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MATHEMATICAL MODELS FOR THERMAL CONDUCTIVITY-DENSITY RELATIONSHIP IN FIBROUS THERMAL INSULATIONS FOR PRACTICAL APPLICATIONS

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Abstract

Relationship between thermal conductivity (λ_t) and density (ρ) of mineral wool insulations has a wide band of measured data, follows some form of median curve which is called reference curve. Departures from this curve are mainly due to the fact that manufacturing conditions change. But the deviation from the curve should be in a limited range because it will affect the quality conformity. With finding $\lambda_t = f(\rho)$ as a reference curve and using routine test results it is possible to note the changes in quality of the products. On the other hand λ_t may be evaluated indirectly using ρ measurements and the established reference curve for a specific production line. For finding the suitable model for $\lambda_t = f(\rho)$, a large amount of data was studied in this article. Different semi-empirical models were presented and statistically compared with the experimental results to evaluate the best one.

Keywords: Thermal conductivity; density; reference curve; mineral wools; insulation

1. Introduction

Many researchers worked on heat transfer in fibrous materials and proposed several odels. Verschoor et al. [1] presented a model for thermal conductivity contributed by radiation. Larkin and Churchill [2] introduced two-flux model and defined total heat flux consists of conduction and radiation. Bankvall [3] theoretically calculated different mechanisms of heat transfer in fibrous materials. Bhattacharyya [4] investigated various form of combining solid and gaseous conduction modes. Tong et al. [5-6] applied two-flux model for calculating radiative heat transfer. Boulet et al. [7] used radiative properties of the medium for calculating transmissive and reflective radiation heat transfer. Zeng et al. [8] developed approximate formulation for coupled conduction and radiation through a medium with arbitrary optical thickness. Andersen and Dyrbol [9] developed a model for heat transfer in fibrous insulation that includes both a two-flux radiation model and the spherical harmonics method. Daryabeigi [10] modeled radiation heat transfer using the modified two-flux

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approximation assuming anisotropic scattering and gray medium [10].

Asllanaj et al. [11] used an angular discrimination technique in order to express the RTE in an inhomogeneous system of linear differential equations [12]. Asllanaj, 2004, investigated transient radiative and conductive heat transfer in a fibrous medium with anisotropic optical properties [13]. Radiative properties of high and low density fiberglass were investigated by Roux [14]. Wang et al., 2006, proposed a random generation growth method for generating micro morphology of fibrous materials based on statistical macroscopic geometrical characteristics and a lattice Boltzman algorithm for solving the energy transport equations [15]. Langlais, Klarsfeld and their co-workers* had a great contribution in this field. They proposed a relationship for conductive heat transfer in air and fibers. This group of researchers worked with several models to analyze the effect of different parameters on radiative heat transfer and studied rigorously other aspects of fibrous insulating materials as well [16]. Veiseh and Hakkaki-Fard [17] proposed a radiation heat transfer model based on Monte Carlo Ray-Trace (MCRT) Method. Veiseh et al. [18] determined air/fiber conductivity of mineral wool insulations using nonlinear estimation methods.

2. Reference Curve

Relationship between thermal conductivity (λ_t) and density (ρ) of mineral wool insulations has a wide band of measured data, follows some form of median curve which is called reference curve. Departures from this curve are mainly due to the fact that manufacturing conditions can never be identical. The reasons may be the difference between methods of manufacture, raw materials, or internal processes and so on. But the deviation from the curve should be limited because it will affect the conformity of the quality. The reference curve applications in mineral wool industry are as follows:

- The conformity of the quality can be maintained with using routine test results and finding their deviation from the reference curve, investigating the reason and taking the proper measure. The more consistent the raw material and production process, the better is conformity. Then the departure from the mean value will be less.
- Thermal conductivity may be evaluated indirectly using the apparent density and its established mathematical correlation of these properties. Manufacturers not having some means of measuring directly the thermal conductivity should rely on measurements of some indirect parameter. In many cases, the density has been found to be the most suitable one. λ_t may be evaluated indirectly using ρ measurement and the established mathematical correlation of these properties (reference curve).

