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ON EFFICIENT DESIGN OF EARTHQUAKE RESISTANT MOMENT FRAMES

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ABSTRACT

This paper introduces a number of simple findings that lead to the efficient design of system based earthquake resisting moment frames. A system based design is defined as one that leads to minimum drift and minimum weight solutions, for code recognized seismic frameworks, without resorting to complicated numerical analysis. These findings are used to form an algorithm, which in turn leads to closed form solutions for system-specific performance-based design of earthquake resisting moment frames. The results of some of these findings may be summarized as follows;

- the efficient design of a representative closed loop sub-frame is one involving beams and columns of equal strength and stiffness,
- a design may be said to be efficient when the demand/capacity ratios of all of its members are as close to unity as possible,
- the magnitude of a mid-span concentrated load may be considered small if it is less than half its plastic collapse value acting alone on the same beam.

Keywords: Earthquakes; moment frames; efficient design; performance control; plastic design

1. INTRODUCTION

The key to successful system-specific Performance Based Seismic Design (PBSD) is to appreciate the essence of the principle that all earthquake resisting systems are expected to withstand controllable, large inelastic displacements, each in its own particular way, while maintaining a certain degree of structural integrity [1, 2]. A system based design focuses attention on the inherent characteristics of the specific structure to fulfill the basic requirements stated above. A traditional design adhering to this principle may entail several

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cycles of complete elastic analysis of a *first-guess system*, followed by complete plastic analysis and code checks of a *continuously-degrading-framework*, until a satisfactory convergence of the pertinent checks and balances is established. The problem is compounded when effects of gravity loads on stiffness degradation and strength deterioration are also considered. The challenge therefore, is to perform numerically massive, often theoretically complicated, analysis needed to estimate the lateral displacements of the structure under different phases of incremental lateral forces, starting from zero to initial yielding, through propagation of plasticity up to and including incipient collapse. While computerized versions of PBSD methodologies have proven their worth in connection with large scale and special order structures, [3, 4] no *direct-design* formulation for routine usage of PBSD has been reported in the literature. However, what has hindered the applications of PBSD to the design of common types of earthquake resisting systems, such as moment frames, is largely due to lack of design oriented analytic tools and familiarity with *direct-response* methods of approach. The difference between conventional investigative *analyses first,* code prescriptive *design next* practices and the relatively new *direct response* methods of design are a matter of experience and extent of familiarity with the inner workings of the subject systems. In the latter design method, both strength and stiffness are induced rather than investigated with respect to certain predetermined criteria. In other words members are selected in relation with predetermined target displacements and prescribed loading, neither displacements nor member strengths are checked again for code compliance. In the case of moment frames, groups of beams and columns are selected in terms of their contributions to the global strength and stiffness of the structure, including P-delta effects, while observing the strong column-weak beam criterion throughout the loading history of the system. *Direct-response* methodologies offer many advantages over the conventional *analyses-first* practices in that, they are much simpler to work with, attract the least amount of seismic input energy, result in minimum drift and weight designs, and provide insight into the nonlinear behavior of the subject systems [5, 6]. Unlike General Purpose Moment Frames (GPMF) that are designed to sustain seismic forces in addition to permanent gravity loads, Earthquake Resisting Moment Frames (ERMF) are primarily designed to withstand seismic forces in combination with small or short term gravity loads. The difference in their performance is attributed to the fact that unlike gravity loading, seismic input energy is a function of the structural properties, to their different modes of failure and the effects of gravity forces on the ultimate lateral carrying capacities of the two systems. In general, large permanent gravity forces adversely effect the lateral performance of all types of moment frames, while moderate to small gravity loads appear to have little to no effect on the ultimate carrying capacity of such structures. These differences are significant in that ERMF perform more predictably and can carry larger earthquake forces than their GPMF counterparts.

2. THE PROPOSED ALOGORITHM

To develop the proposed algorithm, it is assumed that that the entire moment frame including grade beams, is composed of imaginary, rectangular, rigidly connected basic modules which fit into the bays of the structure. Therefore, if each imaginary module, as component part of the moment frame, is designed as an efficient sub-frame, then the entire assembly could also be regarded as an efficient ERMF, in which case, it should be possible to extend the design of one such module to recreate the original system.

The development of the proposed procedure is greatly facilitated by first generating a single row horizontal, Figure1b, or single bay vertical, Figure1c, sub-frame with the same characteristics as the pre-designed efficient module and then extending the sub-frame design to all other parallel sub-frames through direct proportioning. This renders the physical model of the prototype as an upright cantilever, with single story, contiguous sub-frames stacked on top of each other and hinge connected at their common joints *i,j*. Obviously, when the beams of the imaginary sub-frames merge at their common boundaries, the hypothetical hinges disappear, and their physical properties and internal forces become superimposed to regenerate the original frame, e.g., the moments of inertia and the bending moments of the beams of the reassembled structure become $\bar{I}_{i,j} = I_{i,j} + I_{i+1,j}$ and $\bar{M}_{i,j} = M_{i,j} + M_{i+1,j}$ respectively. The proportionate selection of the strength and stiffness of the members of the system, results in the creation of an efficient ERMF where members of geometrically similar groups of beams and columns of identical properties share the same *demand-capacity ratios* regardless of their locations and numbers within the group. The essence of the proposed procedure is described in two parts. Part I, in section 3.1, attempts to establish the pertinent conditions needed for the formulation of an efficient design. Part II, in section 4.1, introduces the required formulae and the manner of implementation of the algorithm through parametric examples.

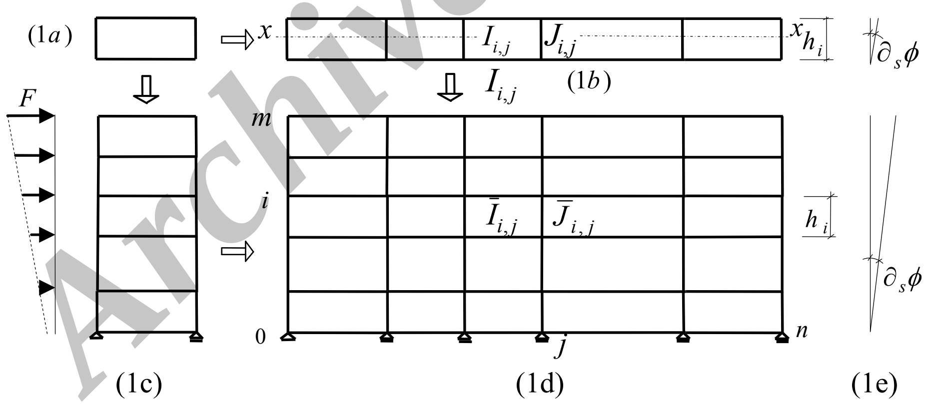


Figure 1. Basic Module, Vertical and Horizontal Sub-frames, ERMF and Drift Angle

2.1 Design conditions

The efficient design of earthquake resisting structures is governed by specific stability and ductility related criteria which may or may not apply to gravity resisting systems [7]. The design of gravity resisting frameworks, on the other hand, may not be required to comply with strict drift and hysteretic conditions. It seems, therefore, expeditious to concentrate effort on system and function specific concerns rather than testing generalities. In an ideal system-oriented design, the essential design criteria, including optimization data, are made part of the selection process rather than comparing results with the same requirements, and iterative processes are replaced with direct numerical substitutions.

