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UNCERTAINTY ANALYSIS OF CORRODED REINFORCED CONCRETE STRUCTURES

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Abstract

In this paper, a new methodology is presented to study the corrosion of RC structures in chloride laden environments under concerned uncertainties. To deal with the involved uncertainties, fuzzy random variables are exploited. An effective computational procedure named as α-level optimization is also introduced and utilized to analysis the FR fundamental equations of corrosion process. The proposed system is applied to predict the various life cycles of corrosion affected RC structures. Moreover, an illustrative example is presented to demonstrate the applicability of proposed method in service life assessment of the RC beams.

Keywords: RC structures; corrosion; chloride; uncertainty; fuzzy; randomness; α-level optimization

1. Introduction

All over the world, RC structures deteriorate because of environmental effects and a great amount of budget is annually spending on maintenance, repair and rehabilitation of these defected structures. Nowadays, chloride-induced corrosion of the reinforcing steel is known to be a major cause of premature rehabilitation of many RC structures [1]. Ingress and attacks of chloride ions on the RC structures are resulted in terrible corrosion damages like the cases in the region of Persian Gulf, Mexican Gulf, and other harsh regions [1-7]. In Iran, many giant RC infrastructures like jetties, shipping ports, refineries, and the like have been constructed in the harsh environment of Persian Gulf and many of them have to be rehabilitated because of the damages due to the chloride-induced corrosion [2]. In this view, the current study tries to study the chloride-induced corrosion and then propose a model to predict the service life of RC structures for future cost-optimization purposes. The proposed

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method considers the uncertainties of the problem both in random and fuzzy characteristics. In this regard, fundamental equations of corrosion process are re-formulated based on fuzzy random variables. To handle theses relationships, α-level optimization (ALO) procedure is utilized to analysis the service life of corroded RC structures in corrosion initiation and corrosion propagation (cracking/unserviceability) periods.

2. Electrochemical Corrosion of Steel in Concrete

Corrosion of the steel embedded in the concrete is generally known to be an electrochemical process as shown in Figure 1. Because of hydroxide ingredients of the concrete like $Ca(OH)_2$, PH of the concrete media would be high [8]. On the other side, due to the high alkalinity of the concrete, the reinforcing steels remain passive because of the formation of the passive oxide layer on the steel-concrete interfacial zone (ITZ). The steel bars remain in passive state until PH of the concrete media is dropped below the specific amount. In this time, the passive oxide layer at ITZ breaks down and the corrosion of steel will be initiated. Here, the role of acidic ions like CO^{2-} and Cl^{-} may be interpreted as decrescent agents of alkalinity of the concrete media to initiate the corrosion of the embedded steel bars. In abstract, Figure 2 shows the minimum prerequisites of the corrosion process [9]. After initiation of the corrosion, in presence of the oxygen, humidity and suitable temperature, the anodic and cathodic reactions of the corrosion continue to produce expansive products. The possible anodic and cathodic reactions of corrosion could be written as follows [8,10]:

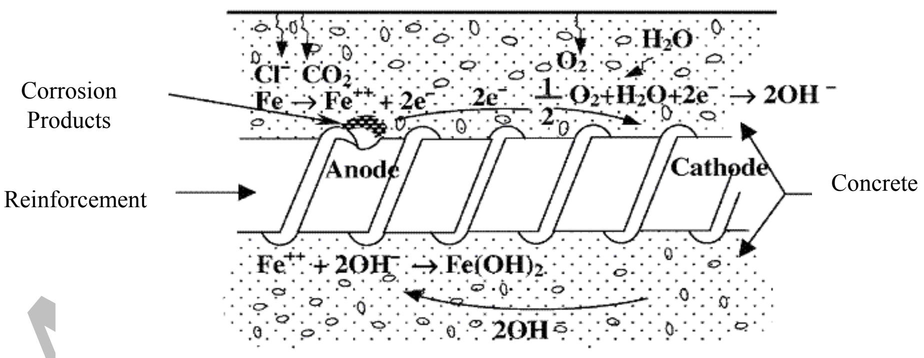


Figure 1. Schematic of electrochemical corrosion of steel in concrete [8]

Anodic reactions:

$$3Fe+4H_2O \rightarrow Fe_3O_4+8H^++8e^- \quad (1)$$

$$2Fe+3H_2O \rightarrow Fe_2O_3+6H^++6e^- \quad (2)$$

$$2Fe+3H_2O\rightarrow Fe_2O_3+6H^++6e^- \tag{3}$$

$$Fe\rightarrow Fe^{2+}+2e^- \tag{4}$$

Cathodic reactions:

$$2H_2O+4O_2\rightarrow 4OH^- \tag{5}$$

$$2H^++2e^-\rightarrow H_2 \tag{6}$$

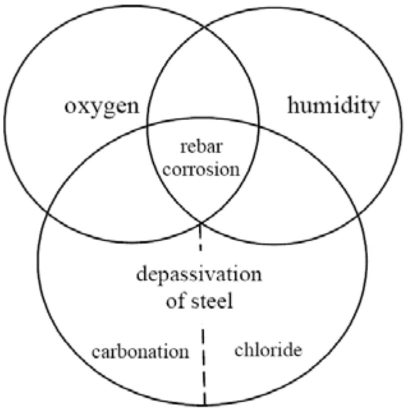


Figure 2. Prerequisites of corrosion

The expansive corrosion products apply stresses on the surrounding cover concrete after filling the porous zones of ITZ. When these stresses are lesser than the tensile strength of the cover concrete, the applied radial forces could not damage the concrete. As soon as the applied forces from corrosion products exceed the tensile strength of the cover concrete, these forces resulted in possible cracking of the concrete cover. Here, this stage is named as corrosion cracking time which is frequently modeled deterministically by means of an ideal thick walled cylinder [11] and reported in previous study [1]. Study of the corrosion damages on RC structures is generally accomplished by a service life model that is described as follows.

