



CALIBRATION OF RESISTANCE FACTORS FOR TORSIONAL REINFORCED CONCRETE BEAMS STRENGTHENED WITH FRP COMPOSITES

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ABSTRACT

This paper presents a statistical analysis for evaluating the four most common torsional analytical models of concrete beams strengthened with fiber reinforced polymer. For this aim, data are collected from experimental works and the results predicted by four analytical models are compared with the experimental results. The most appropriate analytical model is selected and is used for determining resistance factors conducting reliability analysis. The second purpose is to study load and resistance factor design calibration for the selected model. Two possible target reliability levels have been chosen from the literature and, an iterative procedure is used for resistance factors calculation.

Keywords: Torsional strengthening; Statistical analysis; Load and resistance factor design; Composite materials; Calibration; FRP

1. INTRODUCTION

An increasing number of existing bridges and building structures all over the world are in need of repair and strengthening for some reasons such as deterioration, construction or design error, additional load-bearing structural members and changing of the codes. Application of externally bonded high-strength fiber reinforced polymer (FRP) has become increasingly popular in the last decade. FRP usually consists of glass, aramid and carbon fibers in a polymer matrix and possess light weight, high strength, high resistance to corrosion and ease in handling as compared to traditional material such as steel, wood and concrete, [1, 2]. The uses of external FRP reinforcement may be generally classified as flexural strengthening, improving the confinement and ductility of compression members and shear and torsion strengthening. For application of externally bonded FRP in order to

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strengthen the structures, a number of design codes have been developed for external strengthening of reinforced concrete (RC) structures, [3-5]. Only the FIB code has presented a formula for finding the torsional capacity of FRP strengthened RC beams. During recent decade, several experimental studies have been carried out to the torsional strengthening of RC members using FRP materials as external reinforcement. Also, several analytical models have been proposed for predicting the torsional capacity of concrete beams strengthened with FRP. For implementation in design codes, the most appropriate and accurate analytical model must be selected. For this aim, the predictions of these analytical models are compared with experimental data of beams tested by other researchers. The statistical analysis of the selected existing analytical models will present the essential data to calibrate suitable safety factors that can be offered by future design codes. Currently there are two approaches in codes: allowable stress design and probabilistic-based limit states design. Probabilistic limit states design is typically implemented in the load and resistance factor design (LRFD) format. The philosophy of design according to limit states is based on equilibrium between applied loads and resistance of the structure. The safety margin is the difference between two sides of the equilibrium equation, formulated as a limit state function. Load and resistance parameters involve a considerable degree of uncertainty and can be treated as random variables. It makes possible the use of probability and definition of a safety index to obtain a balance between safety and cost. A review of development of reliability-based design can be found in the literature, [6, 7]. Thus, here LRFD format has been selected for determining resistance factors.

During the last few years, several authors have evaluated design equations of concrete beams in flexural and shear strengthened with FRP reinforcement, [8-10]. Also, several studies have been performed for determining resistance factors of the RC members strengthened with FRP; a number of them are mentioned in the following. The earliest effort for reliability-based design of composite strengthening was conducted by Plevris et al. [11]. Their work concerning flexurally strengthened beams examined the sensitivity of reliability to changes in the design variables and calibrated a set of resistance factors for use in design. Reliability-based design of flexural strengthening has been studied by El-Tawil et al. [12] for determining resistance factors of RC and pre-stressed bridge girders. Pham and Al-Mahaidi have studied the reliability analysis of bridge beams strengthened with fiber reinforced polymers. They recommended that the resistance reduction factor for flexure and intermediate span debond should be taken as 0.6; whereas the factor for end debond is 0.5 [13]. Zheng et al. [14] have assessed the resistance factor in the draft version of Chinese Technical code for the application of fiber reinforced polymers in civil engineering for the shear-resistant design of RC beams with U-wrap FRP strengthening from the probabilistic standpoint. As was mentioned before, recent researches of calibration of resistance factors for concrete beams strengthened with FRP have focused on the flexural and shear capacity. The present paper attempts to study LRFD calibration for concrete beams strengthened with FRP in torsion. LRFD calibration uses reliability analysis to select values of resistance factor such that a target probability of failure is achieved for the limit state function. In the first section of this paper, the statistical analysis of available models is carried out. The aim of such statistical analysis is to determine the most appropriate model for implementation in design codes. The experimental database has been carefully selected from the published

literature. Then, the resistance factors are determined with two possible target reliability levels, $\beta_T = 3.0$ and 3.5 .

