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AN ALTERNATE CUT-OFF FREQUENCY FOR THE RESPONSE SPECTRUM METHOD OF SEISMIC ANALYSIS

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ABSTRACT

In response spectrum method of seismic analysis of irregular and complex structures, the response contributions of modes up to rigid frequency are considered for the analysis and the effect of truncated high frequency modes are taken into account using suitable "missing mass" correction methods. In this paper, an alternate method using a cut-off frequency less than rigid frequency for the truncation of modes is proposed. The proposed method is validated with the help of numerical examples by evaluating the response of structures for El Centro (1940) and Fruili (1976) earthquake ground motions.

Keywords: Missing mass; rigid frequency; residual mode; high frequency modes

1. INTRODUCTION

Response Spectrum method is the most attractive and widely used method for the seismic analysis of structures. The accuracy of the response spectrum method is generally good provided that no significant modes have been truncated. The practical difficulties associated with the analysis of large real life structural models leads to the truncation of high frequency modes. Generally, the properties of the first few lower modes are calculated for their use in the analysis and the high frequency modes are ignored. In some cases, the error due to the truncation of modes can be too large to be ignored, especially in the calculation of response of stiff and irregular structural systems. Moreover, some response quantities, which have a significant contribution from the high frequency modes in even not so stiff structures, may also be sensitive to this mode truncation error. In order to check the accuracy of the calculated response when high frequency modes are truncated in a modal superposition, typical building codes specify that the results can be considered accurate if about 90% of the total structural mass participates in the number of modes considered [1-3]. The ninety

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percent criteria even when satisfied may not always result in correct responses in all the elements of an irregular structure [4]. Therefore, for the seismic analysis of irregular buildings using response spectrum method, all the modes up to rigid frequency has to be considered and "missing mass" correction beyond rigid frequency [4, 5].

Techniques have been developed to take the effect of the "missing mass" contained in the uncalculated high frequency modes into account. A comparison between the various "missing mass" correction methods shows that residual mode method is superior to other methods [6-8]. The method approximates the periodic part of the response along with the rigid part of the response. Further the residual mode can be included as modal properties of an additional mode in the dynamic analysis. The ability of the residual mode to approximate the periodic part of the response is not addressed. The modal responses, having a frequency less than rigid frequency also have a rigid content, which gradually diminishes and becomes zero as the frequency further goes down [5,9]. This paper makes an attempt to conduct seismic analysis of structures by truncating the modes at a 'key frequency' having frequency less than rigid frequency and applying a correction using residual mode to take the response contributions of truncated modes into account.

2. MODE SUPERPOSITION METHOD

The equations of motion for an N degrees of freedom(DOF), viscously damped system with classical damping can be written as

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = -\mathbf{M}\mathbf{U}_b \ddot{u}_g \tag{1}$$

where **M**, **C** and **K** are the mass, damping and stiffness matrices, respectively; **U** is the displacement vector; $\mathbf{U}_b$ is the static displacement vector when the base of the structure undergoes a unit deflection in the direction of earthquake; and $\ddot{u}_g$ is the ground acceleration. The displacement **U** of the system can be expressed as the superposition of modal contributions $\mathbf{U}_i$,

$$\mathbf{U} = \sum_{i=1}^{N} \mathbf{U}_i = \sum_{i=1}^{N} \phi_i \mathrm{X}_i \tag{2}$$

where ϕ_i are determined from the general eigen value problem $\mathbf{K}\phi_i = \omega_i^2 \mathbf{M}\phi_i$, X_i = modal coordinates and ω_i is the frequency of the ith mode . By using Eq.(2), (1) can be transformed to a system of uncoupled equations in modal coordinates,

$$\ddot{\mathrm{X}}_i + 2\xi_i\omega_i \dot{\mathrm{X}}_i + \omega_i^2 \mathrm{X}_i = \Gamma_i \ddot{u}_g \text{, } i=1, 2\ldots \mathrm{N} \tag{3}$$