3. Service Life of RC Structures

Figure 3 shows the schematic of proposed service life model. As discussed before, following the initiation of the corrosion, the corrosion reactions resulted in reduction of the reinforcing bars because of the consumption of iron in the progress of coupled anodic-catholic reactions [8]. Considering this phenomenon, the cross section of bars would be reduced with time and consequently the available stresses would be increased. On the other side, the reduction of steel area causes the consequent reduction of flexural strength, shear resistance, steel-concrete bonding strength, flexibility, weakening of ITZ, and etc. Theses consequences will be decrease the general performances of the structure during the corrosion development. Various life cycles of corrosion affected RC structures including corrosion-initiation, corrosion-induced cracking and final stage are illustrated in Figure 3. Each of these cycles is formulized by many researchers based on some assumptions [1,11]. In this study, these periods to be reformulated based on fuzzy random variables afterwards some theoretical clarifications.

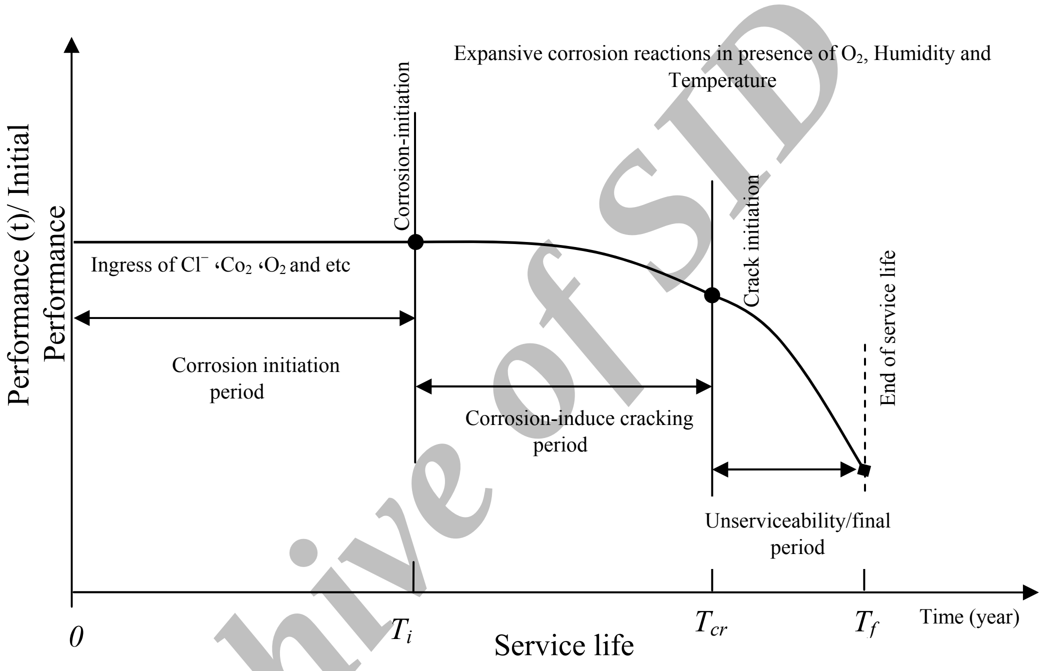


Figure 3. Service life model for RC structures

4. Uncertainty in Corrosion Process

As previously discussed on mechanisms of corrosion, it is perceived that the process of corrosion is naturally uncertain. Beside, the impreciseness of the measurements of the involved parameters, vagueness in linguistic regulations of design and manufacturing of the RC structures superimpose the concerned uncertainties. Up to know the probabilistic methods are successfully applied in service life of the corrosion affected RC structures [1,10-12]. In the recent years, fuzzy techniques are examined for this problem [13,14]. However there is no report on application of a method that reflects two mentioned methods in the field of corrosion of reinforcements. In theory of uncertainties, the method that is considers both fuzziness and randomness of information are often called fuzzy randomness (see Figure 4). This paper tries to examine the applicability of latter method using fuzzy random variables with a computational tools named as α-level optimization.

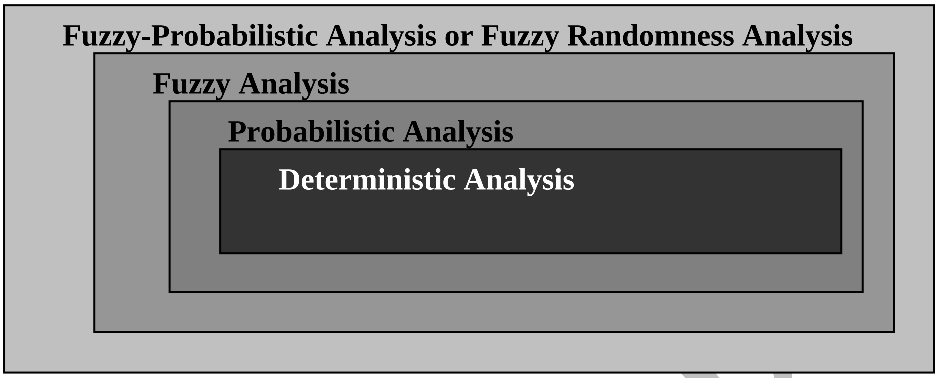


Figure 4. Analysis methods [10]

5. Theoretical Backgrounds

5.1. Mathematics of Fuzzy Randomness

In accordance with the traditional probability theory, the space of the random elementary events Ω and the fundamental set $\underline{X} = \Re^n$ are introduced. Instead of a real-valued realization, a fuzzy realization $\underline{\tilde{x}}(w) = (\tilde{x}_1, ..., \tilde{x}_n) \subseteq \underline{X}$ is assigned to each elementary event $w \in \Omega$ as shown in Figure 5. [15,16]. Now FRV could be defined as

