WAVELET ANALYSIS FOR PROCESSING OF EARTHQUAKE RECORDS

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ABSTRACT

Wavelet analysis is a new mathematical technique and in the recent years enormous interest in application of engineering has been observed. This new technique is particularly suitable for non-stationary processes as in contrast to the Fourier transform (FT). The wavelet transform (WT) allows exceptional localization, both in time and frequency domains. The application of the WT to earthquake engineering is rare. In this paper the WT capability to give a full time–frequency representation of the earthquake record is demonstrated. In this method, the time series of the earthquake record, breaking at the tropical coral reefs and mechanically generated waves in the wave flume demonstrates the ability of the wavelet transform technique to detect a complex variability of these signals in the time–frequency domain. Various spectral representations resulting from the wavelet transform are discussed and their application for earthquake record is shown.

Keywords: Time history; frequency localization; wavelet transform; Fourier transform

1. INTRODUCTION

The earthquake records are the time-domain signals. However, in many cases the most distinguished information is hidden in the frequency spectrum, which provides the energy, associated with a given frequency. The frequency spectrum of the signal can be obtained by the Fourier transform (FT). The FT yields information on how much but not when (in time) the particular frequency components exist. Such information is sufficient in a case of the stationary signals as the frequency content of such signals does not change in time and all frequency components exist all the time. However, in all cases the earthquake waves change in a relatively short period of time.

When waves start to break, the frequency content of signal changes rapidly in time due to the nonlinear interaction between elementary wave components and the resulting energy transfer, and the energy dissipation. In such cases, the FT provides information on the frequency content; however, the information on the frequency localization in time is

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essentially lost in the process. When the time localization of the spectral components is required, the transform of time series, which provides the time–frequency representation of the signal, should be developed. Transform of such type is the wavelet transform, which gives full time–frequency representation of the time series. In contrast to the FT, the wavelet transform allows exceptional localization in the time domain via translations of the so called mother wavelet, and in the scale (frequency) domain via dilations [1-4].

Wavelet transform is a relatively modern technique and in recent years enormous interest in the application of wavelets has been observed. On the other hand, the application of the wavelet transform to earthquake engineering is not frequent. In ref. [5] Gurley and Kareem demonstrated the usefulness of the wavelet transform in studying dispersion of the earthquake load. In Refs. [6-8], discrete and fast wavelet transforms are used for dynamic analysis of structures induced earthquake load. Then the discrete and fast wavelet transform are used for optimization of structures with earthquake loading [9-11]. In the ref. [12], the continuous wavelet transform was developed to analyze the energy balance in the equilibrium spectral subrange of the wind-generated gravity waves. Mori and Yasuda [13] applied the wavelet transform to detect wave growth and breaking in the time series.

In this paper the application of wavelet transform for the processing of earthquake record is discussed and an attempt to produce some useful quantitative results is made. The fundamentals of the WT are given and the difference between FT and WT is demonstrated and the application of the WT for processing of earthquake record is shown.

2. THE FOURIER TRANSFORM

The FT is probably the most popular transform being used, but for better understanding the difference between the WT and the FT, a short overview of the FT is provided. We start with a case of continuous deterministic signal $x(t)$. If the total signal energy, Er, is finite or if:

$$Er = \int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty \tag{1}$$

then $x(t)$ is absolute-integrable over the entire domain and the FT of the $x(t)$ exists as follows:

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-2\pi f t i} dt \tag{2}$$

and

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{2\pi f t i} dt \tag{3}$$

Using the square of the modules of the FT the energy spectral density of the signal is obtained as:

$$S(f) = X(f) * X^*(f) = |X(f)|^2 \tag{4}$$

where the asterisk denotes the complex conjugate. The integration of the spectral density $S(f)$ over the entire spectral domain provides the total energy, Er, of the signal:

$$Er = \int_{-\infty}^{+\infty}\left|X(f)\right|^2 df = \int_{-\infty}^{+\infty}\left|x(t)\right|^2 dt \tag{5}$$

In the FT there is no resolution problem in the frequency domain, as we know exactly what frequencies exist. This perfect frequency resolution in the FT is due to the fact that the window, $e^{2\pi f t i}$ used in this transformation, lasts all the time, from minus infinity to plus infinity. Similarly, there is no time resolution problem in the time domain, since we know the value of the signal at every instant of time. In contrary, we can say that both time resolution in the FT, and the frequency resolution in the time domain are zero, since we have no information about them.

