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SIMULTANEOUS ANALYSIS, DESIGN AND OPTIMIZATION OF STRUCTURES USING FORCE METHOD AND ANT COLONY ALGORITHMS

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ABSTRACT

In this paper, an energy formulation of the force method is developed and the analysis, design and optimization are performed simultaneously using ant colony algorithms. New goal functions are introduced for minimization, and ant colony algorithms are employed for continuous optimization. An efficient method is introduced using ant colony algorithms for designing structures with prescribed member stress ratios. Finally, minimum weight design of truss and frame structures is formulated using ant colony algorithms and applied to some benchmark problems from literature.

Keywords: Continuous ant colony optimization (CACO); Force method; Simultaneous analysis, design and optimization; Fully stressed design (FSD)

1. INTRODUCTION

The computational drawbacks of the existing mathematical programming methods of optimization have forced researchers to rely on heuristic algorithms. Heuristic methods are suitable and powerful for many optimization problems. These methods do not require the derivatives of the objective function and constraints, and use probabilistic transition rules instead of deterministic rules.

Recently simultaneous analysis and design of structures has been performed using genetic algorithms [1-2]. Here, the formulation is modified and optimization is performed using ant colony algorithms.

This article consists of four parts. In first part, analysis is performed by ant colony algorithm using different goal functions for minimization. In second part, a continuous ant colony algorithm is employed for the analysis of truss and frame structures. In third part, a methodology is proposed for design of structures. In fourth part, minimum weight design is formulated and performed using ant colony optimization. For all the above cases, ant colony algorithm is found to be a powerful tool.

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2. ANALYSIS BY FORCE METHOD AND ANT COLONY ALGORITHM

The main aim of this section is to formulate the energy function of a structure and minimize this function using the ant colony algorithm, while satisfying all stated compatibility conditions. The formulation is based on the principle of minimum complementary work.

Suppose $\{\mathbf{p}\}=\{p_1, p_2, \ldots, p_n\}^t$ is the vector of nodal forces, $\{\mathbf{q}\}=\{q_1, q_2, \ldots, q_r\}^t$ contains r redundant forces, and $\{\mathbf{r}\}=\{r_1, r_2, \ldots, r_m\}^t$ comprises of the internal forces of the members. Where, n is the number of nodal forces, r is the number of redundant forces and m is the number of internal forces. From equilibrium we have

$$\{\mathbf{r}\} = [\mathbf{B}_0]\{\mathbf{p}\} + [\mathbf{B}_1]\{\mathbf{q}\} \tag{1}$$

From classical complementary energy concept we have

$$\boldsymbol{U}^{\mathrm{c}} = \frac{1}{2}\{\mathbf{r}\}^{\mathrm{t}}[\mathbf{F}_{\mathrm{m}}]\{\mathbf{r}\} \tag{2}$$

where $[\mathbf{F}_{\mathrm{m}}]$ is the unassembled flexibility matrix of the structure. Substituting $\{\mathbf{r}\}$ from Eq. (1) in Eq. (2) leads to

$$\boldsymbol{U}^{\mathrm{c}} = \frac{1}{2}\begin{bmatrix}\mathbf{p}\\ \mathbf{q}\end{bmatrix}^{\mathrm{t}}[\mathbf{H}]\begin{bmatrix}\mathbf{p}\\ \mathbf{q}\end{bmatrix} \quad \text{where} \quad [\mathbf{H}] = [\mathbf{B}_0 \quad \mathbf{B}_1]^{\mathrm{t}}[\mathbf{F}_{\mathrm{m}}][\mathbf{B}_0 \quad \mathbf{B}_1] \tag{3}$$

Decomposing the matrix $[\mathbf{H}]$ into four submatrices $[\mathbf{H}_{\mathrm{qq}}]$, $[\mathbf{H}_{\mathrm{qp}}]$, $[\mathbf{H}_{\mathrm{pq}}]$, and $[\mathbf{H}_{\mathrm{pp}}]$, we obtain $\boldsymbol{U}^{\mathrm{c}}$ as

$$\boldsymbol{U}^{\mathrm{c}} = \frac{1}{2}(\{\mathbf{p}\}^{\mathrm{t}}[\mathbf{H}_{\mathrm{pp}}]\{\mathbf{p}\} + \{\mathbf{p}\}^{\mathrm{t}}[\mathbf{H}_{\mathrm{pq}}]\{\mathbf{q}\} + \{\mathbf{q}\}^{\mathrm{t}}\{\mathbf{H}_{\mathrm{qp}}\}\{\mathbf{p}\} + \{\mathbf{q}\}^{\mathrm{t}}[\mathbf{H}_{\mathrm{qq}}]\{\mathbf{q}\}) \tag{4}$$

In the classical method, the derivative of $\boldsymbol{U}^{\mathrm{c}}$ with respect to $\{\mathbf{q}\}$ is found and equated to zero, leading to

$$\frac{\partial \boldsymbol{U}^{C}}{\partial \mathbf{q}} = 0 \Rightarrow \{\mathbf{H}_{qp}\}\{\mathbf{p}\} + [\mathbf{H}_{qq}]\{\mathbf{q}\} = \{0\} \Rightarrow \{\mathbf{q}\} = -[\mathbf{H}_{qq}]^{-1}[\mathbf{H}_{qp}]\{\mathbf{p}\} \tag{5}$$

Since $[\mathbf{H}]$ is symmetric, therefore $[\mathbf{H}_{\mathrm{qp}}]^{\mathrm{t}} = [\mathbf{H}_{\mathrm{pq}}]$.

