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SERIAL INTEGRATION OF PARTICLE SWARM AND ANT COLONY ALGORITHMS FOR STRUCTURAL OPTIMIZATION

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ABSTRACT

The main objective of this study is to hybridize particle swarm optimization (PSO) and ant colony optimization (ACO) algorithms to propose an efficient algorithm for optimal designing of truss structures. Two types of serial integration of the algorithms are studied. In the first one, PSO is employed to explore the design space, while ACO is utilized to achieve a local search about the best solution found by PSO. This is denoted as serial particle swarm ant colony algorithm (SPSACA). In the second one, ACO works as the global optimizer while PSO acts as the local one. This is called as serial ant colony particle swarm algorithm (SACPSA). A number of structural optimization benchmark problems are solved by the proposed algorithms. Numerical results indicate that the SPSACA possesses better computational performance compared with the SACPSA and other existing algorithms.

Keywords: Meta-heuristic algorithm; size optimization; particle swarm optimization; ant colony optimization; exterior penalty function; sequential unconstrained minimization technique

1. INTRODUCTION

In the recent decades a number of optimization algorithms based on natural phenomena have been developed. Among these methods meta-heuristic algorithms have impressive features that differs them from the gradient based methods. In the field of structural optimization, genetic algorithms (GA) [1-2], particle swarm optimization (PSO) [3-4] and ant colony optimization (ACO) [5-6] are the most popular algorithms. This class of optimization techniques not only requires no gradient computations but also is simple for computer programming. In the present study PSO, ACO and their combinations are focused.

PSO was inspired by the social behavior of organisms such as bird and fish flocking. As compared to other robust design optimization methods PSO is more efficient, requiring fewer number of function evaluations, while leading to better or the same quality of results [7-8]. Also PSO has some defect such as trapping into local optimum in a complex search

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space and disability to do a good local search around a local optimum. It is known that the PSO may perform better than the other meta-heuristic optimization techniques in the early iterations, but it does not appear competitive when the number of iterations increases [9]. ACO was inspired by the behavior of real ant colonies in finding the shortest paths between food sources and their nest. ACO was combined with PSO to improve the operation of PSO in [10] for solving the continuous unconstrained problems. In this study, two new optimization algorithms are presented by combining of PSO and ACO for optimization of structures. In the first algorithm, a preliminary optimization is achieved by PSO. Then another optimization process is performed by ACO around the best solution found by PSO to finely explore the design space. As this algorithm is a serial integration of PSO and ACO it is denoted as serial particle swarm ant colony algorithm (SPSACA). In the second algorithm, the preliminary search is achieved by ACO while the finer exploration is performed by PSO and therefore this algorithm is termed as serial ant colony particle swarm algorithm (SACPSA).

There are some constraints in structural optimization problems that should be carefully handled. So far, a number of approaches have been proposed by researcher but the penalty function methods have been the most popular constraint-handling techniques due to its simple principle and ease of implementation. In this study, for the both SPSACA and SACPSA strategies, the exterior penalty function method (EPFM) is employed in the framework of the sequential unconstrained minimization technique (SUMT) [11] to handle the constraints.

The numerical results demonstrate the efficiency and robustness of SPSACA compared with the SACPSA, PSO and ACO.

2. OPTIMAL DESIGN PROBLEM

Size optimization of truss structures is defined as minimizing the structural weight that areas of cross-sections of bar members are normally selected as the design variables of the optimization problem. The objective function is the weight of a structure, which is subjected to the stress and the displacement constraints. This can be expressed as follows:

Minimize:
$$w(X) = \sum_{i=1}^{ne} (\gamma_i \, l_i \, A_i) \tag{1}$$

Subject to:

$$\frac{\sigma_i}{\sigma_{all}} - 1 \le 0 \ , \ i = 1,2,\ldots,ne \tag{2}$$

$$\frac{d_j}{d_{all}} - 1 \le 0 \ , \ j = 1,2,\ldots,m \tag{3}$$

$$A_i^L \le A_i \le A_i^U \ , \ i = 1,2,\ldots,ne \tag{4}$$

where w and X are the weight of the structure and the vector of design variables, respectively; γ_i, l_i and A_i are the ith element material density, length and cross sectional area, respectively. Also σ_i, d_j, σ_{all} and d_{all} are the ith element stress, jth node displacement, and their corresponding allowable values, respectively. The lower and upper bounds on the cross-sectional-area of the ith element are represented by A_i^L and A_i^U, respectively.

In this study, EPFM is employed to transform the constrained structural optimization problem into an unconstrained one as described below.

Penalty function methods transform the basic optimization problem into alternative formulations such that numerical solutions are sought by solving a sequence of unconstrained minimization problems. Let the basic constrained optimization problem subject to ng constraints, be of the following form:

$$\text{Minimize:} \quad w(X) \tag{5}$$

$$\text{Subject to:} \quad g_k(X) \le 0, \; k = 1,2,...,ng \tag{6}$$

This problem is converted into an unconstrained minimization problem by constructing a function of the following form:

$$\Phi(X, r_p) = w(X) + p(X) \tag{7}$$

$$p(X) = r_p \sum_{k=1}^{ng} [\max\{0, g_k(X)\}]^2 = r_p \, \mathrm{VC} \tag{8}$$

where Φ, p, r_p and VC are the pseudo objective function, penalty function, positive penalty parameter and violated constraints, respectively.

It can be seen from Eq. (8) that the effects of penalty function only include the violated constraints (VC). By choosing the minor values for the penalty parameter, the effect of constraints in $\Phi(X, r_p)$ decrease and optimization processes cause to minimize objective function with small amount of violated constraints, in other side by choosing the high value for penalty parameter, the effect of constraints in $\Phi(X, r_p)$ increased and the portion of objective function decrease. In [11] it is recommended that if the unconstrained minimization of the $\Phi(X, r_p)$ function is repeated for a sequence of values of the penalty parameter, r_p, the solution may be brought to converge to that of the original problem stated in Eqs. (5-6). These methods are known as sequential unconstrained minimization techniques (SUMT). In the present study, the EPFM is employed in the framework of the SUMT to handle the constraints.

3. META-HEURISTIC OPTIMIZATION ALGORITHMS

In the recent years, some optimization methods that are conceptually different from the traditional mathematical programming techniques have been developed. These methods are denoted as meta-heuristic optimization methods. Most of these methods are based on certain characteristics and behavior of biological, molecular, swarm of insects, and neurobiological systems. In this paper, PSO and ACO meta-heuristics and their combinations are studied.