3. WAVELET TRANSFORM

The WT is similar to the FT as it breaks a signal down into its constituents. Whereas the FT breaks the signal into a series of sine waves of different frequencies, the WT breaks the signal into its wavelets, which are scaled and shifted versions of the so called mother wavelet.

The WT allows exceptional localization both in the time domain via translations of the wavelet, and in the frequency (scale) domain via dilations. The wavelet transforms are proposed in refs. [14-16]. The wavelets are complex or real functions concentrated in time and frequency and having the same shape. In the WT, the signal is multiplied with the wavelet, and the transform is separately computed for different segments of the time domain signal. In general, the WT of the signal, $x(t)$, is defined as a following inner product:

$$WT(\tau,b) = \int_{-\infty}^{+\infty} x(t)g^*(\tau,b)dt \tag{6}$$

The family of continuously translated and dilated wavelets is generated from mother wavelet $g(t)$:

$$g(\tau,b) = \frac{1}{\sqrt{b}}\, g(\frac{t-\tau}{b}) \tag{7}$$

where t is the translation parameter, corresponding to the position of the wavelet as it is shifted through the signal, b is the scale dilation parameter determining the width of the wavelet. The scale $b>1$ dilates (or stretches out) the signals, whereas scale $b<1$ compresses the signal [14]. The wavelet coefficients, $WT(t,b)$, represent the correlation (in terms of the time-scale functions) between the wavelet and a localized section of the signal. If the signal has a major component of the frequency corresponding to the given scale, then the wavelet at this scale is close to the signal at the particular location and the corresponding wavelet transform coefficient, determined at this point, has a

relatively large value. Therefore, the wavelet transform is a sort of microscope with magnification b^{-1} and location given by parameter t, while the optics of the microscope is characterized by the function $g(\tau,b)$. For the wavelet which has the mother wavelet status, the function $g(t)$ must satisfy several properties, such as [15]:

The amplitude $|g(t)|$ must decay rapidly to zero in the limit $|t| \to \infty$. This feature ensures the localization aspect of wavelet analysis. It means that the wavelet $g[(t-\tau)/b]$ has insignificant effect at time $|t| > \tau_{crit}$, where τ_{crit} is a critical time lag.

1. The wavelet $g(t)$ must have zero mean. This condition, known as the admissibility condition, ensures the invertibility of the wavelet transform.
2. The wavelets are regular functions such that $G(w<0)=0$. It means that wavelets need to be described in terms of positive frequencies only.

The original signal can be obtained from the wavelet coefficients through the inverse transform [16]:

$$x(t) = C_\psi^{-1} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} WT(\tau,b)b^{-2}g(\tau,b)d\tau db \tag{8}$$

in which

$$C_\psi^{-1} = \int_{-\infty}^{+\infty} \omega^{-1}|G(\omega)|^2 d\omega \tag{9}$$

and the $G(\omega)$ is the Fourier transform of function $g(t)$.

The wavelet transform should reflect the type of features, which are present in the time series. For time series with sharp steps, a boxcar-like wavelet should be chosen, while for smoothly varying time series a smooth function is more appropriate. However, if the wavelet power spectra are not of the primary interest, the choice of wavelet function is not critical.

One of the most extensively used mother wavelet is the Morlet's wavelet [17]:

$$g(t) = e^{-0.5t^2} e^{ict} \tag{10}$$

Equation 10 represents a plane wave of frequency c, modulated by a Gaussian envelope of the unit width. For calculations in this paper only wavelet Eq. 10 is used. Using the representation Eq. 7, the Morlet wavelet takes the form:

$$g(\tau,b) = \frac{1}{\sqrt{b}} e^{(-0.5(\frac{t-\tau}{b})^2)} e^{(ic\frac{t-\tau}{b})} \tag{11}$$

The frequency nature of the parameter c is clearly seen if we take $c = 2\pi$. Then Eq. 11 becomes:

$$g(\tau,b) = \frac{1}{\sqrt{b}} e^{(-0.5(\frac{t-\tau}{b})^2)} e^{(i\frac{2\pi}{b}(t-\tau))} \tag{12}$$

The term $e^{(i\frac{2\pi}{b}(t-\tau))}$ represents the plane sinusoidal wave of a frequency $2\pi / b$; thus the scale dilation b can be treated as a period.

