# A TWO STAGE METHOD FOR STRUCTURAL DAMAGE IDENTIFICATION USING AN ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM AND PARTICLE SWARM OPTIMIZATION

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## ABSTRACT

An efficient methodology is proposed to detect the multiple damages in structural systems. The methodology consists of two main stages. In the first stage, an exhaustive search is performed using the adaptive neuro-fuzzy inference system (ANFIS) to quickly identify the most potentially damaged elements (MPDE). In the second stage, a particle swarm optimization (PSO) is presented to accurately determine the actual damage extents using the first stage results. In order to assess the performance of the proposed methodology for structural damage detection, two illustrative test examples are considered. The numerical results demonstrate the computational efficiency of the proposed methodology when comparing with those of the methods found in the literature.

**Keywords:** Structural damage detection; adaptive neuro-fuzzy inference system; the most potentially damaged elements; particle swarm optimization; finite element method

## 1. INTRODUCTION

Health monitoring and damage identification is an interesting issue in structural engineering. By using this concept the local damages of a structure can be detected and after rehabilitating the damages, the total age of the structure can increase. In recent years, many methods have been introduced to detect the sites and extents of damages in the structural systems [1-6]. One type of the methods employs the optimization algorithms for detecting the multiple structural damages. Many successful applications of damage detection using the genetic algorithm (GA) have been reported in the literature [7-9]. Although, the use of an optimization algorithm can accurately identify the structural damages, however, they impose much computational effort to the process. In order to reduce the computational cost of the optimization process, some useful techniques can be employed. A useful technique is to reduce the dimension of optimization problem by considering the most potentially damaged elements (MPDE) instead of the total ones [5 and 10]. For this, the adaptive neuro-fuzzy inference system [11-12] can be utilized as an effective tool.

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In this study, an efficient methodology is proposed to accurately detect the sites and extents of multiple structural damages. The proposed methodology has two main phases combining the adaptive neuro-fuzzy inference system (ANFIS) and a particle swarm optimization (PSO) as an optimization solver. In the first phase, the ANFIS is employed to quickly determine the structural elements having the higher probability of damage from the original elements. In the second phase, the reduced damage problem is solved via the PSO to truthfully determine the extents of actual damaged elements. In order to assess the performance of the proposed methodology, two benchmark examples are considered. Numerical results reveal the computational advantages of the proposed methodology for precisely identifying the multiple damages.

## 2. STRUCTURAL DAMAGE DETECTION

Structural damage detection techniques can be generally classified into two main categories. They include the dynamic and static identification methods requiring the dynamic and static test data, respectively. Furthermore, the dynamic identification methods have shown their advantages in comparison with the static ones. Among the dynamic data, the natural frequencies of a structure can be found as a valuable data. Determining the level of correlation between the measured and predicted natural frequencies can provide a simple tool for identifying the locations and extents of structural damages [1 and 4]. Two parameter vectors are used for evaluating correlation coefficients. A vector consists of the ratios of the first $n_f$ natural frequency changes $\Delta F$ due to structural damage, i.e.

$$\Delta F = \frac{F_h - F_d}{F_h} \tag{1}$$

where $F_h$ and $F_d$ denote the natural frequency vectors of the healthy and damaged structure, respectively. Similarly, the corresponding parameter vector predicted from an analytical model can be defined as:

$$\delta F(X) = \frac{F_h - F(X)}{F_h} \tag{2}$$

where $F(X)$ is a natural frequency vector that can be predicted from an analytic model and $X^T = \{x_1,...,x_i,...,x_n\}$ represents a damage variable vector containing the damage extents $(x_i, \ i = 1,...,n)$ of all $n$ structural elements.

Given a pair of parameter vectors, one can estimate the level of correlation in several ways. An efficient way is to evaluate a correlation-based index called the multiple damage location assurance criterion (MDLAC) expressed in the following form [1]:

$$MDLAC(X) = \frac{\left| \Delta F^T . \delta F(X) \right|^2}{(\Delta F^T . \Delta F)(\delta F^T(X) . \delta F(X))} \tag{3}$$

The *MDLAC* compares two frequency change vectors, one obtained from the tested structure and the other from an analytical model of the structure. The *MDLAC* varies from a minimum value 0 to a maximum value 1. It will be maximal when the vector of analytical frequencies is identical to the frequency vector of damaged structure, i.e., $F(X) = F_d$ .