3.1 Ant Colony Optimization (ACO)

ACO is based on the cooperative behavior of real ant colonies, which are able to find the shortest path from their nest to a food source. The method was developed by Dorigo [12]. The ACO process can be explained as follows. The ants start at the home node, travel through the various nodes from the first node to the last node, and end at the destination node in each iteration. Each ant can select only one node in each layer in accordance with the state transition rule [13]. An ant k, when located at node i, uses the pheromone trail τ_{ij} to compute the probability of choosing j as the next node:

$$P_{ij}^{(k)} = \begin{cases} \dfrac{\tau_{ij}^{\alpha}}{\sum_{j \in N_i^k} \tau_{ij}^{\alpha}} & \text{if} \quad j \in N_i^{(k)} \\ 0 & \text{if} \quad j \notin N_i^{(k)} \end{cases} \tag{9}$$

where α denotes the degree of importance of the pheromones and $N_i^{(k)}$ indicates the set of neighborhood nodes of ant k when located at node i.

The neighborhood of node i contain all the nodes directly connected to node i except the predecessor node. This will prevent the ant from returning to the same node visited immediately before node i. An ant travels from node to node until it reaches the destination node. Before returning to the home node, the kth ant deposits an amount of pheromone on arcs it has visited. After all the ants return to the nest, the pheromone information is updated in order to increase the pheromone value associated with good or promising paths. The updating is achieved as follows:

$$\tau_{ij} = (1 - \rho)\tau_{ij} + \Delta\tau_{ij}^{(k)} \tag{10}$$

$$\Delta\tau_{ij}^{(k)} = \frac{Q}{L_k} \tag{11}$$

where $\rho \in (0, 1]$ is the pheromone decay factor; $\Delta\tau_{ij}^{(k)}$ is the amount of pheromone deposited on arc ij by the best ant k. also, Q is a constant and L_k is the length of the path traveled by the kth ant.

When more paths are available from the nest to a food source, a colony of ants will be able to exploit the pheromone trails left by the individual ants to discover the shortest path from the nest to the food source and back [13]. In fact, ACO simulates the optimization of ant foraging behavior.

3.2 Particle Swarm Optimization (PSO)

The PSO has been proposed by Eberhart and Kennedy [14] to simulate the motion of bird swarms. The particle swarm process is stochastic in nature; it uses a velocity vector to update the current position of each particle in the swarm. The velocity vector is updated based on the memory gained by each particle, conceptually resembling an autobiographical memory, as well as the knowledge gained by the swarm as a whole. Thus, the position of each particle in the swarm is updated based on the social behavior of the swarm which adapts to its environment by returning to promising regions of the space previously discovered and searching for better positions over time. Numerically, the position of the *i*th particle, X_i, at iteration $t + 1$ is updated as follows:

$$X_i^{t+1} = X_i^t + V_i^{t+1} \tag{12}$$

Where V_i^{t+1} is the corresponding updated velocity vector given as follows:

$$V_i^{t+1} = \omega V_i^t + c_1 r_1 \left(P_i^t - X_i^t\right) + c_2 r_2 \left(G_{\text{best}}^t - X_i^t\right) \tag{13}$$

where V_i^t is the velocity vector at iteration t, r_1 and r_2 represents random numbers between 0 and 1; P_i^t represents the best ever particle position of particle i, and G_{best}^t corresponds to the global best position in the swarm up to iteration t. The remaining terms are problem dependent parameters; in this paper, cognitive parameter, c_1, and c_2, social parameter, are considered to be equal to 2. Also, ω is the inertia weight which plays an important role in the PSO convergence behavior.

Due to the importance of ω in achieving efficient search behavior the optimal updating criterion is taken as follows:

$$\omega = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{k_{max}} . k \tag{14}$$

where ω_{max} and ω_{min} are the maximum and minimum values of ω, respectively. Also, k_{max}, and k are the number of maximum iterations and the number of present iteration, respectively.

4. SERIAL INTEGRATION OF PSO AND ACO

One of the defect known in PSO is that for swarm which have information about the global best solution (G_{best}), the second and third part of the velocity update equation (Eq. (13)) is zero thus swarm's behavior is on the last motion vector, also because ω is less than one, this part has been damped and the other swarm converge to the best swarm (G_{best}). This is the reason that cause to precocious convergence of algorithm and PSO disable to do a good local search. In order to solve this problem and improve the performance of PSO in [15-16] new terms has been added to the velocity updating equation.

In this study, in order to improve the computational performance of PSO, two hybrid optimization strategies are proposed. In the first one, PSO and ACO are serially integrated and the resulted algorithm is termed as serial particle swarm ant colony algorithm (SPSACA). In the SPSACA the constraints are handled using EPFM in the frame work of

SUMT. The SPSACA works as follows:

At first by using the EPFM and choosing the minor penalty parameter a swarm including n_p particles is randomly selected and PSO is employed to achieve a preliminary optimization task. As the r_p is small, PSO converges to an infeasible solution. In this process the best solution is saved as G_{best} in the memory of the algorithm. In the next step, a new swarm is generated based on elitism. The new elite swarm is created by the means of giving more chance to survive the elite particles. In this case, G_{best} is transformed to the new swarm and the remaining ones are randomly selected as follows:

$$X_j = \boldsymbol{N}(G_{\text{best}}, \alpha G_{\text{best}}) \ , \ \ j = 1,2,\ldots,(n_p - 1) \tag{15}$$

where $\boldsymbol{N}(G_{\text{best}}, \alpha G_{\text{best}})$ represents a random number normally distributed vector with the mean of G_{best} and the standard deviation of αG_{best}.

The produced elite swarm is employed by PSO to achieve another optimization process. In this process according to the SUMT concepts, the penalty parameter is increased as:

$$r_p^{k+1} = 10 r_p^k \tag{16}$$

This procedure is continued until PSO finds a feasible G_{best}. This process is entitled as global search phase (GSP). In the second stage of the SPSACA a finer search is implemented about the feasible G_{best} found by PSO in GSP. A new elite population is created using Eq. (15) and ACO is employed to achieve the optimization task. The best solution found by ACO is considered as the final solution. This later process is termed as local search phase (LSP). The flowchart of the SPSACA is shown in Figure 1.

The second algorithm proposed in this paper, termed as serial ant colony particle swarm algorithm (SACPSA), is similar to the SPSACA with a slight difference. In the framework of SACPSA, ACO is employed in the GSP while PSO is utilized in the LSP. For the both SPSACA and SACPSA, various values of α are examined and the results are reported in the numerical results section.

5. NUMERICAL EXAMPLES

In order to investigate the computational performance of the proposed algorithms, five benchmark structural optimization examples are solved. For the proposed algorithms, a population of 20 individuals is considered.

5.1 A ten-bar planer truss

A 10-bar truss structure is shown in Figure 2. The material density is 0.1 lb/in^3 and the modulus of elasticity is 10,000 ksi. The stress and displacement limitations are ±25 ksi and ±2.0 in, respectively. The design variables can be selected from a range of 0.1 to 35.0 in^2. Two cases are considered: Case (1), P_1=100 kips; and Case (2), P_1= 150 and P_2=50 kips.