Once a wavelet function is chosen, it is necessary to use a set of scales, b, in the wavelet transform Eq. 12 to build up a more complete picture. It is convenient to adopt the scale b as functional powers of two [18]:

$$b_i = b_0 2^{i\delta} \qquad\qquad i = 1,2,...,M \qquad\qquad (13)$$

in which

$$M = \delta^{-1} log_2(\frac{N\Delta t}{b_0}) \qquad\qquad (14)$$

where N is the number of values in the time series, and Δt is the time sampling. The b_0 is the smallest reasonable scale while b_M determines the largest scale. The b_0 should be chosen of such a value to let the equivalent Fourier period to be equal to approximately $2\Delta t$. The scale factor δ should be chosen to provide adequate sampling in scale b. For the Morlet wavelet, δ of about 0.5 is the largest value that still gives a smooth picture of the wavelet spectrum. However, smaller values of δ give finer resolution.

The WT is done in a similar way to the windowed Fourier transform (WFT) analysis in the sense that the signal is multiplied with a window (the wavelet). However, the window width has been changed as the transform is computed for every single spectral component. This is the most significant characteristic of the wavelet transform. In the WFT, the time and frequency resolutions are determined by constant width of the window, i.e. both time and frequency resolutions are constant. Therefore in the WFT case the time–frequency plane consists of rectangles.

4. RESEMBLANCE OF WAVELETS TO FOURIER MODES

For a better understanding of the wavelet transform nature, the relation between the wavelet transform and the more common Fourier transform is the basic one for the interpretation of the results of processing of the signals by the wavelet technique. This relation is not straightforward for arbitrary wavelet. However, in the case of the Morlet wavelet used in this paper, the relationship between WT and FT can be found in a simpler way, mainly due to the periodic character of the Morlet's wavelet [17].

A linear relationship between scale, b, and period, T_0, is as follows:

$$b = \frac{c + \sqrt{c^2 + 2}}{4\pi} T_0 = \alpha T_0 \qquad\qquad (15)$$

where

$$\alpha = \frac{c + \sqrt{c^2 + 2}}{4\pi} \qquad\qquad (16)$$

Again, Eq. 16 indicates that physical dimension of scale, b, is the time. Assuming that $c = 2\pi$, Eq. 16 yields $\alpha = 1.0125$. The scale, b, becomes fully equivalent to the T_0 ($\alpha = 1$) when $c = [(4\pi)^2 - 2]/8\pi \cong 6.2036$. In the application discussed in this paper, the value $c = 6.2036$ is used. Such choice of c value underlines the oscillatory nature of the second term of the Morlet wavelet Eq. 10 which is very suitable for processing of the earthquake record. Thus, for the Morlet wavelet, the scale b and the Fourier period T are nearly identical. However, it should be noted that for other mother wavelets, the Fourier period differs from scale, b.

5. APPLICATION OF WT TO EARTHQUAKE RECORDS

In this section, the application of the wavelet transform to processing of the earthquake record is discussed. In this section two examples are presented. The records of Taft (1952) and El Centro (1940) are decomposed. The number of points of the Taft and El Centro are 730 and 2688, respectively. The time intervals for the records are 0.02 seconds

5.1 Example 1: The Taft record
In this section, FT and WT of the Taft earthquake will be computed. The earthquake record and FT of the Taft are shown in Figures 1 and 2, respectively. WT of the earthquake record in three and two dimension are shown in Figures 3 and 4, respectively. By comparing Figures 2 and 4, it is observed that the highest frequency of the earthquake record is 1.5 (Hz), which is corresponding to scale 0.68. Besides, the frequency range of 1.25-1.8 (Hz) has the most important of FT, is the same as the highest frequency range. On the other hand, we can say that the frequency range of 1.25-1.8 can be the same as the scale rang of 0.55-0.8. Referring to Figures 3 and 4, and the output programming for this purpose, we can distinguish the time of each frequency. For example, the time of the frequency 1.5 which is as correspond to scale 0.68, is 1.3.48 seconds. The time of the frequency rang 1.25-1.8, is 13.2-14.1 seconds. In other words, this range of time is the most dangerous range for all constructions which have the same frequency as the dominant frequency of earthquake. The important frequency of the Taft earthquake is 1.5 Hertz. This frequency equals to frequency of steel frame with the heights of 17m. For this particular construction, the resonance takes place due to the closeness of the frequency of the construction with the frequency of the earthquake at 13.48 seconds after the beginning of the earthquake and in theoretically the construction is destroyed.