The fundamental conditions related to the efficient design of ERMF may be summarized as follows.

- The side-sway of each story shall be a minimum for that story.
- The maximum allowable drift angle shall be the same for all stories.
- The total weight of each constituent module shall be a minimum.
- Gravity loads shall not reduce the ultimate lateral carrying capacity of the frame.
- Columns shall remain stable and elastic throughout all loading stages.
- The structure shall be capable of sustaining large inelastic displacements while maintaining a certain degree of structural integrity.
- Effects of gravity loads on stiffness degradation and strength deterioration shall be considered as part of the prescribed design criteria.
- The solution shall be capable of addressing damage control and meeting predetermined target decisions.
- The design shall satisfy conditions of *uniqueness* and/or *lower bound* solutions during both linear as well as nonlinear stages of loading.
- These issues are addressed in the following studies and their companion examples.

3. PART I –FUNDAMENTALS

The purpose of this section is to focus attention on the more significant aspects of system specific design, rather than generalized analysis. Information gathered from such studies are then directed towards design-specific member selection rather than computer generated outcome. The following areas of interest, in connection with efficient design of basic modules, as component parts of ERMF, are briefly studied in this section.

- Effects of large gravity loads on moment frames
- Basic elastic-plastic response
- Basic plastic limit state response
- Minimum weight-minimum drift association

The outcome of the design process is immensely improved by utilizing the findings of this section, and as a result, unnecessary preliminary computations are routinely avoided. Furthermore, treating the selection of the columns and connections as automatic byproducts of beam design reduces the design effort to a minimum and increases the efficiency of the final product. Minimum weight columns, which are not allowed to contribute to the ductility of the system, are selected in such a way as to support the ductile performance of the rest of the structure at ultimate loading.

However, a revision of the nonlinear response of a single, rectangular, closed loop moment frame or Basic Module (BM), as component part of ERMF, is essential in understanding the applications of the proposed algorithm to the practical design of ERMF.

3.1 Effects of large gravity loads on moment frames

The undermining effects of large gravity loads on ERMF are significant enough to warrant a referenced to the formal study of the subject. While the bending effects of small floor loads *W* have little or no direct influence on the plastic moment of resistance of the BM, the axial effects of *P* and *W* tend to magnify both the lateral displacements as well as the moments generated by the lateral shear forces acting on the system. The question, what constitutes a small or large gravity load is addressed as part of the following discussion. In order to investigate the validity of the belief that small gravity loads do not affect the performance of EMRF, consider the combined effects of loads *V*, *P* and *W* on the ultimate load carrying capacity of the symmetric BM of a. An important characteristic of symmetric BM, is that their total weight is a minimum with respect to symmetric lateral loading [8]. This implies that the subject BM may be more flexible than their regular counterparts. Since these BM are expected to sustain relatively large inelastic displacements during major earthquakes, it becomes crucial to control their global as well as member capabilities at large gravity loads. The softening or loss of stiffness of such BM may be evaluated, to a high degree of accuracy, by the inclusion of the P-delta effects in their plastic carrying capacity computations, [9] expressed as;

$$M^P = \frac{1}{4}\left[\frac{V_P h}{(1+\delta_P^W) f_{cr}} + \frac{W_P L}{4}\right] \tag{1a}$$

Where $\delta_P^W = 0$ for *W=0,* and $\delta_P^W = 1$ for *W≠0.* Both suffix and index *"P"* relate the quantity to plastic collapse. M^P and $N^P > M^P$ are the plastic moments of resistance of the beams and columns of the BM respectively. $f_{cr} = (1 - 2P/P_{cr})$, is the magnification factor described in the next section. It has been shown, [9], that while small floor loads, i.e., $W_{Small} \le 8M^P/L$, have no effects on the lateral carrying capacity of the BM, axial loads *P* tend to reduce its efficiency linearly from full capacity to zero at $P = P_{cr}$. Furthermore, comparing W_{Small} with $W_{Limit} = 16M^P/L$, it may be concluded that in general $W_{Small} \le W_{Limit}/2$. In other words; *the magnitude of a mid-span concentrated load may be considered small if it is less than half its plastic collapse value acting alone on the same beam.* And that *moderate to small gravity loads have little to no effect on the ultimate carrying capacity of ERMF which are designed for code level earthquakes*.

This finding is equally valid for multimember ERMF. Henceforth, the scope of the present study will be confined to *V-W-P* combinations with $P < P_{cr}$ and $W \le W_{Small}$.

3.2 Basic elastic-plastic response

A review of the linear behavior of the basic moment frames of Figure 4 is a priori to appreciating the essence of the proposed algorithm. To this end, the generalized displacement equation of a closed loop or grade beam supported BM in terms of its material and geometric properties, subjected to combined axial and lateral forces has been derived as a closed form formula. This generalized mathematical model which addresses both the elastic as well as plastic deformations of the closed loop system up to and including incipient collapse is then expanded to discuss the basis of the proposed algorithm.

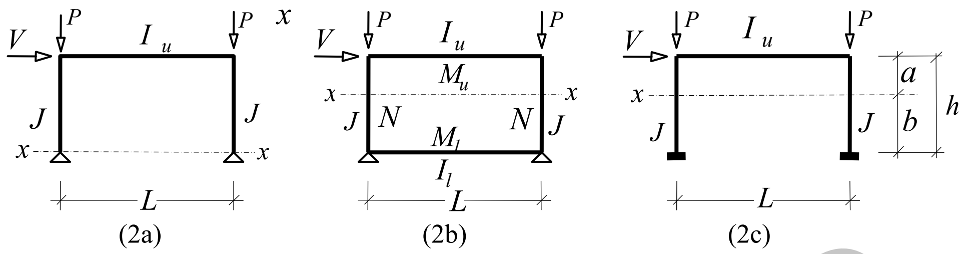


Figure 2. Basic lateral resisting moment frames and boundary support conditions

An generalized study of the plastic deformations of BM with fixed and pinned boundary supported conditions has been reported by the present authors, [10, and 11]. This article is concerned mainly with the elastic-plastic response of grade beam supported multistory moment frames. The drift ratio $\phi_i = \Delta_i / h_i$, of the generalized sub-frame of Figure 4b, subjected to a roof level lateral force $V_i = V$ and axial nodal forces $P_{i,j} = P$ can be expressed as;

$$\phi_{gr} = \frac{V_i}{12Ef_{cr,i,j}h_i}\left[\frac{4(a^3+b^3)}{h_i\sum_{j=0}^{n}(J/h)_{i,j}} + \frac{a^2}{\sum_{j=1}^{n}(\delta_j^P I_u/L)_{i,j}} + \frac{b^2}{\sum_{j=1}^{n}(\delta_j^P I_l/L)_{i,j}}\right] = \frac{V_i}{K_{gr.i,j}f_{cr.i,j}h_i} \tag{2a}$$