## 3. ANFIS FOR DETERMINING THE MPDE

The adaptive neuro-fuzzy inference system (ANFIS) is a process for mapping from a given input to a single output using the fuzzy logic and neuro-adaptive learning algorithms. Figure 1 shows the architecture of a typical ANFIS with two inputs $In_1$ and $In_2$, four rules and one output *out* for the first order Sugeno fuzzy model, where each input is assumed to have two associated membership functions. After several mathematical works, the relation between the inputs and output in the typical ANFIS can be expressed as [11-13]:

$$Out = \sum_{i=1}^{2} \sum_{j=1}^{2} [(\overline{W}_{ij} p_{ij})In_1 + (\overline{W}_{ij} q_{ij})In_2 + (\overline{W}_{ij} r_{ij})] \tag{4}$$

where $\overline{W}_{ij}$ are called normalized firing strengths dependent to membership function parameters and $p_{ij}$, $q_{ij}$ and $r_{ij}$ are consequent parameters. The task of the learning algorithm for this ANFIS architecture is to tune all the parameters to make the ANFIS output matches the training data. The detailed information regarding the ANFIS can be found in Ref [13].
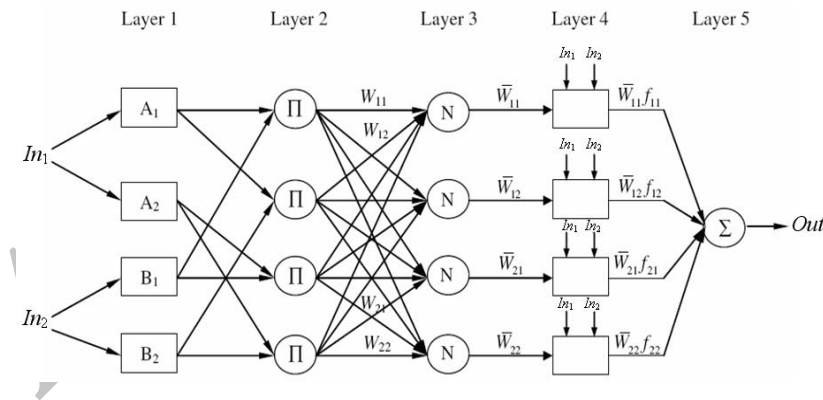


Figure 1. A typical ANFIS architecture for a two-input Sugeno model with four rules

During the last years, the ANFIS has been widely used for different purposes such as prediction, knowledge discovery, medical decision making and disease diagnosis [11-16].The ANFIS concept can also be effectively utilized to determine the most potentially damaged element (MPDE) of an unhealthy structure. For this, some sample structures having the damaged elements are randomly generated based on the damage vector $X$ as the input and the corresponding *MDLAC*(*X*) as the output. Then, an exhaustive search is

performed using the ANFIS within the available input-output data to arrange the structural elements according to their damage potentiality. Essentially, the exhaustive search technique builds an ANFIS network for each damage variable from original ones and trains the network for a little epoch and reports the performance achieved. The step by step summary of the exhaustive search algorithm for determining the MPDE of an unhealthy structure is as follows:

a) Establish the pre-assigned parameters of the intact structure.
b) Randomly generate a number of sample structures having some damaged elements within the allowed space of damage variables $X$.
c) Determine the natural frequencies of the sample structures using a conventional finite element analysis.
d) Estimate the level of correlation between unhealthy structure and each sample structure by evaluating the $MDLAC(X)$ index via Eq. (3).
e) Randomly split the sample structures into two sets with some samples for training and remaining samples for testing the ANFIS, respectively.
f) Build an ANFIS model for each damage variable as the input and the $MDLAC(X)$ as the output. This leads to $n$ ANFIS models equal to the total number of structural elements as:

$$\text{ANFIS model 1}: \quad \overbrace{x_1}^{\text{Input}} \rightarrow \overbrace{MDLAC(X)}^{\text{Output}}$$
$$\text{ANFIS model 2}: \quad x_2 \rightarrow MDLAC(X)$$
$$\text{ANFIS model 3}: \quad x_3 \rightarrow MDLAC(X)$$
$$.$$
$$.$$
$$.$$
$$\text{ANFIS model } \boldsymbol{n}: \quad x_n \rightarrow MDLAC(X)$$

g) Calculate the root mean square error (RMSE) for training and testing sets as:

$$RMSE = \sqrt{\frac{1}{n_t}\sum_{i=1}^{n_t}(ac_i - pr_i)^2} \tag{5}$$

where $ac$ and $pr$ represent the actual and predicted values of the $MDLAC(X)$, also $n_t$ is the number of training or testing samples.

