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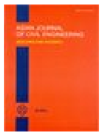

AN INTRODUCTION TO PERFORMANCE CONTROL FOR MOMENT FRAMES OF UNIFORM RESPONSE UNDER LATERAL LOADING

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ABSTRACT

The paper offers an exact, closed form solution for performance-based elastic-plastic design of moment frames under lateral loading. It introduces the concept of uniform response, which in turn, enables the engineer to manually define, predict and to control structural response at pre-selected design stages, such as before and at first yield, any fraction of the failure load or specified drift ratio, up to and including incipient plastic collapse. It is assumed that the moment frames are composed of imaginary, symmetric, rectangular modules that are stacked on top of each other to form vertical subframes of uniform response where individual modules are designed to deform identically and to develop internal stresses of equal magnitude simultaneously throughout the subframe. These subframes respond as structures of uniform strength and stiffness in which members of the same group such as beams and columns, share the same demand-capacity ratios regardless of their location and number of similar elements within the framework. The subframes are eventually integrated to reconstruct the original system. The proposed solutions are *unique* since they satisfy the prescribed yield criteria, static equilibrium as well as the boundary support conditions. While moment frames of uniform response are ideally suited for preliminary performance control studies, the method is further enhanced by the introduction of moment control factors. These factors are used to control the propagation of plasticity within the structure and symbolize the formation or elimination of plastic hinges as needed. In practical terms, cover plates, reduced beam sections or similar technologies may be used to avoid and/or to induce formation of plastic hinges in selected locations. The proposed *drift control* and *moment modification* equations appear to be the only ones of their kind that can analytically estimate the lateral displacements and element moments of such frames, including the P-delta effects, throughout the history of loading of the structure.

Keywords: closed form; moment frames; plastic design; minimum weight; earthquakes; drift control

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1. INTRODUCTION

Performance control (PC) is a modest step towards fulfilling the recommendations of the now famous Vision 2000 document [1] and ASCE 7-05 [2], which in effect, defines performance based engineering as the "*selection of design criteria and structural systems….such that at specified levels of ground motion,….the structure will not be damaged beyond certain limiting states*." While there are no explicit definitions for seismic structural damage and its effects on the performance of the remaining undamaged system, the consensus is that regardless of extent of damage, the framing system should sustain some level of life safety, for a reasonable period of time, before repairs or abandonment. However, it is generally agreed that the overall performance of a ductile structure can be judged based on three controllable performance levels: *immediate occupancy performance level* usually associated with first yield, *life safety performance level* associated with structural behavior after first yield and prior to incipient collapse, and *collapse prevention performance level*, when the system is on the verge of partial or total failure. A survey of the current consensus, e.g. Bozorgnia Y, and V.V. Bertero, [3] and Naeim Farzad [4], indicates that sustaining life safety may be based upon the verification and/or implementation of the following design conditions that:

- The assumed magnitude and distribution of design loads do not underrate the effects of the actual seismic forces on the deteriorating structural system.
- The relative side-sway or inter-story drift of each and every level of the framework is limited to an absolute maximum below which neither local nor partial instability may cause catastrophic failure.
- The degradation of the overall stiffness of the structure, due to formation of plastic hinges, the racking P-delta effects and the redistribution of moments do not compromise the general stability of the system.
- The postulated ductility limits and the yield criteria are not violated.
- The Strong column-weak beam condition is observed throughout the history of loading of the structure.
- The pre-determined sequence and patterns of formation of plastic hinges are enforced, and that boundary support conditions and static equilibrium are satisfied.

In short, the solution of the problem reduces to rational selection of the strength and stiffness or capacity and drift limits respectively of the constituent modules of the system with respect to predetermined control points. In the absence of commonly accepted definitions for lateral structural damage, *allowable stress design* or *load and resistance factor design* limits may be looked upon as first stage damage conditions. However, for the purposes of this article, damage states are looked upon as pre-determined PC points, tentatively grouped into three categories, i.e.

1.1 Linear or elastic range damage

- Defined by exceedance of any pre-assigned or code specified drift limit.
- Exceedance of code level stresses of any member before and including first yield.
- Exceedance of elastic stability limits for any member or group of members.

1.2 Non-linear or post-elastic stages of damage

- Defined by any state of stable equilibrium, within the ductile range of displacements, excluding incipient collapse i.e., short of becoming an active failure mechanism.

1.3 Incipient Collapse

- Defined by the tendency of the structure to become a mechanism, leading to plastic collapse, or to become totally unstable due to partial or complete degradation of the global stiffness.

These life safety and PC conditions suggest that actual structural response should be looked upon as a function of design and construction, rather than theoretical analysis, whence, both strength and stiffness should be induced rather than extracted from numerical investigations. With this in mind, the following modeling concept that captures the characteristic behavior of a single bay, multistory moment frame and its constituent imaginary modules is briefly presented in the next section. The success of the proposed concept may be attributed to the simplifying nature of system transformation into equivalent subframes, [5, 6] and an appreciation of the response of such modules under lateral loading. Perhaps the most appealing aspect of the proposed methodology is that:

- It partially fulfils the theoretical recommendations of the Vision 2000 document.
- It lends itself well to manual and spreadsheet computations.
- It results in minimum weight structures of uniform response (UR).
- It extends the use of UR methodologies to fine-tuned PC.
- Its use can be extended to other types of lateral resisting systems such as braced frames, special truss moment frames, shear walls, hybrid structures etc.

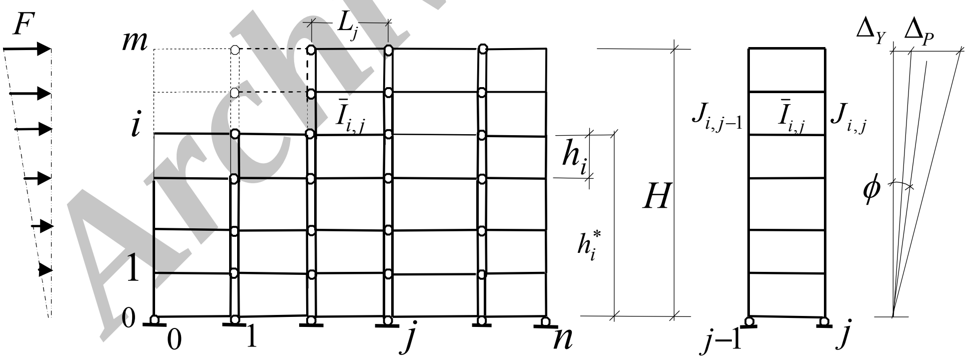
The proposed methodology as presented in this article is based upon two simple but interrelated theoretical developments. The first part consisting of sections 2, 3 and 4 below introduces the concept of *lateral resisting moment frames of UR.* Structures of UR are ideally suited for minimum weight/optimal drift design purposes and lend themselves well to PC treatment throughout the loading history of the frame. The second part consisting of sections 5 and 6 deals with the development of *moment control/modification factors* (MCF) that may be used to fine-tune the PC of the subject frames of UR.