In the same way, we can compute the time of other frequencies of the earthquake record.
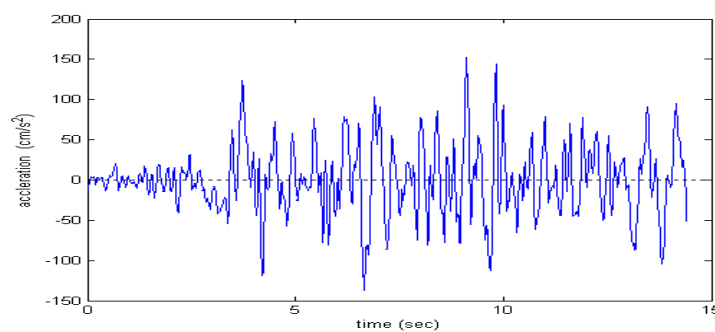
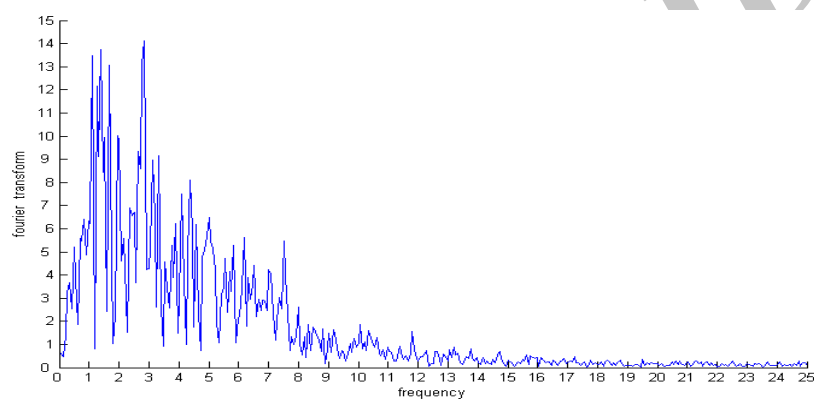Figure 1. The Taft earthquake record
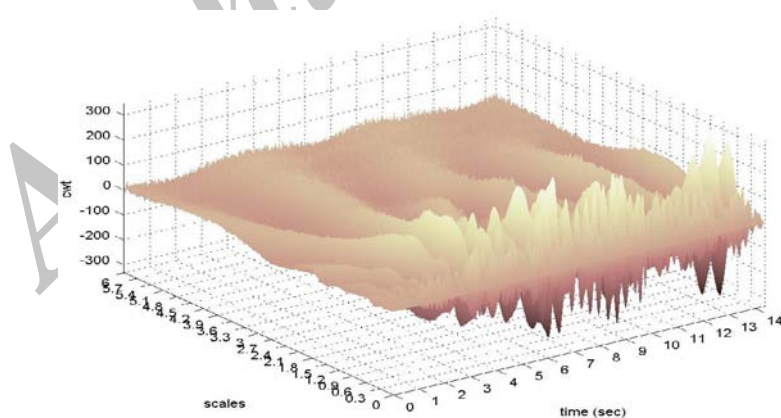

Figure 2. FT of the Taft earthquake record

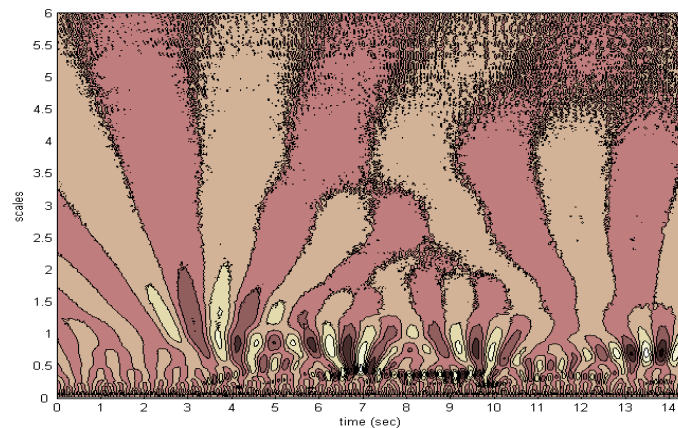Figure 3. Three dimensional WT of the Taft earthquake record