h) Sort the structural elements according to increasing their training $RMSE$ values and select the first $m$ arranged elements, having the least $RMSE$ errors, as the reduced damage vector, denoted here by $X_r^{\text{T}} = \{x_{r1}, x_{r2}, ..., x_{rm}\}$.
i) End of the algorithm.

## 4. DAMAGE IDENTIFICATION USING OPTIMIZATION ALGORITHMS

As noted in section 2, the *MDLAC* index will reach to a maximum value 1 when the structural damage occurs. This concept can be utilized to estimate the damage vector using an optimization algorithm. For this aim, the unconstrained optimization problem with discrete damage variables reduced may be stated as:

$$\text{Find} \qquad X_r^{\text{T}} = \{x_{\text{r1}}, x_{\text{r2}}, ..., x_{rm}\}$$
$$\text{Minimize}: \quad w(X_r) = -MDLAC(X_r) \qquad\qquad (6)$$
$$x_{ri} \in R^d , \quad i = 1,...,m$$

where $R^d$ is a given set of discrete values and the damage extents $x_{ri} (i = 1,...,m)$ can take values only from this set. Also, $w$ is an objective function that should be minimized.

The selection of an efficient algorithm for solving the damage optimization problem is a critical issue. Needing fewer structural analyses for achieving the global optimum without trapping into local optima must be the main characteristic of the algorithm. In this study, a particle swarm optimization (PSO) algorithm working with discrete design variables is proposed to properly solve the damage problem.

*4.1 PSO algorithm*
The particle swarm optimization has been inspired by the social behaviour of animals such as fish schooling, insect swarming and bird flocking. It involves a number of particles which are initialized randomly in the search space of an objective function. These particles are referred to as swarm. Each particle of the swarm represents a potential solution of the optimization problem. The particles fly through the search space and their positions are updated based on the best positions of individual particles in each iteration. The fitness values of particles are obtained to determine which position in the search space is the best. In $k$th iteration, the position and velocity vectors of $n_p$ particles are updated using the following matrix equation [17-18]:

$$\begin{bmatrix} X_i^{k+1} \\ V_i^{k+1} \end{bmatrix} = \begin{bmatrix} 1 - c_1 r_1 - c_2 r_2 & \rho^k \Delta t \\ -\dfrac{(c_1 r_1 + c_2 r_2)}{\Delta t} & \rho^k \end{bmatrix} \begin{bmatrix} X_i^k \\ V_i^k \end{bmatrix} + \begin{bmatrix} c_1 r_1 & c_2 r_2 \\ \dfrac{c_1 r_1}{\Delta t} & \dfrac{c_2 r_2}{\Delta t} \end{bmatrix} \begin{bmatrix} P_i^k \\ P_g^k \end{bmatrix} \quad , \quad i = 1,...,n_p \qquad (7)$$

where $X_i$ and $V_i$ represent the current position and velocity vectors of the $i$th particle, respectively; $P_i$ is the best previous position of the $i$th particle and $P_g$ is the best global position among all the particles in the swarm; $\Delta t$ is the time step value and throughout the present work a unit time step is used; $r_1$ and $r_2$ are two uniform random sequences generated from interval [0, 1]; $c_1$ and $c_2$ are the cognitive and social scaling parameters, respectively and $\rho^k$ is the inertia weight used to discount the previous velocity of particle preserved. The inertia weight $\rho^k$ may be defined to linearly decrease from a maximum value $\rho_{\text{max}}$ to a

minimum value $\rho_{min}$. In the algorithm, the velocity vector is also limited to a lower bound $V^l$ and an upper bound $V^u$. In this study a discrete version of the PSO algorithm is presented. The step by step of a discrete coded-PSO algorithm, limited to search only within a specified set $R^d = \{R_1, R_2..., R_h\}$ is as follows:

**Step 1**: Initialize.
a)    Set counter $k = 0$.
b)    Randomly generate the particle positions $X_i^0$ $(i = 1,...,n_p)$ where each component of a position vector can have an ordinary value 1 or 2 or …or $h$.
c)    Randomly generate the particle velocities $V_i^0$ $(i = 1,...,n_p)$ where each component of a velocity vector can have a real value from $-(h-1)/2$ to $+(h-1)/2$.
d)    Decode the particle positions to their actual values using the set $R^d$ and evaluate the objective function values $w_i^0$ for actual particle positions.
e)    Set $w_i^{best} = w_i^0$ and $P_i^0 = X_i^0$ for $i = 1,...,n_p$.
f)    Set $w_g^{best} = \min(w_i^{best})$ and $P_g^0$ to corresponding $X_i^0$.

**Step 2**: While $k < k_{max}$ or the convergence is not met.
a)    Update the velocity vector $V_i^k$ and position vector $X_i^k$ for all particles using equation (7).
b)    If $V_i^{k+1}$ and $X_i^{k+1}$ for any component exceeds its critical values, then set that component to its minimum and maximum allowable value. Also, round each components of $V_i^{k+1}$ to an integer value.
c)    Decode the particle positions to their actual values using the set $R^d$ and evaluate objective function values $w_i^{k+1}$ using $X_i^{k+1}$ for $i = 1,...,n_p$.
d)    If $w_i^{best} > w_i^{k+1}$ then $w_i^{best} = w_i^{k+1}, P_i^{k+1} = X_i^{k+1}$ for $i = 1,...,n_p$.
e)    If $w_g^{best} > w_g^{k+1}$ then $w_g^{best} = w_g^{k+1}, P_g^{k+1} = X_i^{k+1}$.
f)    Increment $k$.
g)    Save $X_{opt} = P_g^{k+1}$ as the optimal solution and $w_{opt} = w_g^{best}$ as the minimum objective function.
End while.

## 5. TEST EXAMPLES

In order to show the capabilities of the proposed methodology for identifying the multiple structural damages, two illustrative test examples are considered. The first example is a cantilever beam discussed in detail and the second one is a bending plate discussed in brief.

### 5.1 Cantilever beam

A finite element model of a cantilever beam with 15 elements as shown in Figure 2 is considered as the first example [4]. The length, thickness and width of the beam are 2.74, 0.00635, and 0.0760 m, respectively. The mass density is 7860 kg/m³ and the elasticity modulus is 210 GPa. In this example, the first 5 natural frequencies are used for identifying the damage. Damage variables are simulated through a relative reduction of elasticity modulus in each element as:

$$x_i = \frac{E - E_i}{E} \quad , \ i = 1,...,n \tag{8}$$

where $E$ is the original modulus of elasticity and $E_i$ is the final modulus of elasticity of $i$th element. The relative reduction of elasticity modulus 0.30 is induced at elements 4 and 12 of the structure as shown in Figure 2.
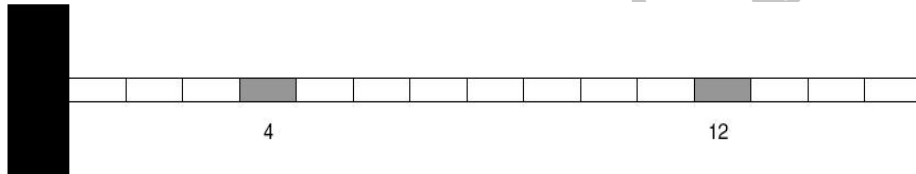
Figure 2. A cantilever beam with 15 elements and two damages induced at elements 4 and 12

### 5.1.1 Building the ANFIS models for determining the MPDE

In order to build the ANFIS models for determining the MPDE of the beam, 200 sample beams are generated randomly based on the damage variables selected from the set $R^d = \{0, 0.1, 0.2, 0.3, 0.4, 0.5\}$ and the first five natural frequencies of the sample structures are computed using the finite element analysis. Then, the *MDALC* index is evaluated for each sample structure. The exhaustive search algorithm described is employed to arrange the beam elements according to their damage potentiality. Figure 3 shows this ordering, where the element number (damage variable) versus its RMSE in training the ANFIS model is depicted. As shown in the figure, the elements 12, 4 and 10 of the beam are the best candidates for selecting them as the MPDE. It should be noted that the process of determining the MPDE takes in a very short clock time. It takes about 5 second by a core™ 2 Duo 2 GHz CPU.