2. MODELING CONCEPT

2.1 Subframe analysis

The theoretical procedure leading to the establishment of predetermined PC stages incorporates the following steps and assumptions: First, in order to simplify the mathematical formulation of the problem, the physical model of a regular $m \times n$ moment frame, depicted in Figure 1b, has been replaced with its own equivalent counterparts, composed of n individual, contiguous, imaginary vertical subframes, such as that shown in Figure 1c, stacked next to each other and hinge connected at their common joints ij, where H is the total structure height, h_i and h_i^* are the i^{th} level story height and story level height from the base respectively. L_j is the span of the j^{th} subframe. J_i and $\bar{I}_{i,j}$ are the i^{th} level column and beam section inertias respectively, of the j^{th} bay subframe. Obviously, the original structure can be reconstructed mentally by merging the imaginary subframes and eliminating the hypothetical hinges. The solution is simplified further by assuming that the structure is enclosed within a virtual rectangle consisting of *m* vertical and *n* horizontal, divisions. The broken lines represent non-existing elements that do not contribute to the overall strength or stiffness of the system. In practical terms, the non-existent members are treated as having zero rigidity or strength, and are signified symbolically by $\delta_{i,j}^{P} = 0$. Integers $j = 0, 1, 2 \ldots n$, and $i = 0, 1, 2 \ldots m$, are non-dimensional coordinates introduced to identify the elements of the structure.

The equivalent frameworks used in this work are not unique, but are the most suitable for the purposes of the present discourse, in the sense that there are other affine frames that eventually lead to the same results, such as geometrically similar horizontal subframes stacked on top of each other [7,8,9]. Perhaps the main advantage of the vertical affine system over its horizontal counterpart is that it facilitates the study of the propagation of plasticity in the vertical direction without resorting to major mathematical complications [10]. The design lateral loading with maximum intensity *F* is represented by the triangular distribution of forces of Figure 1a, where it has been assumed, that the distribution of the loading is independent of the fundamental period of vibrations of the structure. However, since the first natural mode of vibrations of the system controls the distribution of seismic forces, it follows that the lateral displacement profile of the structure should closely resemble the same shape as that of the induced loading. Figure 1d manifests the assertion that relative inter-story sway of each and every level of the framework is confined to the same maximum drift ratio ϕ pre-selected as part of the control process. Three standard boundary support conditions have been envisaged in connection with the rotationally flexible supports provided by the grade beams of the base level modules of Figure 1b below.



(1a) (1b) (1c) (1d)

Figure 1. Moment frame and imaginary vertical sub-frames

The effects of hinged and fully fixed boundary support conditions are discussed later in this paper. The basic function of the grade beams is to control ground level column base rotations and to prevent the formation of plastic hinges at the base of the columns.

2.2 Elastic-plastic response of basic modules

Imaginary/Basic Modules as constituent parts of generalized moment frames have been studied amongst others by the authors [5, 6, 11] to investigate the statics, stability, dynamics and the plastic failure of moment frames under lateral loading. Similar ideas are employed here to study the effects of progressive plasticity, stiffness degradation and use of moment control technologies with a view towards damage control in vertical subframes. Perhaps the most significant aspect of imaginary or representative modules is that they fully describe the response of entire structures of UR without resorting to complicated structural analysis. The role of the imaginary module in describing the response of a regular moment frame is analogous to the functions of the small element $\partial x.\partial y$ devised to develop the constitutive equations of plane-stress problems, e.g. bending of beams. The single bay, vertical subframe of Figure 1c may be construed as being composed of a number of imaginary, symmetrical, rectangular moment frames of equal widths stacked on top of each other and hinge connected at their common joints *(i, j)* and *(i, j-1)*. Figure 2a depicts one such module, where $I_{i,j}$ and $J_{i,j}$ represent the section inertias of its beams and columns respectively. Similarly $M_{i,j}^{P}$ and $N_{i,j}^{P}$ stand for the plastic moments of resistance of the beams and columns of the same module. Obviously when neighboring modules are merged to reconstruct the subframes, their common virtual hinges vanish and the moments of inertia of the beams of the reassembled structure become; $\bar{I}_{i,j} = I_{i,j} + I_{i+1,j}$. Therefore, the roof and grade level beam moments of inertia become $\bar{I}_{m,j} = I_{m,j}$ and $\bar{I}_{0,j} = I_{0,j}$ respectively. By the same token, the plastic moment of resistance of two merging beams becomes; $M_{B,i,j}^{P} = M_{i+1,j}^{P} + M_{i,j}^{P}$.

The flexural performance of a representative idealized module, such as that shown in Figure 2a, is primarily influenced by its geometric and material characteristics, the shear force V_i, beam loading $W_{i,j}/2$ and axial forces $P_{i,j}/2$. An idealized module in the present context is that in which plastic hinges occur at beam-column intersections with *zero offsets* from column center line. In the elastic range, and in the absence of floor loads *W*, the relationship between drift ϕ_Y, corner moments M_Y and shear force V_Y for an idealized module, at first yield can be expressed as;

$$\phi_Y = \frac{V_Y h}{24E}\left[\frac{h}{J} + \frac{L}{I}\right]\frac{1}{(1-P/P_{cr})} = \frac{V_Y}{Khf_{cr}} \text{ and } M_Y = \frac{V_Y h}{4 f_{cr}} \tag{1}$$

respectively, [6,7,8], where E, K and $P_{cr} = Kh$ are the modulus of elasticity, the stiffness, and the critical load, of the idealized module respectively. $f_{cr} = (1 - P/P_{cr})$, is the load magnifying factor generated by the P-delta effect. Drift angle ϕ_Y, and maximum moments M_Y together define the response of the module to external stimuli V_Y and *P* and may therefore be considered as the two most important control parameters of the system.

2.3 Plastic response of basic modules

While the bending effects of floor loads *W* have little or no direct influence on the side-sway and plastic moment of resistance of the module, the axial effects of P and W tend to magnify both the lateral displacements as well as the moments generated by the lateral shear force acting on the system.

However, for all practical purposes, including the use of such moment control technologies as reduced beam sections (RBS), slotted web connections (SWC), added cover plates (ACP), etc., the effects of reduced hinge span $\overline{L} < L$ should be taken into consideration. The reduced values of the hinge spans force the plastic hinges to be formed a small distance *D/2* or $a = (L - \overline{L})/2$ away from the center line of the columns, as depicted in Figures (4a) and (4c). As a result, the plastic beam rotation $\overline{\phi} = (L/\overline{L})\phi = \phi/\varepsilon$ becomes larger than the corresponding drift ϕ of the module. By the same token the plastic moment of resistance at the face of the column becomes $\overline{M}^P = \xi \times M^P$. The ratio $\xi = (L - d_C)/(L - a)$ may be referred to as the MCF. Yield stress over-strength and strain hardening effects are not included in ξ. These conditions affect the response of the module as well as the entire structure. It is assumed that the strong column weak beam condition $N^P > \lambda\xi M^P$ is observed for all values of ξ. In reality ξ is always larger than unity. $\lambda \geq 1.0$ is the plastic over-strength factor of the columns.