Then finding a proper relationship between λ_t and ρ is necessary for each production line and each product for above mentioned purposes. Therefore a group of curves should be developed in a plant for different products with different specific surface area and structure. For procuring the reference curve a mathematical model is needed. Three models were

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studied for finding the most appropriate reference curve using a large amount of experimental data.

3. Models

There are some complicated approaches for numerical modeling of radiation heat transfer in fibrous media. They are time consuming and difficult to apply. Radiative properties of fibers (including index of refraction, extinction coefficient, emissivity …) are needed for them that are difficult and expensive to measure. The radiation heat transfer modeling was the subject of a number of researches [7-17]. This article deals with comparison of three simple and easy-to-use models in order to find the best model for λ_t =f(ρ). The model should be applicable and accessible in the plants and it does not need high educated personnel and radiation properties of the materials.

3.1 Model I

The total thermal Conductivity (λ_t) of a fibrous material is a result of convection, conduction due to air and solid phase and radiation in the material, that is as Eq. (1):

$$\lambda_t = \lambda_{\mathrm{cov}} + \lambda_{con} + \lambda_{rad} \tag{1}$$

Thermal conductivity due to convection is negligible under normal conditions. Conductive thermal conductivity, λ_{con}, is the sum of thermal conductivity of air and fibers. λ_{con} can be defined by Eq. (2).

$$\lambda_{con} = A + B \times \rho \tag{2}$$

A represents the thermal conductivity of air which is independent of density. More than 95% by volume of the mineral wool products consists of air. At temperatures below 200 °C, the greater portion of heat transfer across the insulations occurs by conduction through air. The term $B \times \rho$ describes thermal conduction through the solid material. This mode of heat transfer is important in high density mineral wools.

The radiative thermal conductivity can be derived from Eq. (3):

$$\lambda_{rad} = \frac{C}{\rho} \tag{3}$$

This shows that λ_{rad} is a reciprocal function of ρ. More the density of the material, more the solid shields for preventing radiation are. At low densities, the radiation heat transfer is as important as air and solid conduction heat transfer. The total thermal conductivity can be expressed as Eq. (4) which is called here model I:

$$\lambda_t = A + B \times \rho + \frac{C}{\rho} \tag{4}$$

A, B and C should be found for each production line, product and specific surface. The Datafit software was used for determining these parameters.

As an advantage, model I shows explicitly different modes of heat transfer including radiation, air and solid conduction. Its disadvantage is that it does not consider the thickness effect and temperature effect. Although it is not important in practice because the test usually is performed at the same mean temperature, e.g. 10ºC, and more or less the same thickness.

3.2 Model II

Thermal conductivity due to conductive still air heat transfer and due to solid conduction can be expressed as Eq. (5) and (6) respectively.

$$\lambda_{con,air} = 0.2572 \times T_m^{\ 0.81} \tag{5}$$

$$\lambda_{con,solid} = 0.0527 \times \rho^{0.91} \times \left(1 + 0.13 \times \frac{T_m}{100}\right) \tag{6}$$

Using Eq. (3) for radiative heat transfer, λ_t is given by Eq. (7).

$$\lambda_t = 0.2572 \times T_m^{\ 0.81} + 0.0527 \times \rho^{0.91} \times \left(1 + 0.13 \times \frac{T_m}{100}\right) + \frac{C}{\rho} \tag{7}$$

C should be found for each production line. Experiments show that C is a function of specific surface area per unit volume, temperature, morphology and radiative properties of the material and limiting surfaces (esp. emissivity). This model [16] is an empirical relationship derived from measurements and regression. It is supposed that the chemical constituents of the glass are the same and the fiberizing and forming processes are the same that means the physical properties do not vary. Using model II needs to find C with the use of the measurement results for which Eq. (7) gives the best fit for the curve of $\lambda_t = \mathrm{f}(\rho, T)$. It will be obtained when $f(C)$ of Eq. (8) is minimized.