Where *J,* I_l and I_u are the section moduli of the columns, and, the lower and the upper beams respectively. Δ_i is the roof level lateral displacement. Dimensions *a* and *b,* describe the heights of the *upper* and *lower* parts of the sub-frame measured from their common points of zero moments and separated by the imaginary lines *x*-x. The condition for minimum drift with respect to location of line *x-x* can be expressed as:

$$\frac{\partial\phi}{\partial a} = 0 \quad \text{or} \quad \frac{\partial\phi}{\partial b} = 0 \tag{2b}$$

Introducing $\rho = [\sum_{j=0}^{n}\delta_j^P(J/h)_{i,j} / \sum_{j=1}^{n}(\delta_j^P I_u/L)_{i,j}]$ and $\bar{\rho} = [\sum_{j=0}^{n}\delta_j^P(J/h)_{i,j} / \sum_{j=1}^{n}(\delta_j^P I_l/L)_{i,j}]$ and performing the differentiations (2b) gives; $\frac{a}{h} = \frac{3+\bar{\rho}}{6+\rho+\bar{\rho}}$, $\frac{b}{h} = \frac{3+\rho}{6+\rho+\bar{\rho}}$ and

$$\frac{a}{b} = \frac{3+\bar{\rho}}{3+\rho} \tag{2c}$$

Where $K_{gr,i}$ is the global, un-degraded stiffness and $f_{cr,i} = [1-(\sum_{i=0}^{n} P_{i,j})/P_{cr,i}]$ is the corresponding force magnification functions. $\sum P_{i,j}$ and $P_{cr,i}= K_{gr,i}h_i$ are the total and critical axial loads of the subject sub-frame respectively. Abbreviated suffices *gr, hg* and *fx* refer to grade beam, hinged and fixed support conditions respectively. The generalized Kronecker's Delta δ_j^P, is used to include the effects of formation of plastic hinges at the moment bearing ends of the beams *,* e.g., $\delta_j^P = 1$ for $M_j < M_j^P$ and $\delta_j^P = 0$ for $M_j = M_j^p$. $\delta_j^P = 0$ also implies structural damage and/or loss of stiffness. *E* is the modulus of elasticity of the material of the frame. However, in order to avoid mathematical complications, without loss of generality, a simplified version of Eq. (2a), corresponding to a representative module of the same sub-frame, Figure 4d, is presented for the purposes of this section. The elastic drift ratio $\phi = \Delta / h$, of the representative or basic module of Figure 4b, subjected to similar forces as the sub-frame which it represents, can be expressed as;

$$\phi_{gr} = \frac{V}{12Ef_{cr}h}\left[\frac{2}{J}(a^3+b^3) + L(\frac{a^2}{I_u} + \frac{b^2}{I_l})\right] = \frac{V}{K_{gr}f_{cr}h} \tag{2d}$$

Coefficients ρ and $\overline{\rho}$ now become $(JL/I_u h)$ and $(JL/I_l h)$ respectively. Defining the upper and lower racking stiffness of the basic frame as $S = 3+\overline{\rho}$ and $\overline{S} = 3+\rho$ respectively, allows the corresponding upper and lower beam or corner bending moments M_u and M_l respectively to be expressed as;

$$M_u = \frac{Va}{2f_{cr}} = (\frac{Vh}{2f_{cr}})\frac{S}{S+\overline{S}} \text{ and } M_l = \frac{Vb}{2f_{cr}} = (\frac{Vh}{2f_{cr}})\frac{\overline{S}}{S+\overline{S}} \text{ respectively} \tag{2e}$$

A simple check verifies that the sum of the corner or beam end (or column end) moments is equal to the racking moment $M_R = Vh$ of the frames, i.e.

$$M_R = 2M_u + 2M_l = 2(\frac{Va}{2f_{cr}} + \frac{Vb}{2f_{cr}}) = Vh/f_{cr}\,. \tag{2f}$$

Story level racking moment is, therefore, defined as *the story level shear multiplied by the story height*. Three types of boundary support conditions for lateral resisting moment frames are usually encountered in practice; fully hinged, with flexible grade beams and fully fixed [9]. The complete elastic solution for the basic module with a grade beam is contained in Eqs.(2d)through (2f). The complete elastic solutions to the two companion cases, frames with fully hinged and fully fixed supports, can be derived easily from the same set of equations as for their prototype, e.g., Replacing the partial height *a* with *h* in Eq. (2a), it gives; *b=0* and;

$$\phi_{hg} = \frac{VhL}{12EIf_{cr}}\left[\frac{2+\rho}{\rho}\right] = \frac{V}{K_{hg}f_{cr}h} \tag{3a}$$

as the roof level elastoplastic drift ratio of the hinged support moment frame of Figure(4a). The corresponding beam moments for this case can be written down as;

$$M_u = Va/2f_{cr} = (Vh/2f_{cr})\frac{S}{S+\bar{S}} \text{ and } M_l = \frac{Vb}{2f_{cr}} = (\frac{Vh}{2f_{cr}})\frac{\bar{S}}{S+\bar{S}} \tag{3b}$$

Where, the story racking stiffness factors S and $\bar{S}$ for the fully hinged support moment frame may be defined as $S = 1$ and $\bar{S} = 0$ respectively. Similarly, putting $I_l = \infty$, i.e. $\bar{\rho} = 0$ in Eqs. (1a) and (1b), gives;

$$\phi_{fx} = \frac{V}{12Ef_{cr}h}\left[\frac{2}{J}(a^3+b^3) + L(\frac{a^2}{I_u})\right] = \frac{V}{K_{fx}f_{cr}h} \tag{4a}$$

as the roof level elastic drift ratio of the fixed support moment frame of Figure (4c), where

$$\frac{a}{h} = \frac{3}{6+\rho} \text{ and } \frac{b}{h} = \frac{3+\rho}{6+\rho} \tag{4b}$$

The corner bending moments for moment frames with fixed supports can also be expressed as;

$$M_u = \frac{Va}{2f_{cr}} = (\frac{Vh}{2f_{cr}})\frac{S}{S+\bar{S}} \text{ and } M_l = \frac{Vb}{2f_{cr}} = (\frac{Vh}{2f_{cr}})\frac{\bar{S}}{S+\bar{S}} \text{ respectively} \tag{4c}$$