2. ANALYTICAL MODELS FOR TORSIONAL OF FRP-STRENGTHENED RC BEAMS

In this paper, four analytical models, the model of FIB design code, the model of Hii and Al-Mahaidi (HA) [15], the model of Chalioris (C) [16] and the model of Ameli and Ronagh (AR) [17] have been considered for statistical analysis. Formulations on the models analyzed in this paper are explained in this section.

2.1 Fib model [4]

The ultimate torsional resistance of RC beams with FRP laminate, T_U , consists of the resistance provided by FRP laminate, T_{frp} , and that provided by RC, T_S , as follows,

$$T_U = T_{frp} + T_S \quad (1)$$

Contribution of FRP to the torsion capacity of the beam, T_{frp} , is,

$$T_{frp} = 2bh \frac{t_{frp} w_{frp}}{s_{frp}} E_{frp} \varepsilon_{frpe} \quad (2)$$

where b and h are the width and the height of the cross section, respectively, t_{frp} is the nominal thickness of one ply of FRP laminate, w_{frp} is the width of FRP strip, s_{frp} is the center-to-center distance of FRP strips, E_{frp} is the elasticity modulus of FRP laminate and ε_{frpe} is the effective strain of FRP laminate. T_S is calculated base upon ACI provisions as follows [18],

$$T_S = 2\phi_s A_0 A_t \frac{f_{yv}}{s_t} \cot \theta \quad (3)$$

where $\phi_s = 0.85$ is the partial safety factor of steel strength, A_0 is the cross sectional area bounded by the center line of the shear flow, A_t is the area of one leg of the transverse steel reinforcement (stirrups), f_{yv} is the yield strength of the transverse steel reinforcement, s_t is the spacing of the stirrups and θ is the angle of torsion crack direction with respect to the horizontal line.

2.2 HA model, [15]

The total torsional strength of FRP-strengthened RC beams can be evaluated by the design codes through superposition of contributions of both steel and FRP reinforcement. The proposed general form is shown as:

$$T_U = T_{frp} + T_S \quad (4)$$

In this method, the FRP contribution to the torsional capacity for a strengthened beam with complete wrap or strip, T_{frp} , is as follows:

$$T_{frp} = 2\varepsilon_{frpe} E_{frp} A_{frp} S_{frp}^{-1} A_n (\cot \theta + \cot \alpha) \sin \alpha \quad (5)$$

where A_{frp} is the area of fiber strip/wrap which is equal to $w_{frp} \cdot t_{frp}$, A_n is the area enclosed by the shear flow path which is equal to $0.85x_{oh}y_{oh}$ (see Fig. 1).

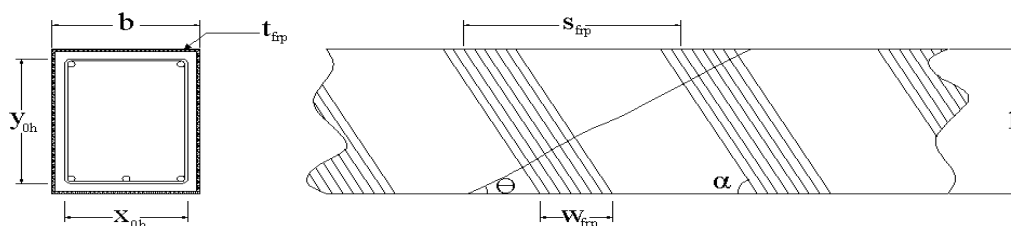


Figure1. Notation for typical torsional strengthening scheme

2.3 C model, [16]

In this model, the calculation of the ultimate torsional moment is based on the well-known softened truss theory. For estimation of ultimate torque strength, the basic equations and considerations of the softened truss model are initially adopted and properly modified to include the influence of FRP. In this approach, the influence of the epoxy-bonded FRP materials as external reinforcement is implemented as an additional component that contributes to the torsional resistance along with the steel reinforcement. So,

$$T_U = 2\sigma_d A_0 t_d \sin \alpha \cos \alpha \quad (6)$$

where σ_d is the principal compressive stress, t_d is the effective thickness of the compression zone in the diagonal compression struts. For calculation of ultimate torsional moment, an efficient trial-and-error algorithm procedure has been developed in this model. The calculation details are described in Ref. [16].

