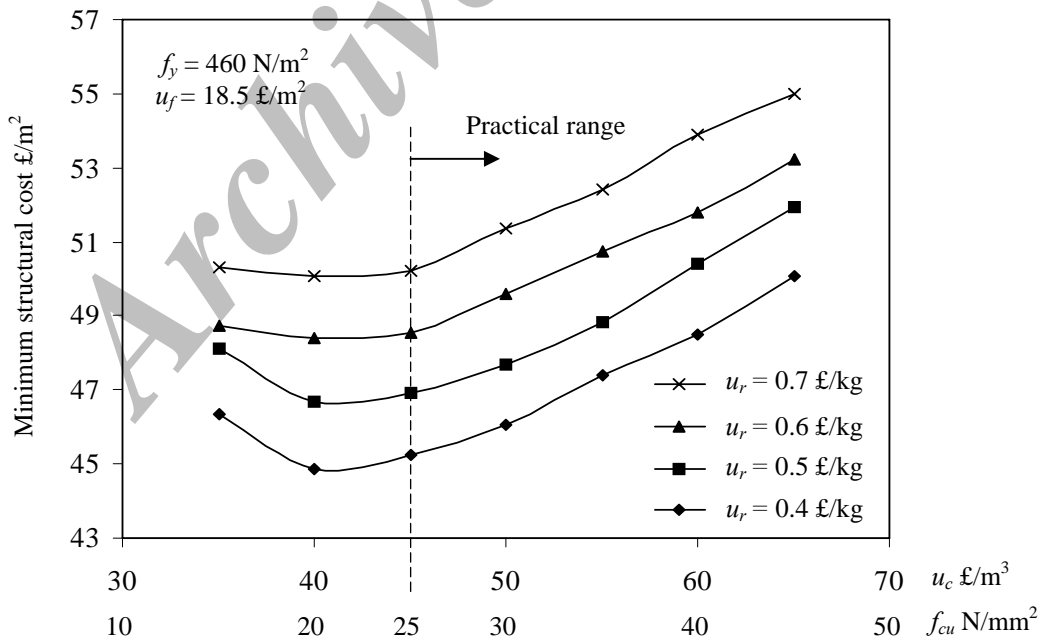


a) $u_c$ changes with respect to $f_{cu}$ linearly (Case 1)
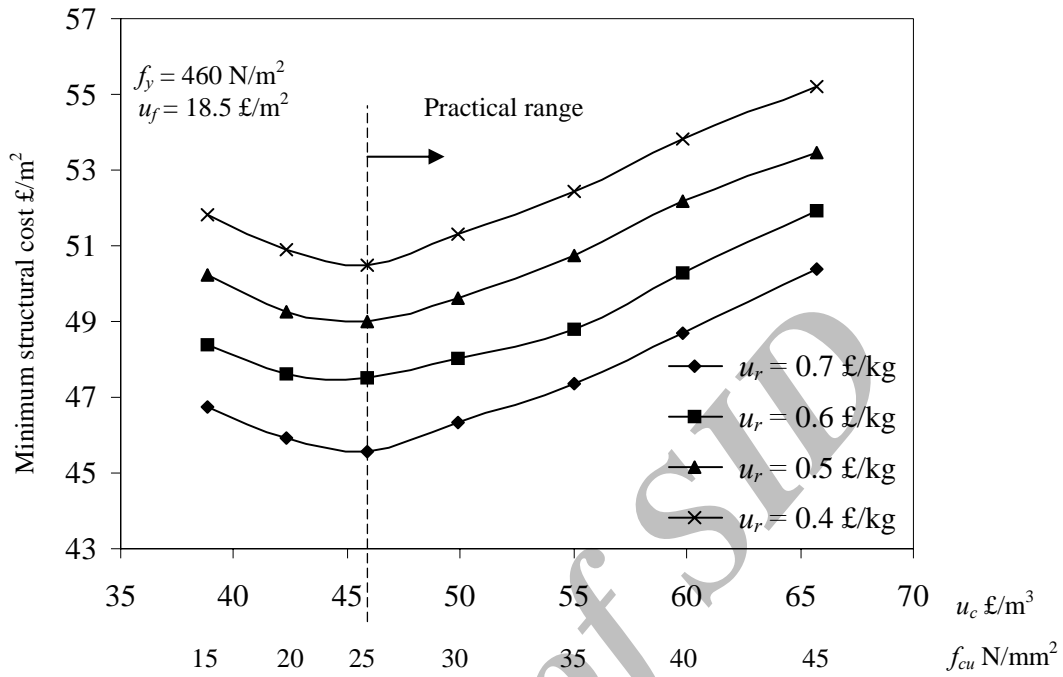
b) $u_c$ changes with respect to $f_{cu}$ depending on the cement content of concrete (Case 2)
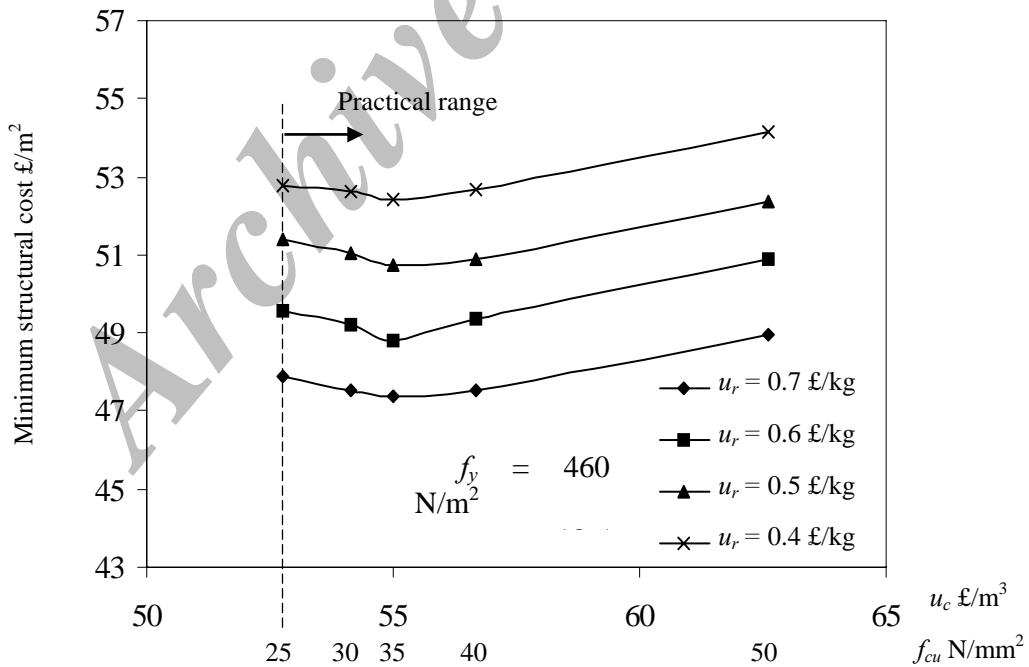


(c) $u_c$ changes with respect to $f_{cu}$ according to Hansons company prices (Case 3)