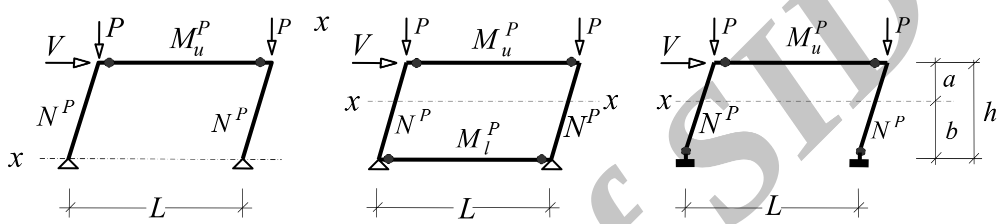
Where, the story racking stiffness factors for the fully fixed support moment frame are defined as $S = 3$ and $\bar{S} = 3+\rho$, comparing Eqs. (2f), (3b) and (4c), it can be seen that in all three cases the sum of the corner or column end moments is equal to the racking moment $M_R = Vh$ of the frames. If the dashed lines passing through the points of zero moment or contra-flexure are treated as the neutral axis of these frames with respect to story level (external) racking moments, then it may be observed that; *the total story level racking moment can be divided into upper and lower, parts in proportion with upper and lower racking stiffnesses of the module respectively, i.e.*

$$M_{RU} = \frac{S}{S+\bar{S}}M_R \text{ and } M_{RL} = \frac{\bar{S}}{S+\bar{S}}M_R \tag{5}$$

Eqs. (3a) and (2d), for the hinged base and grade beam supported modules respectively, can be used directly to compute the corresponding lateral displacements at incipient collapse. The lateral displacements of fixed base portal frame of uniform section can be estimated directly by putting *a=b=h/2* in Eq. (4a).

3.3 Basic plastic limit state response

Plastic limit state design philosophies are based on understanding pertinent failure mechanisms and corresponding load factors at collapse. Therefore, employing the principle of virtual work in connection with the plastic failure mechanisms of figure (5b), and observing that the plastic moments are prevented from forming within columns, i.e., $N^P > M_u^P$ and $N^P > M_l^P$, it gives for the total virtual external and



(3a) (3b) (3c)

Figure 3. Basic Collapse Mechanisms Due To Lateral Shear

internal work quantities; $W_{ext} = V\theta h / f_{cr}$ and $W_{\text{int}} = 2[M_u^P + \delta_{hg}^P M_l^P + \delta_{fx}^P N^{p)}]\theta$ respectively. Virtual work theory states that: $W_{ext} - W_{\text{int}} = 0$, whence,

$$V_P = 2[M_u^P + \delta_{hg}^P M_l^P + \delta_{fx}^P N^{p)}] f_{cr} / h, \tag{6}$$

where, $\delta_{hg}^P = \delta_{fxl}^P = 0$ represents hinged supports with $V_P = 2M_u^P f_{cr} / h$, $\delta_{hg}^P = 0$ and $\delta_{fx}^P = 1$ represent fixed supports with $V_P = 2(M_u^P + N^P) f_{cr} / h$, $\delta_{hg}^P = 1$ and $\delta_{fx}^P = 0$ indicate grade beams at supports with $V_P = 2(M_u^P + M_l^P) f_{cr} / h.$ Substituting for $M_l^P = M^P$, $M_u^P = 2M^P$, $\delta_{hg}^P = 1$ and $\delta_{fx}^P = 0$ in Eq. (6), results in $M^p = V_P h / 6 f_{cr}$, a result previously obtained for the example problem through simple plastic analysis. This indicates that as far as failure theorems are concerned the proposed methodology results in either smaller or equal failure loads compared with those predicted by virtual work analysis. Therefore the proposed algorithm leads to either *lower bound* or *unique* solutions, which are, of course, better suited for both *linear* as well as *non-linear* design purposes [12, 13, 14]. The essence of the proposed design algorithm is illustrated through the following simple parametric examples.

3.4 Illustrative example I-

The purpose of this exercise is twofold; first, to demonstrate that the proposed set of *simple-closed* form formulae, such as (2a), can be used manually to study both the *linear* and *nonlinear* behavior of closed loop moment frames under lateral loading, and that they provide a wealth of design and performance data that neither elastic nor plastic closed forms of analysis can offer on their own. Currently, there are no exact closed form solutions that can estimate global stiffness degradation due to propagation of plastic hinges, nor the de-stabilizing effects of large axial loads. Second, to demonstrate the applications of the *direct response* methodology to the design of efficient ERMS and to show that the proposed solutions can be used in conjunction with ASD, LRFD, PLS and Performance-Based Elastic-Plastic methods to design moment frames of minimum weight and minimum drift subjected to seismic or wind forces, and that in either case both design level strength and stiffness can be induced rather than investigated. Consider the lateral displacements of the BM of Figure2b, subjected to a monotonically increasing lateral force V and constant axial joint forces P .The preliminary design conditions are prescribed as follows; h=L, $I_l = I$, $I_u = 2I$, $J = 3I$, $M_l^P = M^P$, $M_u^P = 2M^P$, $N^P \geq 2.2M^P$ and $\sum P / P_{cr} = 0.10$.

It is required to establish the values I and M^P such that the drift ratio $\phi_Y \leq 0.015$ radians at first yield, (with safety factor =1), and $\phi_P \leq 0.030$ radians at incipient collapse, (with load factor =1).

Since $\delta_\rho^P = \delta_{\bar{\rho}}^P = 1$ before and up to first yield, then substituting for $f_{cr} = 1 - 0.1 = 0.9$, $\rho = (3IL/2Ih) = 3/2$ and $\bar{\rho} = (3IL/Ih) = 3$ in Eqs. (2d) and (2c), it gives; $\phi_Y = [0.0437V_Y h^2 / EI(f_{cr} = 0.9)] = 0.2039M^P h / EI$, as the maximum elastic drift ratio at first yield, and the beam moments $M_l = [9V_Y h / 42(f_{cr} = 0.9)] = M^P$ and $M_u = [12V_Y h / 42(f_{cr} = 0.9)] = 4M^P / 3 < 2M^P$, caused by the corresponding lateral force $V_Y = 42(f_{cr} = 0.9)M^P / 9h$. Obviously, with F_Y or shear at first yield known, the minimum elastic section modulus I_Y, corresponding to an ASD with pre-specified safety factor, (SF=1), and maximum elastic drift ratio ϕ_Y, can be computed as;

$$I_Y = 0.2039M^P h / \phi_Y E = 0.2039M^P h / 0.015h = 13.5933M^P h / E \tag{6a}$$