where, $\Gamma_i = \phi_i^T \mathrm{M}\mathrm{U}_b$, is called the modal participation factor and ξ_i is the damping ratio for the ith mode. In response spectrum method, the contribution of ith mode to nodal displacements $\mathbf{U}$ is given by

$$\mathbf{U}_i = \phi_i X_i = \Gamma_i \phi_i S_{Di} \tag{4}$$

where S_{Di} is the maximum relative displacement of a single degree of freedom system having frequency ω_i and damping ξ_i. The modal response $\mathbf{U}$ can be divided into two parts: the rigid part U^r and the damped periodic part U^p. The modal response U can be expressed as [5, 9],

$$\mathbf{U} = \sqrt{(\mathbf{U}^r)^2 + (\mathbf{U}^p)^2} \tag{5}$$

3. RIGID FREQUENCY AND HIGHER MODES OF VIBRATION

The minimum frequency beyond which the curves for various damping ratios have same values of spectral acceleration is defined as the rigid frequency [10, 11]. This definition of rigid frequency introduces consistency with the physical behavior, but can be prone to individual judgment depending upon a particular record and the sensitivity of its spectral accelerations to the damping ratio in a region near the rigid frequency. The prescription of rigid frequency as 33 Hz, as given by various seismic building codes can also introduce inaccuracies in evaluating the structural response using response spectrum method [10, 12]. The responses in all the high frequency modes having frequency equal to or greater than rigid frequency are in phase to each other and the rigid response always combines algebraically. In this region, the structural frequency is sufficiently higher than the dominant frequencies of the input force and the spectral acceleration becomes equal to the peak ground acceleration, often referred to as the zero period acceleration (ZPA).

At high frequencies, the periodic part of the response becomes negligible and only the rigid part of response remains. Moreover the period of a high frequency mode is very short, so the response in such a mode is essentially static than dynamic. Consider the first '*n*' modes of a *N* degrees of freedom system, having frequencies less than rigid frequency and let the response in these *n* modes be $\mathbf{U}'$, and the response in the remaining modes be $\mathbf{U}_o$. Then,

$$\mathbf{U}' = \sum_{i=1}^{n} \mathbf{U}_i = \sum_{i=1}^{n} \phi_i \mathbf{X}_i \, ; \; \mathbf{U}_o = \sum_{i=n+1}^{N} \mathbf{U}_i = \sum_{i=n+1}^{N} \phi_i \mathbf{X}_i \tag{6}$$

$$\mathbf{U} = \mathbf{U}' + \mathbf{U}_o \tag{7}$$

For high frequency modes, Eqs. (3), (6) and (7) gives

$$\mathbf{M\ddot{U}_0} + \mathbf{C\dot{U}_0} + \mathbf{KU_0} = -\mathbf{MU}_{bo}\ddot{u}_g \, ; \tag{8}$$

where

$$\mathbf{U}_{bo} = \mathbf{U}_b - \sum_{i=1}^{n} \phi_i \Gamma_i \tag{9}$$

Since the residual response in high frequency modes are pseudo static, we can neglect the terms $\ddot{\mathbf{U}}$ and $\dot{\mathbf{U}}$ in Eq. (8), therefore

$$\mathbf{KU_o} = -\mathbf{MU}_{bo}\ddot{u}_g \tag{10}$$

The response of a high frequency mode is essentially static and could be determined by static analysis using Eq. (10) instead of dynamic analysis.

4. MODAL RESPONSE COMBINATION

The peak value of the total response R is estimated by combining the peak modal response of individual modes using modified double sum equation [9,11,12] is given by,

$$R^2 = \sum_{i=1}^{N} R_i^2 + \sum_{i=1}^{N} \sum_{j \neq i} \bar{\varepsilon}_{ij} R_i R_j \tag{11}$$

where R_i is the maximum modal response in *i*th mode, and $\bar{\varepsilon}_{ij}$ is the modified correlation factor defined as,

$$\bar{\varepsilon}_{ij} = \alpha_i \alpha_j + \sqrt{\left[\left(1-\alpha_i^2\right)\left(1-\alpha_j^2\right)\right]}\varepsilon_{ij} \tag{12}$$