### 5.1.2 Damage identification using the PSO

At this stage the reduced damage problem having only 3 damage variables (elements 12, 4 and 10) instead of 15 original ones can be solved via the PSO algorithm. The optimization algorithm with the specifications listed in Table 1 is applied to the problem and the identified damages, expressed in ratios of elasticity modulus reduction, are shown in Figure 4.
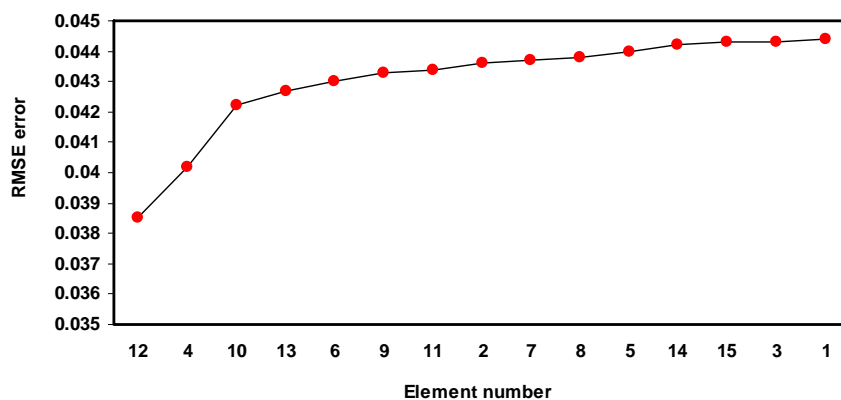
Figure 3. The element ordering with respect to their damage potentiality for 15-element beam

Table 1: The specifications of the PSO algorithm

| Parameter | Description | Value |
|-----------|-------------|-------|
| $np$ | The number of particles | 10 |
| $ni$ | The maximum number of iterations | 10 |
| $c_1$ | Cognitive parameter | 2.0 |
| $c_2$ | Social parameter | 2.0 |
| $\rho_{min}$ | Minimum of inertia weight | 0.40 |
| $\rho_{max}$ | Maximum of inertia weight | 0.90 |
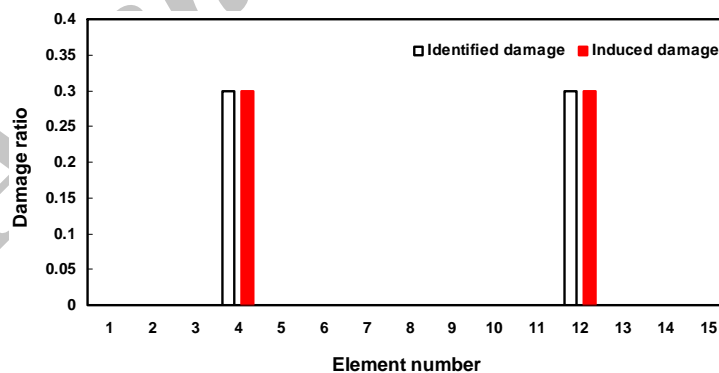


Figure 4. The final identified damage for cantilever beam

It can be observed that the optimization is properly achieved to sites and extents of hypothetical damages, while the method presented in Ref. [4] could not identify the damage extents accurately. The convergence history of the PSO can also be seen in Figure 5 where

the *MDLAC* value versus iteration number during the optimization process is shown. It can be observed that the optimization converges to true damages after only 3 iterations. It means that at optimization stage only 30 FEA are needed to find the damages properly.