Figure 4. Two dimensional WT of the Taft earthquake record

5.2 Example 2: The El Centro record
In this section, FT and WT of the El Centro earthquake will be computed. The earthquake record and FT of the El Centro are shown in Figures 5 and 6, respectively. WT of the earthquake record in three and two dimension are shown in Figures 7 and 8, respectively. By comparing Figures 6 and 8, it is observed that the highest frequency of the earthquake record is 1.7 (Hz), which is corresponding to scale 0.55. Besides, the frequency range of 0.8-2.5 (Hz) has the most important of FT, is the same as the highest frequency range. On the other hand, we can say that the frequency range of 0.8-2.5 can be the same as the scale rang of 0.4-1.25. Referring to Figures 7 and 8, and the output programming for this purpose, we can distinguish the time of each frequency. For example, the time of the frequency 1.7 which is as correspond to scale 0.55, is 1.85 seconds. The time of the frequency rang 0.8-2.5, is 1.4-2 seconds. In other words, this range of time is the most dangerous range for all constructions which have the same frequency as the dominant frequency of earthquake. The important frequency of the Taft earthquake is 1.7 Hertz. This frequency equals to frequency of steel frame with the heights of 17m. For this particular construction, the resonance takes place due to the closeness of the frequency of the construction with the frequency of the earthquake at 1.85 seconds after the beginning of the earthquake and in theoretically the construction is destroyed. In the same way, we can compute the time of other frequencies of the earthquake record.
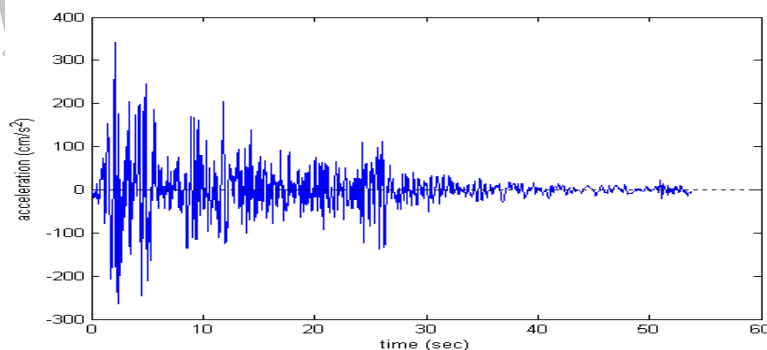
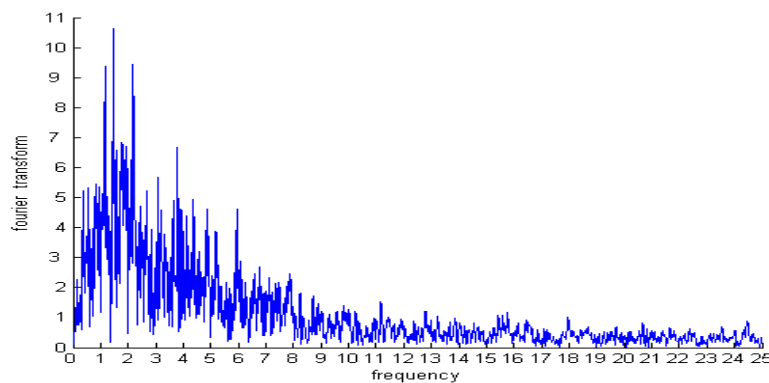Figure 5. The El Centro earthquake record
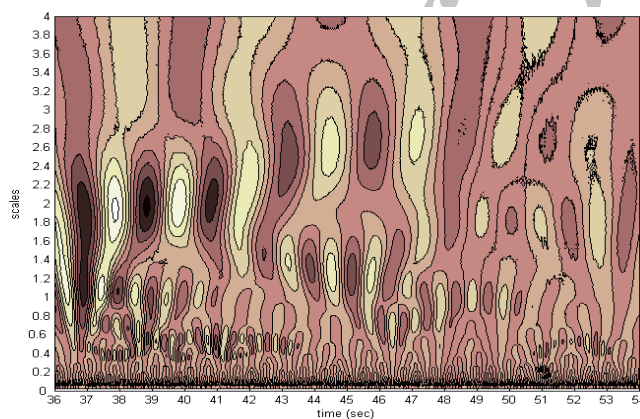