where α_i is the rigid response coefficient in the *i*th mode and ε_{ij} is the correlation coefficient of the damped periodic part of modal responses, given by the well known complete quadratic combination (CQC) rule. For damped periodic modes, $\alpha = 0$, and modified double sum equation reduces to CQC and for $\bar{\varepsilon}_{ij} = 0$, modified double sum method reduces to square root of sum of squares (SRSS). Eqs. (11) and (12) include the effect of rigid response of high frequency modes in the modified correlation coefficient $\bar{\varepsilon}_{ij}$. The rigid response coefficient α_i is defined as [9, 11],

$$\alpha_i = -\frac{\int_0^{t_d} \ddot{x}_i(t)\ddot{u}_g(t)dt}{t_d \sigma_i^{\ddot{x}} \sigma^{\ddot{u}_g}} \tag{13}$$

where $\ddot{x}_i(t)$ is the acceleration response, $\sigma_i^{\ddot{x}}$ and $\sigma^{\ddot{u}_g}$ are the standard deviations of $\ddot{x}_i(t)$

and $\ddot{u}_g(t)$ respectively and t_d is the duration of responses. The value of α gradually reduces from one to zero, from a key frequency f_2 to another key frequency f_1 [5, 9, 11]**.** The key frequency f_2 is the lowest frequency at which the rigid response coefficient becomes 1 and the key frequency f_1 is the highest frequency at which the rigid response coefficient becomes zero. An approximate equation for α_i can be represented by a straight line between the two key frequencies f_1 and f_2 on a semi logarithmic graph, is given by [5, 9],

$$\alpha_i = \frac{\ln f_i / f_1}{\ln f_2 / f_1}\ ,\ 0 \le \alpha_i \le 1 \tag{14}$$

where f_i is the modal frequency in hertz and the key frequencies f_1 and f_2 can be expressed as,

$$f_1 = \frac{S_{A\max}}{2\pi S_{V\max}}\ , \text{Hz.} \tag{15}$$

$$f_2 = (f_1 + 2f^{\,r})/3\ , \text{Hz.} \tag{16}$$

where $S_{A\max}$ = maximum spectral acceleration,

$S_{V\max}$ = maximum spectral velocity and

and $f^{\,r}$ = rigid frequency

5. RESIDUAL MODE METHOD

In this method, the inertia effect of modes having frequencies greater than the rigid frequency is lumped into a 'missing mass' term which yields the 'residual response'. Since the residual response in high frequency modes are pseudo static, for rigid modes, we can neglect the terms $\ddot{\mathbf{U}}$ and $\dot{\mathbf{U}}$ in Eq. (1), and therefore for high frequency modes,

$$\mathbf{KU_o} = -\mathbf{MU}_{bo}\ddot{u}_g \tag{17}$$

where

$$\mathbf{U}_{bo} = \mathbf{U}_b - \sum_{i=1}^{n} \phi_i \Gamma_i \tag{18}$$

U_o = response in high frequency modes and 'n' = number of modes of a N degrees of freedom system having frequencies less than rigid frequency. The term $\ddot{u}_g$ in Eq. (17) is a scalar and can be scaled out from the equation. The solution of resulting equation yields a vector $\boldsymbol{\phi_o}$, which is normalized such that $\boldsymbol{\phi_o}^{\mathbf{T}}\mathbf{M}\boldsymbol{\phi_o} = 1$. The fictitious frequency ω_0^2 corresponding to the residual mode is given by,

$$\omega_{\mathbf{o}}^2 = \phi_{\mathbf{o}}^{\mathbf{T}} \mathbf{K} \phi_{\mathbf{o}} \tag{19}$$

The residual modal vector $\phi_{\mathbf{o}}$ and the corresponding frequency $\omega_{\mathbf{o}}$ can be directly included as modal properties of an additional mode in the dynamic analysis. The system can be analyzed for $n + 1$ modes, where the contributions of high frequency modes are taken into account by the residual mode. Residual mode method takes both rigid and damped periodic part of the response corresponding to the residual mode into account and thus provides an approximate dynamic correction.