*5.2 Bending plate*
A bending plate shown in Figure 6 is selected as the second test example [19]. The structure is a rectangular steel plate of 0.3 by 0.6 m with 3 mm thick, density of 8179 kg/m$^3$ and Young modulus of 200 GPa fixed at one of its short edge. For this test example, three different damage cases defined in Table 2 are studied: Cases 1 and 2 are single damages while Case 3 is a two-site-damage. For this example, the first 5 natural frequencies are also used for identifying the damage.
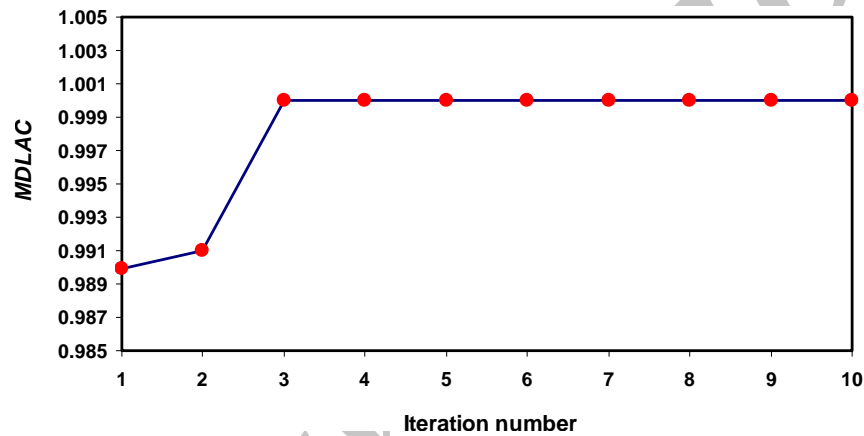
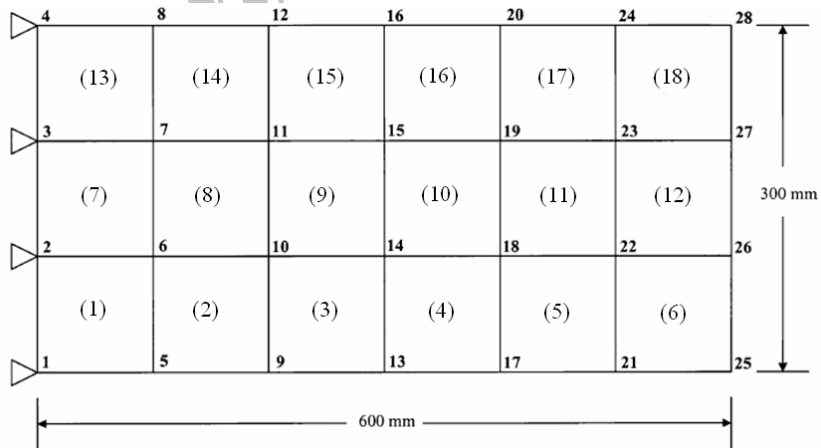Figure 5. The convergence history of the PSO for 15-element beam

Figure 6. A rectangular steel bending plate with 18 elements

Table 2: Three different damage cases induced in 18-element bending plate

| Case 1 | | Case 2 | | Case 3 | |
|---|---|---|---|---|---|
| Element number | Damage ratio | Element number | Damage ratio | Element number | Damage ratio |
| 5 | 0.20 | 10 | 0.20 | 3 | 0.30 |
| - | - | - | - | 15 | 0.30 |

In order to build the ANFIS models for determining the MPDE of the structure, 200 sample plates are randomly generated having damage elements with extents of selected from the set $R^d = \{0, 0.1, 0.2, 0.3, 0.4, 0.5\}$ and then the *MDLAC* of all the sample structures are evaluated. The exhaustive search algorithm described is employed to arrange the plate elements according to their damage potentiality. Figures 7a-c shows the element ordering with respect to their damage probability for damage cases 1 to 3, respectively.