Similarly, if desired, appropriate load factors may be used in conjunction with LRFD to compute the corresponding section moduli in terms of F_Y and ϕ_Y. However, since $M_u = 4M^P / 3 < 2M^P$, it must be increased by $(2 - 4/3)M^P = 2M^P / 3$, before reaching $2M^P$. Therefore replacing the partial height *a* with the total height *h,* and the original stiffness K_{gr} with the degraded stiffness K_{hg} in Eq.(2a) or directly using Eq. (2d) it gives, $2M^P / 3 = \partial Vh / 2\bar{f}_{cr}$ or $\partial V = 4\bar{f}_{cr} M^P / 3h$ as the additional shear force needed to cause plastic collapse at V_P. $\bar{f}_{cr} > f_{cr}$ *is the next or second stage load magnifying factor.* Furthermore since $M_l = M^P$ then $\delta_{\bar{\rho}}^P = 0$, consequently $\bar{\rho} = b = 0$, and Eq. (2a) or more directly (2d) yields; $\partial \phi = \partial Vh^2 / 5.14EI\bar{f}_{cr} = 0.2594M^P h / EI$ as the additional drift angle caused by the additional load ∂V i.e., $\phi_P = \phi_E + \partial\phi = 0.4633 M^P h / EI$, therefore;

$$I_P = 0.4633 M^P h / \phi_P E = 0.2039 M^P h / 0.03h = 15.4433 M^P h / E \tag{6b}$$

Since $I_P > I_Y$, then $I = I_P$ governs. With the stiffness degradation ratio $K_{gr} / K_{hg} = 0.1946 / 0.0437 = 4.45$ known, the new magnifying factor $\bar{f}_{cr}$ can be computed as $\bar{f}_{cr} = [1 - 0.1 \times 4.45] = 0.555,$ which in turn allows the plastic limit state or the collapse load to be estimated at $V_P = V_Y + \partial V = 42 f_{cr} M^P / 9h + 4\bar{f}_{cr} M^P / 3h = 4.94 M^P / h,$ as compared with $V_p = 6 f_{cr} M^P / h = 5.4 M^P / h,$ corresponding to $\bar{f}_{cr} = f_{cr} = 0.9$. It is instructive to observe that with $\bar{f}_{cr} = f_{cr} = 1.0$ the ultimate lateral capacity may be computed as $V_P = V_Y + \partial V = 42 M^P / 9h + 4 M^P / 3h = 6 M^P / h.$ The determination of *I* and M^P automatically establishes the values of *J=3I* and $N^P \geq 1.1 \times 2M^P$ .A comparison of the three different values of V_P, i.e. $V_P(P / P_{cr} = 0) = 6 M^P / h,$ $V_P(P / P_{cr} = Constant) = 5.40 M^P / h$ and $V_P(P / P_{cr} = Variable) = 4.94 M^P / h$, clearly demonstrates the possibilities and advantages of using the proposed formulae over conventional methods of computation.

3.5 Design efficiency, minimum weight-minimum drift association

The basic idea behind the proposed algorithm is the consideration that actual structural response is largely a function of system specific characteristics, rather than mathematical analysis, and that unlike gravity loading, seismic input energy is a function of structural properties and design. Therefore it would be reasonable to expect efficient earthquake resisting structures to be designed in such a way as to adsorb the least possible seismic energy while meeting predetermined performance goals. Total input energy is a function of global stiffness and the way it degrades through different phases of diminishing energy absorption. Structural design efficiency, from a materials consumption point of view, may be linked with its strength and stiffness under specified loading conditions. A structure may be said to be efficient or of minimum weight and minimum drift design if its total weight and drift are minimum under non-plastic conditions-(a non-plastic or partially plastic condition is one in which no partial or global failure mechanism can take place.) By the same token, the system may be regarded as highly efficient or of efficient design if its total weight and drift are a minimum at ultimate loading or at incipient plastic collapse. Supposing that for BM of Figure 2b or 3b $a \neq b$, the imposed demand is $[M_u = (6EI_u \theta_u / L)] > [M_l = (6EI_l \theta_l / L)]$, and that the column design moment $N = M_u$, then the linearized non-plastic total weight function of the BM may be expressed as [15, 16];

$$G = C[LM_l + (2h + L)M_u] \tag{7a}$$

Where, *C* is an arbitrary constant of proportionality. Substituting for M_l from Eq. (2f) into Eq. (7a) and rearranging, it gives;

$$G = C[2hM_u + VhL/2] = C[12EhI_u\theta_u + Vh/2] \tag{7b}$$

Now since $[M_u = (6EI_u\theta_u / L)] > [M_l = (6EI_l\theta_l / L)]$, then for a non-zero solution, Eq.(7b) may be studied with respect to the following limiting conditions;

$$Vh/4 \geq [M_l = (6EI_l\theta_l / L)] > 0 \text{ and } Vh/2 > [M_u = (6EI_u\theta_u / L)] \geq Vh/4 \tag{7c}$$

Comparing inequalities (7c) with Eq.(7b), it follows that the total weight of the BM is a minimum when $M_u = M_l = Vh/4$ and/or $I_u\theta_u = I_l\theta_l$, which can be true only when $I_u = I_l = I$ and $\theta_u = \theta_l$, i.e., when *a*=*b*=*h*/2. Equal upper and lower rotations are associated with minimum drift and points of contra-flexure occurring at mid-height [17, 18]. In other words conditions $I_u = I_l$ and $M_u = M_l$ together result in an efficient solution where both total weight and side-sway are a minimum, i.e. the demand-based non plastic total weight and the corresponding drift ratio become;

$$G = 2C(h+L)M_u \text{ and } \phi = \frac{M_R}{24Ef_{cr}}\left[\frac{1}{\bar{k}} + \frac{1}{k}\right] = \frac{V}{Kf_{cr}h} \tag{8d}$$

respectively, where $k = I/L$ and $\bar{k} = J/h$ represent the simplified relative stiffnesses of the beams and columns of the optimized module. This unique moment frame property leads to the simple but important finding that; *the efficient design of the closed loop basic module is one involving beams and columns of equal strength and stiffness.*

The capacity based total weight can now be expressed as, $G_P = 2C(h+L)M^P$. Since G_P is a constant property of the BM and is valid for all $M_u \leq M^P$ or $M_u / M^P \leq 1$, then efficient conditions may be reached when $G_P - G = 0$. In other words*; a design may be said to be efficient when the demand/capacity ratios of all of its members are as close to unity as possible.* Naturally, if the imaginary modules can be merged to form an efficient sub-frame, then the entire assembly could also be looked upon as an efficient ERMF. The methodology used for Example I, is employed without major modifications to design the multi-bay, multi-story ERMF of the illustrative example II below, which has been devised to illustrate the advantages of incorporating doubly symmetric BM in the design of multi-story ERMF.