$$\mathrm{f}(C) = \sum_{i=1}^{N} \left(\lambda_{model,i} - \lambda_{measur,i}\right)^2 \tag{8}$$

The Microsoft Excel Solver was used to find minimum value of f(C). As the experiments were carried out according to EN 12667 and EN 13162, the mean temperature used was 10°C. Eq. (7) can be rewritten as Eq. (9) called here as model II.

$$\lambda_t = 24.9118 + 0.0721 \times \rho^{0.91} + \frac{C}{\rho} \tag{9}$$

3.3 Model III

Eq. (10) is a combination of Langlais and Klarsfeld semi-empirical relation for conduction heat transfer and Larkin formula for radiation part:

$$\lambda(\rho,T) = 0.2572T_m^{\ 0.81} + 0.0527\rho^{0.91}\left(1+0.0013T_m\right) + \frac{4\sigma T_m^{\ 3}L}{(\frac{2}{\varepsilon}-1)+0.001\bar{\beta}\rho L} \tag{10}$$

Emissivity of boundary surfaces (ε) for lambda measuring instruments commonly used is equal to 0.9. The mean temperature of all experiments was 283.15 *K*. Then the Eq. (10) will be simpler which is given by Eq. (11), and we call it as model III.

$$\lambda_t = 24.9118 + 0.0721\times\rho^{0.91} + \frac{5.1468L}{1.2222+0.001\bar{\beta}\rho L} \tag{11}$$

Extinction coefficient (k_λ) is the sum of absorption coefficient (a_λ) and scattering coefficient ($\sigma_{S\lambda}$) and can be defined as Eq. (12).

$$k_\lambda = a_\lambda + \sigma_{S\lambda} \tag{12}$$

k_λ is a function of temperature, chemical composition of the material and wavelength of the incident radiation. The mass coefficients , $\bar{\beta}$, are given by Eq. (13).

$$\bar{\beta} = a_{\lambda,m} + \sigma_{S\lambda,m} = \frac{k_\lambda}{\rho} = \frac{a_\lambda}{\rho} + \frac{\sigma_{S\lambda}}{\rho} \tag{13}$$

Since the extinction coefficient (k_λ) increases as the density of the material increases, the use of $\bar{\beta}$ has the advantage that it tends to remain more constant than k_λ. $\bar{\beta}$ is a characteristic parameter of the material which depends on chemical composition of the fibers, specific surface of the material, product structure, fiber orientation and temperature. For calculating $\bar{\beta}$ the data for each set of (λ_i, L_i, ρ_i) should be obtained by experiments. $\bar{\beta}$ should be calculated to achieve the best fit for the curve of λ = f(ρ) and experimental data. It will be achieved when f($\bar{\beta}$) is minimized in Eq. (14).

$$\mathrm{f}(\bar{\beta}) = \sum_{i=1}^{N}\left(\lambda_{\mathrm{model},i} - \lambda_{measur,i}\right)^2 \tag{14}$$

$\lambda_{model,i}$ is calculated for each i=1,…,N using Eq. (15).

$$\lambda_{model,i} = 24.9118 + 0.0721 \times \rho_i^{0.91} + \frac{5.1468 L_i}{1.2222 + 0.001\bar{\beta}\rho_i L_i} \tag{15}$$

This model gives satisfactory results for mean temperatures of 10°C and the density of 5-100 kg/m^3. The experiments should be carried out at the same mean temperature and for products which have the same binder content and chemical composition.

4. Apparatus and Materials

The Heat Flow Meter (HFM) apparatus was used for determination of effective thermal conductivity. It was a single-specimen symmetrical device that consists of two heat flow meters. The test method was according to EN12667:2001.

For determining specific surface area the indirect test of fineness index is used that is based upon Darcy's law. The equipment utilized in this test is named Micronaire.

Table 1 shows the product type, the name of producer, Fineness index, and the number of experiments.