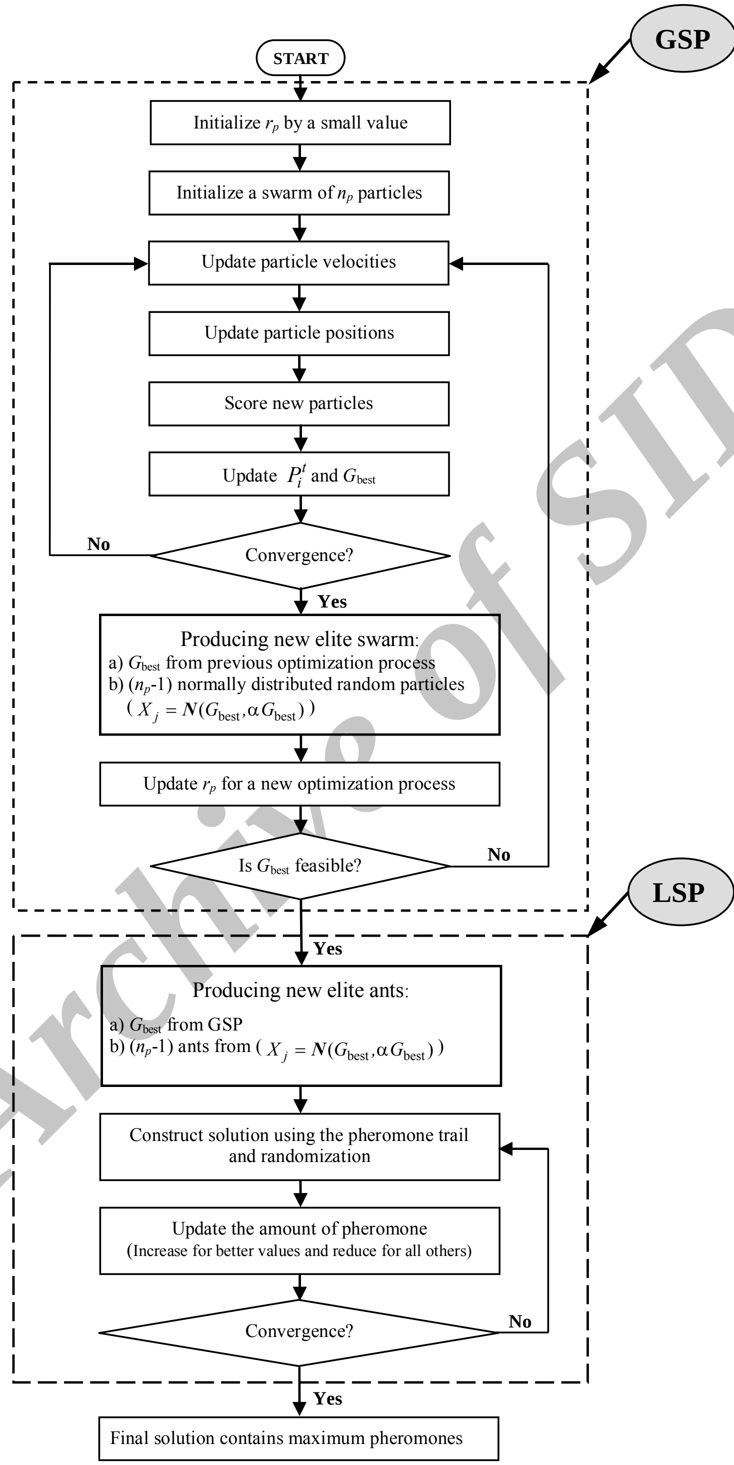


Figure 1. The flowchart of SPSACA

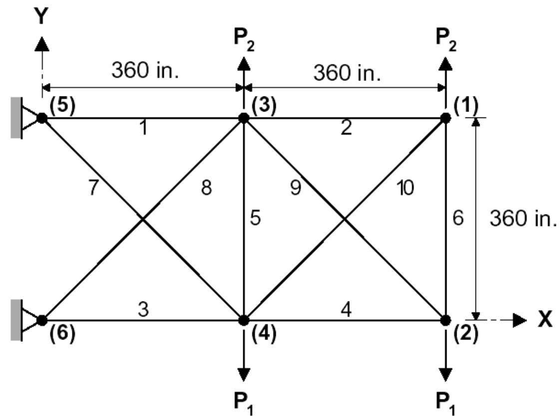


Figure 2. Ten-bar truss structure

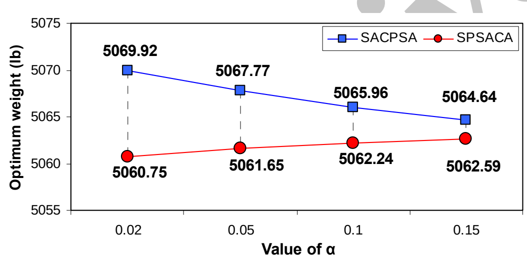
In this example The SPSACA and SACPSA algorithms achieve the best solutions after 500 iterations (10,000 analyses). In Tables 1and 2 the best results obtained in this work is compared with the results of the other works and Figure 3 shows the results of 50 independent runs of SPSACA and SACPSA for the various values of α. The given results indicate that the SPSACA not only finds a better solution but also considerably decreases the number of required structural analyses compared with the other works.

Table 1: Optimal design of the ten-bar planner truss by various methods (Case 1)

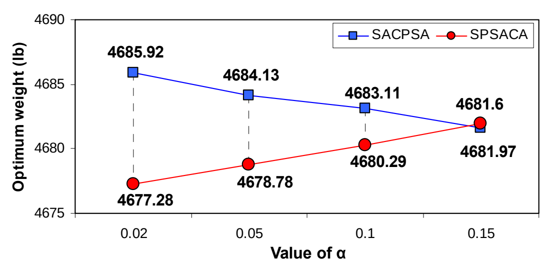
Element group	Li et al. [15]	Present work	
		SACPSA	SPSACA
A_1	30.7040	30.4140	30.4550
A_2	0.1000	0.1010	0.1000
A_3	23.1670	22.3610	23.1330
A_4	15.1830	15.1880	15.2350
A_5	0.1000	0.1009	0.1002
A_6	0.5510	0.5777	0.5500
A_7	7.4600	7.6402	7.4780
A_8	20.9780	21.7730	21.1820
A_9	21.5080	21.3570	21.4470
A_{10}	0.1000	0.1003	0.1000
Weight (lb)	5060.92	5064.64	5060.76
VC	0.0000	0.0000	0.0000
Number of analysis	150,000	10,000	10,000

Table 2: Optimal design of the ten-bar planner truss by various methods (Case 2)

