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DETECTION OF A THROUGH-THICKNESS CRACK BASED ON ELASTIC WAVE SCATTERING IN PLATES
PART II: INVERSE SOLUTION

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ABSTRACT

In the first part of the present paper, a numerical method called spectral finite element method (SFEM) is presented. As shown there, SFEM offered a promising performance in simulating wave scattering phenomena in plates, with less computational effort while obtaining the highest resolution. The forward solution of wave scattering phenomenon based on the differential waveform of highest resolution plays a crucial role for detection of through-thickness crack in plates, as an inverse problem. Therefore, the results of the first part of the paper have been used in the second present part, in which a novel approach in wave scattering based crack detection, according to the concept of Time Delay (Differences) Of Arrival (TDOA) has been presented. In the past decades, TDOA and its related concepts have been applied generally on the problem of localization and pursuing sound sources in robotics, electronics engineering, control engineering, and so on. However, in this study TDOA is employed for crack detection problems for the first time. Introducing TDOA has provided a robust algorithm for crack detection which is highly sensitive to the crack location and size. Using the proposed algorithm, a couple of numerical case studies has been studied, in which the predicted crack location and dimensions are quite encouraging.

Keywords: Time delay (differences) of arrival; elastodynamics; wave scattering analysis; inverse solution

1. INTRODUCTION

Crack detection of various structural components is vital for deciding about their repair or replacement. A crack should be detected in its early state to prevent serious failure of a structural member. Visual inspection may be generally useful for the purpose of crack detection, but for critical and large structures such as railway tracks, slab deck bridges, and aerospace structures, visual inspection may be difficult in practice. Vibration- and wave-

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based methods are important tools for such purposes. There are many research attempts dedicated to develop effective methods for damage detection in structural components in the last three decades. These have resulted in various numerical (e.g., finite element method) [1,2], analytical [3–7], and experimental [8–10] approaches. Furthermore, when it comes to online structural health monitoring, a lot of damage detection strategies fail as being expensive, time consuming or impractical for online situations. Using sensor array techniques which are based on wave propagation in solids have shown better suitability for online structural health monitoring compared to other approaches. A state-of-the-art review in condition monitoring with particular emphasis on structural engineering applications was presented by Carden and Fanning [11]. Furthermore, Fan and Qiao [12] presented a comprehensive review on damage identification methods for beam- or plate-type structures.

Kessler *et al.* [13] surveyed some damage detection algorithms for in situ damage detection of composite materials. They exploited an accurate and easy procedure to determine time of flight (TOF) of a Lamb wave pulse between an actuator and sensor. Kessler's approach was developed to a two-dimensional (2D) domain by Su *et al.* [14]. They used four piezoelectric sensors located near to the vertices of a rectangular CF/EP plate. A nonlinear set of equations was obtained using TOFs and the solution of nonlinear system of equations yielded the coordinates of a point as the crack center. The shortcoming of this method is that just one point is reported as the crack location, which means that the damage severity is unknown. Time reversal imaging is another Lamb wave based damage detection algorithm which uses a concept similar to that of TOFs [15–17]. The main concept in this approach is that for any waves radiating from a source which are subsequently scattered, reflected and refracted by damage, there exists a set of waves that can precisely retrace all the paths and converge in synchrony at the original source, as if time were going backwards [18]. Multiple solutions, complexity and low resolution results are the main disadvantages of this approach. Correlation based algorithms have been recently represented as effective methods for Lamb wave based damage identification [19].

Various methods using artificial intelligence have been proposed to identify different types of damage. Artificial neural networks (ANNs) [20,21] and genetic algorithms (GAs) [22–24] have been widely used to process various types of data carried by Lamb wave signals from different viewpoints. Bayesian Inference is another tool utilized to decrypt the information obtained from sensor arrays to recognize and locate damage [25]. Both ANNs and GAs have well known limitations. For example, an effective ANN is usually trained for a specific type of damage, geometric and boundary conditions, and mechanical properties, so the problem would be quite restrained. GAs are usually time consuming and defining an appropriate cost function and overcoming the ill-posedness is always a matter of concern.

Time Delay (Differences) Of Arrival (TDOA) refers to a concept which is corresponding to sensor arrays and microphone arrays methodologies, for example. TDOA and its related concepts have a rather long history in electronics engineering, robotics, aerospace engineering, and defense technologies. These concepts have been widely studied and developed, especially in 1990s. TDOA-based localization has been often used for sound source localization and tracking [26–28]. In this research, a crack identification approach in a plate using the concept of TDOA-based localization is developed for the first time. The proposed method is based on scattering of elastic waves due to the crack presence. In this approach, the forward problem involves determination of wave scattering phenomenon from the knowledge of plate geometry, its boundary conditions, and loading characteristics. As shown in the first part of the present paper [29], SFEM has shown a promising performance in simulating wave scattering phenomena in plates, with less computational effort while obtaining the highest resolution. As a result, the forward solution of wave scattering phenomenon of highest resolution plays a critical role for detection of through-thickness crack in plates. The predicted crack location and dimensions using the proposed algorithm are quite encouraging.

2. THE CONCEPT OF TDOA FOR CRACK DETECTION

A sensor array is a set of Omni directional sensors which are located in a specific configuration. In this regard, an Omni directional sensor is defined as a sensor which has the same power of signal receiving in all directions with no specific directivity bias. These receivers are commonly microphones and radio antennas. In the present research, sensors play the role of receivers. In other word, a node in a SFEM model may be considered as a sensor/receiver.

The TDOA-based localization scheme requires the time delays measured for the arrival of a wave front to sensors in a sensor array, assuming an arbitrary sensor as the reference sensor. It may be easily shown that in a 2D domain, three sensors or two time delays are needed to theoretically locate a point as the wave origin. In other words, if there is a time delay of $T0_i$ between sensor S_i and sensor S_0 (which is assumed to be the reference sensor), then the loci of all points, which could be the wave origin and produce such a time delay, would be the center of all circles that cross S_0 and are tangent to a circle centered at S_i. The radius of the circle is $C*T0_i$, in which C is the wave velocity. The idea is depicted in Figure 1.

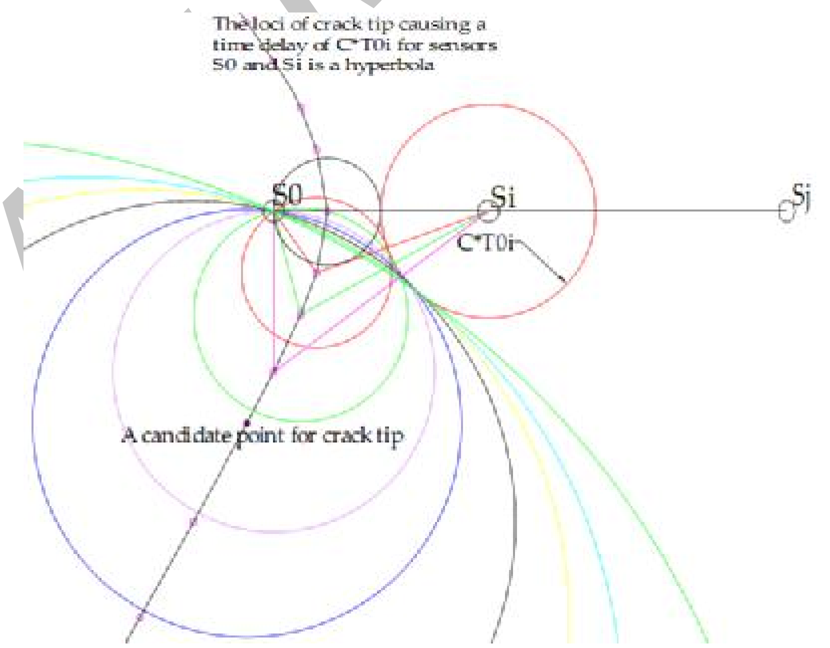

The loci of crack tip causing a time delay of C*T0i for sensors S0 and Si is a hyperbola
S0
Si
Sj
C*T0i
A candidate point for crack tip