Table 1. Product name, the name of manufacturer, fineness index and the number of experiments

Product	**Name of producer**	**Fineness index (micronaire)**	**No. of experiments**
Glass wool	Pashmeshishe Iran	3.0 /5g	212
		5.0 /6g	129
Rock wool	Pashmesang Iran	-	288
Slag wool	Pasa	-	26

5. Assessment of Validation of the Models with Experimental Data

5.1 Glass wool

5.1.1 Micronaire: 3.0 /5g

The best fitted curves for models I, II, and III are presented in Eqs. (16) to (18), and Figures 1 to 3 respectively.

$$\lambda_t = 33.71 - 0.1801 \times \rho + \frac{82.676}{\rho} \tag{16}$$

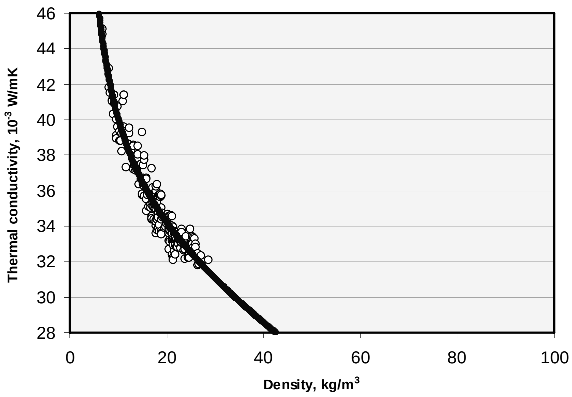

Figure 1. Thermal conductivity versus density of glass wool, micronaire 3.0 /5g, model I

$$\lambda_t = 24.9118 + 0.0721 \times \rho^{0.91} + \frac{154.85}{\rho} \tag{17}$$

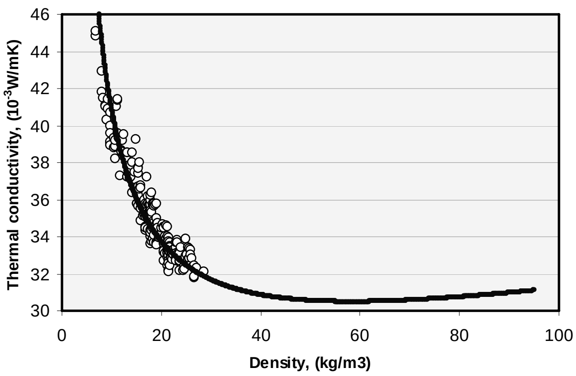

Figure 2. Thermal conductivity versus density of glass wool, micronaire 3.0 /5g, model II

$$\lambda_t = 24.9118 + 0.0721 \times \rho^{0.91} + \frac{5.1468L}{1.2222 + 0.0315\rho L} \tag{18}$$

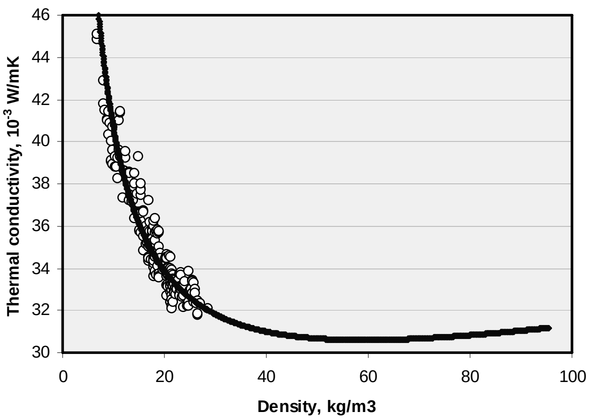

Figure 3. Thermal conductivity versus density of glass wool, micronaire 3.0 /5g, model III

The coefficient of determination (R^2) shows excellent agreement between models I and III with experimental results (Table 2).