Figure 8. The minimum structural cost versus the unit cost of concrete

Table 1. Variation of unit price of concrete with respect to its strength

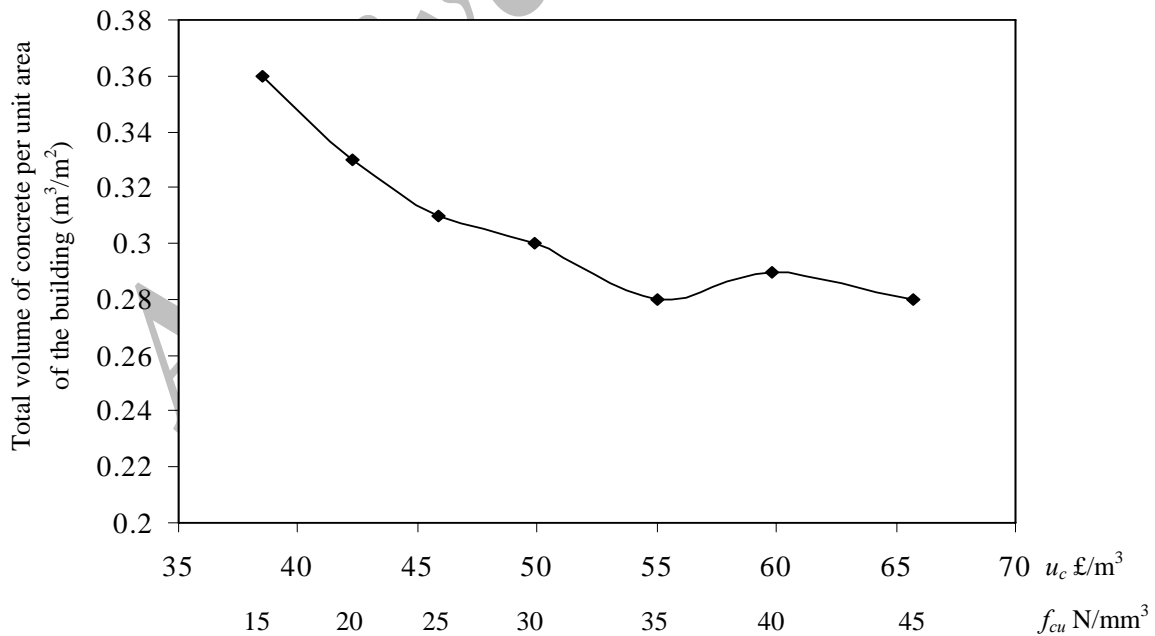| $f_{cu}$ N/mm$^2$ | Neville (Case 2) | | Hanson company (Case 3) |
| --- | --- | --- | --- |
| | Cement content (Kg/m$^3$) | Unit cost of concrete ($u_c$, £/ m$^3$) | Unit cost of concrete ($u_c$, £/ m$^3$) |
| 50 | 513 | | 62.6 |
| 45 | 454 | 65.7 | - |
| 40 | 406 | 59.8 | 56.67 |
| 35 | 355 | 55 | 55 |
| 30 | 315 | 49.9 | 54.16 |
| 25 | 279 | 45.9 | 52.75 |
| 20 | 244 | 42.3 | - |
| 15 | | 38.8 | - |



Figure 9. Variation of the total volume of concrete in the optimum structure per unit area of the building versus the unit cost of concrete and concrete strength
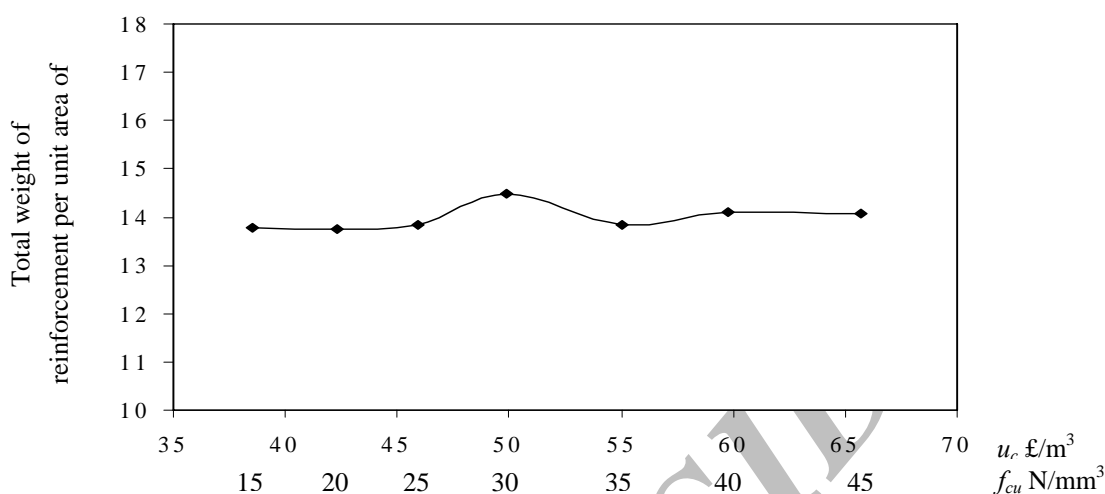
Figure 10. Variation of the total weight of reinforcement in the optimum structure per unit area
of the building versus the unit cost of concrete and concrete strength