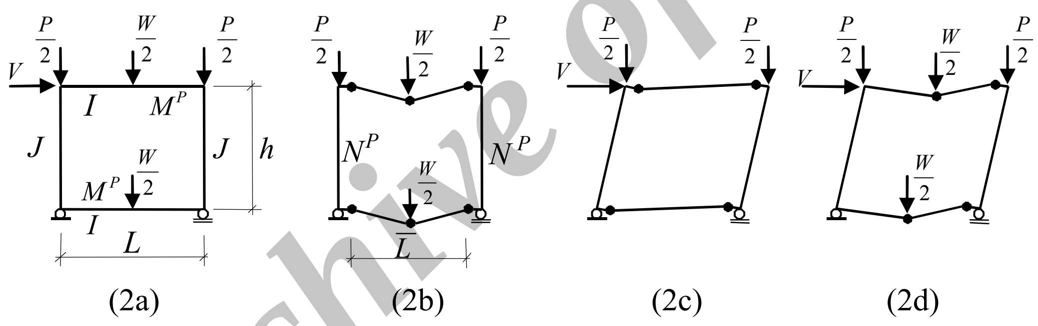


Figure 2. Basic module and plastic failure mechanisms

An important characteristic of modules of UR, [11], is that the weight of each constituent module is a minimum with respect to the demand imposed upon it. This implies that the subject modules may be more flexible than their regular counterparts. Since these modules are expected to sustain relatively large inelastic displacements during major earthquakes it becomes crucial to control their global as well as member instabilities at large axial loads. The softening or loss of stiffness of such modules may be evaluated, to a high degree of accuracy by the inclusion of the P-delta effects in their plastic collapse computations presented below. Using the principle of virtual work, the combined plastic interaction equations of all three modes of plastic collapse may be expressed as;

$$M^P = \frac{\varepsilon}{4(1 + \delta_P^W \delta_P^V \varepsilon)} \left[\frac{V_P h}{f_{cr}} + \frac{W_P L}{2} \right] \tag{2}$$

where, $\varepsilon = \overline{L}/L$, $\delta_P^W = 0$ for $W=0$, and $\delta_P^W = 1$ for $W \neq 0$. Similarly, $\delta_P^V = 0$ for $V=0$, and $\delta_P^V = 1$ for $V \neq 0$. Index *"P"* is meant to relate the quantity to plastic collapse. M^P and N^P are the plastic moments of resistance of the beams and columns of the module respectively. The *P-W-V* interaction Eq. (2) may be expanded to give;

For mode (2b) $\delta_P^W = 1, \delta_P^V = 0, 2a \geq D, V=0$ and $\xi > \frac{L+2a}{L-2a}$

$$M^P = \frac{W_P \varepsilon L}{16} \tag{2a}$$

For mode (2c) $\delta_P^W = 0, \delta_P^V = 1, 2a \geq D, \xi > \frac{L}{L-2a}$ and $W_P \leq \frac{16M^P}{\varepsilon L}$

$$M^P = \frac{\varepsilon V_P h}{4 f_{cr}} \tag{2b}$$

For mode (2d) $\delta_P^W = 1, \delta_P^V = 1, \xi > \frac{L}{L-2a}$ and $V_P \leq \frac{4M^P f_{cr}}{\varepsilon h}$

$$M^P = \frac{\varepsilon L}{8(L-a)}\left[\frac{V_P h}{f_{cr}} + \frac{W_P L}{2}\right] \tag{2c}$$

The rationale pertaining to the mode of inclusion of the P-delta effect, [12], in Eqs.(1) and (2) is presented in the Appendix. The complete solution to Eq. (2), for an idealized module, i.e. $\varepsilon = 1$, in terms of the relative values of forces P, W and V is presented in Figure 3 below. Eqs. (2a), (2b) and (2c) illustrate the interactive effects of loads P and W on the lateral carrying capacity, V, of a typical module as well as the integrated structure. It is clear that while small floor loads $W \leq 8M^P/L$ have no effects on the lateral carrying capacity of the module, axial loads P tend to reduce its efficiency linearly from full carrying capacity to zero at $P = P_{cr}$.

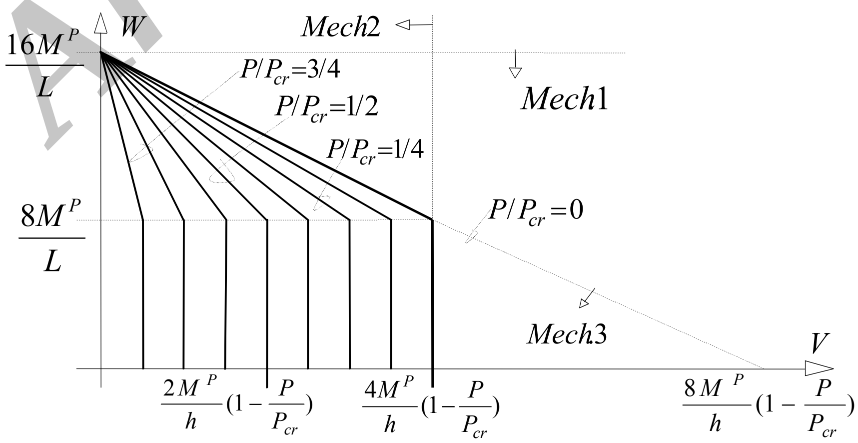


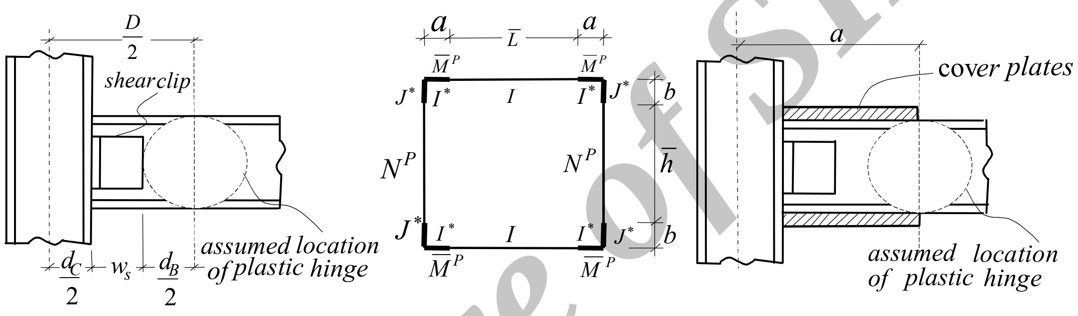
Figure 3. Effects of *P-V-W* interaction on the carrying capacity of a basic module

2.4 Generalization of failure mode 2

The module load displacement relationship, Eq. (1), may now be rewritten in terms of beam and column offset parameters to represent the plastic drift ratio ϕ_P corresponding to incipient collapse mode (2c) or Figure 4c as;

$$\phi_P = \frac{M^P}{6\bar{f}_{cr}E}\left\{\left[\frac{\tau h}{J}+\frac{\varepsilon \bar{L}}{I}\right]+\left[(2+\varepsilon)\xi+2\varepsilon+\eta\right](L-\bar{L})(\frac{1}{2I^*})+[(2+\tau)\gamma+2\tau+\rho](h-\bar{h})(\frac{1}{2J^*})\right\} \quad (3)$$

where, I^*, J^* and $\bar{f}_{cr}$ are the moments of inertia of the modified sections (offset section) of the beams and columns and the modified load reduction factors respectively. The symbols τ, γ and ρ represent the same ratios for the columns as ε, ξ and η do respectively, for the beams.