In the present approach, finding the inverse of $[\mathbf{H}_{\mathrm{qq}}]$ is not required. Instead, $\boldsymbol{U}^{\mathrm{c}}$ from Eq. (3) is minimized by ant colony algorithm.

Previously, it has been stated that the first term of $\boldsymbol{U}^{\mathrm{c}}$ in Eq. (4) is constant and the second and third terms are identical. It can easily be shown that the third and fourth terms of $\boldsymbol{U}^{\mathrm{c}}$ are symmetric and therefore, the second and fourth terms can be omitted and the goal function can be obtained as

$$\boldsymbol{F_U} = \{\mathbf{q}\}^{\mathrm{t}}[\mathbf{H}_{\mathrm{qp}}]\{\mathbf{p}\} \tag{6}$$

Since Eq. (5) holds only in a specific point of search space, and in any other point one cannot omit the second and fourth terms, therefore we use a new goal function as

$$\boldsymbol{F_U} = \begin{bmatrix}\mathbf{p}\\ \mathbf{q}\end{bmatrix}^{\mathrm{t}}[\mathbf{H}]\begin{bmatrix}\mathbf{p}\\ \mathbf{q}\end{bmatrix} = \{\mathbf{p}\}^{\mathrm{t}}[\mathbf{H}_{\mathrm{pq}}]\{\mathbf{q}\} + \{\mathbf{q}\}^{\mathrm{t}}\{\mathbf{H}_{\mathrm{qp}}\}\{\mathbf{p}\} + \{\mathbf{q}\}^{\mathrm{t}}[\mathbf{H}_{\mathrm{qq}}]\{\mathbf{q}\} \tag{7}$$

In order to introduce an other goal function consider the left-hand side of the Eq. (5) that is a zero vector, and should be changed to a scalar. The best is to find its norm. If this norm is zero, all the entries should be zero. Therefore, the goal function can be written as

$$\boldsymbol{F_U} = \mathrm{norm}(\{\mathbf{H}_{\mathrm{qp}}\}\{\mathbf{p}\} + [\mathbf{H}_{\mathrm{qq}}]\{\mathbf{q}\}) \tag{8}$$

In Eqs. (7) and (8), {**P**}, [**H**] and its submatrices are constant; therefore ant colony algorithm finds the best results for {**q**} by minimizing the complementary energy function.

The general complementary energy function (Eq. (2)) can also be used as the goal function of minimization. In this case, there will be no need to calculate the [**H**] matrix and its submatrices. In order to minimize $\boldsymbol{F_U}$, a continuous ant colony algorithm is employed, as described in the subsequent section.

3. CONTINUOUS ANT COLONY OPTIMIZATION

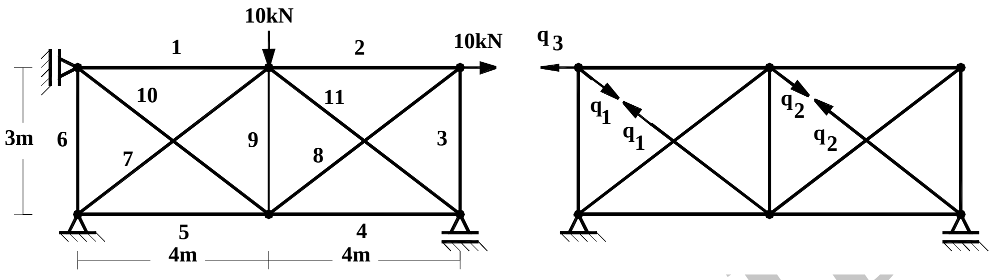
The first AC algorithm was introduced by Dorigo et al. [3]. AC algorithms have been inspired by colonies of real ants, which deposit a chemical substance called *pheromone* on the ground. This substance influences the choices they make: the larger amount of pheromone is on a particular path, the larger probability is that an ant selects the path. Artificial ants in AC algorithms behave in similar way. Thus, these behaviors of ant colony construct a positive feedback loop, and the pheromone is used to communicate information among individuals finding the shortest path from a food source to the nest. Ant colony algorithm simulates this mechanism of optimization, which can find the optimal solutions by means of communication and cooperation with each other.