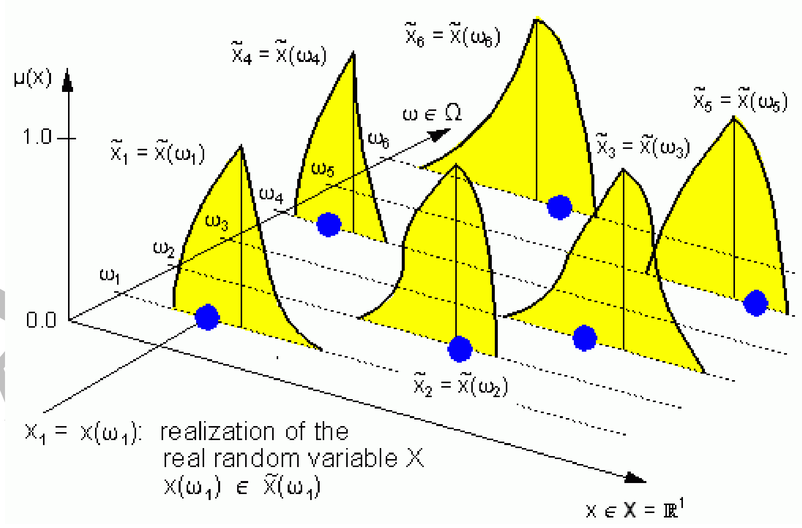


Figure 5. A fuzzy random variable or fuzzy random vector $\underline{\tilde{X}}$

Definition 1: A fuzzy random variables are described as an uncertain mapping of

$$\underline{\widetilde{X}}:\Omega\xrightarrow{\sim}F(\Re^n) \tag{7}$$

5.2. Fuzzy extension principal

Extension principal is introduced to extend the deterministic functions to the corresponding fuzzy functions [17].

Definition 2: The principal is described by following relationships:

$$\text{Extend } y=f(x_1,\ldots,x_n) \text{ to } \hat{B}=F(\hat{A}_1,\ldots,\hat{A}_n): \tag{8}$$

$$\mu_{\hat{B}}(b)=\sup_{\substack{a_1,\ldots,a_n\\ b=f(a_1,\ldots,a_n)}}\min(\mu_{\hat{A}_1}(a_1),\mu_{\hat{A}_2}(a_2),\ldots,\mu_{\hat{A}_n}(a_n))$$

Where $\hat{A}_1,\ldots,\hat{A}_n$ are fuzzy inputs, $\hat{B}$ is fuzzy output, $\mu_{\hat{\circ}}(\bullet)$ is corresponding membership value.

It could be analyzed the fuzzy equations with fuzzy extension principal, however, in practice, the application of this method is restricted to the problems with limited input variables with low level of nonlinearity. In literature, α-level optimization is effectively applied in dealing with the fuzzy randomness [15,16].

5.3. α-level optimization

To solve the uncertain fundamental equations of corrosion in RC structures, α-level optimization procedure is applied as the following steps [10]:

A- Setting value between 0 and 1 for α_k levels,

B- Discreteized the inputs of the model ($\hat{x}_i=\hat{A}_i$) at level α_k,

C- Constructing the crisp sets $A_{i,\alpha k}$, in other word, determine the subsets $\underline{X}_{i,\alpha_k}$,

D- Determining the minimum and maximum values of subsets $\underline{X}_{\alpha_k}$ for all input at α_k levels, or

$$\forall\alpha_{k,i};\quad\begin{cases} find\min X_{\alpha_k,i}=x_{\alpha_{k,i}}r\\ find\max X_{\alpha_k,i}=x_{\alpha_{k,i}}r\end{cases} \tag{9}$$

E- Solving the following bounded optimization problem to find out $\hat{z}_j=\hat{B}_j$,

$$\forall\alpha_{k,i};\text{find}\begin{cases}\underline{\min}_{X_{\alpha_k,i}} f(x_1,\ldots,x_n)=zj_{\alpha_k}r\\ st\quad (x_1,\ldots,x_n)\in X_{\alpha_k,i}\end{cases} \tag{10}$$

$$\forall\alpha_{k,i};\text{find}\begin{cases}\underline{\max}_{X_{\alpha_k,i}} f(x_1,\ldots,x_n)=zj_{\alpha_k}r\\ st\quad (x_1,\ldots,x_n)\in X_{\alpha_k,i}\end{cases} \tag{11}$$

Eventually, Eqs. (8) and (9) will be satisfied by the optimum points $\underline{x}_{opt}$. For each fuzzy result value precisely two optimum points in the crisp subspace $\underline{X}_{\alpha_k}$ belongs to each α-level

α_k. If for all of the prescribed α levels (α_k) the fuzzy result values $\hat{z}_j$ are gained the procedure is accomplished. After completion, the graphical demonstration of input and outputs would represent the fuzzy random distribution (FRD). To infer crisp values of FRD, suitable defuzzification method like the well-known center of gravity method (CGM) (see Appendix I) would be used.

6. Fundamental Equations of Corrosion

6.1. Corrosion-initiation period

As shown in Figure 4, the corrosion initiated after depassivation of reinforcement. In this paper, the effect of chloride ion attack is only considered and the time to reach the level of chloride concentration to the critical value is named as initiation time. Here, the Fick's second law of diffusion [18] is used to develop the fuzzy random fundamental equation:

$$\frac{\partial C(x,t)}{\partial t} = -D_c \frac{\partial^2 C(x,t)}{\partial x^2} \tag{12}$$

$$\tilde{T}_i = \frac{\hat{C}^2}{4\hat{D}_c}\left[erf^{-1}\left(\frac{\hat{C}_s - \hat{C}_{cr}}{\hat{C}_s}\right)\right] \tag{13}$$

Where $\tilde{T}$ FR time for corrosion initiation is, $\hat{C}$ is fuzzy cover depth, $\hat{D}_c$ is fuzzy coefficient of diffusion, $\hat{C}_s$ is fuzzy concentration of chloride at the outer surface of RC member, $\hat{C}_{cr}$ is critical fuzzy chloride concentration to depassive the reinforcement. Note that erf *is* error function that could be obtained from Eq. (14).