(4a) (4b) (4c)

Figure 4. Module offset and assumed position of plastic hinges

Since the length of the modified section is very small compared with the span of the beam, an approximation is suggested that $I^*/I \approx \bar{M}^P/M^P = \xi$ and $I^*/I \approx \bar{N}^P/N^P = \gamma$. Furthermore, if a design decision is made that $J = \mu \times I$, then Eq. (3) results directly in the control quantity I_P at incipient collapse, i.e.

$$I_P = \frac{M^P}{6\bar{f}_{cr}E\phi_P}\left\{\left[\frac{\tau h}{\mu}+\varepsilon \bar{L}\right]+\left[(2+\varepsilon)\xi+2\varepsilon+\eta\right](L-\bar{L})(\frac{I}{2I^*})+[(2+\tau)\gamma+2\tau+\rho](h-\bar{h})(\frac{I}{2J^*})\right\} \quad (4)$$

where, $\eta = (L-d_C)/L$ and M^P is that defined by Eq. (2b) above. A discussion of the effects of variations of ε, τ, ξ and γ on the response of a basic module and therefore an entire structure of UR is presented in the forthcoming sections of this article. Eqs. (2), (3) and (4), together describe completely the elastic-plastic response of the basic module, with offset hinges, throughout its loading history. This information is utilized to better understand and develop the response of vertical subframes under similar loading as presented in the following section.

3. DEVELOPMENT OF VERTICAL SUBFRAMES OF UR

The rationale leading to the development of subframes of UR is instinctively simple-select the relative strength and stiffness of the individual modules in such a way as to induce uniform drift and demand/capacity ratios for all modules of the system. Equal drifts result in inter-modular compatibility at their common joints as well as desirable side-way displacement profiles. Uniform demand/capacity implies providing as much capacity as demand imposed on or attracted by each individual module, in other words, allocating just enough material where it is needed. The basic assumption made here is that $f_{cr.i} = f_{cr}$, and that the effects of hinge offsets can be temporarily ignored without loss of generality. Next, defining the story level shear and raking moments as v_i and $M_i = v_i h_i$ respectively, and comparing the drift equations of two neighboring modules at levels i and $i+1$, it gives:

$$\phi_i = \frac{M_i}{24 f_{cr.i} E}\left[\frac{h_i}{J_i} + \frac{L}{I_i}\right] \text{ and, } \phi_{i+1} = \frac{M_{i+1}}{24 f_{cr.i+1} E}\left[\frac{h_{i+1}}{J_{i+1}} + \frac{L}{I_{i+1}}\right], \text{ respectively.}$$

Now for the condition of uniform drift $\phi_i = \phi_{i+1} = \phi$ to hold, the following rules of proportionality should be observed:

$$\frac{J_i}{J_{i+1}} = \left(\frac{M_i}{M_{i+1}}\right)\left(\frac{h_i}{h_{i+1}}\right), \quad \frac{I_i}{I_{i+1}} = \left(\frac{M_i}{M_{i+1}}\right), \quad \frac{M_i}{M_{i+1}} = \left(\frac{v_i h_i}{v_{i+1} h_{i+1}}\right) \tag{5a}$$

$$\frac{M_i^P}{M_{i+1}^P} = \left(\frac{v_i h_i}{v_{i+1} h_{i+1}}\right) \text{ and } \frac{N_i^P}{N_{i+1}^P} = \left(\frac{v_i h_i}{v_{i+1} h_{i+1}}\right) \tag{5b}$$

The applications of the rules of proportionality are demonstrated in section 6.2 below.

3.1 Closed form solution for subframes of UR under lateral loading

Because of symmetry and imposition of a straight line drift profile, as in Figure (1d), points of inflexion are forced to occur at mid points of all beams and columns, rendering the structure statically determinate and amenable to manual computations and closed form treatment.

Consider the response of an m story subframe, such as that shown in Figure 1c. Assuming that $h_i = h$, $I_m = I$ and $J_{m,j} = J_{m,j-1} = J$, subjected to a triangular distribution of lateral forces of maximum value f, then from static equilibrium of the roof level module $fh = 4 f_{cr} M_{B,m}$. $M_{B,m} = fh/4 f_{cr}$ signifies the beam moments at level m for all types of lateral load distribution with apex value f. As an important characteristic of structures of UR, as f and $M_{B,m}$ reach their maximum values f_P and $M_{B.m}^P = M^P$ respectively at plastic collapse, their relationship $M^P = f_P h\varepsilon / 4 f_{cr}$, also describes the collapse load of the entire system. The complete closed form, lower bound, solution of the subject subframe of UR under triangular distribution of lateral forces is presented as follows;

$$f_i = f(i/m). \quad i^{th} \text{ Level lateral force distribution} \tag{6a}$$

$$v_i = f\left[(m+1)-(i/m)(i-1)\right]/2. \quad i^{th} \text{ Level module shear forces} \tag{6b}$$

$$M_i = v_i h = M_m\left[(m+1)-(i/m)(i-1)\right]/2. \quad i^{th} \text{ Level module moments} \tag{6c}$$

$$M_i^P = 2M^P\left[(m+1)-(i/m)(i-1)\right]. \quad i^{th} \text{ Level module plastic moments} \tag{6d}$$

$$I_i = \left[(m+1)-(i/m)(i-1)\right](I/2). \quad i^{th} \text{ Level module beam inertia} \tag{6e}$$

$$\bar{I}_i = I_m\left[(m+1)-i^2/m\right],\ i \geq 1\ ,\ \bar{I}_0 = I_0 \quad i^{th} \text{ Level beam inertia} \tag{6f}$$

$$J_i = \bar{J}_i = J\left[(m+1)-(i/m)(i-1)\right]/2 \quad i^{th} \text{ Level module and story column inertia} \tag{6g}$$

$$M_{B,i} = (v_i h_i + v_{i+1}h_{i+1})/4 = fh \times \left[(m+1)-(i^2/m)\right]/4,\ i \geq 1 \quad i^{th} \text{ Level beam moments} \tag{6h}$$

$$M_{B,0} = fh(m+1)/2 \quad \text{Grade level beam moments} \tag{6k}$$

$$M_{B,i}^P = M^P\left[(m+1)-(i^2/m)\right],\ i \geq 1,\ M_0^P = M^P(m+1)/2 \quad i^{th} \text{ Level beam plastic moments} \tag{6m}$$

$$M_{C,i,j} = fh \times \left[(m+1)-(i/m)(i-1)\right]/8 \quad i^{th} \text{ Level column moments} \tag{6n}$$

The group of Eq.s (6) completes the exact design analysis of the subject m^{th} level sub frame of UR in accordance with the prescribed conditions. A summary of the results of sets of Eqs. (6) as applied to the solution of the introductory example 1, is presented in Table 1 below.