Here we apply a continuous ant colony algorithm introduced by Chen et al. [4]. Evaporation and some other modifications are added to make it more suitable for our problem. Detailed description for various continues ant colony algorithms may be found in [4-7].

3.1 Example 1

Consider a simple truss as shown in Figure 1. Here, $\boldsymbol{F_U}$ can be formed in terms of three unknowns. The necessary matrices consisting of $\mathbf{B}_0$, $\mathbf{B}_1$ and **F** are constructed considering EA as constant for all the elements. Here, Eq. (7) is considered as the goal function. The variation of $\boldsymbol{F_U}$ versus the number of iterations is illustrated in Figure 2. The exact calculation of $\boldsymbol{F_U}$ leads to 839.8940, whereas, using the present approach $\boldsymbol{F_U}$ leads to

839.8644. Here, {**q**} is calculated as {**q**} = {4.6846, −3.6360, 8.2832}^{t} kN. For the analysis of this structure the number of iterations is taken as 30, and each iteration consists of 50 ants.



(a) (b)

Figure 1. A simple truss and the selected basic structure: (a) A planar truss; and (b) The selected basic structure

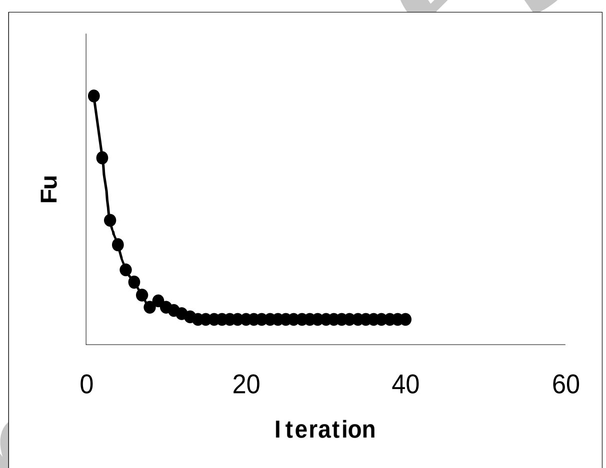


Figure 2. Variation of F_U versus the number of iterations

4. ANT COLONY ALGORITHM FOR DESIGN WITH DIFFERENT MEMBER STRESS RATIOS

Consider a truss identical to the one shown in Figure 1(a), with the vertical and horizontal loads being replaced by 147kN and 245kN, respectively. This truss has been previously designed with the constraints shown in Table I using genetic algorithm [1]. Here, we want to design this truss with new analysis goal function using ant colony algorithm. A basic structure similar to the one shown in Figure 1(b) is selected, where redundant forces consist of two internal forces and one external reaction, denoted by q_1, q_2 and q_3. The complementary energy of the structure is minimized for analysis by the force method.

Table I. Design data for 11-bar planar truss.

Design variables
Redundant and size variables q_1; q_2; q_3; A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8; A_9; A_{10}; A_{11}
Material and section property
Young's modulus is assumed to be constant.
Density of the material: $\rho = 0.00277$ kg/cm^3 =0.1 lb/in^3
$A = 0.4h^2, r = \sqrt{0.4A}$, thickness $t = 0.1h$.
Constraint data
Stress ratios
Case 1 C = {0.9, 0.8, 0.85, 0.8, 0.9, 0.85, 0.95, 0.9, 0.8, 0.9, 0.95}
Case 2 c_i =1; i =1, . . . , 11
For tensile members
$A > F/\left(0.6F_y\right), \frac{L}{r} < 300; r = \sqrt{0.4A}$
For compressive members
$A > F/F_a,\quad Fa = \frac{\left(1-0.5\beta^2\right)F_y}{1.67+0.375\beta-0.125\beta^3};\quad \beta = \frac{L\sqrt{F_y}}{6440r},\quad \frac{L}{r} < 200$
Stress constraints
σ_i<234.43 MPa; i =1, . . . , 11

Having the cross sections A_i $(i =1, \ldots, M)$, analysis is performed using the ant colony algorithm as described in the previous section. Since the main aim is to design, one can obtain cross-section set, A, corresponding to selected values of q (for each ant). $\boldsymbol{U}^{\mathbf{c}}$ is calculated as

$$\boldsymbol{U}^c = \frac{1}{2}\{\mathbf{r}\}^t[\mathbf{F}_m]\{\mathbf{r}\} \text{ where } \{\mathbf{r}\} = \begin{bmatrix}\mathbf{B}_0 & \mathbf{B}_1\end{bmatrix}\begin{bmatrix}\mathbf{p}\\ \mathbf{q}\end{bmatrix} \tag{9}$$

For a truss member $F_m = L/EA$ and for each selected ant q, one can obtain $\{\mathbf{r}\}$ from Eq.