Figure 11 shows the effect of the unit cost of reinforcement on the minimum structural cost for different concrete qualities. As expected, the higher the unit cost of reinforcement, the higher the minimum structural cost for the same characteristic strength of concrete and unit concrete price. The linear variation of minimum structural cost against $u_r$ means that the total volume of concrete and weight of reinforcement for the optimum design for different $u_r$ in the given range are almost fixed. Figures 12 and 13 present the total volume of concrete and the total weight of reinforcement per unit area of building for the optimum structure with respect to $u_r$, respectively. In these Figures $f_{cu}$, $u_r$ and $u_f$ are fixed at 35 N/mm$^2$ 55 £/m$^3$ and 18.5 £/m$^2$, respectively. As it is seen the amount of concrete and reinforcement in optimum structure is almost constant over the studied range of the variation of $u_r$.

Figure 14 shows the variation of the minimum structural cost against the unit cost of concrete for two different kinds of reinforcing steels used for floor slab reinforcement. In this Figure, a linear relationship between concrete strength and its price has been assumed. According to the Foregale Company's quotation, the unit prices of reinforcement for both kinds of steel are identical. As it is seen with increasing the yield strength of reinforcement from 250 N/mm$^2$ to 450 N/mm$^2$, the optimum value of $f_{cu}$ has changed from 175 N/mm$^2$ to 220 N/mm$^2$. This means that with high strength concrete the use of high strength steel with yield strength more than 460 N/mm$^2$ can be more economical.

The unit price of concrete of a given grade can vary in terms of place and time. Figure 15 shows the variation of the minimum structural cost versus the unit cost of concrete when $f_{cu}$ is fixed at 35 N/mm$^2$. The nearly linear variation of the cost of the optimum structure with respect to $u_c$ indicates that the volume of concrete and the weight of reinforcement are almost constant or, in other words, the optimum size of structural elements is almost fixed in the given range of the variation of $u_c$. It has been stated that for simple structural elements the optimum design is relatively insensitive to even large changes in unit costs [21].

In Figure 16 the influence of the characteristic strength of concrete on the minimum

structural cost is presented. For higher concrete strengths, the variation of the minimum structural cost is becoming smaller. Therefore, in this range, any effort to increase the quality of concrete does not result in large savings.
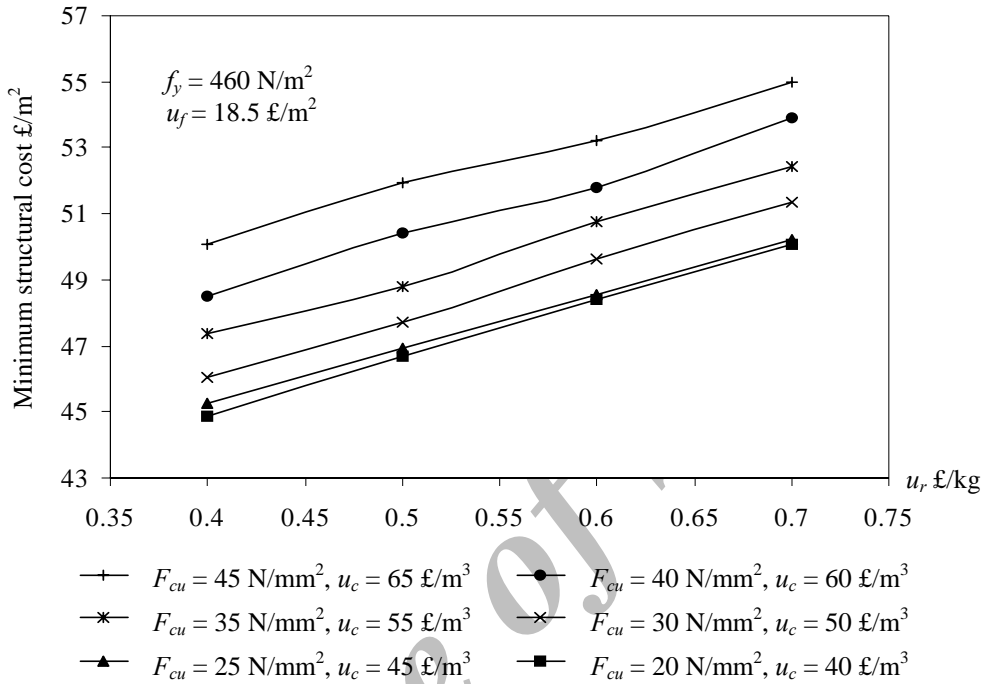