3.2 Plastic collapse of subframes of UR under lateral loading

Because of the deterministic nature of structures of UR, the closed form solutions of section 3.1 may be used directly to describe the deformed shape as well as the distribution of ultimate moments at incipient collapse, in which case *f* should be replaced with f_P in group of Eqs. (6).

In order to investigate the *uniqueness* of the proposed solutions it is necessary to study the *upper bound* characteristics of the structure under the same magnitude and distribution of loading. This is achieved by considering the plastic collapse of the subframe through formation of plastic hinges at all beam ends, including the grade beam. It is assumed that for idealized conditions described by $a = b = 0$ and $f_{cr.i} = f_{cr}$, the columns and beams of the structure tend to rotate through uniform virtual drift angles $\phi = \theta$ and

$\overline{\theta} = (L/\overline{L})\theta = \theta/\varepsilon$ respectively, as shown in Figures 2c and 1d Then, by the use of virtual work theory [13,14], it gives;

$$\text{Total virtual external work} = \sum_{i=1}^{m}(f_i^P / f_{cr}) \times \theta \times h \times i = \frac{f^P \theta \times h}{6 f_{cr}}(m+1)(2m+1). \quad (7a)$$

$$\text{Total virtual internal work} = \sum_{i=0}^{m} \overline{M}_i^P \times \overline{\theta} = \frac{2M^P \theta}{3\varepsilon}(m+1)(2m+1). \quad (7b)$$

Equating the two work equations gives $M^P = f_P h\varepsilon / 4 f_{cr}$ which is the same as the previously obtained global solution, Eq. (2), for an *m* story subframe of UR. Since this solution satisfies the boundary support conditions, static equilibrium and the prescribed yield criterion it is considered *unique* and suitable for design purposes.

The information contained in Eqs. (6) and (7) can be easily extended, without modification to the other subframes of the prototype to create a whole moment frame of UR by matching the stiffness of the modules of the same level to those of the modules studied above, in which case all members of groups of similar elements such as beams, and columns would respond identically to external forces regardless of their numbers within the framework. And, as there are *n* such subframes, the module loading intensity *f* and the shear force v_i would be become; *f=F/n* and $v_i = V_i / n$ respectively.

3.3 Generalized boundary support conditions

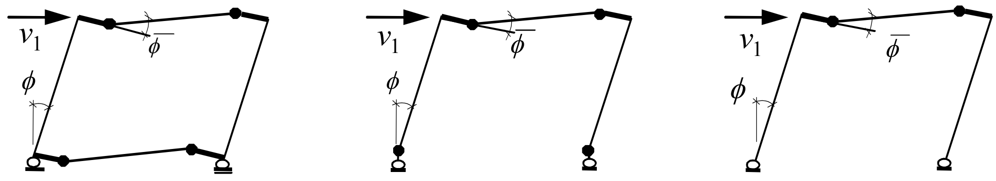
The non-rigid, continuous grade beams of Figure 1b also serve as physical models for the generalization of the column support conditions at the base. Ideally, $I_{0,j} / L_{0,j} = 0$ and $I_{0,j} / L_{0,j} = \infty$ should describe pinned and fully fixed base support conditions respectively. However, in order to avoid mathematical discontinuities, it can be shown, that for a fully fixed, ground level, idealized subframe the elastic drift function ϕ_1 may be expressed as:

$$\phi_1 = \phi = \frac{v_1 h_1 L}{12 f_{cr,1} E I_1}\left[(1+\frac{6}{k})x^2 + (1-3x)\frac{2}{k}\right] \quad (8a)$$

Where, $x = 3/(6+k)$ and $k = J_1 L / I_1 h_1$. Similarly, for a completely pinned, idealized ground level subframe, the elastic drift function ϕ_1 may be expressed as:

$$\phi_1 = \phi = \frac{v_1 h_1 L}{12 f_{cr,1} E I_1}\left(\frac{2}{k}+1\right) \quad (8b)$$

By definition, all modules of the subframes of UR, regardless of their location and boundary support conditions, fail simultaneously under proportional loading.



(5a) (5b) (5c)

Figure 5. Base level module with different boundary support conditions

The virtual work equation may be amended to incorporate the effects of all three boundary support conditions in the same expression, i.e.

$$\frac{v_1 h_1}{f_{cr}} = 2\left[\delta_\lambda^P \lambda\varepsilon + \delta_{grade}^P + 1\right]\frac{M_1^P}{\varepsilon} \tag{8c}$$

Where, δ_{grade}^P=0 and δ_λ^P=0 imply pinned support conditions. $\delta_{grade}^P = 0$, and $\delta_\lambda^P = 1$ refer to fully fixed column end conditions and $\delta_\lambda^P = 0$, and $\delta_{grade}^P = 1$ corresponds to grade beams resisting plastic moments. Substituting for (f_{cr}/ε) from $v_m h_m = 4 f_{cr} M^P/\varepsilon$, it leads to;

$$M_1^P = \left(\frac{v_1 h_1}{v_m h_m}\right)\frac{2M^P}{[1 + \delta_{grade}^P + \delta_\lambda^P \varepsilon\lambda]} \tag{8d}$$

For a regular *m* story subframe with $h_i = h$ and $f_i = f(i/m)$, Eq. (8d) reduces to;

$$M_1^P = \frac{(m+1)M^P}{[1 + \delta_{grade}^P + \delta_\lambda^P \varepsilon\lambda]} \tag{8e}$$

4. GENERATION OF MOMENT FRAMES OF UR

4.1 Methodology

It was shown in the preceding sections that imaginary modules of a moment frame could be proportioned and reassembled in such a way as to develop a vertical subframe of UR. The same principles could easily be extended to generate an entire multi-bay moment frame of UR. This can be achieved by first proportioning the stiffness of the beams and columns of the modules of the remaining subframes in accordance with the rule of equal stiffnesses; $(I_{i,j}/L_j) = (I_i/L)$ and $J_{i,j} = J_i = \bar{J}_i$ respectively, or those of the assembled frame by proportioning in accordance with $(\bar{I}_{i,j}/L_j) = (\bar{I}_i/L)$ and $\bar{J}_{i,j} = J_i = \bar{J}_i$ for *j=0* and *j=n*, and

$\bar{J}_{i,j} = 2J_i = 2\bar{J}_i$ for *j=2,3,…,(n-1),* respectively. Next, the entire set of Eqs. (6) may be used to express the statics of the newly generated frame by simply redefining *f =F/n* in Eq. (6a). By the same token, the plastic collapse load of the entire moment frame can now be expressed by the simple but powerful formula:

$$M^P = F_P h\varepsilon / 4nf_{cr} \qquad (9a)$$

The use of Eqs (7) can also be extended to study the virtual work parameters of the prototype by simply replacing f_i^P with F_i^P in Eq. (7a) and M^P with nM^P in Eqs. (7b) and (8c).