(9), and each r corresponds to a set of cross-sectional areas A, the entries of which appear in the denominator of $\mathbf{F}_m$. Therefore, $\mathbf{F}_m$ is a function of L, E, q and C (i.e. A is eliminated). Thus $\boldsymbol{U}^{\mathbf{c}}$ is a function of q and C only. The pre-selected entries for C may be imposed at this stage. The role of C in finding A in terms of q has thus been shown, and $\boldsymbol{U}^{\mathbf{c}}$ can easily be minimized by the ant colony algorithm. For simplicity and similarity in design, the cross-sections are selected as hollow squares with mean length as h.

$\boldsymbol{U}^{\mathbf{c}}$ should be minimized in which $[\mathbf{F}_m]$ is a function of the unknowns q, C, L and E as

$$[\mathbf{F}_{\mathrm{m}}] = \frac{\mathrm{L}}{\mathrm{EA}} = \frac{\mathrm{L}}{\mathrm{E}\mathbf{f}(\mathbf{r},\mathrm{L},\mathrm{C})} = \boldsymbol{g}(\mathbf{q},\mathrm{C},\mathrm{L},\mathrm{E}) \tag{10}$$

Here we introduce a goal function for truss structures for the purpose of minimization, using ant colony with minimum parameters for efficiency in programming.

$$C_i = \frac{ABS(r_i)}{A_i \sigma_{ai}} \quad \text{and} \quad A_i = \frac{ABS(r_i)}{C_i \sigma_{ai}} \tag{11}$$

ABS standing for the absolute value.

From Eqs. (10) and (11), for each member we have

$$F_{m\,i} = \frac{L_i C_i \sigma_{ai}}{E_i \cdot ABS(r_i)} \tag{12}$$

$$\boldsymbol{U}_i^c = \frac{1}{2} r_i F_{mi} r_i = \frac{1}{2} r_i \frac{L_i C_i \sigma_{ai}}{E_i} \lambda_i \quad \text{where} \quad \left(\lambda_i = \frac{ABS(r_i)}{r_i} \right) \tag{13}$$

For the entire structure we have

$$\boldsymbol{U}^C = \frac{1}{2}\{\mathbf{r}\}^{\mathrm{t}} \begin{bmatrix} \frac{L_1 C_1 \sigma_{a1}}{E_1} & 0 & \cdots & 0 \\ 0 & \frac{L_2 C_2 \sigma_{a2}}{E_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \frac{L_M C_M \sigma_{aM}}{E_M} \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \vdots \\ \vdots \\ \lambda_M \end{Bmatrix} \tag{14}$$

σ_{ai} is the maximal allowable stress for each member. Considering constant values for σ_a and E for all members, the goal function can be written as

$$\boldsymbol{F}_U = \{\mathbf{r}\}^t \begin{bmatrix} L_1C_1 & 0 & \cdots & 0 \\ 0 & L_2C_2 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & L_MC_M \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \vdots \\ \vdots \\ \lambda_M \end{Bmatrix} \tag{15}$$

In this study, $\boldsymbol{F}_U$ is minimized by ant colony algorithm. For the introduced functions, [**q**] is unknown and L_i, C_i, σ_{ai} and E_i are specified for each member.

The magnitudes of A_i are determined considering the selected values for c_i and design constraints (buckling,…). The calculation of this example performed by ant colony algorithm and results are presented in Table II. The results obtained from GA, are also provided for comparison in Table II.

Table II. Results for the 11-bar planar truss (Cases 1 and 2)

Case 1	
	A={17.46,17.64,5.63,4.44,4.44,5.63,15.63,6.94,6.75,6.94,15.63} cm^2 W (AC) =1238.17 N W (GA) =1347.30 N
Case 2	
	A={18.58,14.46,5.63,4.44,4.44,5.63,15.63,6.94,5.63,6.94,15.63} cm^2 W (AC) =1206.65N W (GA) =1225.03 N

For this example, 50 ants have been created. The convergence is achieved after 40 iterations.

It can be observed that the weight of the truss in Case 2 is reduced compared to Case 1 because of higher magnitudes of member stress ratios. The present method results in lower weight and smaller number of iteration compared to GA. Results showed that all the pre-selected c_i values are attained, and the convergence of the analysis/design process is guaranteed.