Table 2. Comparison of models for glass wool micronaire 3.0 /5g

Model	R^2	SEE
I	0.91	0.78
II	0.85	0.99
III	0.89	0.90

5.1.2 Micronaire: 5.0 /6g

The best fitted curves for models I, II, and III are presented in Eqs. (19) to (21), and Figures 4 to 6 respectively.

$$\lambda_t = 26.09 + 0.415 \times \rho + \frac{118.53}{\rho} \tag{19}$$

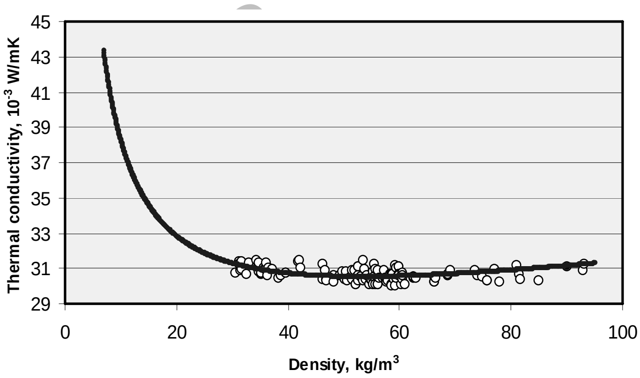

Figure 4. Thermal conductivity versus density of glass wool, micronaire 5.0 /6g, model I

$$\lambda_t = 24.9118 + 0.0721 \times \rho^{0.91} + \frac{152.60}{\rho} \tag{20}$$

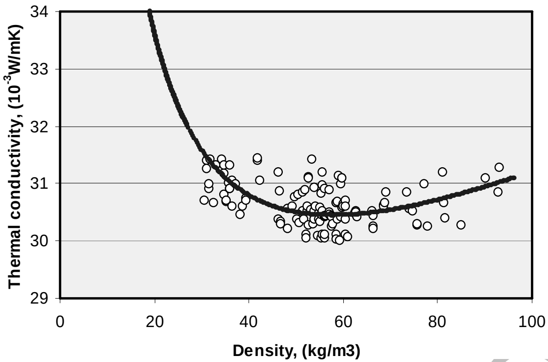

Figure 5. Thermal conductivity versus density of glass wool, micronaire 5.0 /6g, model II

$$\lambda_t = 24.9118 + 0.0721 \times \rho^{0.91} + \frac{5.1468L}{1.2222 + 0.0332\rho L} \tag{21}$$

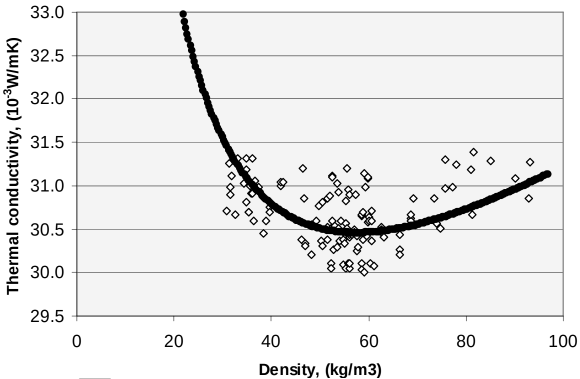

Figure 6. Thermal conductivity versus density of glass wool, micronaire 5.0 /6g, model III

Determination coefficient (R^2) shows a poor agreement between the models and experimental results (Table 3).

Table 3. Comparison of models for glass wool, micronaire 5.0 /6g

Model	**R^2**	**SEE**
I	0.32	0.30
II	0.20	0.37
III	0.21	0.32

5.2 Rock wool

The best fitted curves for models I, II, and III are presented in Eqs. (22) to (24), and Figures 7 to 9, respectively.

$$\lambda_t = 27{,}008 + 0{,}04382 \times \rho + \frac{194{,}719}{\rho} \tag{22}$$

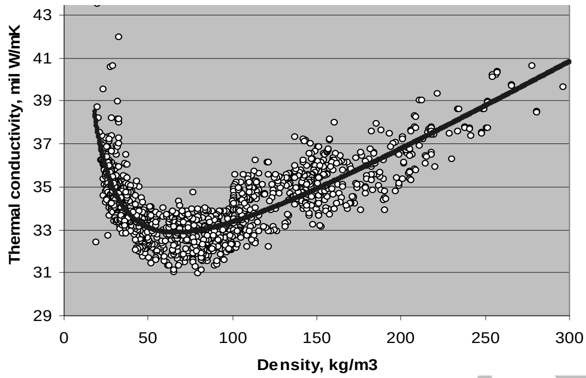

Figure 7. Thermal conductivity versus density of rock wool, model I

$$\lambda_t = 24.9118 + 0.0721 \times \rho^{0.91} + \frac{268.84}{\rho} \qquad (23)$$