Figure 1. Representing the concept of TDOA for the sensors S_0 and S_i

The context which is shown in Figure 1 could be expressed in a mathematical form as

$$T0_i = \frac{\| O - S_0 \| - \| O - S_i \|}{C} \tag{1}$$

in which O is the location vector for a candidate point for wave origin, and $\| O - S_i \|$ stands for $\sqrt{(x_0 - x_{S_i})^2 + (y_0 - y_{S_i})^2}$. It should be noted that Eq. (1) denotes a hyperbola. In other words, each pair of sensors reveals the nonlinear equation of its associated hyperbola. Now, considering sensor S_j paired with the reference sensor S_0, one may obtain another nonlinear equation (i.e., another hyperbola) which is defined by Eq. (2) and drawn in **Error! Reference source not found.**

$$T0_j = \frac{\| O - S_0 \| - \| O - S_j \|}{C} \tag{2}$$

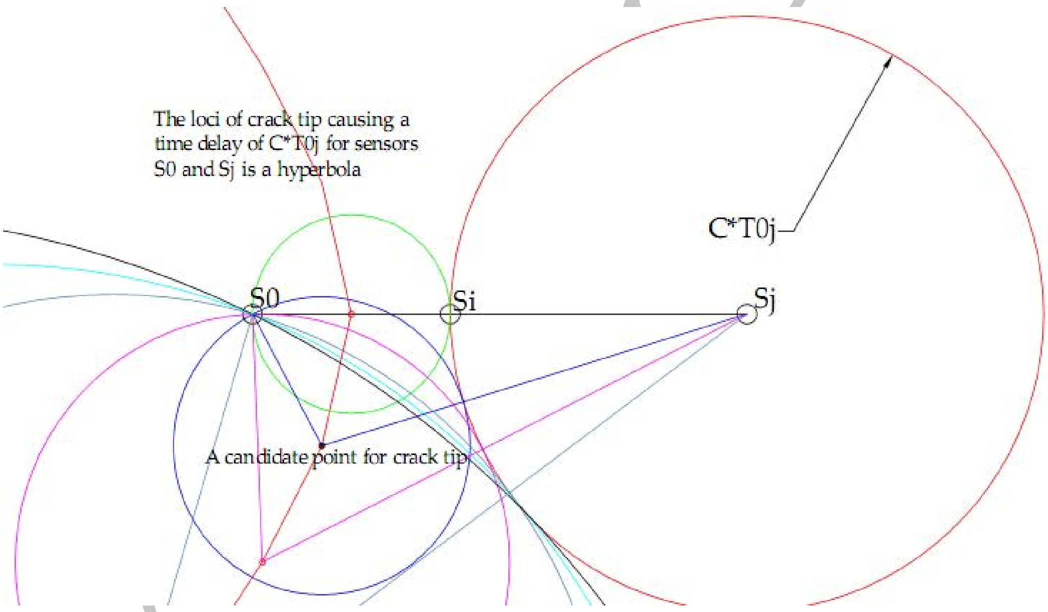

The loci of crack tip causing a time delay of C*T0j for sensors S0 and Sj is a hyperbola
C*T0j
S0
Si
Sj
A candidate point for crack tip

Figure 2. Representing the concept of TDOA for the sensors S_0 and S_j

The hypothetical solution of the problem is then the point at which these two hyperbolas intersect. This could be observed in Figure 3.

The solution obtained by solving system of TDOA equations would be credible provided that all time delays are measured very accurately. In fact, such solution is highly sensitive to the measurement error of TDOAs. Designing effective methods to precisely estimate TDOAs has not worked so far. In other words, all these methods could not guarantee to provide enough precise TDOAs such that when used in the nonlinear system of TDOA equations, a credible solution could be achieved. Hence, to overcome this problem and reduce the sensitivity of solution to TDOAs, a larger sensor array should be exploited. In such case, the number of TDOA hyperbolas is more than minimum required (two in 2D case). Moreover, because of various types of measurement errors in evaluating TDOAs, these hyperbolas may not converge in one point but in a specific region. Therefore, the problem of wave source localization becomes an optimization process which finds a location at which some cost function related to TDOA equations is optimized.

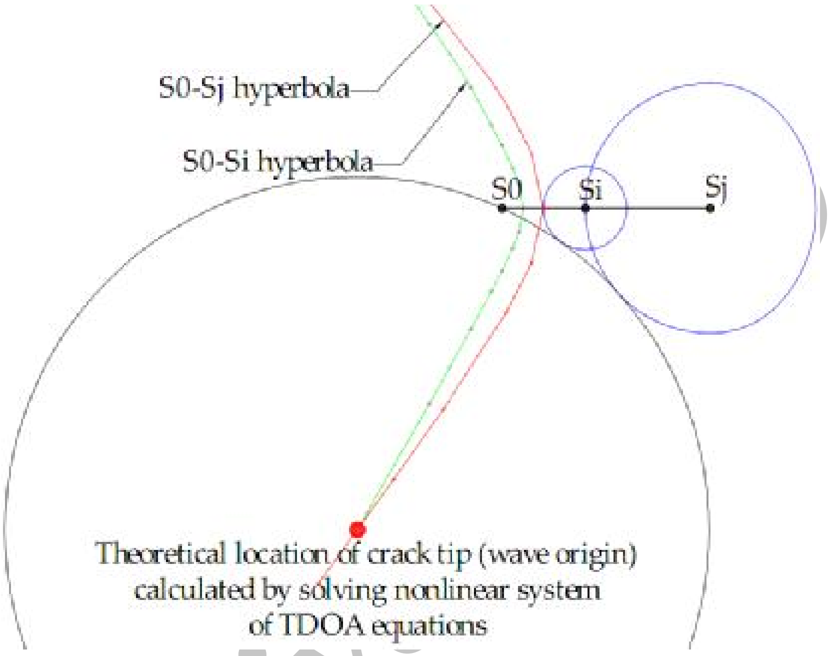

S0-Sj hyperbola
S0-Si hyperbola
S0
Si
Sj
Theoretical location of crack tip (wave origin)
calculated by solving nonlinear system
of TDOA equations

Figure 3. Solving nonlinear system of TDOA equations for the hypothetical location of wave origin

In this study, we have focused on minimizing the error between theoretical TDOAs and measured TDOAs. In this regard, the error may be represented by the following relation

$$\text{Error Function}(\{S_0, S_i\}, O) = T0_i - \frac{\| O - S_0 \| - \| O - S_i \|}{C}, \; i = 1, 2, \ldots, N_S \tag{3}$$

in which O is the location vector for a candidate point for wave origin, S_i indicates the location vector for sensor S_i, S_0 is the location vector for reference sensor, and N_S denotes the number of sensors. In addition, $T0_i$ denotes measured time delay between sensor S_i and the reference sensor, and $\frac{1}{C}(\| O - S_0 \| - \| O - S_i \|)$ is computed time delay between the same sensors. The solution for wave origin should be a location that minimizes the error criteria for all sensors. Consequently, a cost function is defined as follows

$$\text{Cost Function} = \sum_{i=1}^{N_S} \left(T0_i - \frac{\| O - S_0 \| - \| O - S_i \|}{C} \right)^2 \tag{4}$$

in which the sum of squares of error criteria for all sensors is minimized in the optimization process, from which the argument O may be the estimated location of wave origin. Using larger sensor arrays, the optimization process acquires more accurate solutions. This characteristic helps when there is uncertainty in evaluating TDOAs.

3. ESTIMATING TDOAS

As mentioned before, obtaining enough accurate TDOAs is a great challenge. This challenge becomes more severe when wave propagation in solids is our concern. Dissimilar to in-air wave propagation, Lamb waves propagate in a frequency dependent velocity in a solid plate, which is called dispersion phenomena. Furthermore, wave attenuation occurs in Lamb wave propagation in solids which makes the estimation of TDOAs more complicated.