3.6 Illustrative example II-

Assuming $f_{cr} = 0.9$, compare the total weight and the drift angles of the BM of example I with that of a similar BM with beams of equal average strengths $M^P_{average} = (M^P + 2M^P)/2 = 3M^P/2$ and equal average section inertias,

$I_{average}=(I+2I)/2=3I/2$, at incipient collapse. The total weight of the example BM at incipient collapse may be estimated as; $G=C[2\times1.1hM_u+L(M_u+M_l)]=7.40ChM^P$. Similarly, the total weight of the BM with equal beam strengths may be estimated as; $G=C[2\times1.1hM_u+L(M_u+M_l)]=2C[1.1h+L]M^P_{average}=6.30ChM^P$.

The drift angle of the BM of Example I was computed as $\phi_P=0.4633M^ph/EI$. The corresponding drift angle for the BM with equal beam section inertias can be expressed as, $\phi_P=0.2778M^ph/EI$. Clearly, doubly symmetric basic modules perform more efficiently than their singly symmetric and non symmetric counterparts. The results of this finding are extended to the design of multimember ERMF in the forthcoming sections of this paper.

4. PART II-DEVELOPMENT OF THE ALGORITHM

In the proposed design methodology both strength and stiffness are to be induced rather than investigated with respect to predetermined criteria. What is needed, therefore, is an analytic tool that is based on the findings of Part I and can estimate the elastic-plastic deformations of the subject frame through a single, seamless expression.

4.1 Frame displacements

The generalized drift Eq. (2a), can be modified [19, 20] to represent the drift equation of an imaginary efficient horizontal sub-frame, such as that shown in figure 1b, say at level $i=m$, under a monotonically increasing roof level lateral load, i.e.

$$\partial_s\phi_{m,s}=\frac{\partial_s M_{R.m,s}}{12Ef_{cr.m,s}}\left[\frac{1}{\sum_{j=0}^{n}\bar{k}_{m,j}}+\frac{1}{\sum_{r=1}^{n}\delta_r^{s-1}k_{m,r}}\right]=\frac{\partial_s V_{m,s}}{K_{m,s}h_m} \tag{9a}$$

Where, the term $\sum_{r=1}^{n}\delta_r^{s-1}k_{m,r}$ has been introduced to signify the order of formation of plastic hinges within the beams of the imaginary sub-frame. The numerical vale of "*s*", the number of iterations or different values of *k*, increases with decreasing order of *k,* whence by definition, the smaller the symbol "*s"* the stiffer the beam it represents. In mathematical terms, $\delta_r^{s-1}=1$ for $r>s-1$, and $\delta_r^{s-1}=0$ for $r\le s-1$. In physical terms $\delta_r^{s-1}=0$ for $M_{B.m,s}=M^P_{B.m,s}$ and implies structural damage and/or loss of stiffness with respect to member "i,s". $\delta_r^{s-1}=1$ for $M_{B.m,s}<M^P_{B.m,s}$. $f_{cr.m,s}=[1-(\sum_{j=0}^{n}P_{m,j})/P_{cr.m,s}]$, is the force

magnification function. $\sum_{j=0}^{n} P_{m,j}$ and $P_{cr.m,s} = K_{m,s} h_m$ are the total axial load and the critical axial load of level m at s^{th} response stage respectively. $K_{m,s}$ is the total stiffness of the m^{th} level sub-frame at s^{th} response stage. Symbol ∂_s signifies increment at s^{th} consecutive iteration. The variations of the magnifying factor can be expressed in terms of the stiffness degradation function as;

$$f_{cr.m,s} = [1 - (\frac{K_{m,1}}{K_{m,s}}) \sum_{j=0}^{n} P_{m,j}] \quad (9b)$$

4.2 Rules of proportionality

Next, in order to satisfy the minimum constant drift criterion for all sub-frames of the structure, it would be sufficient to impose the condition $\partial_s \phi_{s,m} = \partial_s \phi_{s,i} = \partial_s \phi_s$ in Eq.(9) for all "i". With "s", $\partial_s \phi_s$ and $\partial M_{R.m,s} = \partial_s V_{m,s} h_m$ known, the response of the m^{th} level sub-frame, in terms of $I_{m,j,s}$ and $M_{m,j,s}$ can be studied for all values of $s \leq n$. These results can then be extended to all other sub-frames, in accordance with the following rules of proportionality;

$$\sum_{j=0}^{n} \bar{k}_{i,j} = (\frac{f_{cr.i,s}}{f_{cr.m.s}})(\frac{M_{R.i,s}}{M_{R.m,s}}) \sum_{j=0}^{n} \bar{k}_{m,j} \text{ and } \sum_{r=1}^{n} \delta_r^{s-1} k_{i,r} = (\frac{f_{cr.i,s}}{f_{cr.m.s}})(\frac{M_{R.i,s}}{M_{R.m,s}}) \sum_{r=0}^{n} \delta_r^{s-1} k_{m,r} \quad (9c)$$

Assuming the moment magnification factor, $f_{cr.i,s}$, remains constant for the same "s", then it may be shown that the most expedient nonzero solution to the pair of Eqs. (9c), in terms of selected relative stiffness factors of the sub-frames, could be expressed as; $\bar{k}_{i,j} = (M_{R.i,s} / M_{R.m,s}) \bar{k}_{m,j}$ and $k_{i,j} = (M_{R.i,s} / M_{R.m,s)}) k_{m,j}$, or more directly in terms of the desired section moments of inertia as;

$$J_{i,j} = (M_{R.i,s} / M_{R.m,s})(h_i / h_m) J_{m,j} \text{ and } I_{i,j} = (M_{R.i,s} / M_{R.m,s}) I_{m,j} \quad (9d)$$

respectively. Similarly, the desired distribution of sub-frame beam and column bending moments, $M_{i,j}$ and $N_{i,j}$ can also be expressed as;

$$N_{i,j} = (M_{R.i,s} / M_{R.m,s}) N_{m,j} \text{ and } M_{i,j} = (M_{R.i,s} / M_{R.m,s}) M_{m,j} \quad (9e)$$

Introducing the proportionality factors $\eta_i = (M_{R.i,s} + M_{R.i+1,s}) / M_{R.m,s}$ and $\mu_i = (M_{R.i,s} / M_{R.m,s})$ the desired section properties of the members of the actual frame can now be established as;

$$\bar{I}_{i,j} = I_{i,j} + I_{i+1,j} = \eta_i I_{m,j} \text{ and } \bar{J}_{i,j} = \mu_i (h_i / h_m)\ J_{m,j} \tag{9f}$$

Similarly, the bending moments of the members of the actual frame can be evaluated as;

$$\overline{M}_{i,j} = M_{i,j} + M_{i+1,j} = \eta_i M_{m,j} \text{ and } \overline{N}_{i,j} = \mu_i N_{m,j} \tag{9g}$$