It is instructive to note that in the practical ranges of application, (L=h to $L \approx 4h$), small variations of ξ, from approximately 0.70 to 2.0, and ε from approximately 0.6 to 0.90, could significantly affect the magnitudes of M^P per Eq. (8) as well as ϕ per Eq. (3), depending on the type of moment modification method selected for the project.

4.2 Introductory Example 1

Generate an ideal, regular, $(m = 6) \times (n = 4)$, grade beam supported, moment frame of UR, with $L_1 = L_2 = L = h,\ L_3 = 1.5L$ and $L_4 = 2L$, subjected to a triangular distribution of lateral forces $F_i = F(i/m)$, and axial nodal forces $P_{i,j}$, such that $f_{cr.i} = f_{cr} = 0.875$, with the proviso that $J = (\mu = 1.1) \times I$, and that the maximum drift angle ϕ_Y does not exceed 0.02 radians at incipient collapse. Assume that spans L_3 and L_4 are ideally RBS augmented, such that $M_{i,j}^P = M_i^P$ for all *j*.

Solution: For ideal conditions $\varepsilon = \xi = 1$ and the ultimate carrying capacity of the uppermost level module of the vertical subframe, such as that shown in Figure 1c or 5a can be expressed as $M^P = F_P h / 4nf_{cr} = F_P h / 14,$ which also represents the unique collapse load of the subject frame under the prescribed loading. It simply follows that $N^P = \lambda M^P$.

From Eq. (1) the representative values of *I* and *J* for the m^{th} level module can be computed as; $I = F_P h^2 (1 + 1/\mu) / 24nE\phi_Y = 0.9943 F_P h^2 / E$ and $J = \mu I = 1.1I$ respectively.

The complete solution of the uppermost module is contained in the four vales M^P, N^P, I and J computed above. No additional drift calculations are necessary since the value of ϕ_Y was built into the design of *I* and *J* through the governing load-displacement equation.

For uniform response, the relative stiffness of the four spans of the structure should be the same i.e. $I_1 / L_1 = I_2 / L_2 = I_3 / L_3 = I_4 / L_4 = I / L$, therefore $I_1 = I_2 = I,\ I_3 = 1.5I$ and $I_4 = 2I.$ therefore there is no need to repeat the same calculations for the remaining subframes of the structure.

The complete design of the introductory example 1 is summarized in Table 1 below.

The results of this section are used in section 6.3 to illustrate the applications of fine tuning to moment frames of UR.

Table 1: Summary of numerical solutions of the sub frame of Example 1

i	f_i/f	v_i/f	v_ih_i/fh	I_i/I	$\bar{I}_i/I$	J_i/I	$M^P_{B,i}/M^P$	Δ/h
6	6/6	6/6	6/6	6/6	6/6	6.6/6	6/6	0.12
5	5/6	11/6	11/6	11/6	17/6	18.7/6	17/6	0.10
4	4/6	15/6	15/6	15/6	26/6	28.6/6	26/6	0.08
3	3/6	18/6	18/6	18/6	33/6	36.3/6	33/6	0.06
2	2/6	20/6	20/6	20/6	38/6	41.8/6	38/6	0.04
1	1/6	21/6	21/6	21/6	21/6	23.1/6	21/6	0.02

4.3 Introductory Example 2

Compare the total material weight of the subframe of UR of the previous example with that of a similar subframe of uniform sections, under similar loading and identical plastic collapse load.

Solution- Denoting the plastic moment of resistance of the sections of the new subframe by M_E^P, then from the virtual work equation, $91fh/6 = 14M_E^P$, corresponding to failure pattern 6e. It gives; $M_E^P = 13fh/12$. To compare the efficiencies of the two systems, their total weight functions in terms of their section properties may be expressed as;

$$G = C[\textstyle\sum_{i=0}^{6} LM_i^P + 2h\sum_{i=1}^{6} N_i^P] \tag{9b}$$

Where, C is an arbitrary constant of proportionality. Assuming $\lambda = 1$ and substituting for $L = h$ and $M_i^P = N_i^P = M_E^P$ into Eqn. (9b), it gives; $G_{US.} = 19CLM_E^P = 247CL^2 f/12$. Similarly, the total weight function of the subframe of UR can be computed as; $G_{UR} = 4 \times 91CLM^P/6 = 182CL^2 f/12$. As expected the system of UR is 1/3 lighter than its counterpart composed of uniform sections.

5. ON MOMENT CONTROL FACTORS

Moment control/modification factors have been introduced to effectively delay or accelerate, or to eliminate or induce formation of plastic hinges within selected beams of steel subframes. There are roughly ten pre-qualified and several proprietary, patented beam-to-column connections custom designed for earthquake resisting moment frames. The effects of natural offsets on the performance of moment frames, has been addressed, amongst others

[15] by S.C.Goel [16, 17] and his associates. The effects of four most prevalent such connections on moment control are briefly discussed in this section.

5.1 Case 1-Natural hinge offsets

The plastic failure modes of Figures 2b, 2c and 2d indicate that for kinematically admissible mechanisms to occur, beam end plastic hinges should form at approximately a minimum distance $D/2=(d_B+d_C)/2$ away from the centerline of columns as in Figure 4a. These short segments of length *D/2* act as rigid links of infinite moments of resistance, suggesting that for realistic design purposes, the static equilibrium condition $\varepsilon\times\xi=1-d_C/L\approx 1$ and the yield criterion $\infty\geq\xi\geq 1$ should be taken into consideration. And, if by definition the offset distance $\varepsilon=(\overline{L}/L)=(L-D)/L$, then $\xi=(L-d_C)/(L-D)$. Obviously, for all positive values of *D*, $\infty>\xi>1$ leads to the theoretically admissible *unique* solution $M^P=V_P h\varepsilon/4f_{cr}$. $V_P=4M^P f_{cr}/h\varepsilon$, indicates that by virtue of $\varepsilon<1$, a higher load carrying capacity for the subject module than that computed for its idealized counterpart.

The classical assumption $a=d_C=0$ or $\varepsilon=\xi=1$, satisfies both the yield criterion as well as the static equilibrium condition described above, but results in the lower bound solution $M^P=V_P h/4f_{cr}$. Eq. (3) suggests that for normal ranges of $\xi>1$, the actual global plastic drift of the subframe could decrease by as much as 15%. In other words, ignoring the effects of *D* could lead to an underestimation of module strength and overestimation of its lateral displacement.

Welded un-reinforced-bolted web (WUF-B), Welded, un-reinforced-welded web (WUF-W), Welded free flange, and Bolted un-stiffened end plate (BUEP) pre-qualified connections may be categorized under case 1 discussed in this section.

5.2 Case 2-RBS connections

RBS connections are frequently used to reduce beam end plastic moments in order to prevent formation of plastic hinges in columns. However, in the realms of performance analysis, RBS are viewed as means of controlling the propagation of plasticity in a moment frames of UR.