As shown in Figures 7a-c, the MPDE for damage case 1 are elements 5, 11 and 17; for damage case 2 are elements 10, 8, 9 and 17; and for damage case 3 are elements 10, 15 and 3. It is reveled that the damage variables are reduced from 18 to 3 or 4 numbers. The PSO is now employed to solve the reduced damage detection problem having only three or four damage variables. The damage identification for Cases 1 to 3 is shown in Figures 8a-c, respectively. It can be observed that the optimization process converges truthfully to the sites and extents of damages.
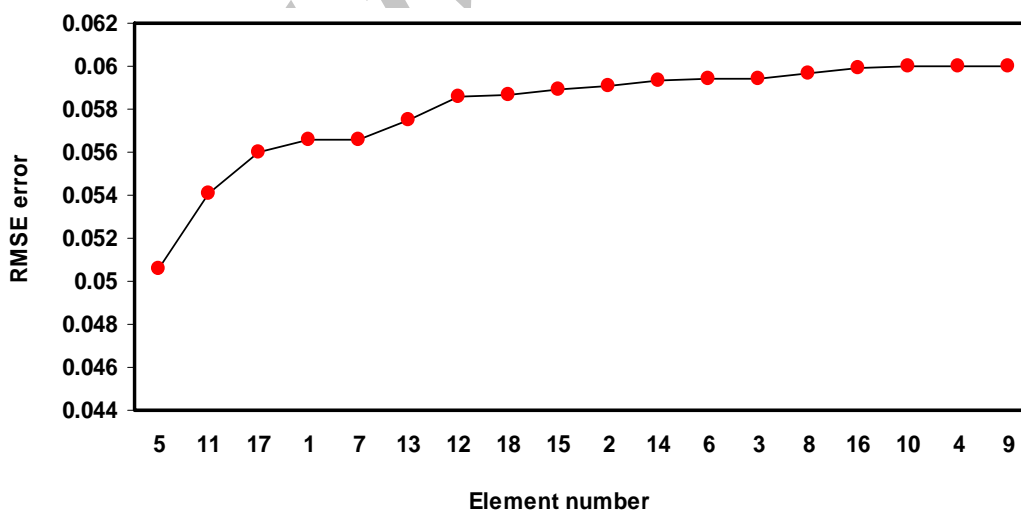


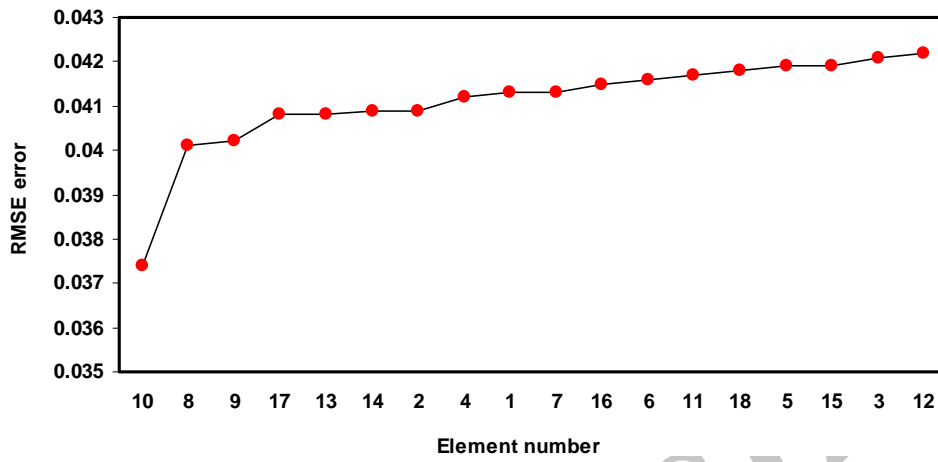Figure 7a. The element ordering with respect to their damage potentiality for Case 1

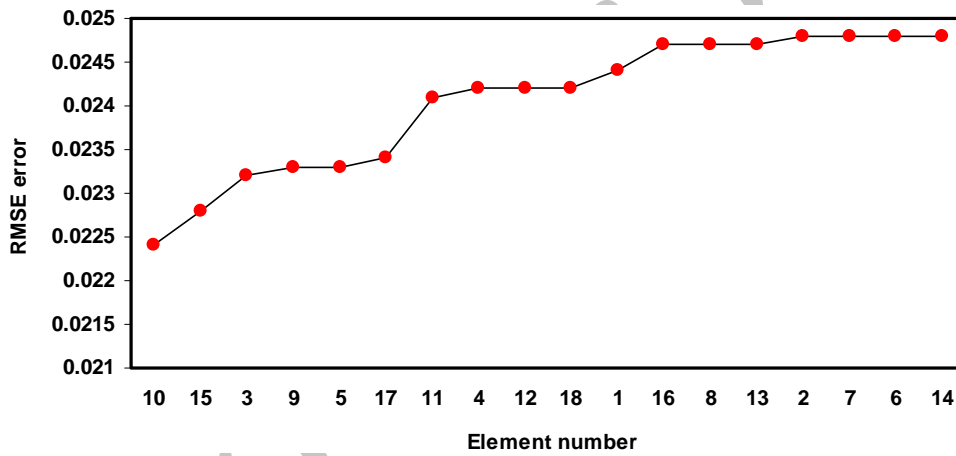Figure 7b. The element ordering with respect to their damage potentiality for Case 2