Figure 6. FT of the Taft earthquake record

Figure 7. Three dimensional WT of the El Centro earthquake record
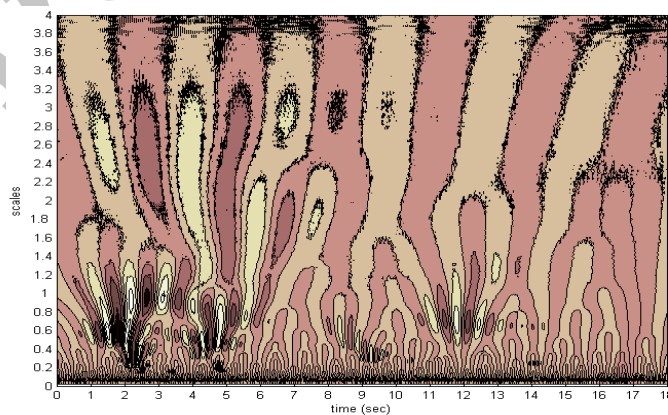
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Figure 8(a). Two dimensional WT of the El Centro earthquake record (time 0-18 Sec.)
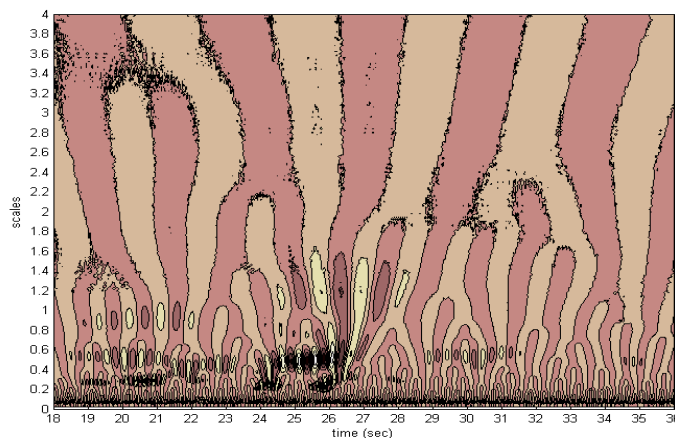

Figure 8(b). Two dimensional WT of the El Centro earthquake record (time 18-36 Sec.)
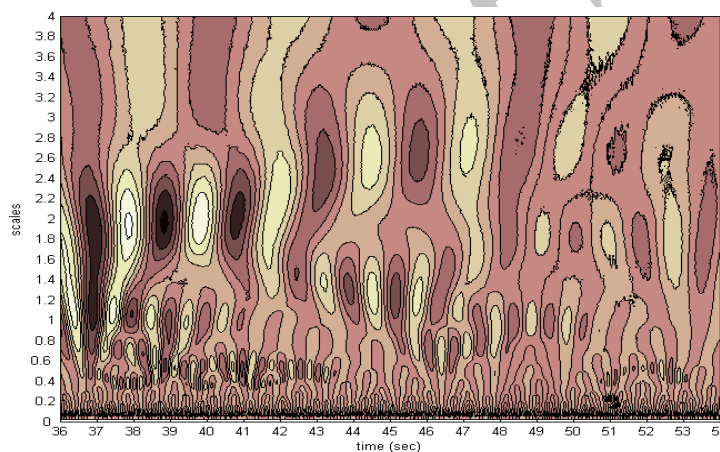

Figure 8(c). Two dimensional WT of the El Centro earthquake record (time 36-54 Sec.)

6. CONCLUSIONS

The wavelet transform technique is particularly suitable for non-stationary signals. In contrast to the Fourier transform, the wavelet transform allows exceptional localization, both in the time domain via translation t of the wavelet, and in the frequency domain via dilations scales b, which can be changed from minimum to maximum, chosen by the user. The two examples demonstrate the WT's capability to give a full time–frequency representation of the earthquake record. The processing of the time series resulting from recording the El Centro and Taft records demonstrates the ability of the WT technique to

detect the complex variability of these signals in time–frequency space.

The variations of the frequencies corresponding to the maximum of the wavelet transform at particular time are very similar to the recorded frequencies. It is interesting to note that wavelet transform shows the influence of the some low frequency components which are not clearly seen in the classical energy spectrum. Finally, the WT method helps to examine a process of the disintegration of mechanically generated waves.

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