2.4 AR model, [17]

Ameli and Ronagh proposed a model based on compression field theory of Collins and

Mitchell (1980). Using such model, they have presented an analytical method for finding ultimate torsional strength of FRP-strengthened RC beams, T_U , which is as follows:

$$T_U = 2A_0 \sqrt{\frac{\Delta N}{p_0} \left(\frac{A_t f_t}{s_t} + \frac{A_{frp} f_{frp}}{s_{frp}} \right)} \quad (7)$$

where ΔN is the total tension forces, p_0 is the perimeter enclosing gross area of concrete, A_t is the area of stirrup, f_t is stress at stirrup, A_{frp} is the area of FRP, f_{frp} is stress at FRP. For calculation the ultimate torsional moment, an efficient trial-and-error algorithm procedure has been developed in this model. The calculation details are described in Ref. [17].

3. EXPERIMENTAL DATABASE

The four models explained in pervious section have been selected to predict the torsional capacity of concrete beams strengthened with FRP. The validity of these analytical models is checked comparing the models predicted result with the experimental results concerning FRP strengthened RC beams under torsion. The experimental data used herein are obtained from the available literature, [19-25]. It must be noted that parts of the experimental program and result of the torsional beams have already been published by the corresponding author, [26, 27]. Common torsional strengthening configurations include full and strip wrapping along the entire beam, jacketing of three sides and, side bonding on two sides with or without anchorages (Figure 2). In this work, to assemble a consistent database, all beams selected of full and strip wrapping along the entire beam. Geometrical and reinforcement characteristics of the beams are presented in Table 1.

4. EVALUATION OF THE ANALYTICAL MODELS

In the first step of evaluation of the models, the torsional value that obtained from the experimental data base, T_{exp} , is compared to the torsional value predicted by each of the models, T_{pred} . The ratio of experimental torsional strength to predicted results,

$\mu = \frac{T_{exp}}{T_{pred}}$, is calculated for each specimen of the database. Verification of the torsional

models equations is shown in Figures 3–6 by plotting the predicted torsion strength versus the experimental values. In each plot, a straight line with 45° angle (μ) is drawn which indicates the exact accurate prediction. Table 2 summarizes the statistical values of μ ratio.

Minimum (Min) and maximum (Max) values, the average (Avg) (which shows a global safety factor associated with design procedure), standard deviation (Std) and coefficient of

variation (Cov) are also indicated.

Table 2 shows that all four methods give average ratio of μ greater than 1.0. The table also shows that a few differences can be noticed between the analytical models in predicting the torsional strength of concrete beams strengthened with FRP. The average ratio of μ is ranged between 1.08 and 1.21 with a variation range of more than 12% between the most and the least predictions. The largest scatter is obtained by the FIB model (Cov=31%) and the least scattered model is C (Cov=24%). Table 2 shows that the HA model gives the most accurate prediction as the average ratio of μ and Cov is 1.08 and 25%, respectively. These values are adopted as statistical values used to account for the uncertainties in predicting resistance.

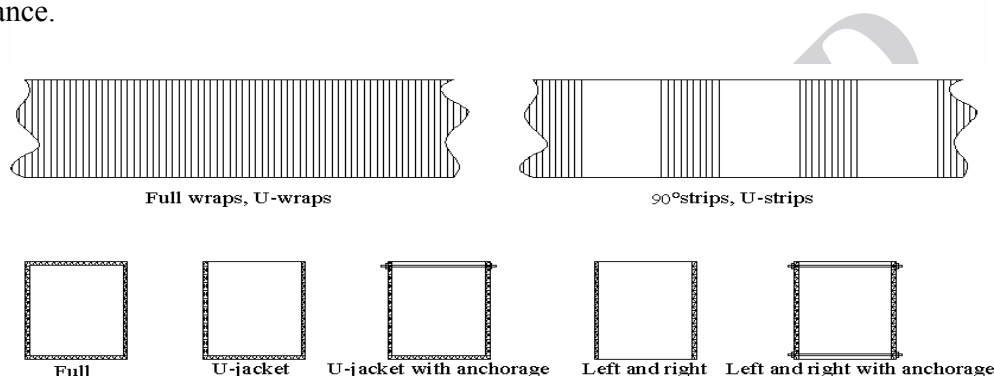


Figure 2. Common strengthening schemes that can be used for torsion

Table 1: Geometrical and reinforcement characteristics of the FRP-strengthened RC beams