Figure 7c. The element ordering with respect to their damage potentiality for Case 3
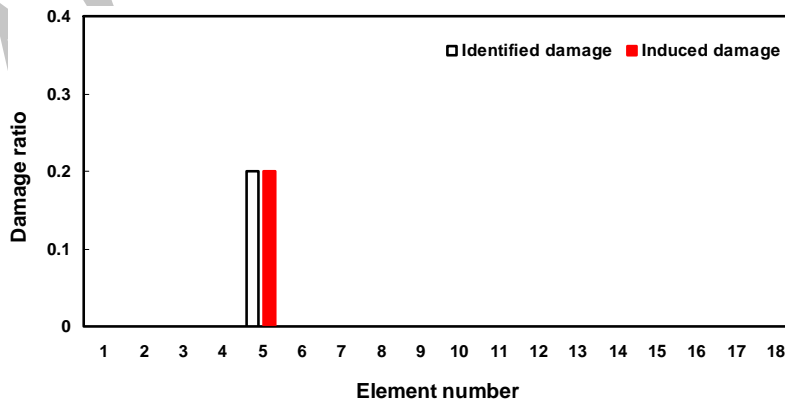


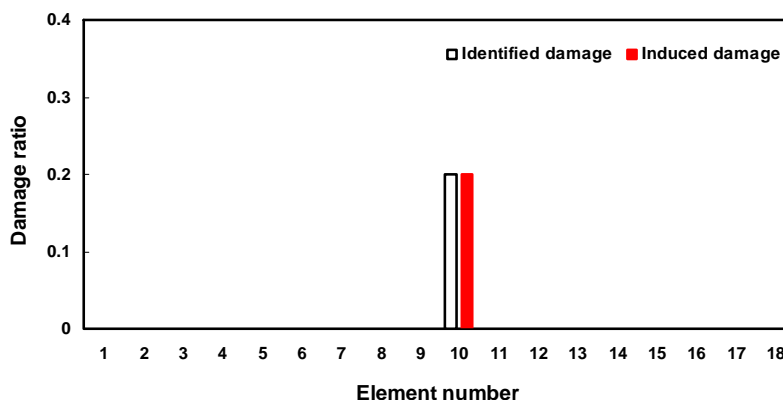Figure 8a. The final identified damage of bending plate for Case 1

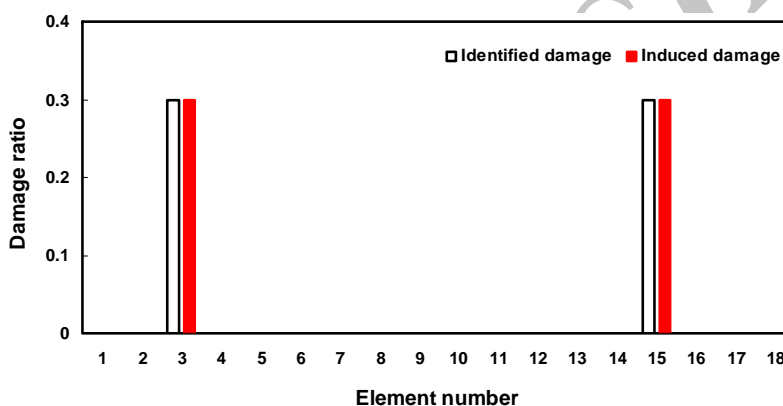Figure 8b. The final identified damage of bending plate for Case 2



Figure 8c. The final identified damage of bending plate for Case 3

## 6. CONCLUSIONS

In this study, a two-stage procedure is proposed to properly identify the sites and extents of multiple damages in structural systems. In the first stage, an exhaustive search algorithm based on the adaptive neuro-fuzzy inference system (ANFIS) is utilized to recognize the most potentially damage elements (MPDE) of a damaged structure. In the second stage, the damage identification problem, having a lower numbers of damage variables compared to original ones, is transformed into an optimization problem. The optimization problem is solved using a particle swarm optimization (PSO) to identify the actual damages. In order to show the effectiveness of the proposed methodology, two illustrative test examples are considered. The numerical results demonstrate that the combination of the ANFIS and PSO can produce an efficient tool for correctly detecting the locations and sizes of damages induced.

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