It was proposed to calculate the arrival time of a Lamb wave signal to a sensor by defining a threshold [30]. This method [30] may give better results when preceded by signal filtering and normalization. Subtracting such arrival times, we have employed this method to estimate TDOAs. However, using threshold for determining the rising edge of signals is prone to a significant error if the threshold is not appropriately selected or in a case where signals are weak and/or damaged by ambient noise. Another traditional method for direct determination of TDOAs is to find the maximum of cross correlation function of two signals [31]. Cross correlation function for two signals s_i and s_j which are respectively captured by sensors S_i and S_j, is determined as given below

$$R_{ij}(t) = \int_{-\infty}^{+\infty} s_i(t)\, s_j(t+t)\, dt \tag{5}$$

In other words, $R_{ij}(t)$ may have its maximum value when argument t shifts s_j so that the two signals show the best overlapping with each other. In such a case, t would be TDOA for signals s_i and s_j. It should be noted that if signals have a periodic nature, then $R_{ij}(t)$ will also show a periodic behavior. This means that there might be some local peaks which have almost the same magnitude or even larger than that of expected time delay. Furthermore, Lamb waves propagated in a plate are affected by wave attenuation and dispersion which means that the wave form of the signal recorded by sensor S_i may be significantly different from that of sensor S_j. In addition, multimode property of Lamb waves makes the situation far worse. All these situations indicate that if cross correlation method is used, it must be preceded by some initial preparations like signal filtering and/or normalization, selective mode excitation [32], preliminary signal processing [33] and so forth.

4. PREPARATION OF TDOA CONCEPT FOR CRACK LOCALIZATION

In this research, the main target is to develop a new algorithm for crack detection in plates, based on processing of Lamb waves. For this purpose, by finding some specific points, say crack's tips, a suitable estimation of crack configuration is proposed. The following assumptions are considered in the proposed algorithm:

(a) The structure under investigation is a plate with plane stress condition.
(b) The boundary conditions of the plate are known.
(c) The material's behavior is small-displacement linear elastic, whose known properties are assumed isotropic and homogeneous.
(d) The existing crack is a straight line whose sides have no interaction with each other.

As already discussed, TDOA-based localization schemes have been mainly used for sound source localization (SSL) in air. In this section, some points and challenges about TDOA-based crack detection algorithms are discussed. The first issue is that in SSL, sound source itself propagates waves within the domain, while a crack is not a wave source by itself. Figure 11 of the first part of this paper shows a cracked aluminum plate in which Lamb waves are being propagated and then reflected, scattered and refracted by the crack and boundaries. The crack is aligned vertically and has a length of 100 millimeters. The bottom tip of the crack locates at x = 300 mm and y = 475 mm. It can be observed from this figure that the crack may be recognized as a secondary wave origin. This idea becomes more remarkable when the response of the uncracked aluminum plate is subtracted from that of cracked one (see Figure 4). Consequently, if subtracted signals are taken into consideration, it is just like the crack is the wave origin itself.

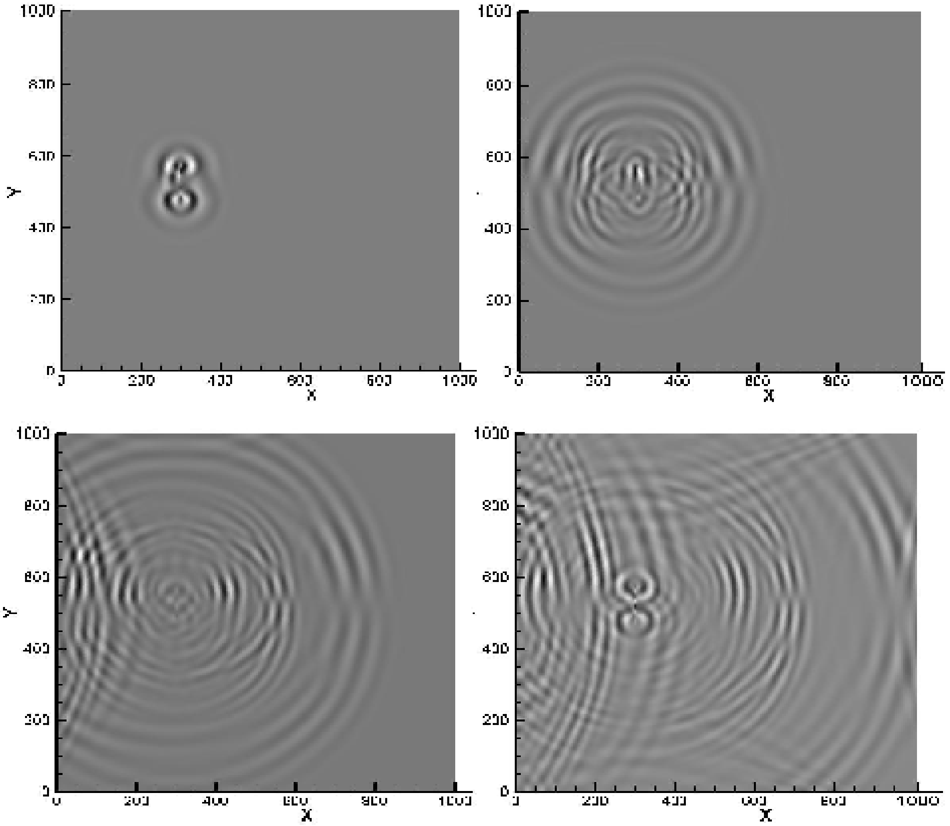


Figure 4. Subtracted horizontal displacement contours of a cracked aluminum plate plotted in four different time steps

In a solid plane, different Lamb wave modes are generated by a general excitation like a concentrated dynamic force, while this problem does not appear in SSL. Each Lamb mode has its own dispersive property, and may be symmetric or asymmetric [34]. Dispersive curves show how a Lamb mode's propagation velocity depends on frequency. This dependency in a thin plate may be written as follows

$$\frac{\tan(qh)}{\tan(ph)}=-\left(\frac{4k^2qp}{(q^2-k^2)^2}\right)^{\pm 1} \tag{6}$$

where

$$p=\frac{w^2}{c_L^2}-k^2 \quad , \quad q=\frac{w^2}{c_T^2}-k^2 \quad , \quad k=\frac{w}{c_p} \quad , \quad c_g=\frac{dw}{dk} \tag{7}$$

in which w denotes the wave circular frequency, c_L and c_T are velocities of longitudinal and transverse modes respectively, c_p and c_g indicate the phase and group velocities, k denotes the wave number, and the plate's thickness is $2h$. The sign ± 1 is used to preserve the application of the equation for both symmetric and asymmetric modes, among which the exponent +1 applies for symmetrical modes while the exponent -1 is for asymmetrical ones [32]. In addition to Lamb wave modes, Love wave (or shear horizontal mode) is produced and propagated in a thin plate excited by a general excitation. In summary, the above discussions may be concluded in two issues: propagation of several wave modes in a plate and dispersion, which is the dependence of wave propagation velocity on frequency.

Signal processing and its interpretation would be much more sophisticated when multiple wave modes exist in signals, especially when these wave fronts are reflected by boundaries, inclusions, and obstacles. To overcome this problem, some approaches such as using special actuators and/or specific methods of excitation have been reported [35]. Moreover, those localization methods that use the first wave front in signals to detect the flaw are much less prone to these complications, although a large amount of information which is available in signals may be left unused in such methods.

When Lamb waves interact with a crack, reflected waves do not maintain the original wave properties. In other words, the frequency content of the reflected waves may not remain unchanged either. Therefore, the propagation velocity of damage induced waves may be different from that of diagnostic waves. The frequency content and other properties of damage induced waves depend on the damage properties. As the damage properties are unknown, the frequency content and subsequently the propagation velocity of damage induced waves would remain unknown. Some researchers have dealt with the selection of a diagnostic signal which minimizes wave dispersal [36]. It is recommended that a narrow bandwidth input signal like a windowed Tone-burst be used as diagnostic signal. Following these instructions, one may expect that dispersion effects are minimal and might be able to pick a range for propagation velocity of damage induced waves. As a result, by using differential signals and appropriate alignment of sensors, both crack's tips may be found, which results in crack configuration. It is worthwhile remarking that crack orientation is not a matter of concern in this research. Moreover, any other shapes than rectangular or square plates may be investigated employing the proposed algorithm.