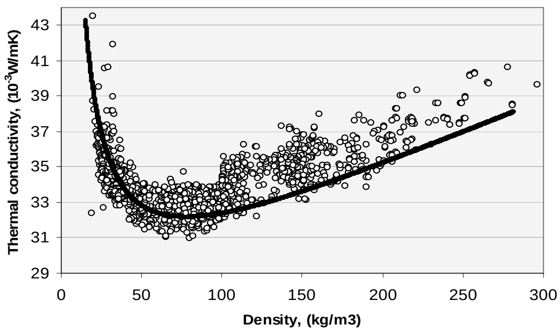

Figure 8. Thermal conductivity versus density of rock wool, model II

$$\lambda_t = 24.9118 + 0.0721 \times \rho^{0.91} + \frac{5.1468L}{1.2222 + 0.01838\rho L} \qquad (24)$$

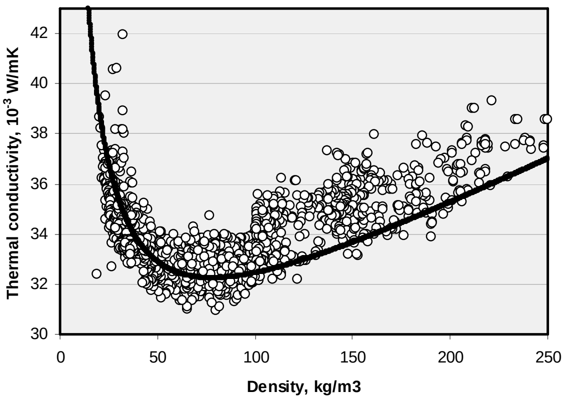

Figure 9. Thermal conductivity versus density of rock wool, model III

The statistical parameters of R^2 and SEE for these samples are presented in Table 4.

Table 4. Comparison of models for rock wool

Model	R^2	SEE
I	0.65	1.0
II	0.33	1.4
III	0.68	1.3

The deviation of λ values is due to difference in manufacturing conditions including the orientation of the fibers and shot content.

5.3 Slag wool

The best fitted curves for models I, II, and III are presented in Eqs. (25) to (27), and Figures 10 to 12 respectively.

$$\lambda_t = 24.688 - 0.0407 \times \rho + \frac{355.32}{\rho} \tag{25}$$

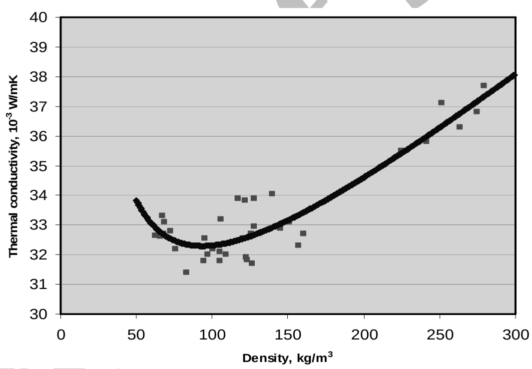

Figure 10. Thermal conductivity versus density of Slag wool, model I

$$\lambda_t = 24.918 + 0.0721 \times \rho^{0.91} + \frac{267.99}{\rho} \tag{26}$$

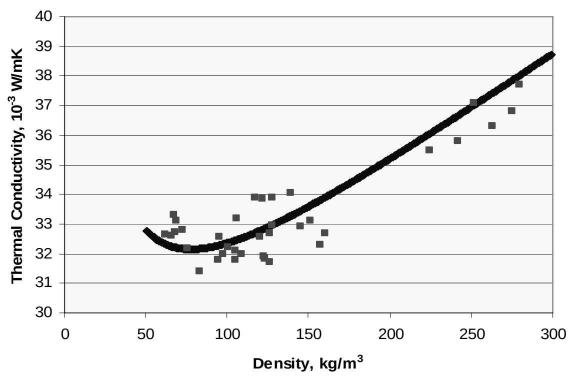

Figure 11. Thermal conductivity versus density of slag wool, model II

$$\lambda_t = 24.9118 + 0.0721 \times \rho^{0.91} + \frac{5.1468L}{1.2222 + 0.0190\rho L} \quad (27)$$