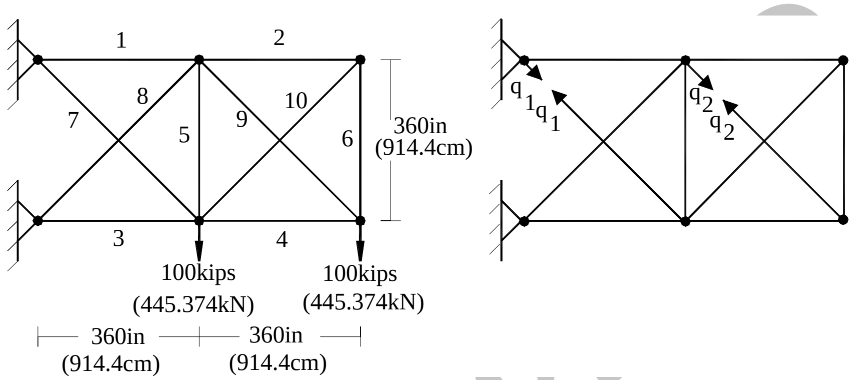
5. OPTIMAL DESIGN USING ANT COLONY ALGORITHM

Optimality criteria method (OCM) is one of the earliest optimization approaches [8]. Fully stressed design (FSD) is a kind of OCM which leads to correct optimal for statically determinate structures under a single load condition. In FSD all the members are supposed to be subjected to their maximal allowable stresses. Such a design can not always be achieved for an indeterminate structure with fixed geometry. Even by changing the geometry, a FSD may not be attained. Here, an AC formulation of FSD is presented without using direct analysis in the process of optimization. For this purpose, a truss type of structure is considered, and the complementary energy is written as

$$\boldsymbol{U}^c = \sum_{i=1}^{M} \frac{1}{2} \frac{P_i^2 L_i}{E_i A_i} = \sum_{i=1}^{M} \frac{1}{2} \frac{\gamma_i P_i^2 L_i A_i}{\gamma_i E_i A_i^2} = \frac{1}{2E\gamma} \sum_{i=1}^{M} \sigma_i^2 W_i \tag{16}$$

For constant values of E and γ, the minimum weight can be achieved only when the stresses in all the members are identical, and the corresponding term moves out of the summation. One may ignore the constraint of weight, and look for a structure that is fully stressed. The goal function for FSD design can be considered as Eq. (14) without c_i.

As an example, consider the structure shown in Figure 3. The member size constraint provided in Table III, leads to a design for which not all the members are fully stressed. Table IV contains the results of this example are obtained using the present ACO algorithm and GA for three cases.



(a) (b)

Figure 3. A simple truss and the selected basic structure: (a) A planar truss; (b) The selected basic structure

Table III. Design data for a 10-bar planar truss

Design variables
Size variables $A_1;A_2;A_3;A_4;A_5;A_6;A_7;A_8;A_9;A_{10}$ (and q_1; q_2)
Material property and constraint data
Young's modulus: E =1e7 psi=6.895e7MPa.
Density of the material: ρ = 0.1 lb/in^3 =0.00277 kg/cm^3
For all members: $A_i \geq 0.1$ in^2; i =1, . . . , 10
Stress constraints
(a) FSD
Case 1: $\lvert\sigma_i\rvert \leq 25$ ksi(172.375MPa); i =1, . . . , 10
Case 2: $\lvert\sigma_i\rvert \leq 25$ ksi; i =1, . . . , 8, 10 and $\lvert\sigma_9\rvert \leq 50$ ksi (344.75MPa)
(b) Weight minimization
Case 3: $\lvert\sigma_i\rvert \leq 25$ ksi; i =1, . . . , 8, 10 and $\lvert\sigma_9\rvert \leq 50$ ksi (344.75MPa)

In this example, the internal forces in members 7 and 9 are taken as redundant forces, forming the initial ants of the AC. The fitness function for Case 2 and Case 1 is the complementary energy as introduced before (Eq. (14) or Eq. (15)).

In Case 3, we want to design a FS structure with the least possible weight, therefore the problem becomes more involved, since different cases may arise with the condition of FS, and one naturally wants the one with the smallest weight.

Choosing a function in the form of $\boldsymbol{f}=\boldsymbol{W}+\alpha \boldsymbol{U}^c$ does not help, since penalty functions are commonly selected as

$$\boldsymbol{f}=A+\alpha B \tag{17}$$

for which ultimately f converges to A, and B approaches to zero. Therefore, α is often selected as a big number. The main difficulty arises when for α, the minimum value of $\boldsymbol{f}$ does not correspond to the minimum $\boldsymbol{U}^c$. In this case, $\boldsymbol{W}$ is minimum while the corresponding $\boldsymbol{U}^c$ is not minimum, i.e. the analysis of the structure is not completed yet. Small α will not guarantee the minimality of $\boldsymbol{U}^c$ and a big α does not lead to minimum $\boldsymbol{W}$. Therefore, a new formulation is required.

In a formulation, we alter the second term of Eq. (17) such that its minimum value becomes zero. Then one can use a formulation similar to the common penalty function.