5. NUMERICAL CALCULATIONS

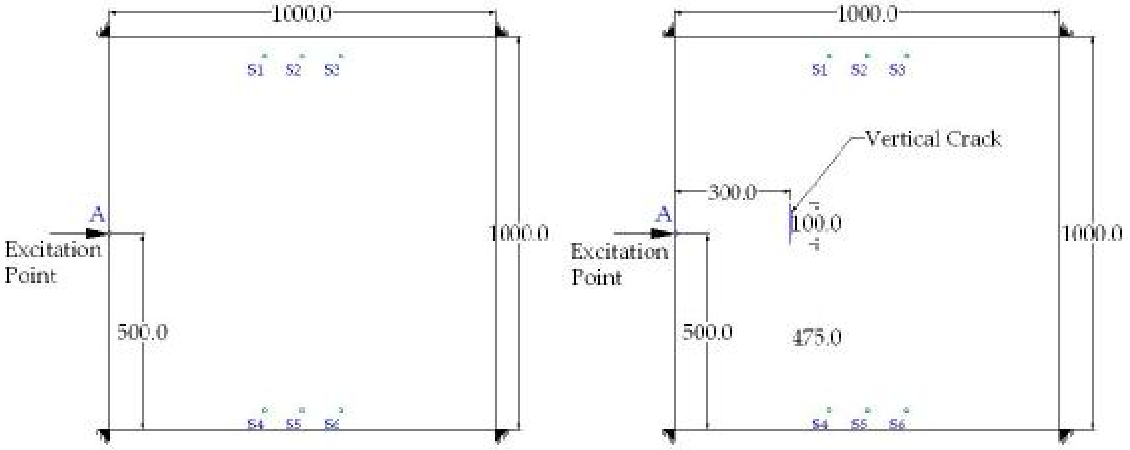
In this section the SFEM based code developed in the first part of the present paper is used for wave propagation analysis in cracked and intact/uncracked plates. Afterwards, the TDOA-based crack detection algorithms of the present second part are used for crack localization. All quantities are measured in SI units. It should be noted that in the first example, the minimum number of sensors have been used and no optimization process has been applied. Damage induced wave velocity has been assumed to be known due to the negligible effect of dispersion. TDOAs have been extracted from normalized signals defining appropriate thresholds. Since there is no noise mixed with signals, TDOAs have been estimated with relatively high precision. In other words the main objective in the first example is to show that TDOA-based crack localization shows encouraging performance even when the number of sensors is minimal and the effect of dispersion is neglected. The only limitation in such situation is that TDOAs must be obtained with high accuracy. In the second example, the wave velocity is considered to be unknown and a rather large sensor array is exploited for crack localization. TDOAs have been extracted from normalized signals using cross correlation method which is a more applicable method although associated with larger estimation errors due to dispersion. To overcome the estimation error of TDOAs, an optimization process is used in this second TDOA-based crack detection algorithm. The main goal in the second example is to show the fact that when TDOAs are expected to contain rough errors, the precision of solution could be guaranteed combining TDOA concept with optimization. The TDOA-Optimization crack detection algorithm is verified in the second example.

5.1 Crack detection in an aluminum square plate

In the first example, a 2D square plate is considered. The Aluminum plate with the geometric configuration of $1000\text{mm}\times 1000\text{mm}\times 1\text{mm}$ is shown in Figure 5. Mechanical properties are assumed to be: Young modulus E = 72.9 GPa, Poisson’s ratio $n = 0.33$ and mass density $r = 2700\,\text{kg/m}^3$. The embedded crack has a length of 100mm and is a through-thickness crack. Sensors are shown in Figure 5 and are labeled by S_1 through S_6. The coordinates of sensor positions are given in Table 1 assuming the bottom left corner of the plate as the origin of coordinates system.

To discretize the plate using SFEs, a regular structured mesh including 40×40 elements of 36-nodes of $25\times 25\ \text{mm}^2$ is employed. This SFE mesh has a total number of degrees of freedom (DOFs) equal to 80802. All four edges of the plate are free while its four vertices are constrained in both directions (see Figure 6). The crack has been modeled in a way that four elements have been aligned along the crack [37].

A concentrated excitation force signal of 100N magnitude has been applied at point A. Figure 8 of the first part of the paper shows the signal in both time and frequency domains. The amplitude of the concentrated force is a Hann windowed five cycle sinusoidal Tone-burst with a central frequency of 100 kHz.

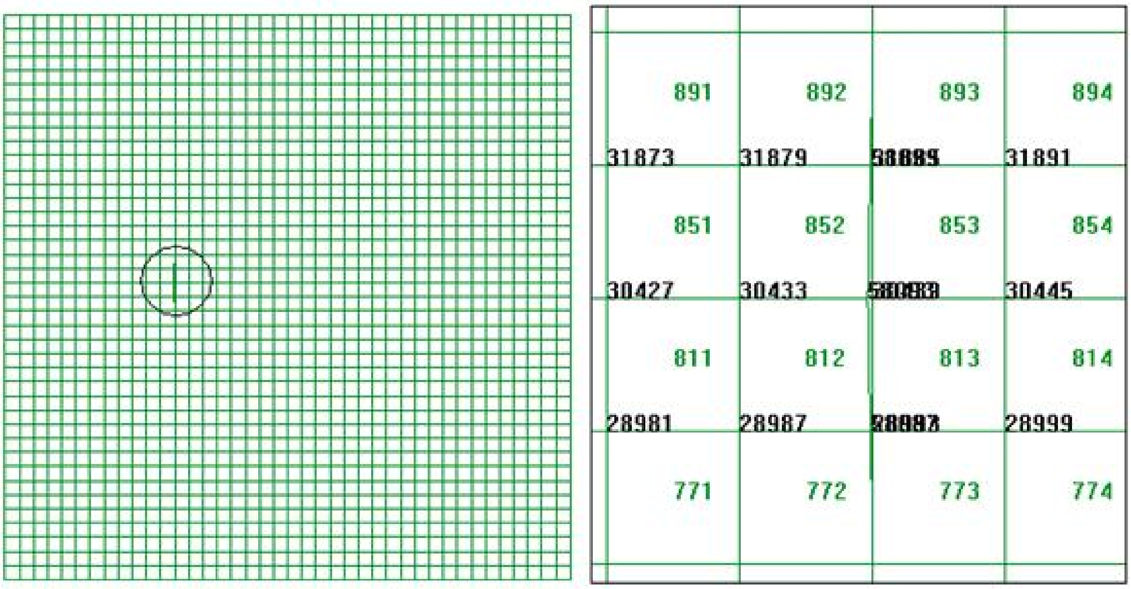

1000.0
S1 S2 S3
A
Excitation Point
500.0
1000.0
S4 S5 S6
1000.0
S1 S2 S3
Vertical Crack
300.0
100.0
A
Excitation Point
500.0
475.0
1000.0
S4 S5 S6

(a) (b)

Figure 5. Geometry, loading, boundary conditions, and sensor alignment of the plate of the first example (length unit is in millimeters): (a) uncracked plate, (b) cracked plate