Beam ^a	Cross section (mm×mm)	Diameter of stirrups and rebars (mm)	Stirrup and rebar yield stress (MPa)	Stirrup Spacing (mm)	Fiber thickness (mm)	Fiber modulus of elasticity (MPa)	Configuration of FRP ^b
H(F1)	140×260	6, 12	450, 600	120	0.176	240,000	Carbon - W-S [w f/s f = 0.26]
H(F2)	140×260	6, 12	450, 600	120	0.176	240,000	Carbon - W-S [w f/s f = 0.38]
G(C1)	150×350	6.35, 11.3, 16	409, 457	70	0.165	240,000	Carbon -W-C
G(C2)	150×350	6.35, 11.3, 16	409, 457	70	0.165	240,000	Carbon - W-S [w f/s f = 0.5]
G(C4)	150×350	6.35, 11.3, 16	409, 457	70	0.165	240,000	Carbon - W-S [w f/s f = 0.67]
G(C5)	150×350	6.35, 11.3, 16	409, 457	70	0.165	240,000	Carbon - W-S [w f/s f = 0.4]
G(G1)	150×350	6.35, 11.3, 16	409, 457	70	0.154	73,000	Glass -W-C
G(G2)	150×350	6.35, 11.3, 16	409, 457	70	0.154	73,000	Carbon - W-S [w f/s f = 0.5]
Z(L4)	150×250	6.5, 10	256, 446	120	0.111	235,000	Carbon - W-S [w f/s f = 0.5]
Z(L5)	150×250	6.5, 10	256, 446	120	0.111	235,000	Carbon - W-S [w f/s f = 0.5]
Z(L6)	150×250	6.5, 10	256, 446	120	0.111	235,000	Carbon - W-S [w f/s f = 0.4]
Z(L7)	150×250	6.5, 10	256, 446	120	0.111	235,000	Carbon - W-S [w f/s f = 0.4]
A(CFE)	150×350	6, 16	251, 502	80	0.165	243,789	Carbon -W-C

A(CFE2)	150×350	6, 16	251, 502	80	0.165	243,789	Carbon -W-C
A(CFS)	150×350	6, 16	251, 502	80	0.165	243,789	Carbon - W-S [w f/s f = 0.5]
A(GFE)	150×350	6, 16	251, 502	80	0.154	72,723	Glass -W-C
A(GFE2)	150×350	6, 16	251, 502	80	0.154	72,723	Glass - W-S [w f/s f = 0.5]
A(GFS)	150×350	6, 16	251, 502	80	0.154	72,723	Glass -W-S
P(A90S4)	279×279	9.53, 9.53, 12.7	420, 460	152.4	0.353	72,000	Glass - W-S [w f/s f = 0.5]
P(A90W4)	279×279	9.53, 9.53, 12.7	420, 460	152.4	0.353	72,000	Glass -W-C
C(RaS-FS150(2))	100×200	5.5, 8	350, 560	130	0.11	230,000	Carbon - W-S [w f/s f = 0.5]
C(RbS-FS200(1))	150×300	5.5, 8	350, 560	100	0.11	230,000	Carbon - W-S [w f/s f = 0.5]
M(ACW1)	150×350	8, 10	480, 352	80	0.176	240,000	Carbon -W-C
M(ACW2)	150×350	8, 10	480, 352	80	0.176	240,000	Carbon -W-C
M(BCW1(1))	150×350	8, 14	480, 397	80	0.176	240,000	Carbon -W-C
M(BCW1(2))	150×350	8, 14	480, 397	80	0.176	240,000	Carbon -W-C
M(BCW1(3))	150×350	8, 14	480, 397	80	0.176	240,000	Carbon -W-C
M(BCS1)	150×350	8, 14	480, 397	80	0.176	240,000	Carbon - W-S [w f/s f = 0.5]
M(BCS2)	150×350	8, 14	480, 397	80	0.176	240,000	Carbon - W-S [w f/s f = 0.5]
M(ACW2)	150×350	8,10	480, 352	80	0.176	240,000	Carbon -W-C
M(BCW2)	150×350	8, 14	480, 397	80	0.176	240,000	Carbon -W-C
M(CCW2)	150×350	10, 16	352, 404	80	0.176	240,000	Carbon -W-C

^a A=Ameli et al. [19], C=Chalioris [20], G=Ghobarah et al. [21], H=Hii and Al-Mahaidi [22], M=Mohammadizadeh and Fadaee [23], P=Panchacharam and Belarbi [24], Z=Zhang et al. [25].

^b W: Wrapping; C: Continuous sheets; S: Strips [width wf at spacing sf (in mm)].