Theoretical considerations for a plausible collapse failure mechanism that also satisfies the yield criterion $\alpha M^P<M^P$, require that $M_\alpha^P=V_P h\varepsilon/4f_{cr}\alpha$, provided that $a\geq D/2$ and $(L-2a)/(l-D)<\alpha<1$. α, is the moment reduction coefficient. In RBS design distance *D/2* ranges from 3’-6”corresponding to minimum *L*=10’-0*”* to 6’-0”corresponding to $L\geq$40’-0”, indicating that the ratio ε could range from 0.7 to 0.9. Consider the case *L*=40’-0’’, $\alpha=0.875$ and $a=4'\text{–}0''$ i.e. $\varepsilon=0.8$, where the effective collapse load reduces to $V_P=0.9125(4f_{cr}M^P/h)$, i.e. 9 % less than that anticipated for the corresponding regular module. In conclusion, RBS connections may tend to increase the plastic drift of the structure by as much as 10% for modules spanning less than 20’-0”.

Reduced web (RW) proprietary connections may also be categorized under case 2 discussed above.

5.3 Case 3-Slotted web connections

The proprietary Slotted Web connections for steel moment frames were developed originally to prevent damage to beam-column junctions during strong ground shaking. However, because of their well established hysteretic characteristics, they can also be used as reliable PC devices.

The arguments pertaining to this case are practically the same as those elaborated for Case 1 above with the difference that the total offset distance D may be computed as $D = d_C + 2w_s$, where w_s is defined as the width of the connecting welded shear plate. Assuming that; $d_C = 1'\text{–}0''$, the minimum total offset distance is $D_{\min} = 3'\text{–}6''$, and that the minimum control or over-strength factor for the section containing the welded shear plate $\xi \approx 1.15$, then $\xi = (L - d_{C)}/(L - D)$, yields the limiting value of the corresponding span length as $L_{\min.} \approx 20'$. SW connections have little or no effect on the plastic drift of the system.

5.4 Case 4-ACP connections

Added cover plates, also known as flange plates, are the oldest and most effective method of moment enhancement in steel beams. Addition of cover plates, Figure 4c, also provides simple and economic means of delaying formation of plastic hinges in all types of steel moment frames. The challenge in using cover plates is to determine the necessary length and thickness of the added material to achieve pre-selected levels of moment control. Comparing the performance of an ACP connection with that of a regular connection described under case1 above, it can be seen that three essential conditions should be addressed. First, for the cover plates to be effective the total offset distance a should be larger than $D/2$ defined for the latter case, i.e. $a > D/2$. Second, the moment control factor should be larger than unity i.e. $\xi > 1$. For instance if, L=30'-0", $d_C = 1'\text{–}2''$ and $\xi = 1.2$, then using $\xi > (L - d_C)/(L - 2a)$, the required cover lengths become $a = 3'\text{–}0''$.

Welded flange plate (WFP), bolted flange plate (BFP), Stiffened end plate (SEP), double split tee (DST), Added haunched sections (AHS) and proprietary Side plate connection may be categorized under case 4 of this section.

6. PERFPRMANCE CONTROL

By definition, PC means the ability to design a structure in such a way as to expect predetermined modes of response at certain extents of damage, loading stage and/or limiting drift angles. It has been shown that structures of UR, by virtue of their well defined demand-capacity relationships throughout their loading history lend themselves well to PC treatment. UR analysis may be looked upon as limiting value envelopes for minimum member strength and stiffness, below which the solutions would be deficient and in violation of the stated design rules. In other words any decrease in the UR element strengths and/or stiffness would adversely affect both the ultimate carrying capacity as well as the global drift of the structure. On the other hand, if for any reason such as practical sizing and selection, any member strength and/or stiffness is increased beyond its UR value, the change would only enhance the structural integrity of the system.

6.1 PC. with respect to a condition of reference

Typically, a structure of UR can be controlled in relationship with numerous general and four distinct but interrelated points of reference, V_Y, V_P, ϕ_Y and ϕ_P. The two general ranges of reference may be identified as: $\phi \le \phi_Y$ and $\phi_Y \le \phi \le \phi_P$ or $V \le V_Y$ and $V_Y \le V \le V_P$.

The following parametric solution is presented to demonstrate the use of single points of reference such as ϕ_Y to ensure drift control in terms of selected section inertias. For instance, if W=0, $\varepsilon = 1$, $\phi_Y = 0.02$ radians, $P/P_{cr} = 0.1$ and $J = 1.1I$, then Eq. (1) may be used to control the lateral displacements of the idealized module of Figure 2c and subsequently the entire structure at first yield by selecting the corresponding section inertias as:

$$I_Y = \frac{V_Y h}{(.02)\times 24E}\left[\frac{h}{1.1}+L\right]\frac{1}{(1-0.1)} = \frac{2.315\times V_Y h}{E}\left[\frac{h}{1.1}+L\right] \qquad (10)$$

A particular case of this solution was presented under section 4.2, the introductory example 1 above.

6.2 PC. within ductility range

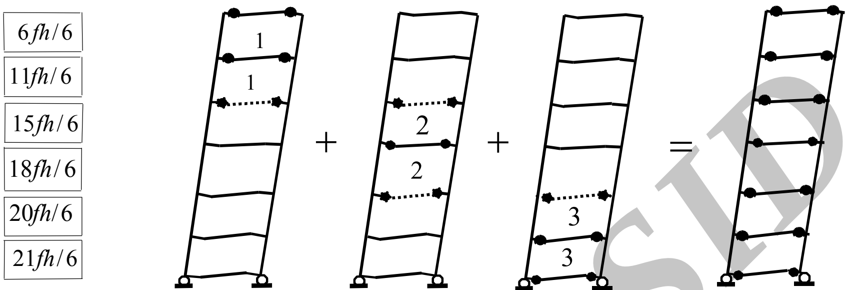
Supposing structural damage could be associated with propagation of plasticity within the members of the structure and that as design requirement, it is needed to control such damage with reference to specific physical conditions. Generally, ductility is defined as the relationship between two well defined reference conditions-the onset of plasticity at first yield and incipient plastic collapse. For instance, if the state of incipient plastic collapse with a certain number of plastic hinges is selected as a reference condition, then other intermediate states of loading may be referred to as states of partial damage associated with a percentage of the total number of plastic hinges needed to generate a failure mechanism. The challenge therefore is to control the number and sequences of formation of plastic hinges using simple moment modifiers in such a way as not to compromise the ultimate load carrying capacity of the structure.

The applications of the proposed method of approach are demonstrated by the following simple examples, where the plastic collapse load of an idealized six story subframe of UR, such as that shown in Figure 6e, is computed as $f_P = M^P / 4h$.

6.3 Introductory Example 3

Use the results of the introductory example 1, section 4.2, to induce a three-stage sequential formation of plastic hinges in the subject vertical subframes. Assume the column offset distance b=0 and that $\bar{f}_{cr.i} = 1$.