6. PROPOSED METHODOLOGY

The modal responses, having frequency less than rigid frequency also have a rigid content. There is a transition from peak ground acceleration to rigid spectral acceleration as the frequency goes down from rigid frequency $f^{\,r}$ to the key frequency f_2. Further there is a transition from rigid spectral acceleration to amplified periodic spectral acceleration from key frequency f_2 to key frequency f_1. At key frequency f_1 the rigid content reduces to zero. In mid-frequency region between f_1 to $f^{\,r}$ the responses consist of two parts; the damped periodic part and the rigid part. The mid-frequency region can be further divided into two sub-regions; region between, f_1 and f_2 and the region between f_2 and $f^{\,r}$. In the region between f_2 and $f^{\,r}$ of the spectrum, the modal responses move in phase with the ground motion. The damped periodic portion of response in this region is negligible and not considered in the derivation of rigid response coefficient. The variation of rigid response coefficient in the mid frequency region for El Centro(1940) and Fruili(1976) earthquake ground motions for 5% damping are shown in Figures 1 and 2 respectively. In the region between f_2 and $f^{\,r}$, the periodic part of the response is negligible.

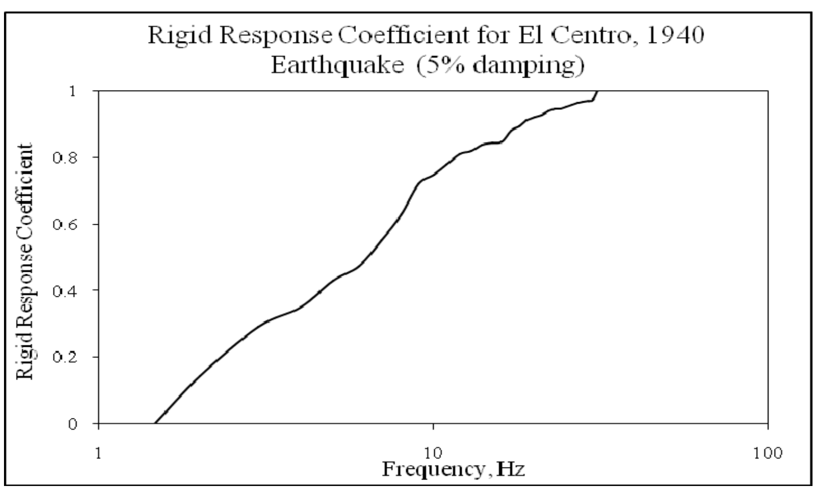

Figure 1. Variation of correlation coefficient for El Centro, 1940 earthquake

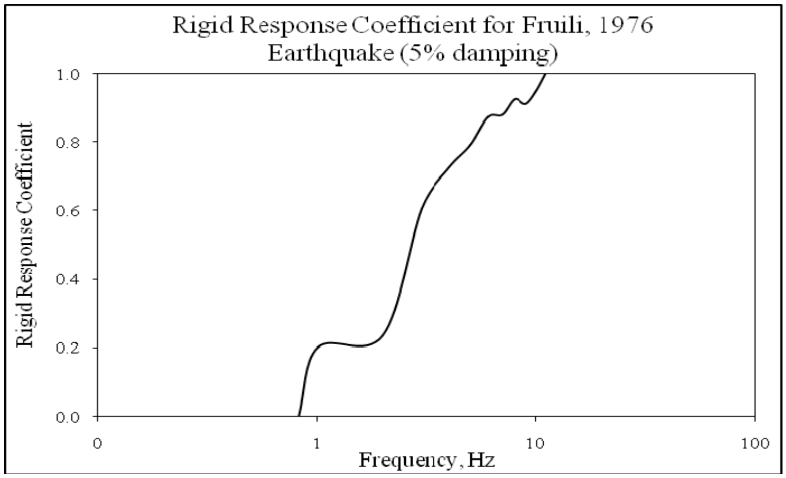

Figure 2. Variation of correlation coefficient for Fruili, Italy, 1976 earthquake