Table 2: Statistical values of μ factor

μ	Min	Max	Avg	Std	Cov (%)
Fib model	0.8	1.72	1.21	0.38	31
HA model	0.65	1.99	1.08	0.27	25
C model	0.74	1.52	1.2	0.29	24
AR model	0.65	1.62	1.16	0.31	27

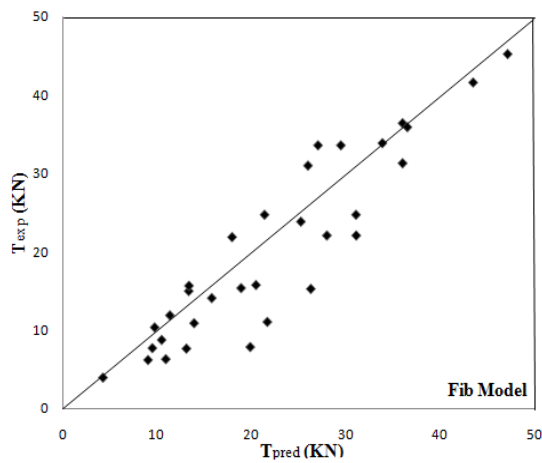


Figure 3. Predictions of FIB model for

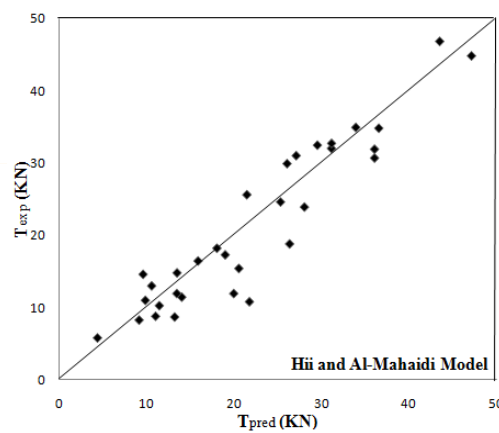


Figure 4. Predictions of HA model for

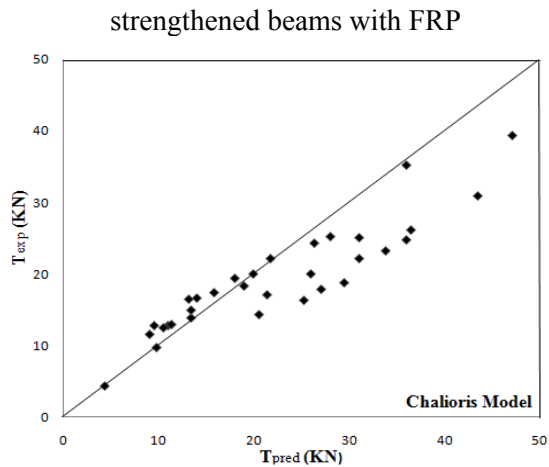


Figure 5. Predictions of C model for strengthened beams with FRP

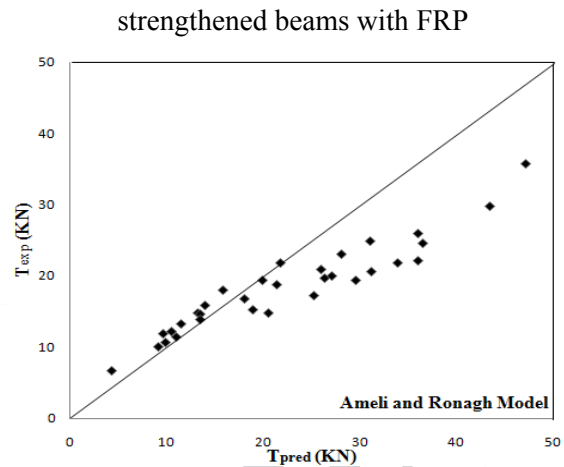


Figure 6. Predictions of AR model for strengthened beams with FRP

The capacity of a strengthened beam is based on the adopted FRP configuration. The torsional value is predicted for each kind of configuration. Full wrapping along the entire beam with one layer (FW1), strip wrapping along the entire beam with one layer (SW1) and full or strip wrapping along the entire beam with more than one layer (FSW) are considered for predictive performance analytical models and compared with all databases (AD). Figures 7-10 show variation of μ with the FRP configuration. Horizontal dashed line presents the mean of μ corresponds to the AD and the vertical dashed lines present Min and Max values of μ correspond to each configuration. Also, the sizes of the black areas show the number of the specimens. Figure 8 shows that the FW1, SW1 and FSW configurations have close values for the mean value of μ in HA model. Figure 10 exhibits the FW1, SW1 and FSW configurations different values for the mean value of μ in AR model.