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt \tag{14}$$

6.2. Corrosion-induced cracking time

As briefly discussed above, with the application of internal stresses exerted of corrosion products, the cover concrete would be cracked. The time in which this happens, depends on the mechanical properties of concrete and reinforcing steel, geometric dimensions, environmental conditions, etc. Deterministic mathematical relationship for this time period was formulated based on thick cylinder model considering some assumptions like [19]:

- A porous zone is assumed to exist around the ITZ which is assumed to be uniform and the pressure application over the surrounding concrete is also assumed to be uniform.
- The concrete is assumed to be a homogeneous, isotropic and linear elastic material
- Due to the complex nature of the problem, the used model is restricted to the stresses resulting from the expansion of corrosion products only.

- Mechanical properties of the rust products, viz. modulus of elasticity and Poisson's ratio are considered to be same and other assumption (refer to the [19]) , etc.

Therefore, the fuzzy random time to cover cracking ($\tilde{T}_{cr}$) would be re-formulated from deterministic ones 1 based on the above mentioned assumptions as follows [10]:

$$\tilde{T}_{cr} = \frac{\tilde{W}^2_{crit}}{2\tilde{k}_P} \tag{15}$$

Where, $\tilde{W}_{crit}$ is FR amount of corrosion products (mg/mm), and $\tilde{k}_P$ is FR rate of rust production. $\tilde{W}_{crit}$ can be defined as [10]

$$\tilde{W}_{crit} = \rho_{rust}\left(\pi\left[\frac{\hat{C}f_t^{'}}{E_{ef}}\left(\frac{\tilde{a}^2+\tilde{b}^2}{\tilde{b}^2-\tilde{a}^2}+\nu_c\right)+d_0\right]\left(\hat{D}_b+\frac{\hat{W}_{st}}{\rho_{st}}\right)\right) \tag{16}$$

Where $\hat{D}_b$ is fuzzy diameter of rebar, ν_c is poison ratio of concrete, d_0 is thickness of steel-concrete interface layer, ρ_{rust} is density of rust, ρ_{st} is density of steel, $\tilde{a} = (\hat{D}_b + 2d_0)/2$, $\tilde{b} = \hat{C} + (\hat{D}_b + 2d_0)/2$, $\tilde{W}_{st} = \alpha_r \tilde{W}_{crit}$ and $\tilde{k}_P$ may be expressed as [10]

$$\tilde{k}_P = 0.105(1/\alpha_r)\pi\hat{D}_b \cdot \hat{i}_{cor} \tag{17}$$

Where, $\hat{i}_{corr}$ is the annual fuzzy mean corrosion rate (μA/cm^2) and α_r (ratio of molecular weight of iron to the molecular weight of the corrosion products.

Figure 6 shows values of α_r, ratio of molecular weight of iron to the molecular weight of the corrosion products in comparison with the $\alpha 1$, ratio of volume of expansive corrosion products to the volume of iron consumed in the corrosion process for various corrosion products.

6.3. Failure time period

In Figure 3, T_f is defined as ultimate service life of RC structures. In this paper, rebar area reduction and flexural strength of defected RC member during corrosion process, a simple time dependent degradation model is accepted as follows:

6.3.1. Cross-section area degradation [10]

$$\tilde{A}_s(t) = \frac{\pi}{4}\sum_{j=1}^{n}\left[\hat{D}_{b0j} - 0.0115\,\alpha_p \hat{i}_{corr}(t-\tilde{T}_i)\right] \tag{18}$$

Where $\tilde{A}_s(t)$ is FR time-reduced area of rebar, n is the number of rebars, $\hat{D}_{boj}$ is fuzzy initial diameter of *j-th* bar (mm), 0.0115 is a factor which converts μA/cm^2 to mm/year, t is the time elapsed (years), and α_p is a coefficient for pitting corrosion.

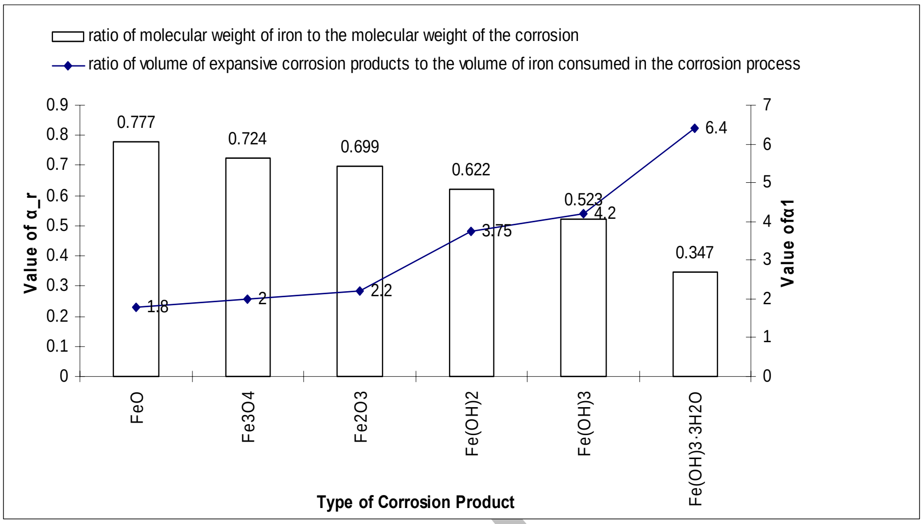


Figure 6. Evalution of α_r, and α_1 for values corrosion products

6.3.2. Flexural strength degradation [10]

$$\tilde{M}_n(t) = \tilde{A}_s(t) f_y \left(d - \frac{1}{2} \frac{\hat{A}_s(t) f_y}{0.85 f'_c \hat{B}} \right) \tag{19}$$

Where, f_y is yield strength of reinforcement, d is effective depth, f'_c is 28 days compressive strength of concrete, and $\hat{B}$ is the fuzzy width of the compression face of the RC member cross-section that is shown in Figure 7.