4.3 Illustrative example III

The purpose of this simple, parametric example is to demonstrate that the group of seemingly complicated Eqs. 9 can result in direct design values without resorting to tedious analysis. Consider the response of a regular (m=4)×(n=5), uniform story height h, moment frame subjected to a uniform distribution of lateral forces $F_i = F_P$ at incipient collapse, with continuous beams of uniform moment of inertia, I_i and plastic moment of resistance M_i^P at each level "*i*". Assuming, $I_{m=4} = I$, $M_{m=4}^P = M^P$ and $\phi_i = \phi$ are known, compute I_i, M_i^P and the ultimate carrying capacity, $F_{P,}$ of the structure. If ϕ_i is constant, then I_i and M_i^P can be computed using Eqs. (9d) and (9e) as multiples of I_m and M_m^P respectively, in proportion with their racking moments $M_{R,i} = V_i h_i$. The story level racking moments can be computed as; $M_{R,4} = F_P h$, $M_{R,3} = 2F_P h$, $M_{R,2} = 3F_P h$ and $M_{R,1} = 4F_P h$. It is instructive to note that $\sum_{i=1}^{m=4} M_{R.i} = 10F_P h$ is equal to the overturning moment of the external forces about the base of the structure. Whence, Eq.(9d) gives for the imaginary sub-frame beam moments of inertia; $I_4 = I$, $I_3 = (2F_P h / F_P h) I_4 = 2I$, $I_2 = (3F_P h / F_P h) I_4 = 3I$ and $I_1 = (4F_P h / F_P h) I_4 = 4I$, consequently the actual design moments of inertia for the beams of the prototype become; $\bar{I}_4 = I_4 + 0 = I$, $\bar{I}_3 = I_4 + I_3 = 3I$, $\bar{I}_2 = I_3 + I_2 = 5I$, $\bar{I}_1 = I_2 + I_1 = 7I$ and $\bar{I}_0 = I_1 + 0 = 4I$. Similarly it can be shown that the plastic moments of resistance of the beams of the imaginary sub-frames are; $M_4^P = M^P$, $M_3^P = 2M^P$, $M_2^P = 3M^P$ and $M_1^P = 4M^P$, based on which the actual design plastic moments of resistance of the beams of the original frame can be computed as; $\overline{M}_4^P = M_4 + 0 = M^P$, $\overline{M}_3^P = M_4^P + M_3^P = 3M^P$, $\overline{M}_2^P = M_3^P + M_2^P = 5M^P$, $\overline{M}_2^P = M_2^P + M_1^P = 7M^P$ and $\overline{M}_0^P = M_1^P + 0 = 4M^P$. Next observing that the roof level sub-frame resists its own racking moment $F_P h$ through formation of $[2\times(2\times 5 = \text{No. of beams})]$ =20=No. of plastic binges, caused by plastic moments M^P, then by static equilibrium $F_P h = 20M^P$, as an indication of the ultimate lateral capacity of the system. Using the principles of the virtual work method of plastic analysis for this particular problem gives;

$$W_{ext.} = \sum_{i=1}^{m=4}(M_{R.i} = V_i h_i) = 10F_P h \text{ and } W_{\text{int.}} = \sum_{i=1}^{m=4} 20M^P \times (M_{R.i} / M_{R,m)} = 20M^P \times 10 \quad (9f)$$

Equating the results of the two work equations, directly verifies the previously obtained solution, $F_P = 20M^P / h$ without resorting to additional computations.

4.4 Sub-frame moments

Eq.(9a) is composed of two components; drift due to beam rotations, $\partial_s \phi_{Beam.m,s}$, and drift due to column bending, $\partial_s \phi_{Col.m,s}$, , i.e.

$$\partial_s \phi_{Beam,m,s} = \frac{\partial_s M_{R.m,s}}{12Ef_{cr.m,s}\sum_{r=1}^{n}\delta_r^{s-1}k_{m,r}} \text{ and } \partial_s \phi_{Col,m,s} = \frac{\partial_s M_{R.m,s}}{12Ef_{cr.m,s}\sum_{j=0}^{n}\bar{k}_{m,j}} \quad (10a)$$

Substituting for $\partial_s \phi_{beam.m,s} = M_{m,j} / 6Ek_{m,j}$ and $\partial_s \phi_{Col.m,s} = N_{m,j} / 12E\bar{k}_{m,j}$ in Eq.(10a) and rearranging, gives the moment redistribution or plasticity progression Equations of the beams and columns of the subject sub-frame at any given response stage as;

$$\partial_s M_{m,j,s} = \frac{\partial_s M_{R.m,s} k_{m,j}}{4f_{cr.m,s}\sum_{r=1}^{n}\delta_r^{s-1}k_{m,r}} \text{ and } \partial_s N_{m,j,s} = \frac{\partial_s M_{R.m,s}\bar{k}_{m,j}}{f_{cr.m,s}\sum_{j=0}^{n}\bar{k}_{m,j}} \quad (10b)$$

respectively. The pair of Eqs.10 are statically consistent at all joints *m,j* of the subject sub-frame, i.e. $\partial_s M_{m,j,s} + \partial_s M_{m,j+1,s} + \partial_s N_{m,j,s} = 0$. The sequential plastic hinge formation, Eq. (11a), of the beams of the m^{th} level sub-frame can be expressed as;

$$\partial_s F_{m,s} = \frac{4M^p}{k_{m,s}h_m} f_{cr.m,s}(1 - \frac{k_{m,s}}{k_{m,s-1}})\sum_{r=1}^{n}\delta_r^{s-1}k_{m,r} \quad (11a)$$

The validity of Eq.(11a) can be verified by comparing the sum of the incremental forces $\partial_s F_{m,s}$, from zero up to incipient collapse, with the collapse load corresponding to $f_{cr.m,s} = 1$, estimated by the virtual work analysis, Eq. (9f) i.e.

$$F_p = \sum_{s=1}^{n}\partial_s F_{m,s} = \frac{4M^p}{h_m}\sum_{s=1}^{n}(\frac{1}{k_{m,s}} - \frac{1}{k_{m,s-1}})\sum_{r=1}^{n}\delta_r^{s-1}k_{m,r} = \frac{4nM^P}{h_m} \quad (11b)$$

Which is in full agreement with the corresponding virtual work result, $F_P = 20M^P/h$, described above. A detailed verification of Eqs. (11a) and (11b) is presented in the Appendix.