In this method, one does not need $\boldsymbol{U}$, and optimization can be performed employing only $\boldsymbol{U}^c$, where $\boldsymbol{U}^c$ can be written in the form of Eq. (4). For the analysis, $\mathbf{q}$ should be selected such that Eq. (8) becomes zero. In this case, we can write

$$\boldsymbol{F}(\mathbf{q},\mathrm{A})=\mathbf{W}(\mathrm{A})(1+\alpha\,\mathrm{norm}([\mathbf{H}_{\mathrm{qp}}]\{\mathbf{p}\}+[\mathbf{H}_{\mathrm{qq}}]\{\mathbf{q}\})) \tag{18}$$

Here, the input is $\{\mathbf{q}\}$, and having $\{\mathbf{q}\}$ from Eq. (18), the magnitude of $\boldsymbol{F}$ can be calculated and its minimum for a large value of α will correspond to minimum $\boldsymbol{W}$. If a structure contains other constraints, then these should be normalized and added to the above function with a penalty coefficient. Therefore, the final formulation of the problem for the two cases of discrete and continuous cross sections, are as follows:

$$\text{Find}\quad (\mathbf{q},\mathrm{A});\mathrm{A}\in\{\mathrm{S_d},\mathrm{S_c}\}\qquad \mathbf{W}(\mathrm{A})=\sum_{i=1}^{M}A_iL_i\rho_i$$
$$\text{Min}\quad \boldsymbol{F}(\mathbf{q},\mathrm{A})=\mathbf{W}(\mathrm{A})(1+\alpha\,\mathrm{norm}([\mathbf{H}_{\mathrm{qp}}]\{\mathbf{p}\}+[\mathbf{H}_{\mathrm{qq}}]\{\mathbf{q}\}))+\sum_{m=1}^{nc}\max[0,\mathbf{g}_{\mathrm{m}}(\mathrm{A})] \tag{19}$$

Where S_d and S_c are the discrete and continuous cross sections, respectively. $\boldsymbol{g}_m(A)$ correspond to violations of constraints, which include stress constraints, displacement constraints and buckling constraints. Their magnitudes can be written in the form of the absolute value of existing value to permissible value minus one.

From Table IV, it is noticeable that if in a structure the maximum allowable stresses of all members are equal, then FSD leads in a minimum weight design. Otherwise, we have to add a penalty function (that contains the weight of the structure) to the goal function for the purpose of minimization using ant colony algorithm.

Table IV. Results for the 10-bar planar truss (Cases 1-3)

Case 1 (FSD)	
Present work (ACO)	
	A={51.42,0.64,51.80,25.55,0.64,0.64,36.77,36.19,36.19,0.64} cm^2
	W =1595.88 lb=7.11 KN
Genetic Algorithm	
	A= {51.16, 0.64, 52.06, 25.35, 0.642, 0.64, 37.16, 35.81, 35.81, 0.84} cm^2
W =1593.5 lb=7.09 KN	
Case 2 (FSD)	
Present work (ACO)	
	A= {26.06, 25.35, 77.16, 0.64, 0.64, 25.35, 72.64, 0.64, 0.64, 35.87} cm^2
	W =1732.68 lb=7.71 KN
Genetic Algorithm	
	A= {26.58, 25.03, 76.64, 0.77, 0.64, 25.03, 71.87, 1.10, 0.64, 35.35} cm^2
W =1723.5 lb=7.6661 KN	
Case 3 (Weight minimization)	
Present work (ACO)	
	A= {51.22, 0.71, 51.93, 25.10, 0.64, 0.71, 36.97, 35.93, 17.74, 0.97} cm^2
	W =1450.15lb=6.46 KN
Genetic Algorithm	
	A= {50.26, 1.68, 53.03, 24.58, 0.64, 1.54, 38.52, 35.48, 23.16, 2.19} cm^2
W =1519.2 lb=6.75 KN	

In the following, two examples are presented and the results are compared to those of the existing approaches.

5.1 Example 1: A 10-bar planar truss

Optimal design of 10-bar truss, shown in Figure 3, is performed. Table V contains the necessary information and a displacement constraint that is added to show the efficiency of present method. Material properties can be found in Table III. In this example, two cases are considered, the first case is for discrete and the second case corresponds to continuous sections. In both cases, A and q are design variables, but in discrete case, we employed a code for sections. Using the formulation of the previous section and minimizing Eq. (19), the results are obtained as shown in Tables VI and VII.

Table V. Design data for the 10-bar planar truss

Nodal displacement constraint in all directions of the co-ordinate system
$\|\sigma_i\| \leq 2$ in (5.08 cm); i =1, . . . , 4
List of the available profiles
Case 1: (Discrete sections)
A_i = {1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.5, 13.5, 13.9, 14.2, 15.5, 16.0, 16.9, 18.8, 19.9, 22.0, 22.9, 26.5, 30.0, 33.5} in^2
A_i = {10.4516, 11.6129, 12.8387, 13.7419, 15.3548, 16.9032, 16.9677, 18.5806, 18.9032, 19.9354, 20.1935, 21.8064, 22.3871, 22.9032, 23.4193, 24.7741, 24.9677, 25.0322, 26.9677, 27.2258, 28.9677, 29.6128, 30.9677, 32.0645, 33.0322, 37.0322, 46.5806, 51.4193, 74.1934, 87.0966, 89.6772, 91.6127, 99.9998, 103.2256, 109.0320, 121.2901, 128.3868, 141.9352, 147.7416, 170.9674, 193.5480, 216.1286} cm^2
Case 2: (Continuous sections)
$0.1 \leq A_i \leq 35$ in^2 (225.8960) cm^2; i =1, . . . , 10