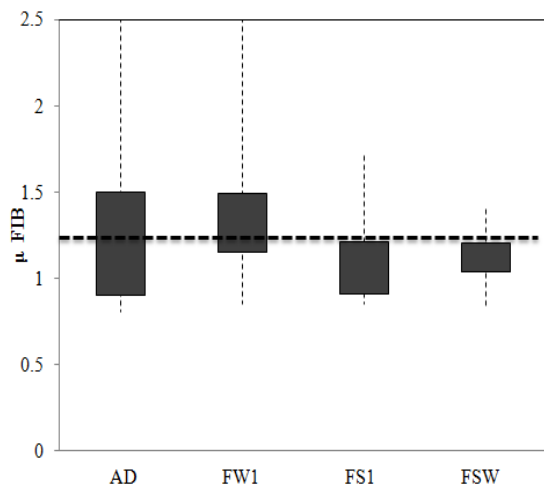


Figure 7. Variation of μ with FRP

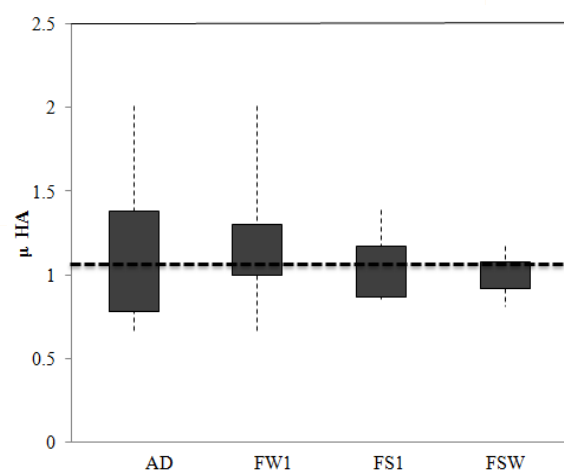


Figure 8. Variation of μ with FRP

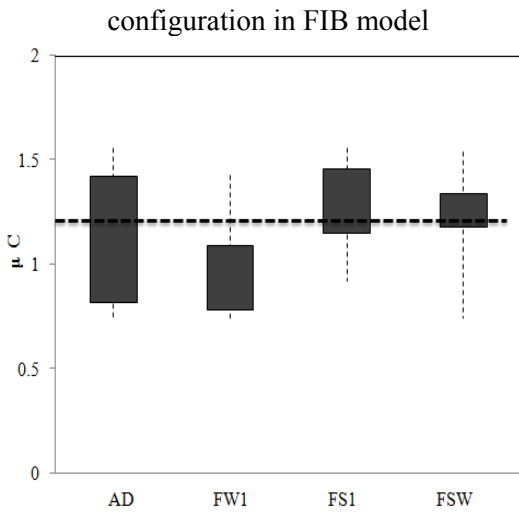


Figure 9. Variation of μ with FRP configuration in C model

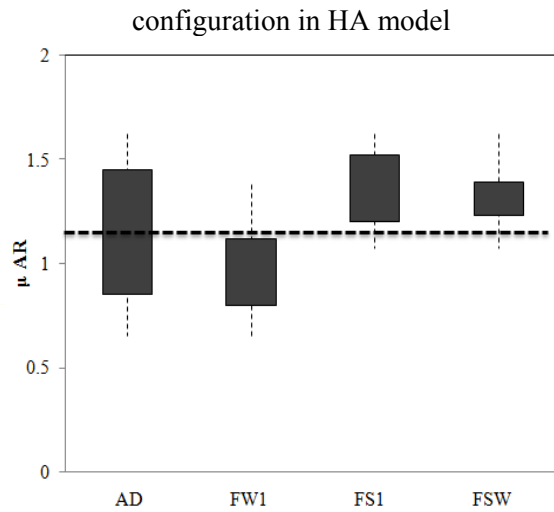


Figure 10. Variation of μ with FRP configuration in AR model

5. CALIBRATION PROCEDURE

One of the primary objectives of this paper is calibration of resistance factors for the best analytical model, HA, in LRFD format which is a relatively new format in structural engineering. The fundamental function in LRFD can be expressed as,

$$\phi(R_d) \geq \sum \gamma Q_d \tag{8}$$

where ϕ is the resistance factor, R_d is the factored resistance which is equal to $\mu \cdot T_{UHA}$ in which T_{UHA} is the ultimate torsional resistance of RC beams with FRP laminate resulted from HA model, γ is the load factor, Q_d is the maximum of combination of factored dead and live load effects expressed by the following equation,

$$Q_d = \gamma_D Q_D + \gamma_L Q_L \tag{9}$$

where Q_D is the characteristic load effects caused by dead load, Q_L is the characteristic load effects caused by live load, γ_D is the partial safety factor of dead load, γ_L is the partial safety factor of live load.