In all missing mass correction methods other than residual mode method, the damped periodic part of the response of the truncated modes is neglected. Residual mode method tries to approximate the damped periodic part of the response along with the rigid part. The residual mode method includes the dynamics of the residual mode and can be included as an additional mode in the modal analysis. Therefore modal analysis can be conducted by truncating the modes having frequencies higher than the key frequency f_2, instead of rigid frequency f^r and applying 'missing mass' correction using a residual mode to take the effect of truncated modes into account. Table 1 show that the value of f_2 calculated using Eq. (16) is less than the actual value of f_2. The value of f_2 calculated using the approximate expression (Eq. (16)) varies with the actual value of f_2. The modal response combination rule using modified correlation coefficient is sensitive to the value of f_2. The value f_2 depends on the value of f^r and f_1 (Eq. (16)). Furthermore the fixing up of rigid frequency can be prone to individual judgment and can be lead to the use of different rigid frequencies by different engineers.

Table 1: Comparison of the value of f_2, Hz

Sl. No.	**Earthquake**	**f^r, Hz**	**f_2, Hz (Numerical method)**	**f_2, Hz (Equation (16))**
1	El Centro, 1940	33	27	22.59
2	Friuli, Italy, 1976	15	12	10.23

The residual mode approximates both periodic and rigid part of the response in the zone between f_2 and f^r depending on the modal mass contributions of the modes. The proposed method helps to reduce the error due to the approximation involved in the calculation of f_2 using Eq. (16) and the use of different f_2 values by different engineers. The method reduces the cut-off frequency from rigid frequency to key frequency f_2, having frequency less than rigid frequency. The proposed method simplifies the seismic analysis of structures having significant contributions from higher modes. The following numerical examples will confirm this point.

7. NUMERICAL EXAMPLES

To evaluate the accuracy of the proposed method numerically, a response spectrum analysis using El Centro (1940) earthquake ground motion is performed on the following numerical examples. Numerical example 3 is further analyzed using Friuli (1976) earthquake ground motion. The key frequencies f_1, f_2 and rigid frequency f^r for El Centro (1940) ground motion and Friuli (1976) earthquake ground motion are shown in Table 1. Equations (15) and (16) are used to evaluate the values of f_1, and f_2. The variation of rigid response coefficient for five percentage damping with respect to frequency is shown in Figures 1 and 2. For the analysis, higher modes having frequency more than key frequency f_2 are truncated and "missing mass" correction is applied using residual mode method. The error is calculated by comparing the peak modal responses calculated using the proposed method, with the peak modal responses calculated using modal analysis with all modes.

7.1 Numerical example 1

Consider a two DOF system with story stiffness and the floor masses as shown in Figure 3. The frequencies, of this system are 14.84Hz and 25.33Hz respectively and the modal damping is five percentage. The mass participation percentage for different modes for this system is given in Table 2. El Centro (1940) ground motion is used for earthquake excitation. The frequency of second mode is greater than key frequency f_2 and less than the rigid frequency. Therefore, the response of second mode contains a rigid as well as a damped periodic part. For the analysis, second mode is truncated and "missing mass" correction is applied using residual mode method. The frequency corresponding to the residual mode vector is 25.33Hz, same as the frequency of the second mode. The results of modal analysis with all modes and residual mode method are same (Table 3). The residual mode method approximates the damped periodic part along with the rigid part of the response corresponding to the residual mode.

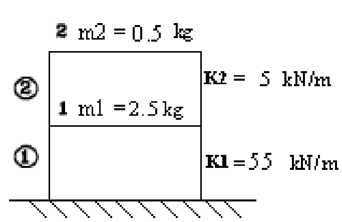

Figure 3. 2-DOF system, numerical example 1

Table 2: Frequencies, damping ratio and mass participation percentage for the 2 DOF system shown in Figure 3

Variable	Including all modes		Truncated modes	
	Mode 1	Mode 2	Mode 1	Residual mode
Frequency (Hz)	14.84	25.33	14.84	25.33
Damping ratio	0.05	0.05	0.05	0.05
Mass Participation %	42.0	58.0	42.0	58.0