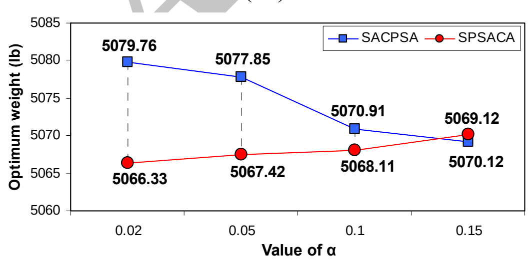
Element group	Li et al. [15]	Present work	
		SACPSA	SPSACA
A_1	23.3530	24.6640	23.7580
A_2	0.1000	0.1000	0.1000
A_3	25.5020	24.4470	25.1930
A_4	14.2500	14.6670	14.1550
A_5	0.1000	0.1000	0.1000
A_6	1.9720	1.9804	1.9723
A_7	12.3630	12.5910	12.4240
A_8	12.8940	12.6920	12.8160
A_9	20.3560	19.9370	20.3720
A_{10}	0.1010	0.1000	0.1000
weight	4677.30	4681.81	4677.29
VC	0.0000	0.0000	0.0000
Number of analysis	150,000	10,000	10,000



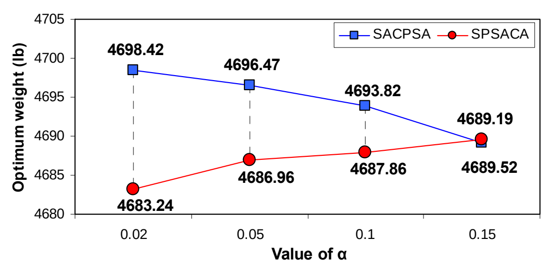
(a1)



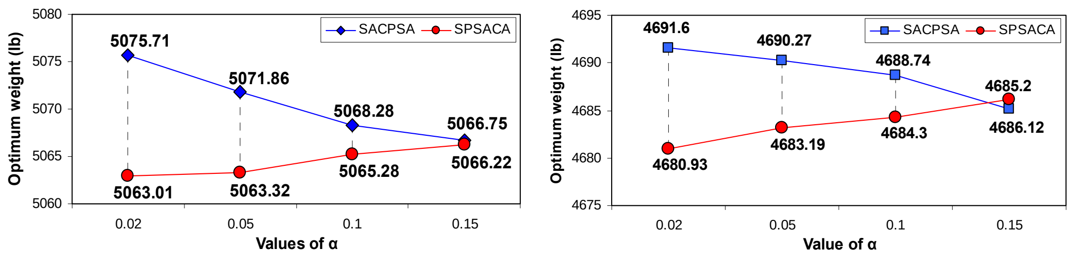
(a2)



(b1)



(b2)



(c1) (c2)

Figure 3. Result of 50 independent runs of the SACPSA and SPSACA for Ten-bar planar truss for various α: (a1) Best, (b1) Worst and (c1) Average weights (Case 1) & (a2) Best, (b2) Worst and (c2) Average weights (Case 2)

The results shown in Figure 3 reveal that in the case of SPSACA the best result are obtained when $\alpha = 0.02$ and by increasing the value of α the algorithm loses its exploitation ability and converges to heavier structures. In the case of the SACPSA reverse of this observation is true.

5.2 A seventeen-bar planar truss

The 17-bar planar truss, shown in Figure 4, has been studied by Khot and Berke [17] and Li et al. [15]. The material density and the modulus of elasticity are 0.268 lb/in^3 and 30,000 ksi, respectively. The stress and displacement limitations are ±50 ksi and ±2.0 in, respectively. No design variables' linking is used and there are seventeen independent design variables. The minimum cross-sectional area of the members is 0.1 in^2.

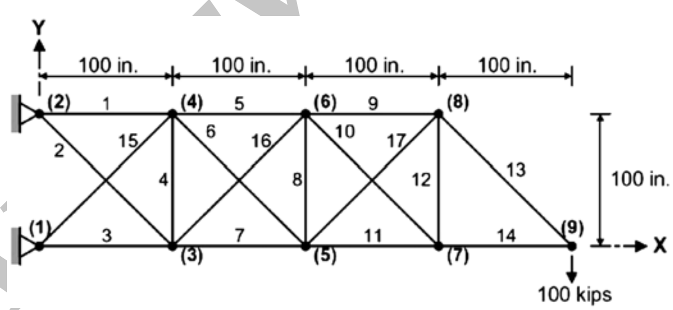


Figure 4. A seventeen-bar planar truss

The optimal results are compared with the solutions reported by Khot and Berke [17] and Li et al. [15] in Table 3. Figure 5 shows the results of 50 independent runs of SPSACA and SACPSA for the various values of α. The optimal design obtained by SPSACA is slightly better than both of the previously reported results but its required analyses is considerably lower than that of them.

Table 3: Optimal design of the Seventeen-bar planner truss by various methods

Element groups	Khot [17]	Li et al. [15]	Present work	
			SACPSA	**SPSACA**
A_1	15.930	15.890	15.8050	15.9320
A_2	0.100	0.103	0.1566	0.1000
A_3	12.070	12.090	13.0120	12.0700
A_4	0.100	0.100	0.1000	0.1000
A_5	8.067	8.063	7.9980	8.0656
A_6	5.562	5.591	5.4687	5.5639
A_7	11.930	11.910	11.7420	11.9310
A_8	0.100	0.100	0.1032	0.1000
A_9	7.945	7.965	7.9384	7.9478
A_{10}	0.100	0.100	0.1000	0.1000
A_{11}	4.055	4.076	4.1834	4.0584
A_{12}	0.100	0.100	0.1029	0.1000
A_{13}	5.657	5.670	5.4291	5.6515
A_{14}	4.000	3.998	3.9973	4.0004
A_{15}	5.558	5.548	5.3498	5.5611
A_{16}	0.100	0.103	0.1093	0.1000
A_{17}	5.579	5.537	5.6361	5.5747
Weight (lb)	2581.89	2581.94	2584.80	2581.88
VC	0.0000	0.0000	0.0000	0.0000
Number of analysis	30,000	150,000	10,000	10,000