LRFD calibration uses reliability analysis to calculate the values of the resistance factor based on available statistical data. For this aim, the first order-second moment reliability method is applied to calculate reliability index, β . For calculation of β , the statistical characteristics of the dead and live loads and design variables have been considered, then,

the average reliability indexes for different load effect ratios ($\lambda = \frac{Q_L}{Q_D}$), i.e., 0.25, 0.5, 1, 1.5,

2, 2.5 in strengthened beams have been found. In order to make the evaluation general, two specimen groups, i.e. A and B, are selected. The value of random variables for groups A and B are adopted from Ref. [21] and Ref. [22], respectively. Load effect ratio, λ , has a significant influence on reliability level, as shown in Figures 11 and 12. Averaging all reliability indexes gives the global average reliability indexes of 2.87, 2.69 for groups A and B, respectively. So, averaging reliability index for group A and group B result in 2.78. In the present work, FERUM software is used for doing such calculations. Currently, FERUM is located at the internet address <http://www.ce.berkeley.-edu/FERUM>. The statistical properties including distribution type, mean, bias (mean/nominal) and coefficient of variation (Cov) of the random variables are required for the calculation of reliability analysis. Table 3 lists the statistical properties found in the literature.

The basic idea of the procedure is to select resistance factor based on minimizing the difference between the reliability index, β , and the target reliability, β_T . The least-squares average, $LSA = n^{-1} \sum_i (\beta_i - \beta_T)^2$, is used for getting the best resistance factor. Two different target reliability levels, $\beta_T = 3.0$ and 3.5 have been considered. These two levels have been chosen from the literatures, $\beta_T = 3$ from [28] and $\beta_T = 3.5$ from [1], to represent the range of target reliability indexes. An iterative procedure is used for calculation of LSA based on assumed values of the resistance factors. In this paper, first, the value of ϕ has been considered 0.5 to 1 in increments of 0.1, and then, the minimum value of LSA is adopted. The LSA is plotted as a function of ϕ in Figures 13 and 14. The lowest point on the relationship curve is the optimum point that would maintain the smallest error for all cases.

Table 3: Statistical Properties of Variables

variables	Groups name	Nominal value	Mean/Nominal	Coefficient of Variation	Probability Distribution
b(mm)	A	150	1	0.03	Normal [29]
	B	140			
h(mm)	A	350	1	0.03	Normal [29]
	B	260			
$W_{frp}(mm)$	A	100	1	0.02	Normal [14]
	B	50			
$s_{frp}(mm)$	A	100	1	0.02	Normal [14]
	B	195			
$E_{frp}(MPa)$	A	235000	1.1	0.12	Weibull [14]
	B	240000			
$t_{frp}(mm)$	A	0.165	1.02	0.05	Lognormal [1]
	B	0.176			
Q_D	A	1.2	1.05	0.1	Normal [6,30]
	B				
Q_L	A	1.6	1	0.25	Extreme 1 [6,30]
	B				
μ	A	1.08	1	0.25	Normal