Table VI. Optimal design comparison for the 10-bar planar truss (discrete)

Method	**Weight: lb (KN)**	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Kaveh and Rahami [1]	5490.738 (24.4228)	33.5	1.62	22.90	14.2	1.62	1.62	7.97	22.9	22.00	1.62
Shih [9]	5491.71 (24.4271)	33.50	1.62	22.90	15.50	1.62	1.62	7.97	22.00	22.00	1.62
Rajeev [10]	5613.84 (24.9704)	33.50	1.62	22.90	15.50	1.62	1.62	14.20	19.90	19.90	2.62
Present work	5517.72 (24.5702)	33.50	1.62	22.90	14.2	1.62	1.62	11.5	22.00	19.90	1.62

Table VII. Optimal design comparison for the 10-bar planar truss (continuous)

Method	Weight: lb (KN)	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Kaveh and Rahami [1]	5061.90	30.67	0.1	22.87	15.34	0.1	0.46	7.48	20.96	21.70	0.1
Schmit and Farshi [11]	5089.0 (22.6359)	33.43	0.1	24.26	14.26	0.1	0.1	8.39	20.74	19.69	0.1
Schmit and Miura [12]	5076.85 (22.5818)	30.67	0.1	23.76	14.59	0.1	0.1	8.59	21.07	20.96	0.1
Schmit and Miura [12]	5107.3 (22.7173)	30.57	0.37	23.97	14.73	0.1	0.36	8.55	21.11	20.77	0.32
Venkayya [13]	5084.9 (22.6176)	30.42	0.13	23.41	14.91	0.10	0.10	8.70	21.08	21.08	0.19
Gellatly and Berke [14]	5112.0 (22.7382)	31.35	0.10	20.03	15.60	0.14	0.24	8.350	22.21	22.06	0.1
Dobbs and Nelson [15]	5080.0 (22.5958)	30.50	0.1	23.29	15.43	0.1	0.21	7.65	20.98	21.82	0.1
Rozzo [16]	5076.66 (22.5810)	30.73	0.1	23.93	14.73	0.1	0.1	8.54	20.95	21.84	0.1
Khan and Willmert [17]	5066.98 (22.5379)	30.98	0.1	24.17	14.81	0.1	0.41	7.547	21.05	20.94	0.1
Presemt work	5095.46 (22.6899)	30.86	0.1	23.55	15.01	0.1	0.22	7.63	21.65	21.32	0.1

5.2 Example 2: An eight story frame

An eight story frame is considered as shown in Figure 4. This structure was optimized using discrete cross sections by Camp et al. [18] and Nanakorn and Meesomklin [19]. Here we apply a continues optimization using the empirical cross section (A) - moment of inertia (I) relationships of [20] as follows :

$$I = \begin{cases} 4.592A^2 & 0 \le A \le 15 \\ 4.638A^2 & 15 < A \le 44 \\ 256.229A - 2300 & 44 < A \le 100 \end{cases} \tag{20}$$

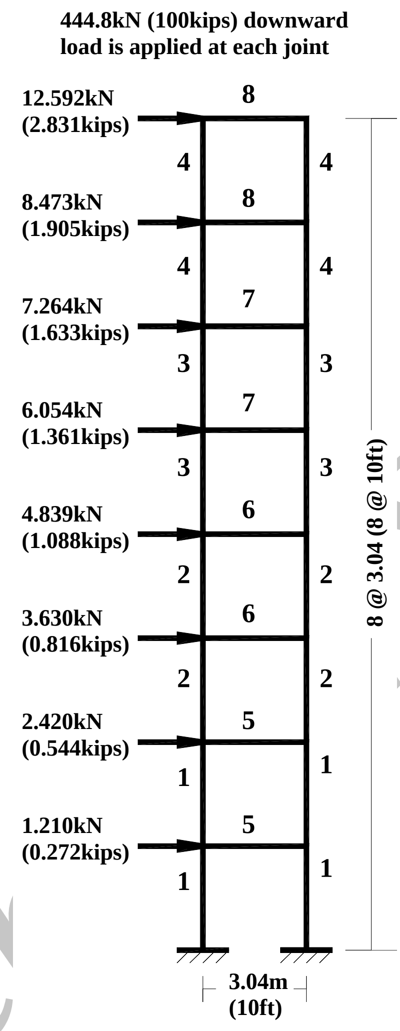


Figure 4. An eight story frame and grouping of its members.