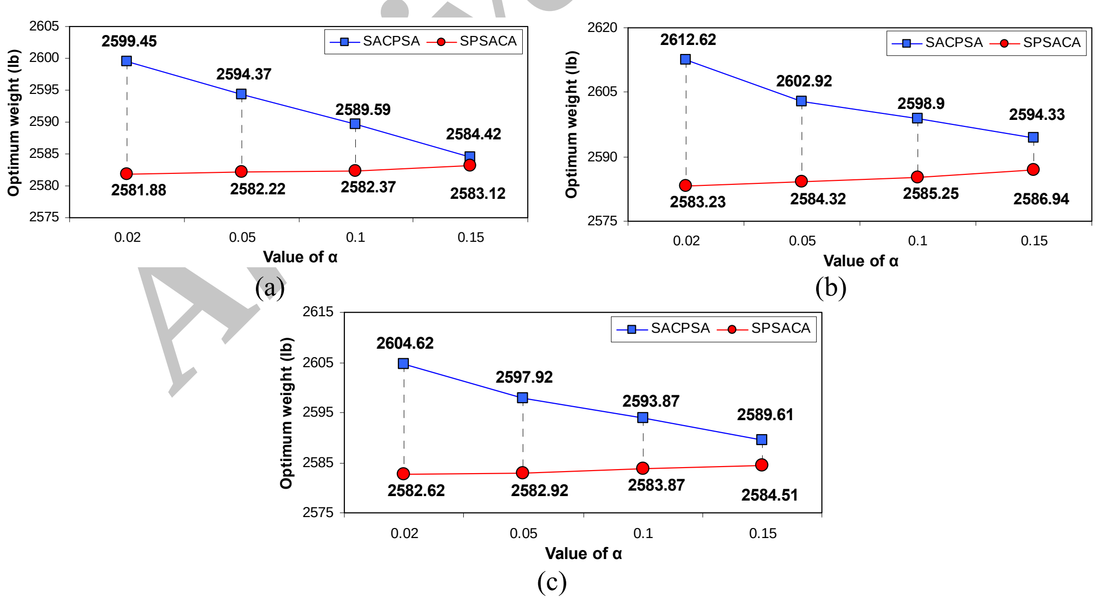


Figure 5. Result of 50 independent runs of the SACPSA and SPSACA for Seventeen-bar planar truss for various α: (a) Best, (b) Worst and (c) Average weights

Figure 5 indicates that for $\alpha = 0.02$ the SPACA found the best result. Also it is observed that increasing the value of α results in decreasing the exploitation ability of the algorithm. For the SACPSA the reverse of this idea is true.

5.3 A twenty five-bar spatial truss

A 25-bar space truss, shown in Figure 6, has been optimized by many researchers. The material density is 0.1 lb/in^3 and modulus of elasticity is 10,000 ksi. This space truss is subjected to the two loading conditions shown in Table 4.

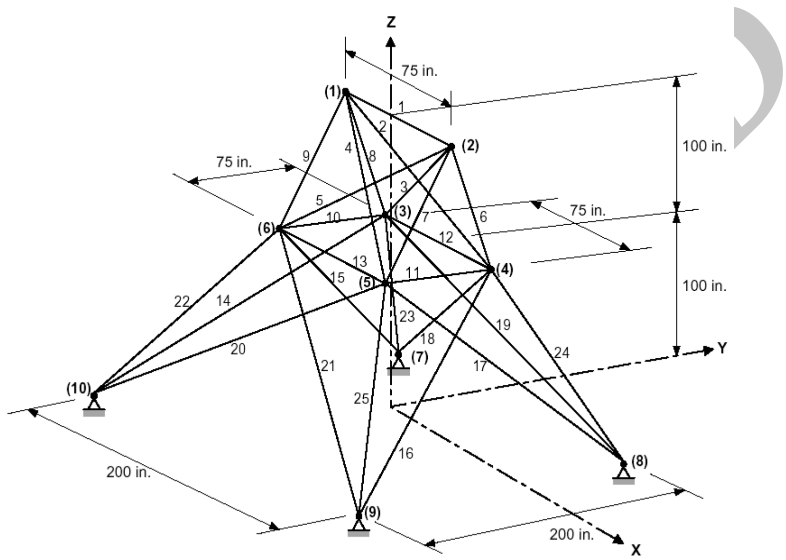


Figure 6. A twenty five-bar spatial truss

Table 4: Loading conditions for the twenty five-bar spatial truss

Node	Case 1			Case 2		
	P_X	P_Y	P_Z	P_X	P_Y	P_Z
1	0.0	20.0	–5.0	1.0	10.0	–5.0
2	0.0	–20.0	–5.0	0.0	10.0	–5.0
3	0.0	0.0	0.0	0.5	0.0	0.0
6	0.0	0.0	0.0	0.5	0.0	0.0

The truss members are subjected to the compressive and tensile stress limitations shown in Table 5. Displacement limitation is ±0.35 in and the minimum and maximum cross-sectional areas of all members are 0.01 in^2 and 3.4 in^2, respectively. The member groups are as follows:

(1) A_1, (2) A_2–A_5, (3) A_6–A_9, (4) A_{10}–A_{11}, (5) A_{12}–A_{13}, (6) A_{14}–A_{17}, (7) A_{18}–A_{21}, (8) A_{22}–A_{25}

Table 5: Member stress limitations for the twenty-bar spatial truss

Element group	Compressive stress (ksi)	Tensile stress (ksi)
A_1	35.092	40.0
A_2–A_5	11.590	40.0
A_6–A_9	17.305	40.0
A_{10}–A_{11}	35.092	40.0
A_{12}–A_{13}	35.092	40.0
A_{14}–A_{17}	6.759	40.0
A_{18}–A_{21}	6.959	40.0
A_{22}–A_{25}	11.082	40.0

Table 6 gives a comparison between the solutions reported in the literature and the present work.

Table 6: Optimal design comparison for the twenty-bar spatial truss

Element group	Li et al. [15]	Kaveh and Talatahari [18]	Present work	
			SACPSA	SPSACA
A_1	0.010	0.010	0.0101	0.0100
A_2–A_5	1.970	1.993	1.9520	1.9935
A_6–A_9	3.016	3.056	3.0440	2.9792
A_{10}–A_{11}	0.010	0.010	0.0102	0.0100
A_{12}–A_{13}	0.010	0.010	0.0119	0.0100
A_{14}–A_{17}	0.694	0.665	0.6770	0.6822
A_{18}–A_{21}	1.681	1.642	1.6890	1.6784
A_{22}–A_{25}	2.643	2.679	2.6530	2.6679
Weight (lb)	545.19	545.16	545.41	545.18
VC	0.0000	0.0001	0.0000	0.0000
Number of analysis	30,000	12,500	10,000	10,000

It is observed that the SPSACA obtains an optimal structure which is slightly better than previously found designs by the other researchers but its required analyses is lower than that of them. Figure 7 shows the results of 50 independent runs of SPSACA and SACPSA for the various values of α. These results indicate that in the case of SPACA if $\alpha = 0.02$ the best result can be obtained also by increasing the value of α the algorithm converges to heavier structures. In the case of the SACPSA the reverse of this observation is true.