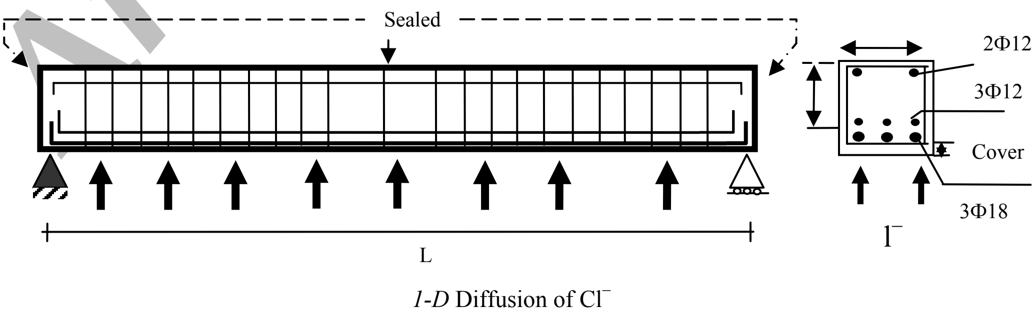


Figure 7. Schematic of a simple supported RC beam under chloride ions attack

7. Proposed Methodology

The proposed methodology is presented in Figures 8 and 9. Figure 8 shows the preprocessing interface of proposed system while Figure 6 demonstrates the computational core of suggested methodology. It should be indicated that the proposed system is implemented in Matlab R.2006b environment and all computations are carried out using programmed scripts.