Table 1: Sensor coordinates of the first example

	S_1	S_2	S_3	S_4	S_5	S_6
X	400	500	600	400	500	600
Y	950	950	950	50	50	50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31873
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31891
851
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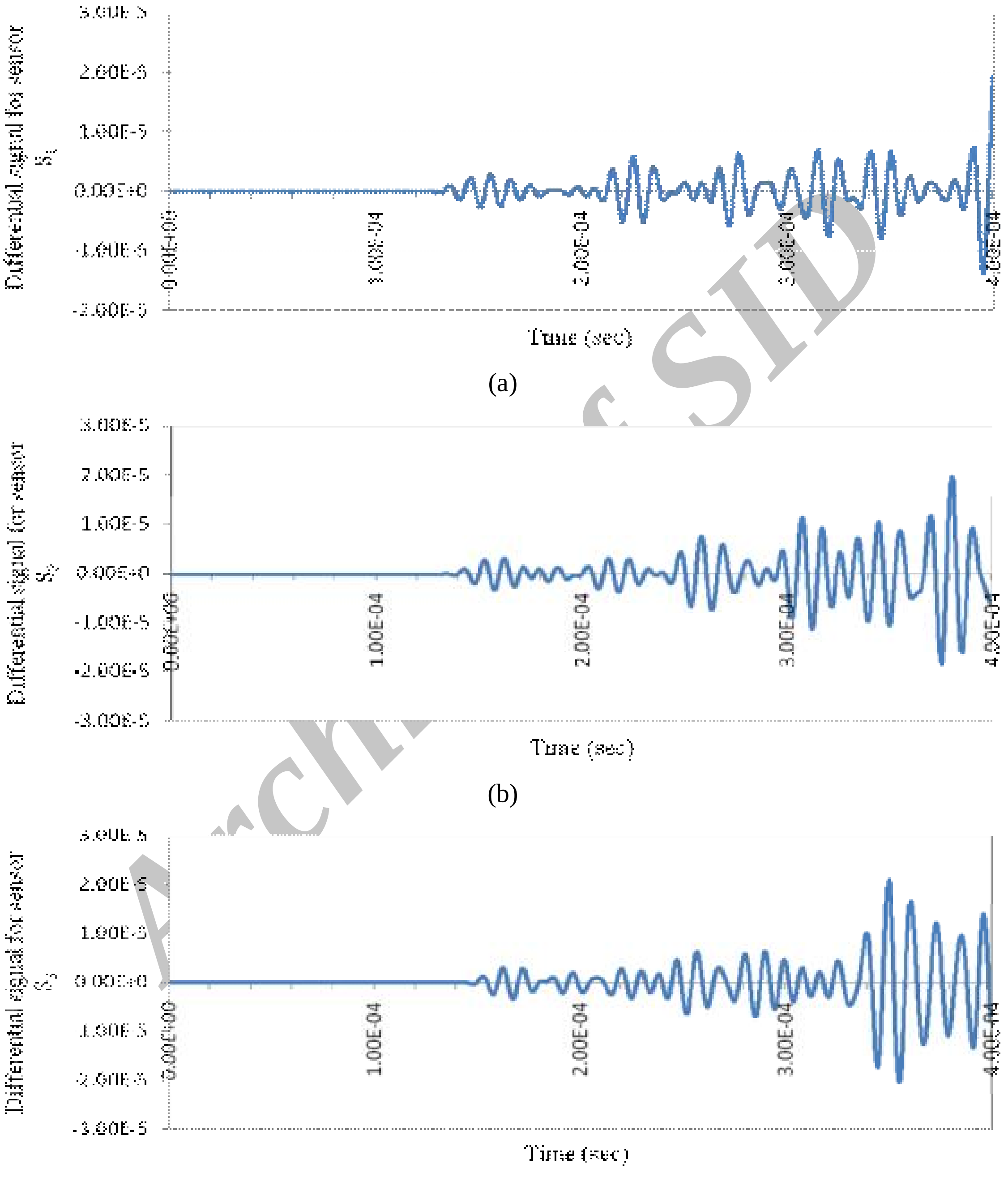
853
854
30427
30433
30445
811
812
813
814
28981
28987
28999
771
772
773
774

(a) (b)

Figure 6. A structured mesh on the cracked plate of the first example shown using GID: (a) square domain with an embedded crack, (b) detailed view of cracked zone

Response signals have been recorded at sensor locations for both uncracked and cracked plates, and then the differential signals have been obtained by subtracting uncracked signals from cracked ones. Differential signals are shown in Figure 7 for the three sensors.



(a)

(b)

(c)

Figure 7. Differential signals for (a) sensor S_1, (b) sensor S_2, and (c) sensor S_3

These signals have been subjected to preparation and normalization first, and then the arrival time of the first wave front which is the S_0 Lamb mode has been extracted by defining an appropriate threshold. Afterwards, TDOAs have been calculated and then four nonlinear TDOA equations have been obtained. For each crack tip, there are two nonlinear TDOA equations. In addition, it is assumed that damage induced wave propagation velocity to be known and equal to the incident wave velocity in this problem. This assumption is not generally valid as discussed before, however it has imposed relatively negligible error in this example due to appropriate selection of diagnostic signal which is expected to minimize wave dispersal. The nonlinear system of equations has been solved using the modified Powell's hybrid algorithm [38] and crack tips have been found as the result. Table 2 shows the results obtained from the nonlinear system of TDOA equations, and Figure 8 depicts the same results graphically.

Table 2: Crack detection results of the first example by solving TDOA nonlinear equations

	Lower crack tip		**Upper crack tip**	
	Actual	**Estimated**	**Actual**	**Estimated**
X	300	293.68	300	310.71
Y	475	477.98	575	589.48

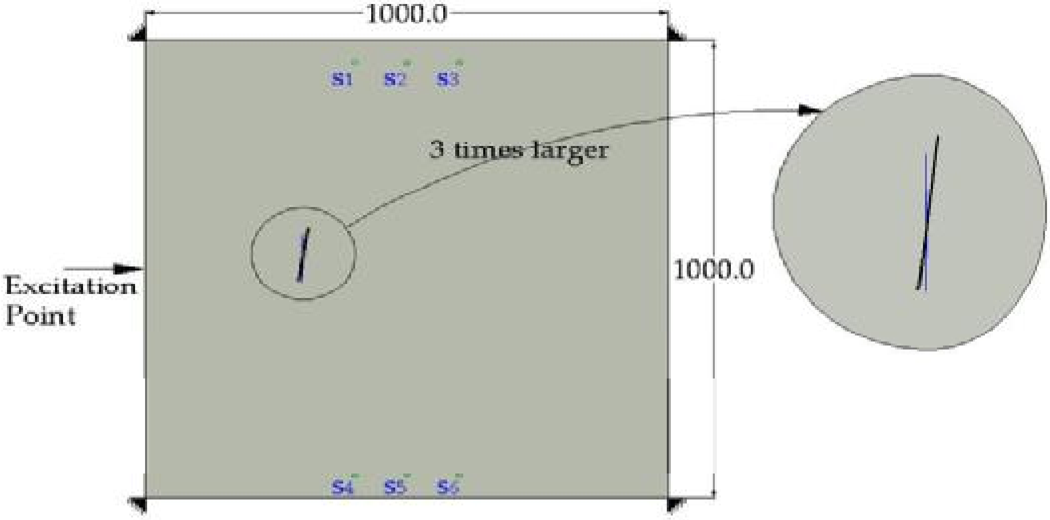


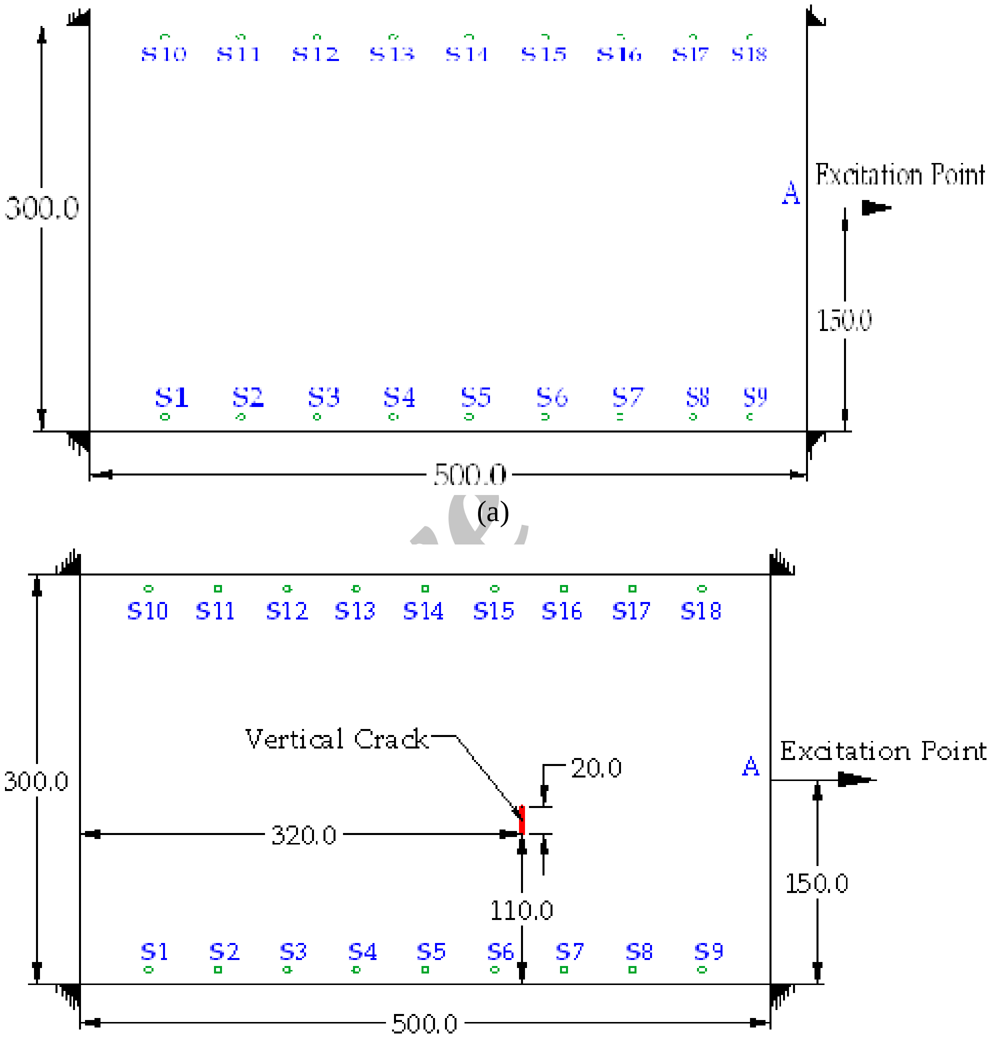
Figure 8. Crack detection result of the first example by solving TDOA nonlinear set of equations. The solid thick black line denotes the prediction, while the solid thin blue line indicates the real existing crack of SFEM model