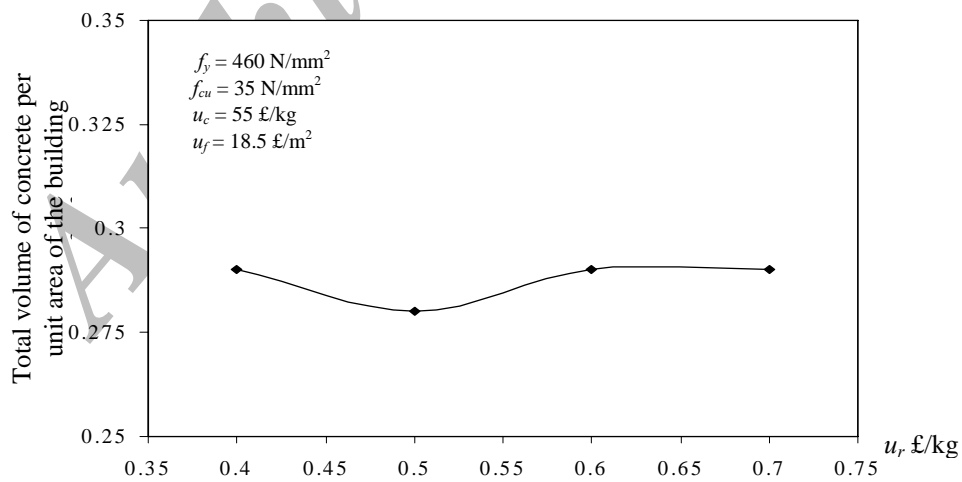Figure 11. The minimum structural cost versus the unit cost of reinforcement