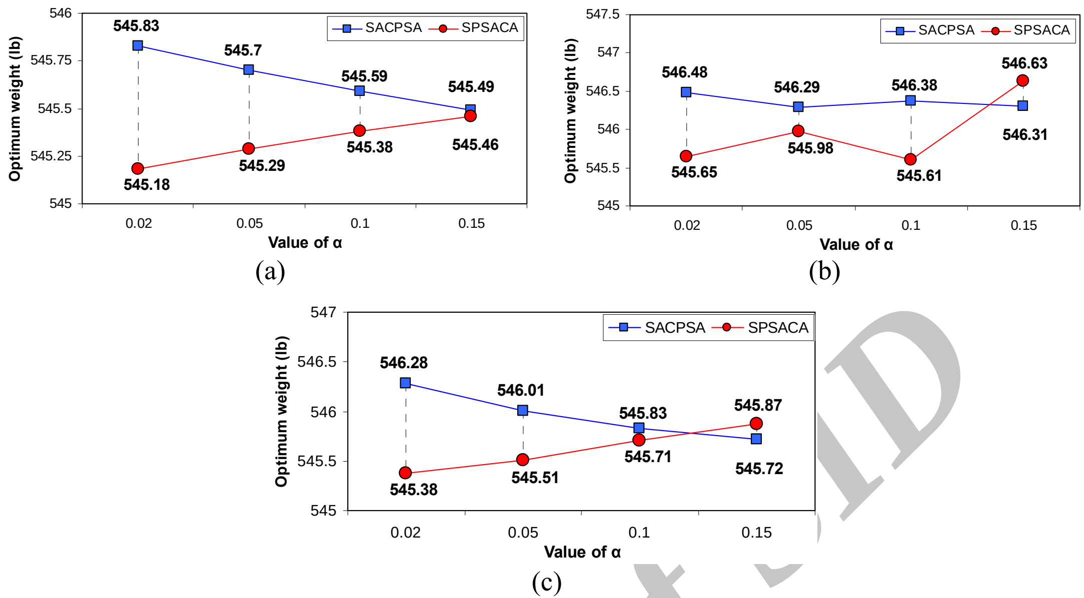


Figure 7. Result of 50 independent runs of the SACPSA and SPSACA for Twenty five-bar spatial truss for various α: (a) Best, (b) Worst and (c) Average weights

5.4 SA seventy two-bar spatial truss

A 72-bar spatial truss structure is shown in Figure 8. The material density is 0.1 lb/in^3 and the modulus of elasticity is 10,000 ksi. The members are subjected to the stress limits of ±25 ksi and the maximum displacement of top nodes in each direction is limited to ±0.25 in. This structure is subjected to the two loading conditions as follows:

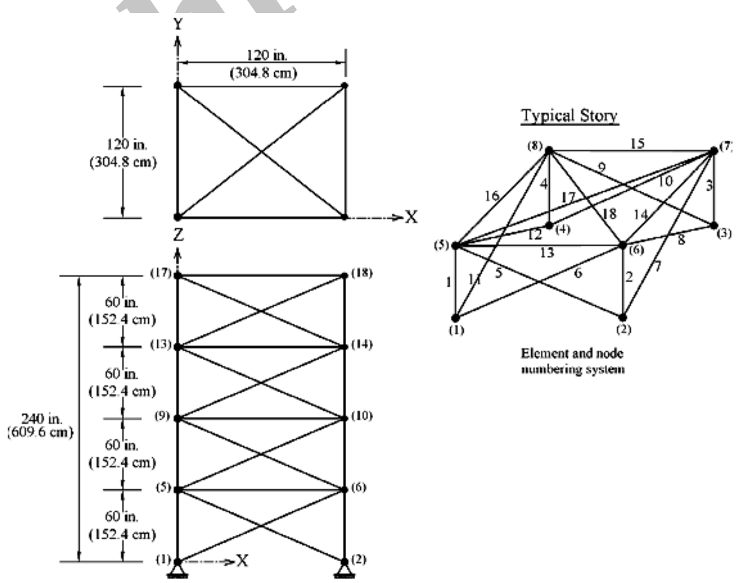


Figure 8. A seventy two-bar spatial truss

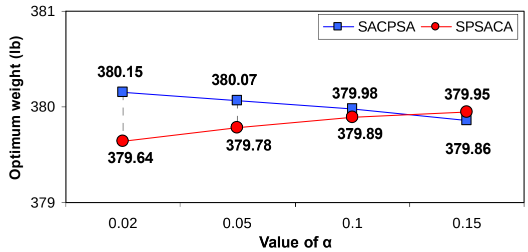
Case 1; Node 17: P_X = 5.0 ksi, P_Y = 5.0 ksi and P_Z = –5.0 ksi. **Case 2;** Nodes 17, 18, 19 and 20: P_Z = –5.0 ksi.

The structural elements are classified into 16 groups as follows:
(1) A_1-A_4, (2) A_5-A_{12}, (3) A_{13}-A_{16}, (4) A_{17}-A_{18}, (5) A_{19}-A_{22}, (6) A_{23}-A_{30}, (7) A_{31}-A_{34}, (8) A_{35}-A_{36}, (9) A_{37}-A_{40}, (10) A_{41}-A_{48}, (11) A_{49}-A_{52}, (12) A_{53}-A_{54}, (13) A_{55}-A_{58}, (14) A_{59}-A_{66}, (15) A_{67}-A_{70}, (16) A_{71}-A_{72}.

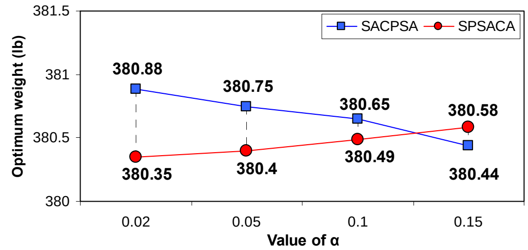
The lower and upper bounds on the cross-sectional area of each element are 0.1 in^2 and 4.0 in^2, respectively. In Table 7 a comparison between the solutions reported in the literature and the present work is presented.

Table 7: Optimal design comparison for the seventy two-bar spatial truss

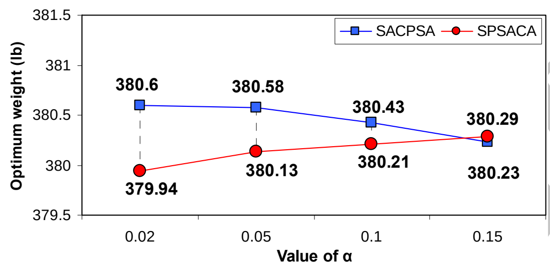
Element groups	**Camp et al. [19]**	**Kaveh and Talatahari [18]**	**Present work**	
			SACPSA	**SPSACA**
A_1	1.8577	1.9042	1.8577	1.8822
A_2	0.5059	0.5162	0.5059	0.5114
A_3	0.1000	0.1000	0.1000	0.1000
A_4	0.1000	0.1000	0.1000	0.1000
A_5	1.2476	1.2582	1.2476	1.2571
A_6	0.5269	0.5035	0.5269	0.5136
A_7	0.1000	0.1000	0.1000	0.1000
A_8	0.1012	0.1000	0.1012	0.1000
A_9	0.5209	0.5178	0.5210	0.5300
A_{10}	0.5172	0.5214	0.5172	0.5200
A_{11}	0.1004	0.1000	0.1004	0.1000
A_{12}	0.1005	0.1007	0.1009	0.1000
A_{13}	0.1565	0.1566	0.1565	0.1566
A_{14}	0.5507	0.5421	0.5507	0.5465
A_{15}	0.3922	0.4132	0.3922	0.4080
A_{16}	0.5922	0.5756	0.5922	0.5648
weight	379.84	379.66	379.86	379.64
VC	0.0000	0.0000	0.0000	0.0000
Number of analysis	-	13,200	10,000	10,000



(a)



(b)



(c)

Figure 9. Result of 50 independent runs of the SACPSA and SPSACA for Seventy two-bar spatial truss for various α: (a) Best, (b) Worst and (c) Average weights

The solution obtained by SPSACA is slightly better than that of other researchers but its required analyses is lower than that of them. Figure 9 shows the results of 50 independent runs of SPSACA and SACPSA for the various values of α. These results demonstrate that in the case of SPSACA by increasing the value of α the algorithm converges to heavier structures while for the SACPSA reverse of this observation is true.