5.2 Crack detection in a rectangular steel plate

Figure 9a shows the uncracked steel plate which is under investigation in the second example. The plate has a length of 500mm and a width of 300mm and its thickness is 1mm. Mechanical properties are assumed to be: Young modulus E = 200 GPa, Poisson's

of SID

ratio $n = 0.29$, and mass density $r = 7850\,\mathrm{kg/m^3}$. The cracked plate is displayed in Figure 9b. The crack has a length of 20mm and is a through-thickness crack shown by a red line in the figure. Figure 9 also shows the employed sensors labeled by S_1 through S_{18}. The coordinates of sensor positions are given in Table 3, assuming the bottom left corner of the plate as the origin of coordinates system. In this example, signals have been recorded at sensor positions in both uncracked and cracked models and differential signals have been used to calculate TDOAs.


S10
S11
S12
S13
S14
S15
S16
S17
S18
300.0
A
Excitation Point
150.0
S1
S2
S3
S4
S5
S6
S7
S8
S9
500.0
Vertical Crack
20.0
320.0
110.0

(a)

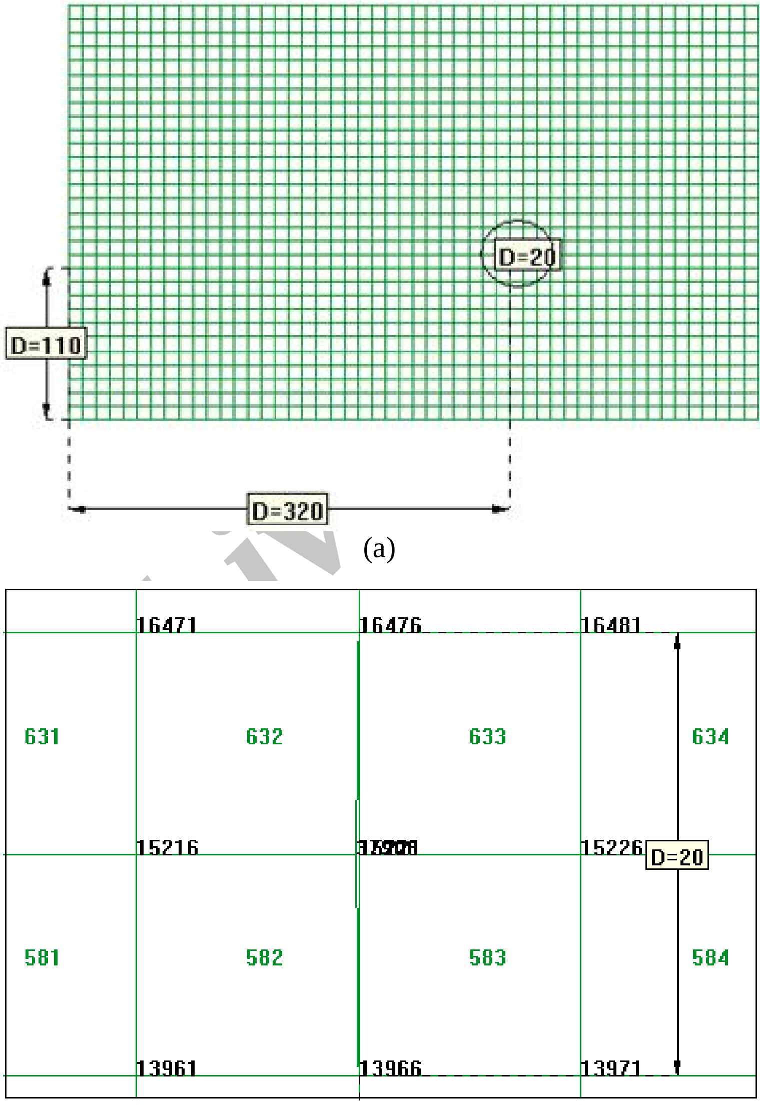
(b)

Figure 9. Geometry, loading, boundary conditions, and sensor alignment of the plate of the second example (length unit is in millimeters): (a) uncracked plate, (b) cracked plate

Both uncracked and cracked plates have been modeled by a structured spectral mesh consisting of 1500 SFEs (see Figure 10a). Each element is a 36-node SFE. In addition, Figure 10b illustrates the crack zone in an enlarged view.

Table 3: Sensor coordinates of the second example

Sensor ID	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**	**S_6**	**S_7**	**S_8**	**S_9**
X	50	100	150	200	250	300	350	400	450
Y	10	10	10	10	10	10	10	10	10
Sensor ID	**S_{10}**	**S_{11}**	**S_{12}**	**S_{13}**	**S_{14}**	**S_{15}**	**S_{16}**	**S_{17}**	**S_{18}**
X	50	100	150	200	250	300	350	400	450
Y	290	290	290	290	290	290	290	290	290

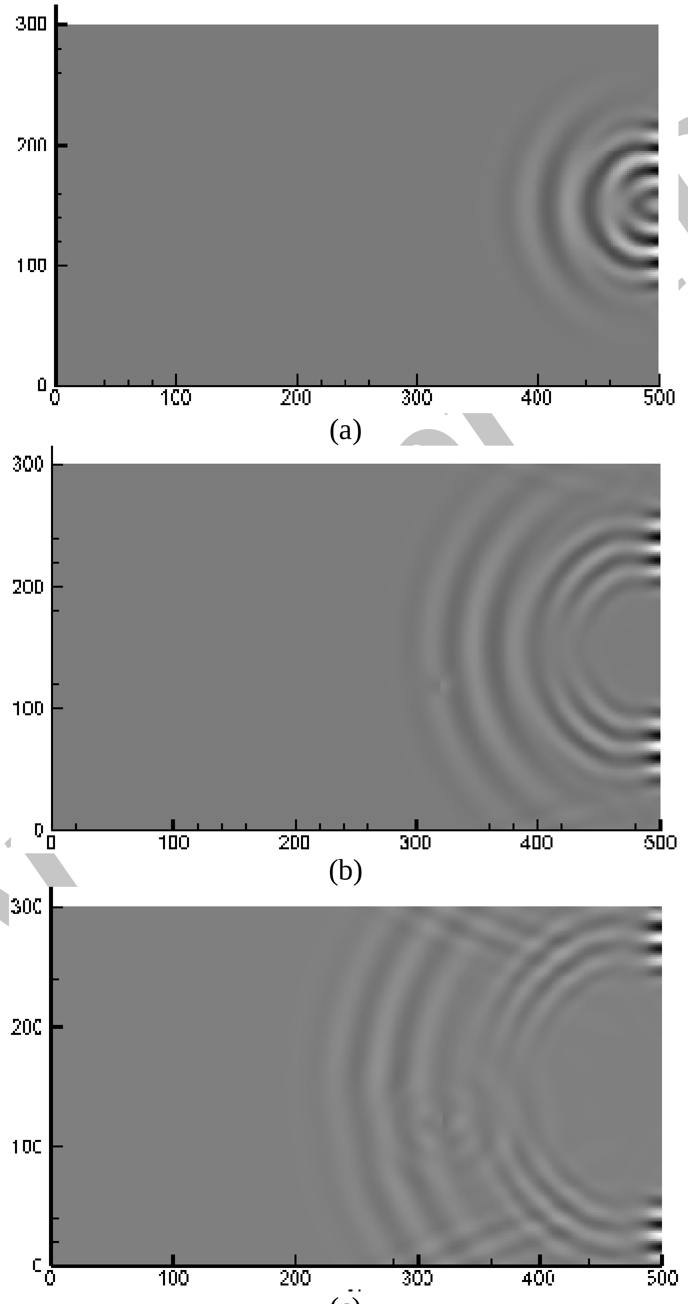

D=20
D=110
D=320
16471
16476
16481
631
632
633
634
15216
15226
D=20
581
582
583
584
13961
13966
13971

(a)