The 24 members of the structure are categorized into 8 groups as indicated in Figure 4. Lateral displacement at the top story is the only design constraint that must be less than 5.08cm (2in). The modulus of elasticity, E=200 GPa (29×10^3 ksi) and the material density, ρ=76.8 kN/m^3 (2.83×10^{-4} kip/in^3). As the axial internal forces effects are negligible in frame structures, we consider only the bending moments in elements. Table VIII contains the selected cross sections for each group. The final displacement at top story is 5.08cm (2.00 in) that shows the efficiency of the method.

Table VIII. Results for eight story frame.

Group number	Optimal cross section (cm^2)
1	42.71413 (6.620703 in^2)
2	37.76298 (5.853274 in^2)
3	35.22271 (5.459531 in^2)
4	30.905 (4.790285 in^2)
5	58.47396 (9.063481 in^2)
6	43.45519 (6.735569 in^2)
7	49.84243 (7.725593 in^2)
8	34.64226 (5.369561 in^2)
Total Weight (kN)	22.45467 (5.049294 kip)

6. CONCLUDING REMARKS

In this article some new formulations are presented for simultaneous analysis, design and optimization of structures using ant colony algorithms. These methods employ basic ideas from energy and complementary energy, and employ continuous ant colony algorithm. AC performs analysis of the structures without using classical methods which require the direct solution of equations taking more computational time for calculation. Design is performed providing prescribed stress ratios for members, and as a special case, an efficient approach is examined for fully stressed design using AC. Formulation in terms of energy concepts permits the efficient application of AC in optimization. The examples studied in this paper for simultaneous analysis, design and optimization, illustrate the capability and the accuracy of the present methods.

REFERENCES

1. Kaveh A, Rahami H. Analysis, design and optimization of structures using force method and genetic algorithm. *International Journal for Numerical Methods in Engineering*, **65**(2006)1570-84.
2. Kaveh A, Rahami H. Nonlinear analysis and optimal design of structures via force method and genetic algorithm. *Computer and Structures*, **84**(2006)770-8.

3. Dorigo M, Stützle T. Ant colony optimization. The MIT Press: Cambridge, 2004.
4. Chen L, Shen J, Qin L, Fan J. A Method for Solving Optimization Problem in Continuous Space Using Improved Ant Colony Algorithm. CASDMKM, LNAI **332**7(2004)61-70.
5. Socha K. ACO for Continuous and Mixed-Variable Optimization. ANTS, LNCS **3172**(2004)25-36.
6. Pourtakdoust H, Nobari H. An Extension of Ant Colony System to Continuous Optimization Problems. ANTS, LNCS **3172**(2004)294-301.
7. Kong M, Tian P. A Direct Application of Ant Colony Optimization to Function Optimization Problem in Continuous Domain. ANTS, LNCS **4150**(2006)324-31.
8. Kirsch U. *Optimum Structural Design, Concepts, Methods and Applications*. McGraw-Hill: New York, 1981.
9. Shih CJ, Yang YC. Generalized Hopfield network based structural optimization using sequential unconstrained minimization technique with additional penalty strategy. *Advances in Engineering Software*, **33**(2002)721-9.
10. Rajeev S, Krishnamoorthy CS. Discrete optimization of structures using genetic algorithms. *Journal of Structural Engineering, ASCE*, **118**(1992)1233-50.
11. Schmit Jr LA, Farshi B. Some approximation concepts for structural synthesis. *AIAA Journal*, **12**(1974)692-9.
12. Schmit Jr LA,, Miura H. Approximation concepts for efficient structural synthesis. NASA CR-2552. NASA: Washington, DC, 1976.
13. V.B. Venkayya, Design of optimum structures. *Computers and Structures*, **1**(1971)265-309.
14. Gellatly RA, Berke L. Optimal structural design. AFFDLTR-70-165, Air Force Flight Dynamics Laboratory, Wright-Patterson AFB, OH, 1971.
15. Dobbs MW, Nelson RB. Application of optimality criteria to automated structural design. *AIAA Journal*, **14**(1976):1436-43.
16. Rizzi P. Optimization of multi-constrained structures based on optimality criteria. AIAA/ASME/SAE 17th Structures, Structural Dynamics, and Materials Conference, King of Prussia, PA, 1976.
17. Khan MR, Willmert KD, Thornton WA. An optimality criterion method for large-scale structures. *AIAA Journal*, **17**(1979)753-61.
18. Camp C, Pezeshk S, Cao G. Optimized design of two-dimensional structures using a genetic algorithm. *Journal of Structural Engineering, ASCE*, **124**(1998):551-559.
19. Nanakorn P, Meesomklin K. An adaptive penalty functions in genetic algorithms for structural design optimization. *Computer and Structures*, **79**(2001)2527-39.
20. Khan MR. Optimality criterion techniques applied to frames having general cross-sectional relationships. *AIAA Journal*, **22**(1984):669-76.