Figure 12. Variation of the total volume of concrete in the optimum structure per unit area of the building versus the unit cost of reinforcement
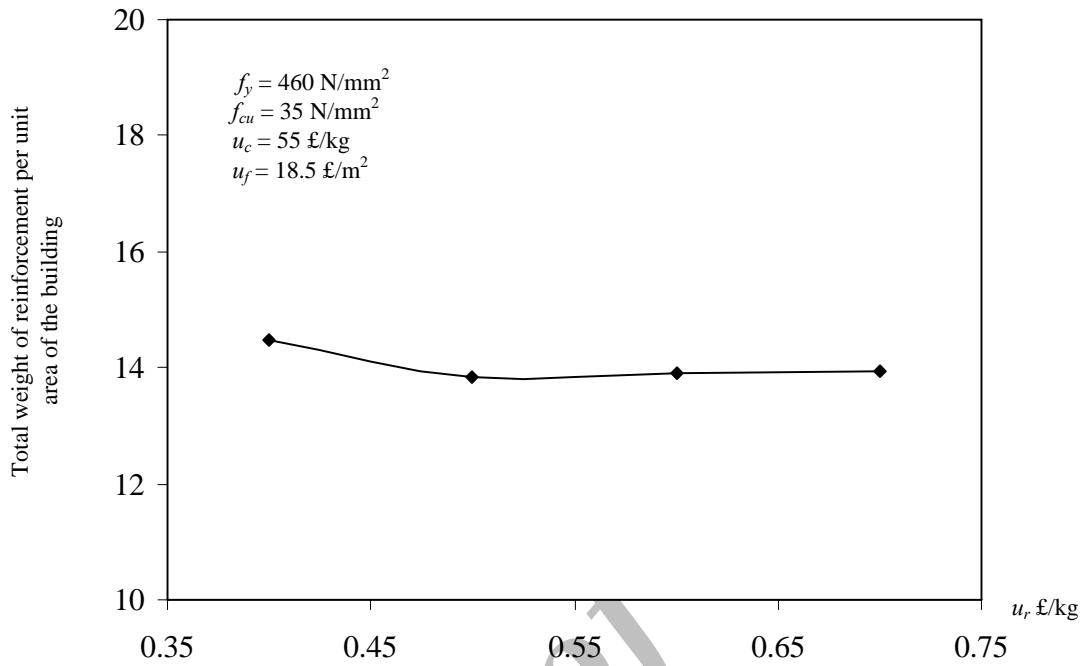
Figure 13. Variation of the total weight of reinforcement in the optimum structure per unit area
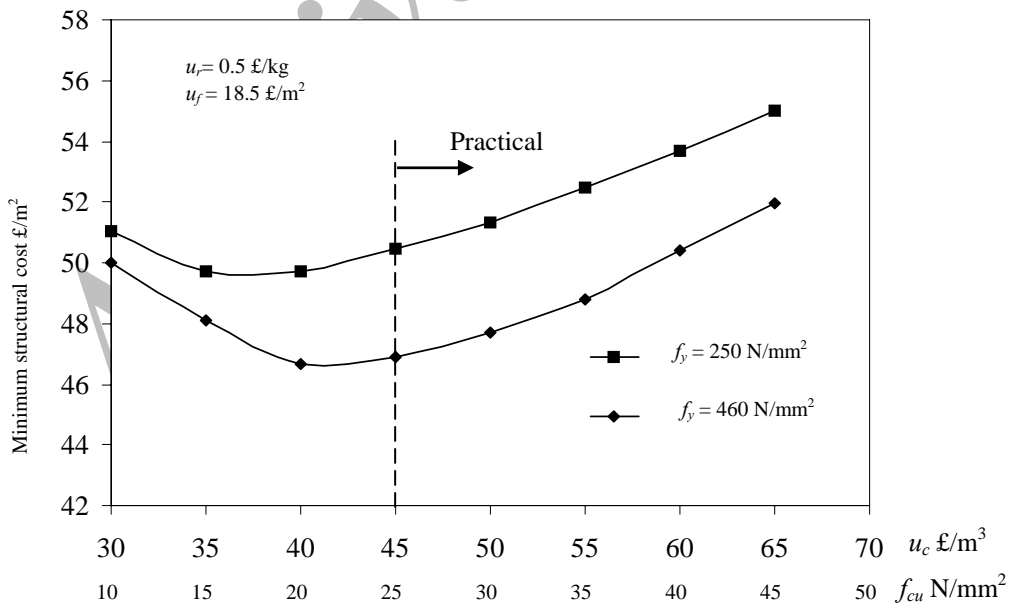of the building versus the unit cost of reinforcement