(b)

Figure 10. A structured mesh on the cracked steel plate of the second example depicted using GID: (a) rectangular domain with an embedded crack, (b) detailed view of cracked zone

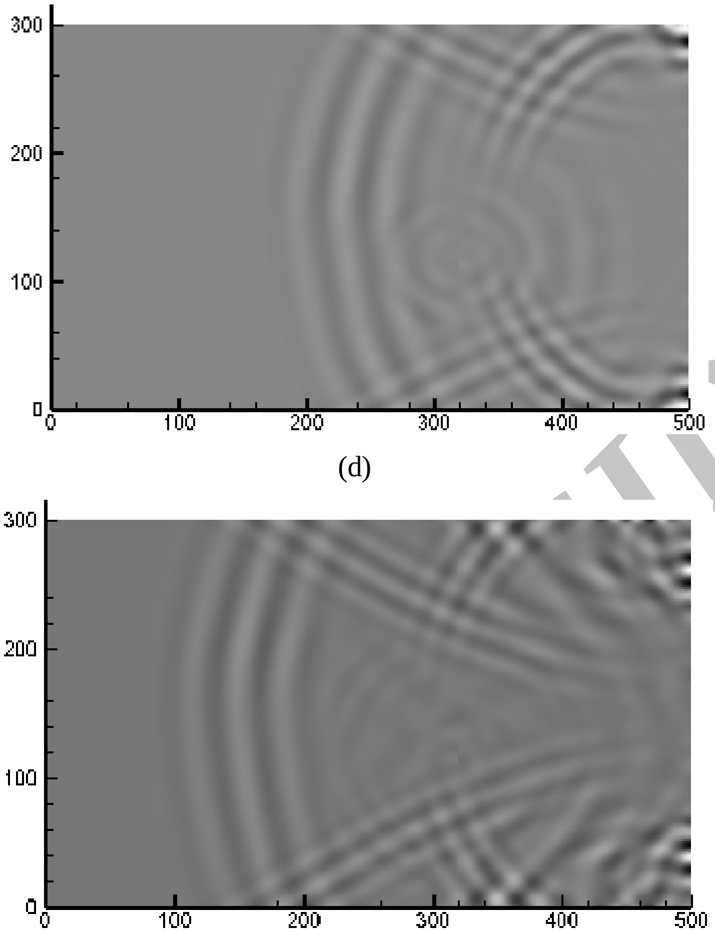
A concentrated force of 1kN magnitude has been applied at point A as shown in Figure 9. The amplitude of the concentrated force is a Hann windowed five cycle sinusoidal Tone-burst with a central frequency of 150 kHz. Figure 11 plots horizontal displacement contours for the entire steel plate in five different time steps, plotted by TECPLOT software. Furthermore, Figure 12 illustrates the differential horizontal displacement contours plotted in the same five different time steps as Figure 11.



(a)

(b)

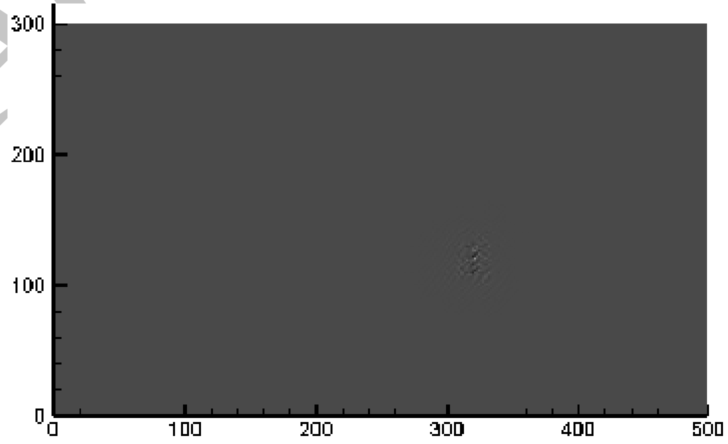
(c)

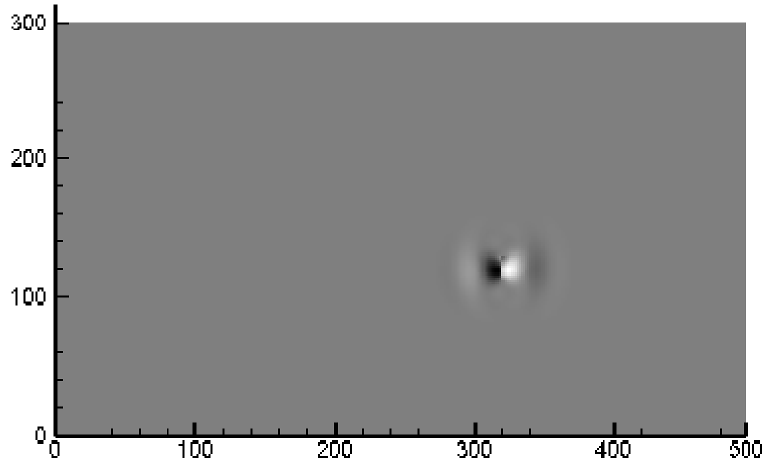


(d)

(e)

Figure 11. Horizontal displacement contours of cracked steel plate drawn in five various time steps of (a) 30 μsec, (b) 45 μsec, (c) 60 μsec, (d) 67.5 μsec, and (e) 82.5 μsec




300
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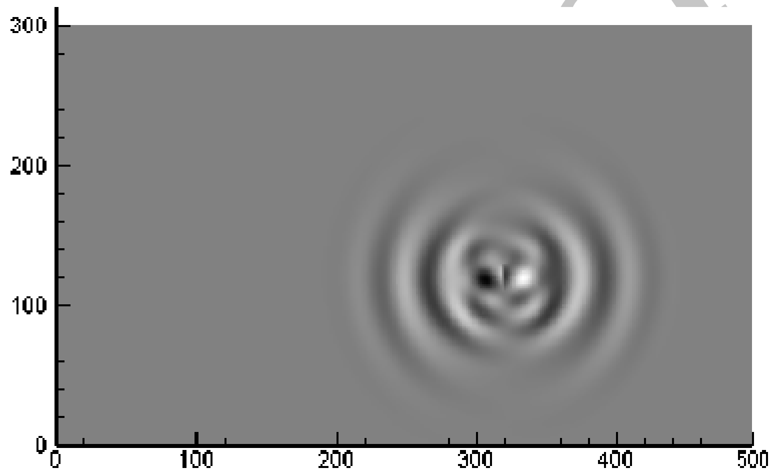
400
500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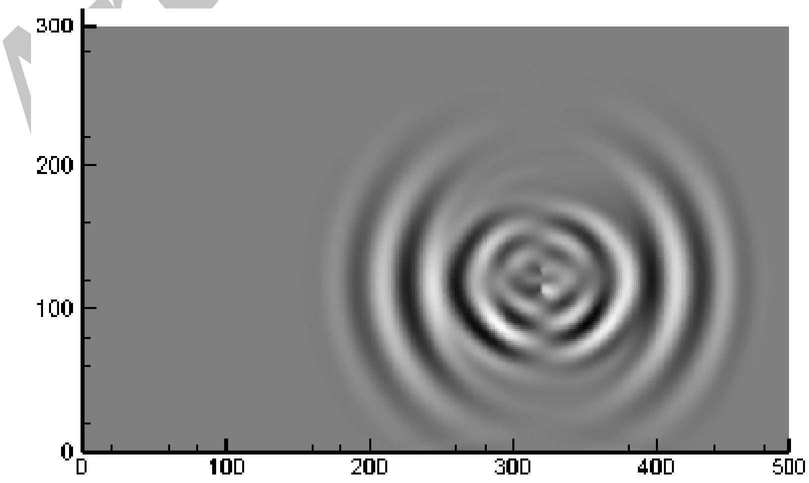
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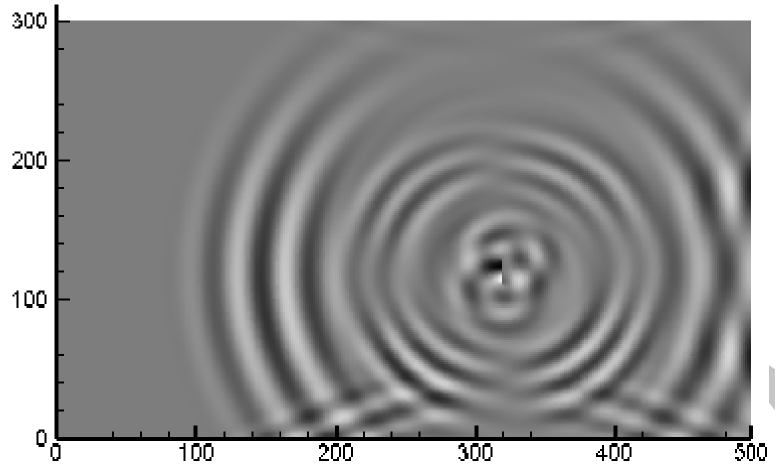