4.5 Illustrative example IV

Consider the lateral displacements of the moment frame of Example III, subjected to a monotonically increasing triangular distribution of lateral force $F_i = F(i/m)$ and constant $\sum P_i / P_{cr.i} = 0.10$. The frame geometry is defined as follows; $h_1 = 1.5h$, $h_2 = 1.25h$, $h_3 = h_4 = h$, $L_1 = L = h$, $L_2 = 1.25L$, $L_3 = 1.50L$ and $L4 = 2L$. The 4th level sub-frame column section inertias are defined as $J_0 = J_4 = J = 1.1I$, and $J_1 = J_2 = J_3 = 2J = 2.2I$. The uniform plastic moments of resistance of the beams and columns of the 4th level sub-frame are selected as $M_m^P = M^P$, $N_{m,0}^P = N_{m,4}^P = N^P$ and $N_{m,1}^P = N_{m,2}^P = N_{m,3}^P = 2N^P$ with the provision that, $N^P \geq 1.1`M^P$. It is required to establish the values I and M^P such that the drift ratio $\phi_Y \leq 0.020$ radians at first yield and $\phi_P \leq 0.030$ radians at incipient collapse. Since $k_1 = k_2$, then $s =$ the distinct number of different values of k =4. It follows therefore, that

$$k_1 = k_2 = I/L,\ k_2 = I/1.25L = 0.8I/L,\ k_3 = I/1.50L = 0.6667I/L,\ k_4 = I/2L = 0.5I/L.$$

Table 1: Summary-example IV, numerical solutions of Eqs. (9a),(9b) and (12)

s	**J**	$K_s/(EL/h^3)$	$F_s/(M^P/h)$	$\phi_s/(M^P h/EI)$	$f_{cr.s}$	$f_{cr.s}F_s$	$f_{cr.s}\phi_s$
1	1,2	49.7512	15.8666	0.2583	0.9000	14.28	0.4273
2	3	31.2500	1.9667	0.0509	0.8408	1.65	0.0428
3	4	20.7900	1.1667	0.0455	0.7607	0.89	0.0346
4	5	9.9010	1.0000	0.0818	0.4975	0.50	0.0409
			20.0000	0.4365		17.32	0.5456

The complete elastic-plastic solution of the roof level sub-frame, as well as the governing values of the global drift angle and the ultimate crying capacity of the subject ERMF are presented in table 1 below. The validity and uniqueness of the proposed solution is corroborated by the sum of the monotonically increasing lateral force F_s (last row, 4th column). It can also be seen that stiffness degradation reduces the ultimate carrying capacity from $F_P = 20M^P/h$ to $F_P = 17.32M^P/h$, and increases the drift angle from $\phi_P = 0.4365M^P h/EI$ to $\phi_P = 0.5456M^P h/EI$ - phenomena that can not be detected by conventional closed form solutions. From line s=1 the maximum section inertia corresponding to first yield, $F_Y = F_1$, can be computed as;

$I_Y \geq 0.2583 M^P h / 0.02E = 12.92 M^P / E$ and the corresponding value at incipient collapse, $F_P = F_4$, as; $I_P \geq 0.4365\ M^P h / 0.03\,E = 14.55\,M^P h / E$. Since $I_P > I_Y$ then $I = 14.55 M^p h / E$ and $I = 14.55 M^p h / E.$ With I_m, J_m and M_m^P known, Eqs. (9f) and (9g) can be used to compute the corresponding values for all other sub-frames and the members of the ERMF by direct proportioning. The story level racking moments and the complete parametric design of the subject ERMF is summarized in Table 2 below.

Table 2: Summary-example IV, numerical solutions of Eqs. (9f) and (9g)

i	$M_{R.i} / Fh$	μ_i	η_i	$\bar{I}_i = \eta_i I_m$	$\bar{J}_i = \mu_i h_i J_m / h_m$	M_i^P / M^P	N_i^P / N^P
4	1.0000	1.0000	1.0000	1.0000I	1.1000I	1.0000	1.0000
3	1.7500	1.7500	2.7500	2.7500I	1.9250I	2.7500	1.7500
2	2.8125	2.8125	4.5625	4.5635I	3.8672I	4.5625	2.8125
1	3.3750	3.3750	6.1875	6.1875I	5.5689I	6.1875	3.3750
0	------	-------	3.3750	3.3750I	--------	3.3750	--------

5. CONCLUSIONS

A system-based algorithm has been proposed that results in minimum weight minimum drift moment frames under purely lateral loading. The algorithm is general and may be extended, with some modifications, to the efficient design of other types of earthquake resisting systems such as eccentric and concentric braced frames, special truss moment frames, shear walls, hybrid systems, etc., under pre-selected distributions of lateral loading. The methodology reduces the otherwise complicated task of structural optimization to direct design through basic analysis and recommended rules of general application. The proposed procedures are particularly suited to manual and spreadsheet computations. The proposed algorithm leads to a number of useful design formulae that have not appeared in standard texts and literature. Several generic examples were presented to demonstrate the applications of these formulae. The proposed design formulae provide a wealth of analytic information that may not be readily available through conventional methods of design. The ultimate failure load solutions, thus obtained, are *unique* and suitable for preliminary plastic design treatment in that they include P-Δ and stiffness degradation effects due to monolithically increasing lateral forces satisfy the prescribed yield criteria as well as the boundary support and static equilibrium conditions. The applications of this article are limited in nature to non-slender systems, as the assumption of uniform drift my not be compatible with higher modes of natural vibrations.

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APPENDIX

Substituting for $\partial_1 M_{R.m,s=1} = \partial_1 V_{m,s=1} h_m = \partial_1 F_{m,s=1} h_m = \partial_1 F_{m,1} h$, $\partial_1 M_{.m,s=1} = M^P$ and $\delta_r^{s=1} = \delta_r^{1-1=0} = 1$ in Eq.(11) after some rearrangement, it gives the amount of force needed to produce the first set of plastic hinges in the *stiffest* beams of the m^{th} level sub-frame;

$$\partial_1 F_{m,1} = \frac{4M^p}{k_{m,1} h_m} f_{cr.m,1} \sum_{r=1}^{n} k_{m,r} \tag{12}$$

Next bearing in mind that by virtue of Eq.(12) moments generated in the x^{th} beam ($x>s$) of any level can be expressed in terms of the maximum moments of the stiffest beam of that level i.e. $\partial_s M_{m,s=x} = (k_{m,s=x} / k_{m,s}) M_i^P$ and that the sequence of formation of the plastic hinges of any level is the same as the sequence of decreasing order of stiffness of the beams of the same floor, then the plastic moment of resistance of the stiffest element s=1 and moment of resistance of the next stiffest element s=2 can be computed as M^P and $(k_{m,s=2} / k_{m,s=1}) M^p < M^P$ respectively. Therefore, the balance of bending moment needed to elevate the moment of resistance of beam s=2 to M^p can be computed as $[1-(k_{m,s=2} / k_{m,s=1})] M^p$, whence the amount of additional force required to generate plastic hinges at the ends of the next stiffest beam may be generalized as Eq. (11b).