Solution- The object of the exercise is to force the first group of plastic hinges to form in the top two imaginary modules of the subframe as shown in Figure 6b, followed by the second and third groups of hinges forming sequentially in the middle two and lowermost two modules respectively. Figures 6b, 6c and 6d depict the pre-planned sequences of formation of the plastic hinges after neighboring modules are merged together. Depending on the statement of the problem two control options and their combinations are available- whether the ultimate carrying capacity of the system may be altered in a positive or negative sense. RBS modifications tend to decrease while ACP connections tend to increase the plastic collapse capacity of ductile structures. By the same token RBS treatments tend to compromise while ACP tend to enhance both local and global drift ratios. However, the two systems may also be combined to control the ultimate response of the system as desired.



(6a) (6b) (6c) (6d) (6e)

Figure 6. Controlled plasticity in sub frame of uniform resistance

The total external work corresponding to system of forces of Eq. (6a) is given by Eq. (7a) as $91fh\theta/6$, which is the same as the sum of the module racking moments shown in Figure 6a. Now if the plastic moments of resistance of the three groups of modules were to be modified by factors, $\xi_1 = 0.8$, $\xi_2 = 0.9$ and $\xi_3 > 1.0$, in order to force the required sequence of formation of hinges, then the premature formation of plastic hinges in the subject modules would tend to reduce the load carrying capacity of the original structure of UR. The internal work Eq. incorporating the plastic hinge rotations of the imaginary modules in terms of the three moment modifiers may be expressed as;

$$\xi_1\left(\frac{6}{6}+\frac{11}{6}\right)4M^P\theta+\xi_2\left(\frac{15}{6}+\frac{18}{6}\right)4M^P\theta+\xi_3\left(\frac{20}{6}+\frac{21}{6}\right)4M^P\theta \tag{11}$$

The virtual work equation for the subject subframe yields;

$$f=\left(17\xi_1+33\xi_2+41\xi_3\right)4M^P/91h \tag{12}$$

Eq. (12) is the governing PC expression of the system. It describes the following conditions;

For $\xi_1=\xi_2=\xi_3=1$ $f=4M^P/h$, as expected.

For $\xi_1=0.8$ and $\xi_2=\xi_3=1$. $f=(0.9626)4M^P/h$, i.e., a minor drop in the collapse load.

For $\xi_1=0.8$, $\xi_2=0.9$ and $\xi_3=1$. $f=(0.9264)4M^P/h$, i.e., a minor drop in the collapse load.

Each loading state corresponding to ξ_1 or ξ_2 may be targeted as a design stage for which a value for I and J may be computed as provided for by Eq. (6e). However in order to compensate for the reduced carrying capacity of the structure, without altering the pre-selected sequence of formation of the hinges, the last moment modifier ξ_3 should be selected in such a way that $f \geq 4M^P/h$ .Substitution of this critical value of f in Eq. (12) gives; $\xi_3 \geq 1.0795$. Alternatively, the same sequencing may be enforced by selecting the following combinations of the moment modifiers.

For $\xi_1 = 1.0$ and $\xi_2 = \xi_3 = 1.1$: $f = (1.0873)4M^P/h$, i.e. a minor increase in the collapse load.

For $\xi_1 = 1.0$, $\xi_2 = 1.1$ and $\xi_3 = 1.2$; $f = (1.1264)4M^P/h$, i.e. further increase in the collapse load.

Since the combined application of the three moment modifies does not reduce the ultimate carrying capacity nor the global stiffness of the subject subframe, the expected drift angle should remain within the specified limit of $\phi_Y \leq 0.02$ radians. In other words, fine tuning or plastic hinge sequencing can be implemented in such a way as not to reduce the performance level of structures of UR.

7. CONCLUDING REMARKS

A simple, manual method of Performance-based elastic-plastic design for moment frames of UR, under lateral loading has been developed. The proposed formulae appear to be the only ones of their kind that can predict the elastic-plastic displacements of moment frames under combined axial and lateral forces. The method enables the engineer to define, predict and to control the structural response of multistory multi bay moment frames at pre-selected stages without resorting to complicated numerical analysis or computer generated results. Reference loading stages have been identified as those causing first yield, any fraction of the failure load or specified drift ratio, up to and including incipient collapse. Several generic examples have been provided to illustrate the applications of the proposed formulae.

It has been shown that the selection of the properties of the constituent modules of the moment frame can be controlled mathematically in such a way that members of the same group of elements such as beams and columns could share the same demand-capacity ratios regardless of their numbers and locations within the structure. Most importantly, moment frames of UR can be fine-tuned, using moment modifiers, to respond as directed by the design engineer.

The concept of applying the proposed method of design to the PC of earthquake resisting moment frames becomes attractive when its analytic simplicity and inherent benefits are considered. One of the most beneficial aspects of performance controlled structures of UR is that their theoretical total material weight is a *minimum* and that their lateral displacements vary linearly along the height during both the elastic as well as ductile phases of loading. The proposed concept becomes more feasible realizing that structures of UR behave as statically determinate systems and lend themselves well to manual as well as spreadsheet computations.

Perhaps the most important message contained in this article is the suggestion that actual structural response should be looked upon as a function of design and construction, rather than theoretical analysis, whence, both strength and stiffness should be induced rather than extracted from numerical computations, stability and failure patterns should be imposed rather than investigated. The proposed procedure not only satisfies all current code requirements in a much simpler and economical manner, but also suggests a more realistic approach towards structure specific designs in the future.

Finally it is hoped that the applications of the proposed structure and loading specific design methodologies will be extended to other types of earthquake resisting systems such as braced frames, shear walls, hybrid structures etc.

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APPENDIX

Eq. (1) indicates that the local flexural side-sway of a typical, frame model, under purely lateral forces, may be expressed in terms of two distinct components as;

$$\Delta = \left[\frac{V}{K_C} + \frac{V}{K_{B.}} \right] = \frac{V}{K_.} \tag{A1}$$

where, $K_{C,} K_{B.}$ and K represent the stiffnesses of the columns and the beams and the overall stiffness respectively of the subject closed loop frame. The P-delta effects of the axial forces P_j may be expressed in the form of an equivalent shear force $V' = (\sum_{j=1}^{n} P_j \Delta)/h$ acting in the same sense as V. Eq. (A1) may now be rewritten as; $\Delta K = V + V' = V + \Delta(\sum_{j=1}^{n} P_j)/h$, which after rearrangement becomes;

$$\Delta = \frac{V_i}{K(1 - \sum_{j=0}^{n} P_j / Kh)} = \left[\frac{V}{f_{cr.} K_C} + \frac{V}{f_{cr.} K_B} \right] = \frac{V}{f_{cr} K} \tag{A2}$$

This implies that both $K_{C.}$ and K_B are influenced by $f_{cr.}$. Similarly, the plastic collapse load of the subframe may be computed as $Vh + \sum_{j=0}^{n} P_j \Delta = 4nM^P$. Substituting for Δ from Eq. (A2) and rearranging gives,

$$M^P = Vh / 4nf_{cr} \tag{A3}$$