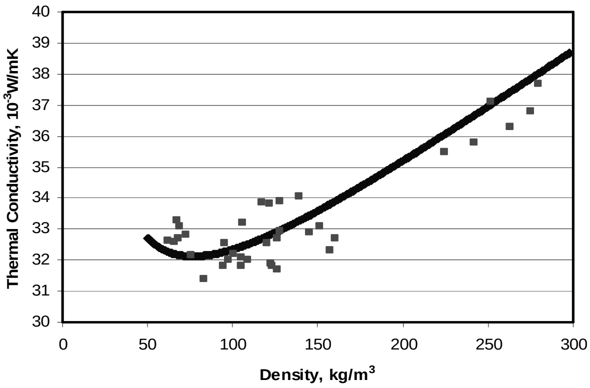

Figure 12. Thermal conductivity versus density of slag wool, model III

The statistical parameters of R^2 and SEE for these samples are presented in Table 5.

Table 5. Comparison of models for slag wool

Model	R^2	SEE
I	0.84	0.66
II	0.78	0.78
III	0.78	0.78

As it can be seen the correlation of model I is better than two other models. Models II and III have the same statistical values.

5.4 Models evaluation

The comparison of the statistical evaluation results showed that in most cases, all models have given the same correlation results. Results showed that model I can only be used for a limited range of lower densities as it does not have a U shape curve in some cases. Hence it is not a good definition for λ–ρ relationship because with increasing of density, heat transfer increases via solid conduction and this model does not consider this. Then it is proposed to use model II and III, that in all cases they have a U shape curve. Between these two models, model III is preferable because it considers the thickness and mean temperature's effects. In general model III is recommended for finding reference curve.

6. Conclusions

In this article three models were evaluated in order to find the most suitable reference curve for λ_t and $f(\rho)$ relation in mineral wool products. In this regard a large number of experiments were carried out to determine the conductivity of three kinds of mineral wools including glass wool, rock wool, and slag wool in a wide range of densities. Then three semi-empirical models were presented and statistically compared with the experimental measurements. Results showed that the proposed model III which considers the thickness and mean temperature's effects has the best fitness with the experimental data for different density ranges and products, so it is recommended for obtaining suitable reference curves.

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Notations

k_λ	Extinction Coefficient
L	Thickness of insulation, mm
N	Number of measurements
R^2	Coefficient of Determination
SEE	Standard Error of the Estimate
T_m	Mean temperature, (K)
a_λ	Absorption coefficient
$\bar{\beta}$	Extinction coefficient, m^2/kg
ε	Emissivity of boundary surfaces
λ_{con}	Air and fiber thermal conductivity, (10^{-3}W/m.K)
$\lambda_{con,air}$	Still air thermal conductivity, (10^{-3}W/m.K)
$\lambda_{con,solid}$	Solid thermal conductivity, (10^{-3}W/m.K)
λ_{cov}	Convective thermal conductivity, (10^{-3}W/m.K)
λ_{measur}	Total thermal conductivity obtained by measurements, (10^{-3}W/m.K)
$\lambda_{\mathrm{mod}el}$	Total thermal conductivity predicted by model, (10^{-3}W/m.K)
λ_{rad}	Radiative thermal conductivity, (10^{-3}W/m.K)
λ_t	Effective or total thermal conductivity, (10^{-3}W/m.K);
ρ	Apparent density of the medium, (kg/m^3)
σ	Stefan-Boltzmann constant, (σ = 5.67 × 10^{-8} W. m^{-2} K^{-4})
$\sigma_{S\lambda}$	Scattering coefficient