Table 3: Spring force (N) of 2 DOF system, shown in Figure 3

Analysis	Element 1	Element 2
Modal analysis (All modes)	11.58	2.84
Residual mode method (cut off frequency f_2) Error	11.58 0.0 %	2.84 0.0 %

7.2 Numerical example 2

A 5 DOF model of a structure with a relatively stiff base supporting a flexible tower shown in Figure 4 is considered for analysis using the proposed method. The story stiffness and the floor masses of the system are given in Figure 4. The frequencies, modal damping and the mass participation percentage for different modes for this system are given in Table 4. El Centro (1940) ground motion is used for earthquake excitation. The third, fourth and fifth modes having frequency higher than key frequency f_2 are truncated for the analysis and "missing mass" correction is applied using residual mode method. The frequency corresponding to the residual mode vector is 32.29Hz. The results are compared with the peak modal responses calculated using modal analysis with all modes. The error involved in the calculation of spring force using each method is shown in Table 5. The error involved in the calculation of response in the elements is less than 1 percentage.

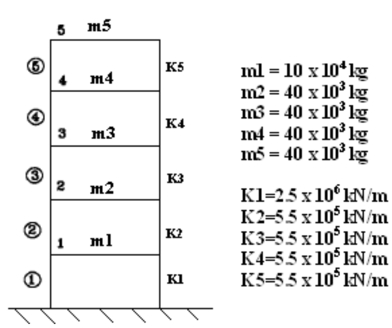

Figure 4. 5-DOF system, numerical example 2

Table 4: Frequencies, damping ratio and mass participation percentage for 5 DOF system, shown in Figure 4

Including all modes			
Mode number	**Frequency (Hz)**	**Damping ratio**	**Mass participation %**
1	6.17	0.05	63.32
2	17.61	0.05	13.27
3	25.89	0.05	16.01
4	30.69	0.05	6.89
5	35.39	0.05	0.48

Table 5: Spring force (kN) of 5 DOF system shown in Figure 4

Analysis	Element 1	Element 2	Element 3	Element 4	Element 5
Modal Analysis (All modes)	1146.89	887.06	732.92	525.36	275.19
Residual Mode method (cut off frequency f^r) Error	1146.89 0.0 %	887.06 0.0 %	732.92 0.0 %	525.36 0.0 %	275.19 0.0 %
Residual Mode method (cut off frequency f_2) Error	1143.12 0.33 %	883.29 0.42 %	739.68 0.92%	524.72 0.12%	271.94 0.71%

7.2 Numerical example 3

A 5 DOF model of a structure with a relatively stiff base supporting a flexible tower shown in Figure 5 is considered for analysis using the proposed method. The story stiffness and the floor masses of the system are given in Figure 5. The frequencies, modal damping and the mass participation percentage for different modes for this system are given in Table 6. El Centro (1940) ground motion is used for earthquake excitation. The third, fourth and fifth modes having frequencies higher than key frequency f_2 are truncated for the analysis and “missing mass” correction is applied using residual mode method. The frequency corresponding to the residual mode vector is 27.61Hz. The results are compared with the peak modal responses calculated using modal analysis with all modes. The error involved in the calculation of spring force using each method with respect to modal analysis with all modes is shown in Table 7. The error involved in the calculation of response using residual mode method is negligible.

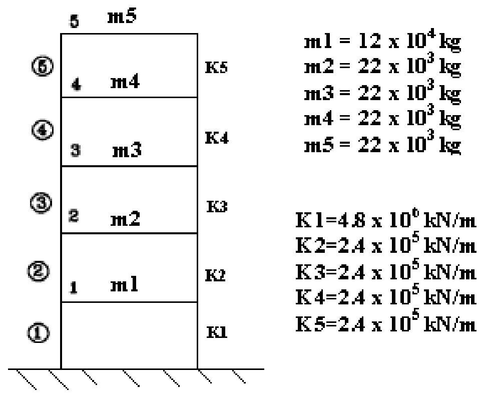

Figure 5. 5-DOF system, numerical example 3

Table 6: Frequencies, damping ratio and mass participation percentage for the 5 DOF system shown in Figure 5