Figure 14. The minimum structural cost versus the unit cost of concrete for two kinds of steel
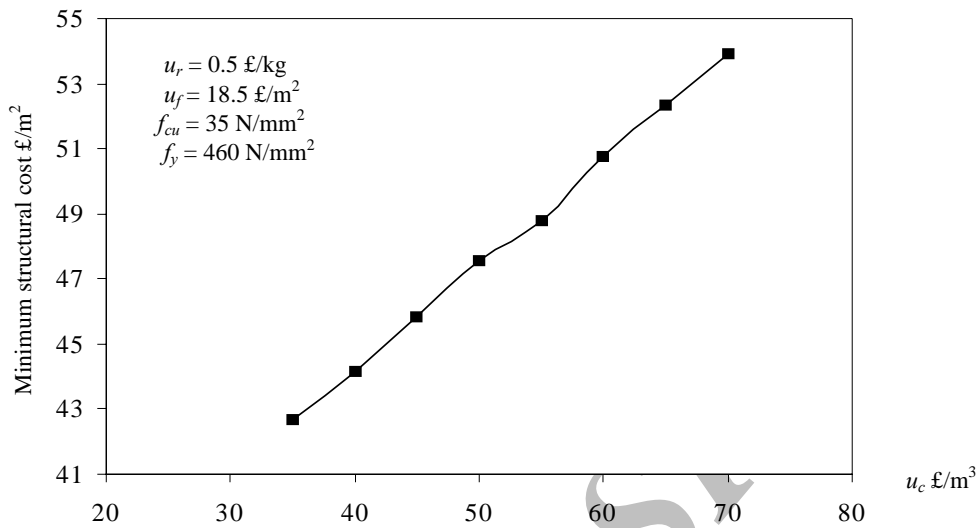used for floor slab reinforcement

Figure 15. Variation of the minimum structural cost versus the unit cost of concrete of a given grade
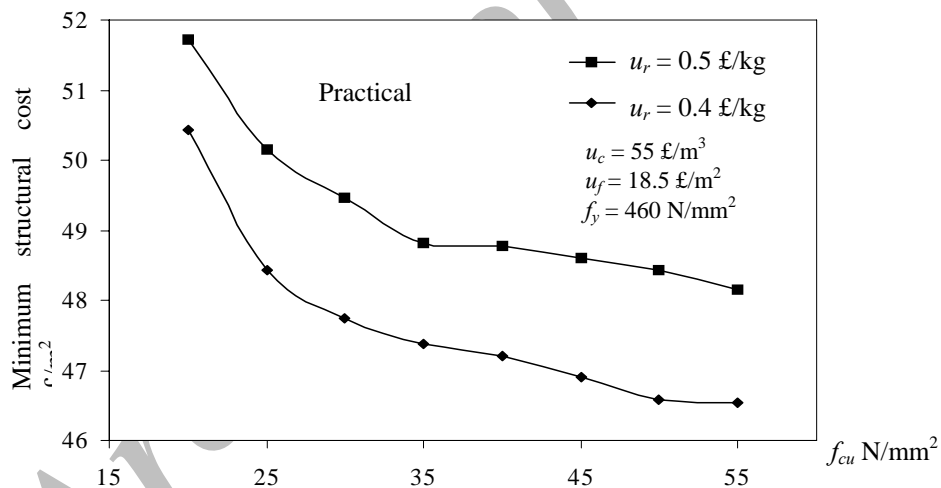


Figure 16. The minimum structural cost versus the characteristic strength of concrete

## 8. Conclusions

Cost optimization of RC flat slab buildings using a multilevel optimization procedure was presented. The procedure includes finding the cross-sectional dimensions and reinforcement of the concrete elements. Sensitivity of the optimum design to unit cost components and strength of materials was studied. The main conclusion can be summarized as follows:

- Cost of floor slabs is the major structural cost components which can result in a

significant cost saving in optimization of RC flat slab buildings.

- Although the cost of the optimum structure changes with the variation of the unit cost of materials but, in a practical range of the variation of the unit costs the amount of concrete and reinforcement is almost fixed in the optimum structure. This fact indicates that the optimum solution does not change significantly depending on when and where the building is to be built.
- The increase in characteristic strength of concrete does not necessarily produce a more economical structure; the optimum characteristic strength of concrete depends on the assumed relationship between the unit cost and the strength of concrete.
- To have an economical design by increasing the characteristic strength of concrete the characteristic strength of steel is also need to be increased.

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