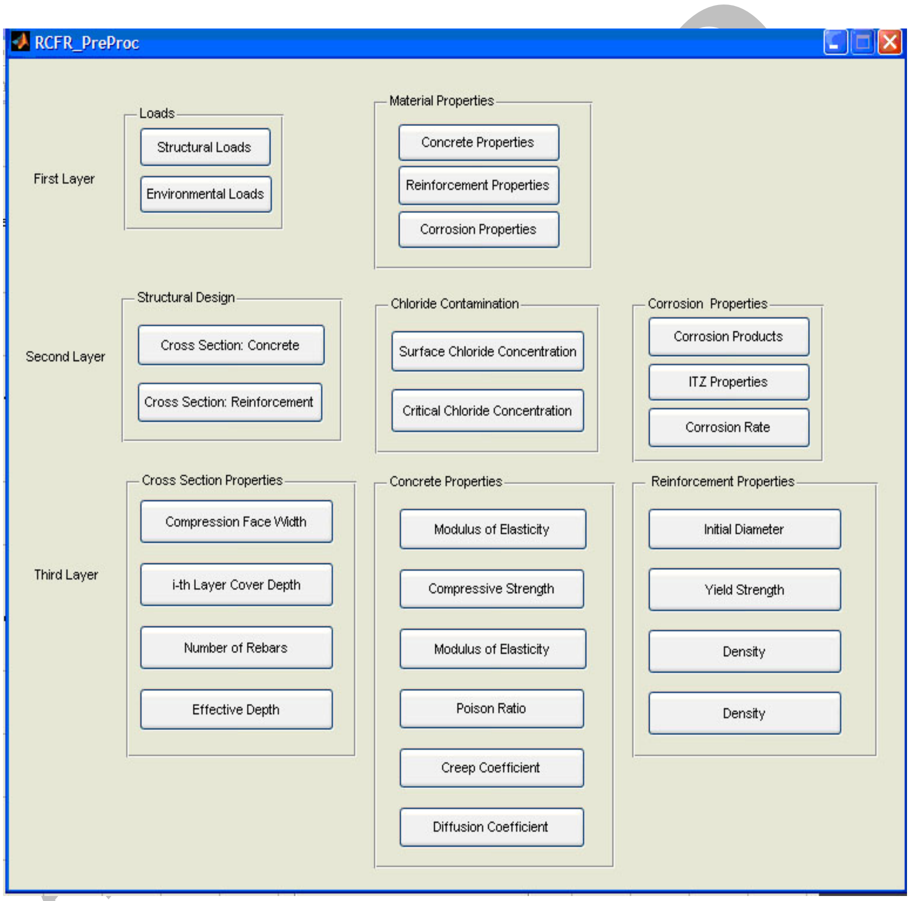


Figure 8. Preprocessing unit of proposed system for service life prediction

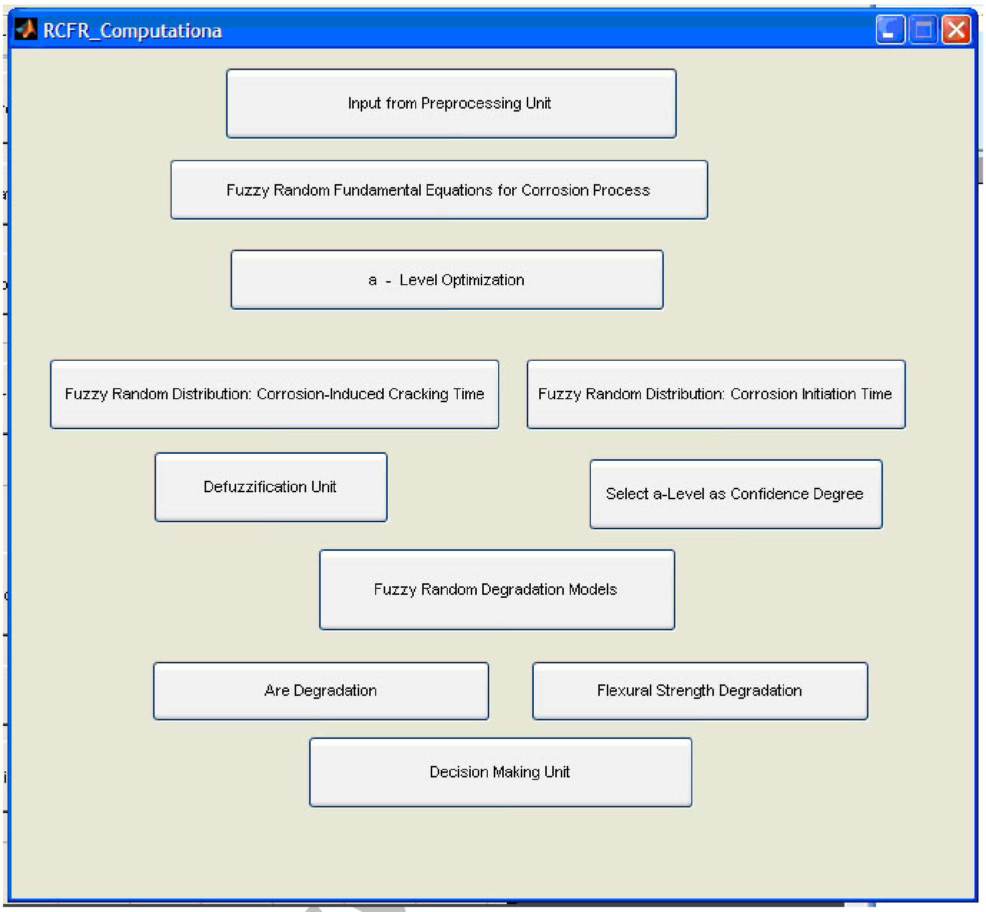


Figure. 9. Computational core of proposed system for estimating the service life

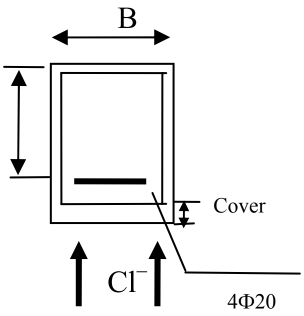
8. Durability Assessment Example

Consider a RC beam which is subjected to the chloride ions attack (Figure 10). Verify the service life of this beam. Exposure condition, properties of materials and chloride contaminations are addressed as labels on Figure 10.

Solution: To develop the service life, proposed methodology is applied as follows:

8.1. Input fuzzy random variables

Fuzzy basic variables of corrosion i.e. cover depth, diffusion coefficient, reinforcing bar diameter, effective modulus of elasticity, chloride surface concentration and chloride critical concentration are constructed based of corresponding probability distributions and plotted in Figure 11.



Concrete

E_c=27 Gpa
E_{ef}=9 Gpa
B=150 cm
d=68.73
Cover= 44 mm
f'_c=190 kg/cm^2
w/c=0.5
D_c=100 mm^2/year
υ_c=0.18
Ψ_c=2.0

Chloride Contamination (Cl^-)

C_s=0.25
C_{cr}=0.02

Exposure

T= 35 C
RH= 60%
Wet.-dry. ratio= 0.7

Reinforcement

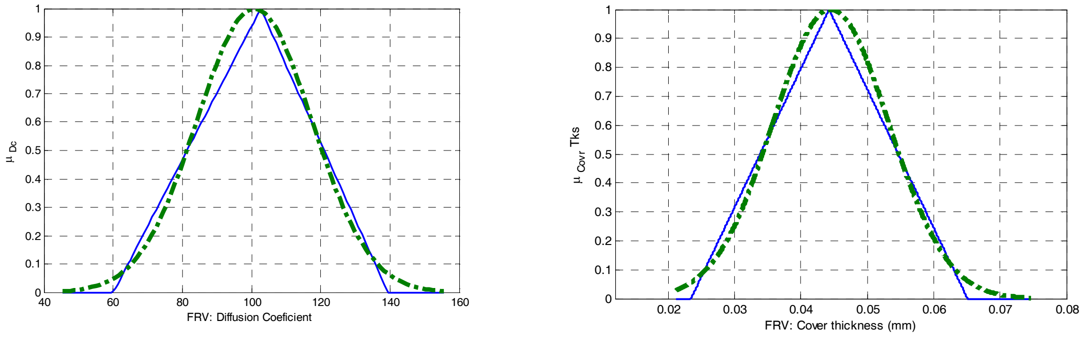
Φ=20 mm
f_{yd}=3100 kg/cm^2
ρ_{st}=7850 kg/m^3