Including all modes			
Mode Number	**Frequency (Hz)**	**Damping ratio**	**Mass participation %**
1	5.71	0.05	40.55
2	16.44	0.05	6.21
3	25.21	0.05	4.51
4	30.95	0.05	8.00
5	33.23	0.05	40.73

Table **7**: Spring force (kN) of 5 DOF system, shown in Figure 5 for El Centro (1940)

Analysis	**Element 1**	**Element 2**	**Element 3**	**Element 4**	**Element 5**
Modal Analysis (All modes)	907.32	600.35	502.90	362.58	190.32
Residual Mode method (cut off frequency f^r) Error	907.32 0.0 %	600.35 0.0 %	502.90 0.0 %	362.58 0.0%	190.32 0.0%
Residual Mode method (cut off frequency f_2) Error	897.31 1.10 %	597.25 0.51 %	507.69 0.95 %	362.36 0.06 %	188.25 1.09%

The 5 DOF system is further analyzed using Fruili (1976) ground motion. The second, third, fourth and fifth modes having frequencies higher than key frequency f_2 are truncated for the analysis and "missing mass" correction is applied using residual mode method. The results are compared with the peak modal responses calculated using modal analysis with all modes. The error involved in the calculation of spring force using each method with respect to modal analysis with all modes is shown in Table 8. The error involved in the calculation of response using residual mode method is less than 10 percentage.

Table 8: Spring force (N) of 5 DOF system, shown in Figure 5 for Fruili (1976)

Analysis	**Element 1**	**Element 2**	**Element 3**	**Element 4**	**Element 5**
Modal analysis (All modes)	98.39	54.22	43.93	31.02	16.06
Residual mode method (cut off frequency f^r) Error	98.35 0.04 %	51.26 5.45 %	44.90 2.20 %	33.24 7.15%	17.66 9.96%
Residual mode method (cut off frequency f_2) Error	98.35 0.04 %	51.26 5.45 %	44.90 2.20 %	33.24 7.15%	17.66 9.96%

The mass participating in the truncated modes in numerical examples 1, 2 and 3 are 58 percentage, 53.24 percentage and 23.38 percentage respectively. In all the above examples, the modes up to 22.59Hz is considered for the analysis using El Centro (1940) earthquake ground motion, instead of 33 Hz and "missing mass" correction using residual mode is applied beyond 23Hz. Numerical example 3 is further analyzed for Fruili (1976) earthquake ground motion. All the modes up to 10.23Hz are considered for the analysis using Fruili (1976) earthquake ground motion, instead of 15 Hz and "missing mass" correction using residual mode is applied beyond 10.23Hz. The examples show that the error involved in the calculation of response using the proposed method is less than 10 percentage. Therefore structures having significant contribution from high frequency modes can be analyzed by truncating modes above key frequency f_2 and applying "missing mass" correction using residual mode to take the contribution of the truncated modes into account. The residual response corresponding to the residual mode can be combined with the modal response of other modes using the modal response combination rules. The proposed method simplifies the analysis procedure for the seismic analysis of irregular building structures having significant contributions from higher modes.

8. CONCLUSIONS

Response spectrum method of seismic analysis gives accurate results when the modal responses are calculated for all the modes. It is impractical to calculate all the modes for systems with large degrees of freedom. Therefore the response contributions of modes up to rigid frequency is included in the analysis and "missing mass" correction techniques are applied to account for the response contributions of uncalculated high frequency modes. This paper proposes an alternate method for the seismic analysis of building structures in which the modes can be truncated at a cut off frequency f_2, less than the rigid frequency and applying a correction using a residual mode to take the response contributions of uncalculated higher modes into account. The residual mode approximates the damped periodic part also, along with the rigid part of the response of the uncalculated modes. This method helps to nullify the approximation involved in the calculation of f_2. The residual mode can be included as an additional mode in the analysis and the corresponding response can be combined according to the modal response combination rules. The proposed method is simpler to implement and can be used in the seismic analysis of irregular building structures.

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