5.5 A 120-bar dome truss

A 120-bar dome, shown in Figure 10, was studied in [20-21]. All members of the dome are linked into seven groups, as shown in Figure 10. The minimum cross sectional area of all members is 0.775 in^2.

The allowable tensile and compressive stresses are imposed according to the AISC ASD (1989) [22], as follows:

$$\sigma_{ia} = F_{cr} \quad \text{for} \quad \text{compression} \quad \text{stress} \tag{17}$$

$$F_{cr} = \begin{cases} \left[\left(1 - \dfrac{\lambda_i^2}{2C_c^2}\right)F_y\right] \Big/ \left[\dfrac{5}{3} + \dfrac{3\lambda_i}{8C_c} + \dfrac{\lambda_i^3}{8C_c^3}\right] & \lambda_i < C_c \\ \dfrac{12\pi E}{23\lambda_i^2} & \lambda_i \geq C_c \end{cases} \tag{18}$$

$$\sigma_{ia} = 0.6F_y \quad \text{for} \quad \text{tensile} \quad \text{stress} \tag{19}$$

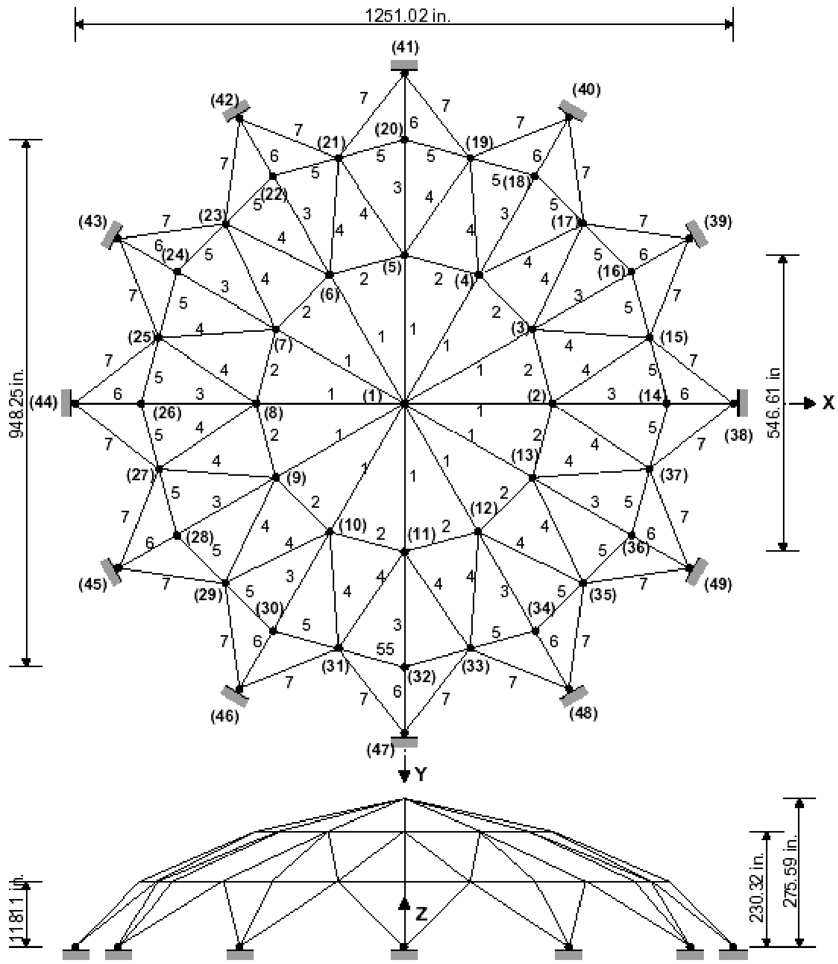


Figure 10. One hundred twenty-bar dome

where E and F_y are the modulus of elasticity and yield stress of steel, respectively; C_c is the slenderness ratio dividing the elastic and inelastic buckling regions ($C_c = \sqrt{2\pi^2 E/F_y}$); λ_i is the slenderness ratio ($\lambda_i = kL_i/r_i$); The modulus of elasticity is 30,450 ksi and the material density is 0.288 lb/in^3 .The yield stress of steel is taken as 58.0 ksi. On the other hand, the radius of gyration (r_i) can be expressed in terms of cross-sectional areas, i.e., $r_i = aA^b$. Here, a and b are the constants depending on the types of sections adopted for the members such as pipes, angles, and tees. In this example, pipe sections (a = 0.4993 and b = 0.6777) were adopted for bars.

The dome is considered to be subjected to vertical loading at all the unsupported joints. These are taken as −13.49 kips at node 1, −6.744 kips at nodes 2 through 13, and −2.248 kips at the rest of the nodes. A comparison between the optimal solutions reported in the literature and the present work is given in Table 8 and Figure 11 shows the results of 50 independent run of the proposed algorithms for the various values of α.

Table 8: Optimal design comparison for the one hundred twenty-bar dome

Element group	Lee et al. [21]	Seyedpoor et al. [23]	Present work	
			SACPSA	SPSACA
A_1	3.296	3.040	3.0240	3.0199
A_2	2.786	2.298	2.2610	2.2603
A_3	3.872	3.133	3.0730	3.0706
A_4	2.570	1.983	1.9660	1.9652
A_5	1.149	0.775	0.7770	0.7753
A_6	3.331	2.810	2.8001	2.7903
A_7	2.781	2.370	2.3481	2.3480
weight	19893.34	16370.23	16221.33	16205.1
VC	0.0	0.0	0.0	0.0
Number of analysis	35000	1700	1600	1600