Corrosion Product

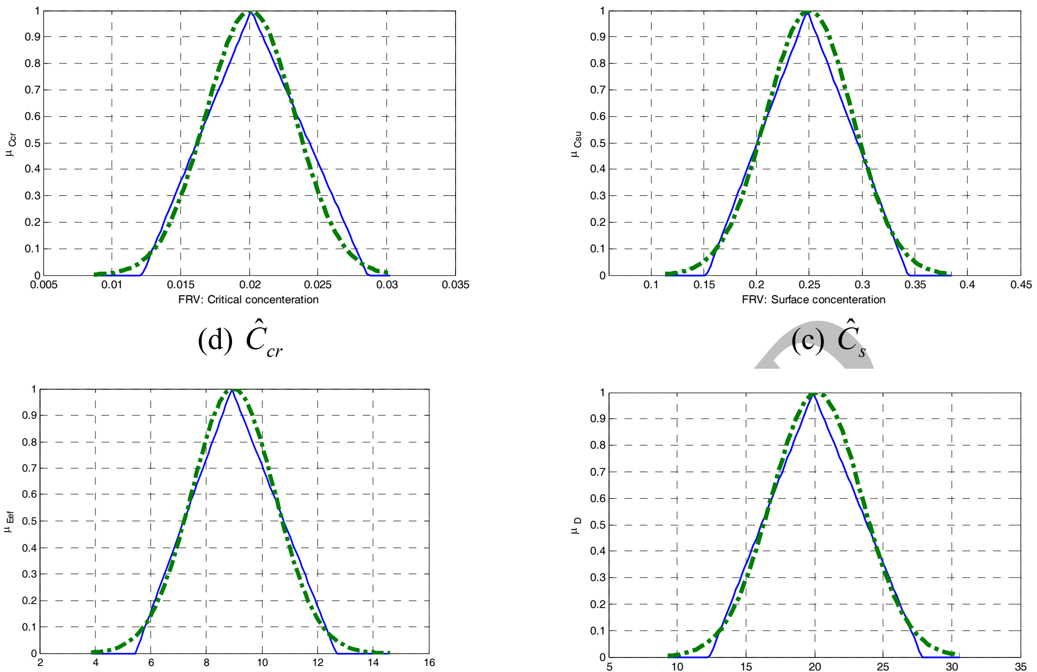
Type=n_1Fe(OH)3,+ n_2Fe(OH)2
d_{ITZ}=12.5 μm
ρ_{rust}=3600 kg/m^3

Figure 10. Sample RC beam for service life assessment

In order to devolve the service life, proposed methodology is applied as follows: Fuzzy basic variables of corrosion i.e. cover depth, diffusion coefficient, reinforcing bar diameter, effective modulus of elasticity, chloride surface concentration and chloride critical concentration are constructed based of corresponding probability distributions and plotted in Figure 11.



(b) $\widetilde{D}_c$ (a) $\hat{C}$



(d) $\hat{C}_{cr}$ (c) $\hat{C}_{s}$

(f) $\hat{E}_{ef}$ (e) D_b

Figure 11. Input variable

8.2. Fuzzy random distributions of $\widetilde{T}_i$ and $\widetilde{T}_{cr}$

Exploiting of α-level optimization on the fundamental equations of corrosion in the corrosion initiation and corrosion-induced cracking phases, FRD of $\widetilde{T}_i$ and $\widetilde{T}_{cr}$ are gained as shown in Figures 12 and 13 respectively. Defuzzifing the FRD of both $\widetilde{T}_i$ and $\widetilde{T}_{cr}$ yields the crisp values for corrosion initiation and corrosion cracking time periods as represented in Table 1.

Table 1. Crisp values of $\widetilde{T}_i$ and $\widetilde{T}_{cr}$

Life cycle periods		**Value (days)**	**Cumulative Value (days)**
Corrosion Initiation	T_i	1475	1475
Corrosion Crack Initiation	T_{cr}	143	1618

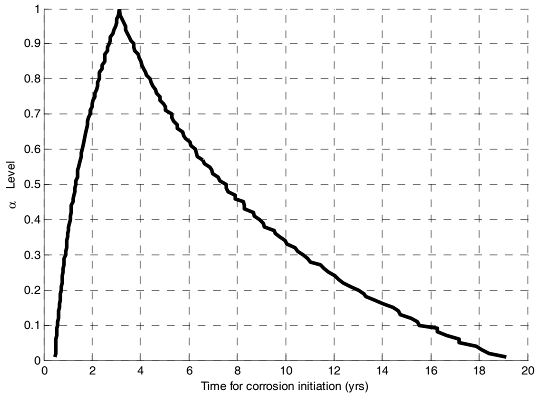


Figure 12. FR distribution of $\widetilde{T}_i$

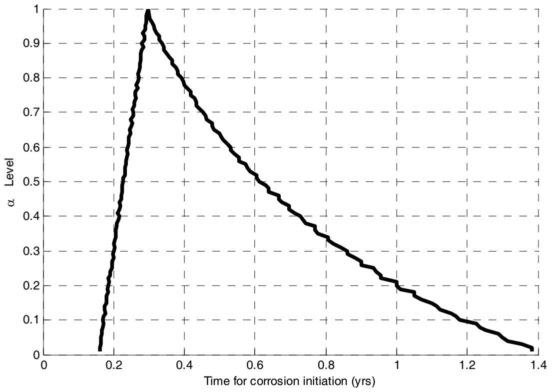


Figure 13. FR distribution of $\widetilde{T}_{cr}$

8.3. Degradation of bar cross-section area and flexural capacity

Pursuing proposed method, the life-performances of corrosion affected sample beam will be gained. Figures 14 and 15 have been shown the time-dependent degradation of reinforcement area and flexural capacity respectively. Table 2 summarizes the sample RC beam's life cycles in various confidence levels.

Table 2. Assessed service life of corroded RC beam

Life cycle	T_f						
	Confidence Limit			-5%	+5%	-95%	+95%
End of Service Life	Criterion	(1)	30% A_s Loss.	14.38	38.44	16.73	17.31
		(2)	60% M_n Loss	11.07	27.72	13.43	14.00
		(3)	Min (1,2)	11.07	27.72	13.43	14.00
	Reliability			Pessimistic		Optimistic	

It can be perceived that the optimistic service life of the sample beam will be between 13.43 to 14.00 years and pessimistic service life can be in the 11.07 to 27.72 years interval.