Figure 12. Differential horizontal displacement contours of the cracked steel plate plotted in the same five different time steps as Figure 11

In order to consider more realistic condition in this problem, the velocity of waves scattered by damage has been assumed to be unknown. Consequently, the optimization problem is introduced as given below

$$\text{Cost Function} = \sum_{i=1}^{N_S} \left(T0_i - \frac{\| O - S_i \| - \| O - S_0 \|}{c} \right)^2 \tag{8}$$

subject to

$$0.0 \le x_0 \le 500.0\text{mm},\ 0.0 \le y_0 \le 300.0\text{mm},\ 5000.0 \le c \le 5500.0\text{m/s} \tag{9}$$

in which x_0 and y_0 are the coordinates of the crack tip and c is the damage induced wave velocity. The unknown wave velocity could be restrained as discussed before. The restrain bounds have been selected based on the propagation velocity of S_0 Lamb mode in the central frequency of diagnostic signal. In this problem, the velocity restrain is a bit broad and it can be determined narrower. Differential signals have been normalized first and then TDOAs have been extracted using cross correlation method. The result of this process is reported in Table 4.

Table 4: Extracted TDOAs of the second example using cross correlation method (1E-7 sec)

Sensor ID	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**	**S_6**	**S_7**	**S_8**	**S_9**
Time delay	0	86	169	245	309	345	341	297	231
Sensor ID	**S_{10}**	**S_{11}**	**S_{12}**	**S_{13}**	**S_{14}**	**S_{15}**	**S_{16}**	**S_{17}**	**S_{18}**
Time delay	0	78	151	212	260	285	283	252	202

Estimated TDOAs have been used in the cost function and the optimization problem has then been solved using modified Levenberg-Marquardt algorithm [38]. The results of optimization process are shown in Table 5. Moreover, the obtained results are graphically illustrated in Figure 13 where the crack zone is enlarged in another window. As it is indicated in this figure, estimated crack (dotted green line) is almost identical to the actual crack (solid red line) and the estimation error is very small due to the relatively large number of sensors. To get a better sense on what TDOAs mean, TDOA hyperbolas are also plotted in Figure 13. Light brown points are the optimal answers of the optimization process which produce the minimum errors and minimize cost function due to error criteria of Eq. (3).

Table 5: Crack detection results of the second example obtained by optimization process

	Estimated		**Actual**	
	Upper crack tip	**Lower crack tip**	**Upper crack tip**	**Lower crack tip**
X (mm)	320.281	320.066	320.000	320.000
Y (mm)	129.066	112.126	130.000	110.000
c (m/s)	5344.070	5349.193	-	-

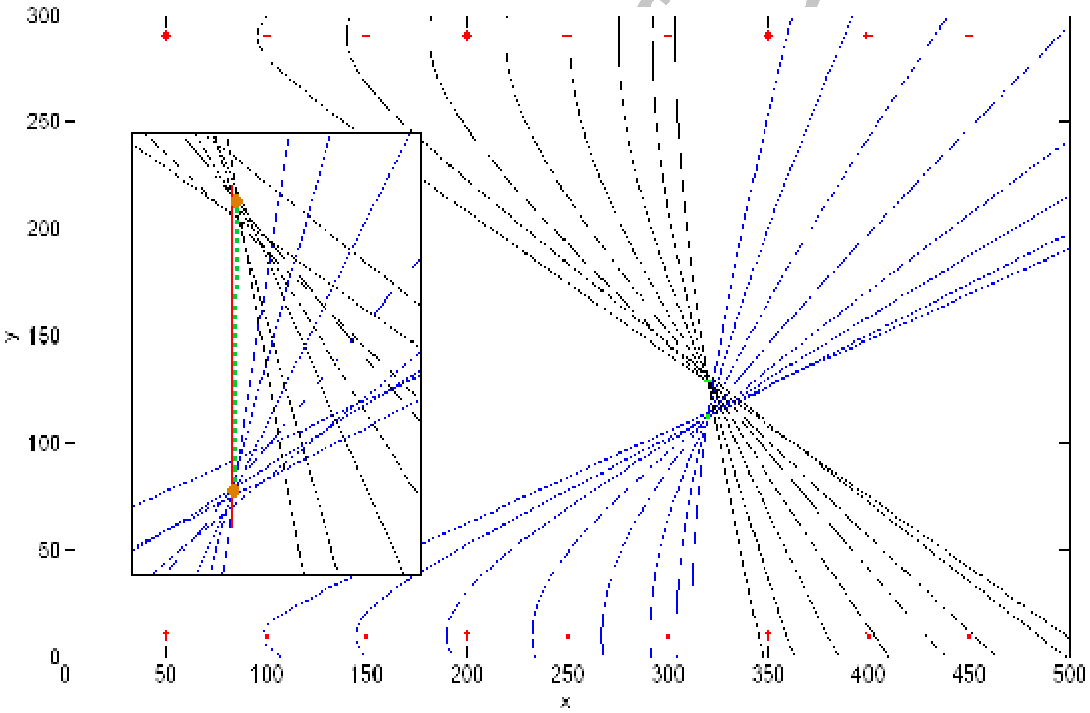

Figure 13. Estimation of crack location using the TDOA-Optimization scheme

6. DISCUSSIONS AND CONCLUDING REMARKS

In this paper, detailed concept of TDOA was introduced as a tool for solving the inverse problem of crack detection in isotropic, homogeneous elastic plates, based on the application of results of the first part of the present paper. In this regard, two TDOA-based crack detection algorithms were represented and exploited. The first algorithm is according to solving the nonlinear system of TDOA equations. The solution of each set of nonlinear equations is a crack tip. This algorithm is very fast and economic because the number of sensors is minimal and the total computation cost in this method is just the process of solving nonlinear system of equations. However, this method requires high precision TDOAs as mentioned before. Since obtaining accurate TDOAs is not possible in every circumstance, the second algorithm is proposed to overcome this shortcoming. The second algorithm is more time consuming than the first algorithm as a larger sensor array must be used and the solution is the outcome of an optimization process. However, the speed of the second algorithm is still suitable for online crack detection purposes because a classical optimization scheme could be used. Both crack detection algorithms were successful in localization of the crack as shown by the results of the two examples; however, the TDOA-optimization method revealed better results and reliability. Moreover, it was indicated that TDOA-based crack detection algorithms as Lamb wave based crack detection methods could be extremely sensitive to the damage location and size and they can produce excellent results provided that TDOAs are calculated exactly enough.

It should be noticed that the proposed algorithms may precisely locate the crack, requiring rather small amount of inputs. The methods do not depend on the geometric properties of the domain, and may be formulated for 3D anisotropic and inhomogeneous media. Moreover, the orientation of the crack is not a limitation and the solution is obtained fast enough to introduce these algorithms as online crack detection algorithms. As online structural health monitoring techniques, these algorithms do not require large computation costs, and the domain does not need to be structurally analyzed more than once (in spite of lots of other algorithms especially those which work based on artificial intelligence). The main shortcoming of the present approach is the estimation error of TDOAs. If TDOAs are not obtained accurately enough, this approach might lead to significant errors and ultimately fail. In this research, the error of the solution due to damaged TDOAs was controlled combining the concept of TDOA with optimization.

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