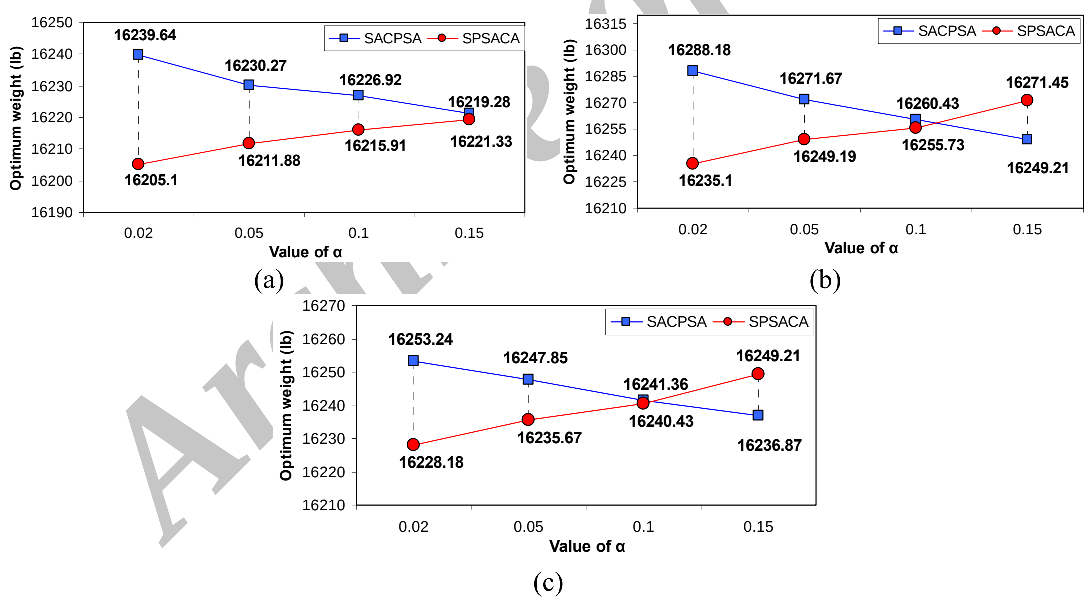


Figure 11. Result of 50 independent runs of SACPSA and SPSACA for One hundred twenty-bar dome for various α: (a) Best, (b) Worst and (c) Average weights

It is observed that the results of SPSACA are better than that of the other proposed algorithms. As well as the examples 1 to 4 in this example also in the cases of SPSACA and SACPSA the best results associates with $\alpha = 0.02$ and $\alpha = 0.15$, respectively. By increasing the value of α the exploitation ability of SPSACA decreases while for the SACPSA the reverse of this observation is true.

6. CONCLUSIONS

In this paper, PSO and ACO are serially integrated and two combinatorial metaheuristic algorithms are proposed. These algorithms denoted as serial particle swarm ant colony algorithm (SPSACA) and serial ant colony particle swarm algorithm (SACPSA) are employed in the framework of the sequential unconstrained minimization techniques (SUMT). In the SPSACA global search is achieved by PSO while the zone search around the best solution found by PSO is implemented by ACO. In the SACPSA the global and local searches are performed by ACO and PSO, respectively. In order to handle the constraints the exterior penalty function method (EPFM) is used. By increasing the number of iterations, the SUMT gradually increases the penalty parameter and lead to decrease the constraint violation. The efficiency of the SPSACA and SACPSA are tested for optimum design of five planar and spatial pin connected structures. The numerical results demonstrate that SPSACA is not only leads to better solutions but also requires fewer number of structural analyses compared to other algorithms.

REFERENCES

1. Kaveh A, Kalatjari V. Genetic algorithm for discrete sizing optimal design of trusses using the force method, *International Journal for Numerical Methods in Engineering*, **55**(2002) 55–72.
2. Salajegheh E, Gholizadeh S. Optimum design of structures by an improved genetic algorithm using neural networks, *Advances in Engineering Software*, **36**(2005) 757–67.
3. Gholizadeh S, Salajegheh E. Optimal design of structures for time history loading by swarm intelligence and an advanced met model, *Computer Methods in Applied Mechanics and Engineering*, **198**(2009) 2936–49.
4. Kaveh A, Talatahari S. A hybrid particle swarm and ant colony optimization for design of truss structure, *Asian Journal of Civil Engineering*, **9**(2008) 329-48.
5. Camp CV, Bichon BJ. Design of space trusses using ant colony optimization, *Journal of Structural Engineering*, **130**(2004) 741–51.
6. Camp CV, Bichon BJ, Stovall SP. Design of steel frames using ant colony optimization, *Journal of Structural Engineering*, **131**(2005) 369–79.
7. Eberhart R, Shi Y. Engineering optimization with particle swarm. *In: IEEE swarm intelligence symposium,* Indianapolis, 2003, pp. 53–57.
8. Hassan R, Cohanim B, de Weck O, Venter G. A comparison of particle swarm optimization and the genetic algorithm. *In: 1st AIAA Multidisciplinary Design Optimization Specialist Conference*. No. AIAA-2005-1897.
9. Angeline P. Evolutionary optimization versus particle swarm optimization: philosophy and performance difference, *Proceeding of the Evolutionary Programming Conference*, San Diego, USA, 1998.
10. Shelokar PS, Siarry P, Jayaraman VK, Kulkarni BD. Particle swarm and ant colony

algorithms hybridized for improved continuous optimization, *Applied Mathematics and Computation*, **188**(2007) 129-42.

11. Vanderplaats GN. Numerical optimization techniques for engineering design: with application. 2nd ed. NewYork: McGraw-Hill, 1984.
12. Dorigo M. Optimization, learning and natural algorithms (in Italian), PhD Thesis. Dipartimento di Elettronica, Politecnico di Milano, IT, 1992.
13. Dorigo M, Caro DG, Gambardella LM. Ant algorithms for discrete optimization, *Art if Life*, **5**(1999) 137-72.
14. Eberhart RC, Kennedy J. A new optimizer using particle swarm theory. *In: Proceedings of the sixth international symposium on micro machine and human science*, Nagoya, Japan, 1995.
15. Li LI, Huang ZB, Liu F, Wu QH. A heuristic particle swarm optimizer for optimization of pin connected structures, *Computers and Structures*, *85*(1997) 340-9.
16. Gholizadeh S. Optimum Design of Structures by an Improved Particle Swarm Algorithm, *Asian Journal of Civil Engineering*, 11(2010) 777-93.
17. Khot NS, Berke L. Structural optimization using optimality criteria methods. *In: Atrek E, Gallagher RH, Ragsdell KM, Zienkiewicz OC, editors. New directions in optimum structural design*. New York: John Wiley, 1984.
18. Kaveh A, Talatahari S. Size optimization of space trusses using Big Bang–Big Crunch algorithm, *Computers & Structures*, **87**(2009) 1129–40.
19. Camp CV. Design of space trusses using Big Bang–Big Crunch optimization, *Journal of Structural Engineering*, **133**(2007) 999-1008.
20. Soh CK, Yang J. Fuzzy controlled genetic algorithms search for shape optimization, *Journal of Computing in Civil Engineering*, **10**(1996) 143-50.
21. Lee KS and Geem ZW. A new structural optimization method based on the harmony search algorithm, *Computers & Structures*, **82**(2004) 781–98.
22. American Institute of Steel Construction (AISC).: 1989, Manual of steel construction allowable stress design, 9th ed., Chicago, IL.
23. Seyedpoor SM, Gholizadeh S, Talebian SR. An efficient structural optimization algorithm using a hybrid version of particle swarm optimization with simultaneous perturbation stochastic approximation, *Civil Engineering and Environmental Systems*, **27**(2008) 295–313.