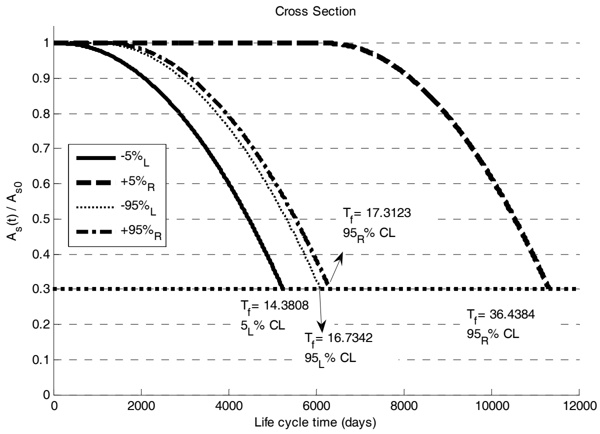


Figure 14. Deterioration of bar area during corrosion

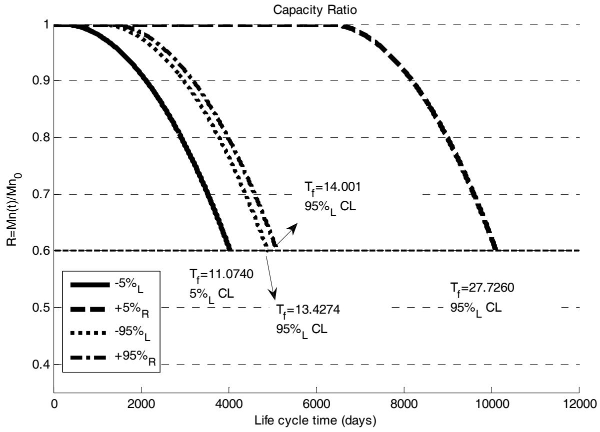


Figure 15. Deterioration of beam flexural strength during corrosion

9. Conclusion

In this paper, corrosion of reinforcements in concrete is considered as an uncertain problem. To deal with the involve uncertainties, fuzzy variables are applied to the concerning relationships and α-level optimization is introduced as an effective method other than fuzzy extension principle to handle the fundamental equations of corrosion process. The designed system is utilized in service life assessment of a corroded RC beam under chloride attacks. Corrosion initiation, corrosion-induced cracking and final stages of the beam life are calculated using the proposed methodology that shows the applicability of the proposed system.

References

1. Sobhani J, Ramezanianpour AA. Chloride-induced corrosion of RC structures, *Asian Journal of Civil Engineering*, No. 5, **8**(2007)531-47.
2. Ramezanianpour AA, Pourkhorshidi AR. National code of practice for concrete durability in the Persian Gulf and Oman Sea (proposed), BHRC Publication, Tehran, Iran, 2004.
3. Novoshchenov V. Corrosion of reinforced concrete in the Persian gulf region**,** *Material Performance*, No. 1, **34**(1995)51-4.
4. Alizadeh R, Ghods R, Chini M, Hoseini M, Ghalibafian M, Shekarchi M. Effect of curing conditions on the service life design of RC structures in the Persian gulf region, *Materials in Civil Engineering*, No. 1, **20**(1998)2-8.
5. Bahar SA, Attiogbe EK, Kamal H. Investigation of corrosion damage in a reinforced concrete structure in Kuwait, *ACI Materials Journal*, No. 3, **95**(1998)226-31.
6. Pech Canul MA, Castro P. Corrosion measurements of steel reinforcement in concrete exposed to a tropical marine atmosphere, *Cement and Concrete Research***,** No. 3, **32**(2002)491-98.
7. Costa A, Appleton J. Case studies of concrete deterioration in a marine environment in Portugal, *Cement and Concrete Composite*, **24**(2002)169-79.
8. Ahmad S. Reinforcement corrosion in concrete structures, its monitoring and service life prediction–A review, *Cement and Concrete Composite*, **25**(2003)459-71.
9. Roelfstra G, Hajdin R, Adey B, Bruhwiler E. Condition evolution in bridge management systems and corrosion-induced deterioration, *JournaL of Bridge Engineering*, No. 3, **9**(2004)268-77.
10. Sobhani J. A soft computing approach for service-life prediction of RC bridge deck in chloride-laden environments, PhD dissertation, Amir Kabir University of Technology, Tehran, Iran, 2007.
11. Liu Y, Weyers RE. Modeling the time-to-corrosion cracking in chloride contaminated reinforced concrete structures, *ACI Materials Journal*, No. 6, **95**(1998)675-81.
12. Li CQ, Melchers RE, Zheng JJ. Analytical model for corrosion-induced crack width in reinforced concrete structures, *ACI Materials Journal*, No. 4, **103**(2006)479-87.
13. Anoop MB, Rao KB, Gopalakrishnan S. Conversion of probabilistic information into fuzzy sets for engineering decision analysis, *Computers and Structures*, **84**(2006)141-55.
14. Anoop MB, Rao KB. Application of fuzzy sets for remaining life assessment of corrosion affected reinforced concrete bridge girders, *Journal of Performance of Constructed Facilities*, No. 2, **21**(2007)166-71.
15. Möller B, Graf W, Beer M. Fuzzy structural analysis using α-level optimization, *Computational Mechanics*, **26**(2000)547-65.
16. Möller B, Beer M. Engineering computation under uncertainty–Capabilities of non-traditional models, *Computers and Structures*, No. 10, **86**(2008)1024-41.

17. Savoia M. Structural reliability analysis through fuzzy number approach, with application to stability, *Computers and Structures*, **80**(2002)1087-102.
18. Clifton JR. Predicting the service life of concrete, *ACI Materials Journal*, No. 6, **90**(1993)611-17.
19. Bhargava K, Ghosh AK, Mori Y, Ramanujam S. Analytical model for time to cover cracking in RC structures due to rebar corrosion, *Nuclear Engineering and Design*, **236**(2006)1123-39.

Appendix (I)

Center of gravity method

$$y^*_{CG} = \frac{\int_V y \cdot \mu_{B'}(y)dy}{\int_V \mu_{B'}(y)dy} \qquad (20)$$

Where y^*_{CG} is defuzzified value for FRD of B'