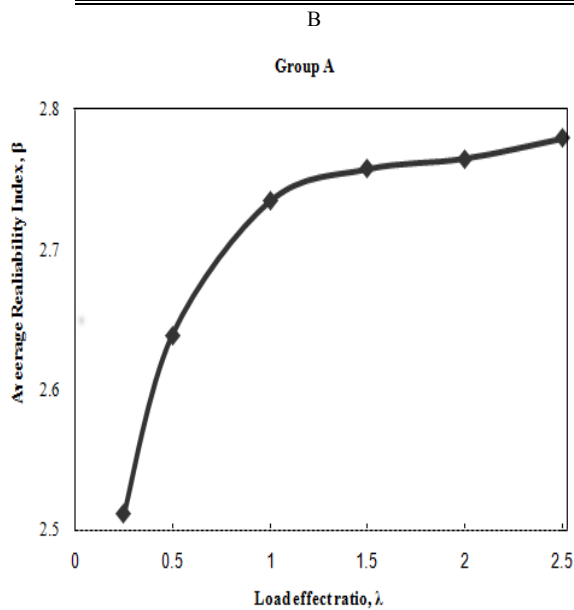


Figure 11. Load Effect Ratio for group A

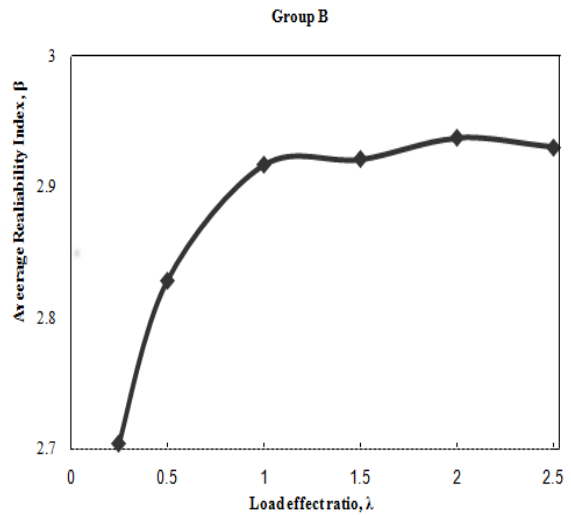


Figure 12. Load Effect Ratio for group B

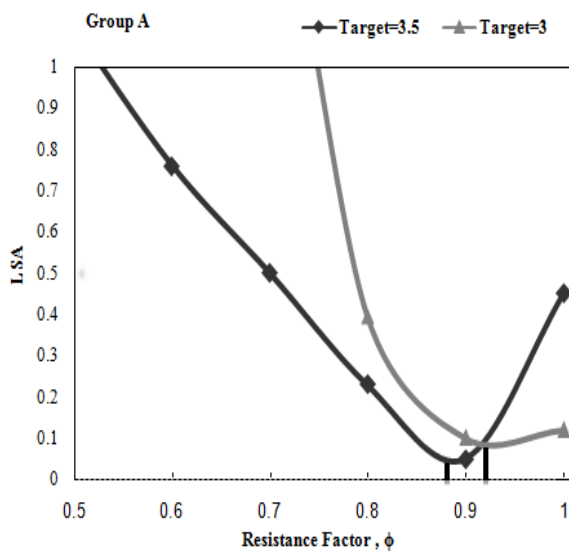


Figure13. Least-squares analysis to determine ϕ for group A

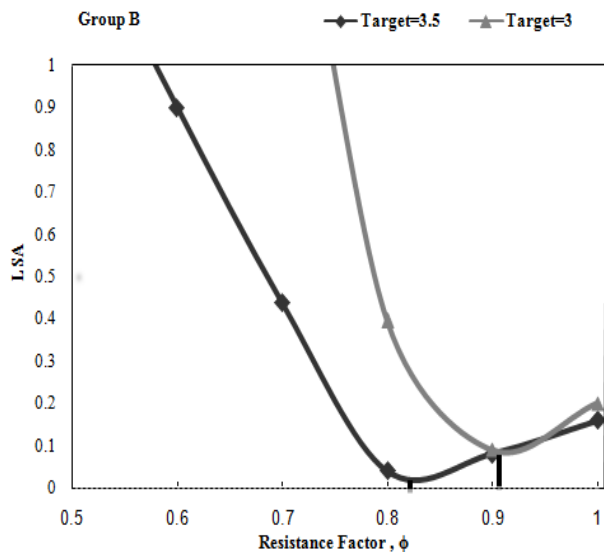


Figure14. Least-squares analysis to determine ϕ for group B

Considering target reliability level $\beta_T = 3.0$, ϕ would be equal to 0.92 and 0.91 for A and B groups, respectively, and considering target reliability level $\beta_T = 3.5$, ϕ would be equal to 0.88 and 0.825 for A and B groups, respectively. Calculations show that ϕ for the beam should be approximately 0.915 and 0.8525 for target $\beta_T = 3.0$ and 3.5, respectively.

6. CONCLUSION

This paper has conducted a statistical analysis to evaluate the four most common torsional analytical models for concrete beams strengthened with FRP. An experimental database available in the literature has been applied to find the best analytical model. Resistance factors for use in a LRFD format are developed for the best analytical model of concrete beams strengthened with FRP in torsion. Resistance factor, ϕ , has been calibrated for two different target reliability levels. Several conclusions can be drawn through such assessment as follows:

1. Four analytical models including the model of FIB design code, HA model, C model and AR model have been considered for statistical analysis. The validity of these analytical models is checked through comparing the results predicted by the models with the experimental results for RC beams under torsion strengthened with FRP. From a statistical point of view, the HA model seems to be the best analytical models since it provides a proper accurate prediction with the average ratio of $\mu = 1.08$ and $Cov = 25\%$.

2. A reliability analysis has been also carried out to calculate reliability index, β . Load effect ratio, λ , has a significant influence on the reliability level for strengthened beams. Therefore, reliability indexes are calculated for different load effect ratios, i.e., 0.25, 0.5, 1, 1.5, 2, 2.5. The results indicate that the average reliability index, β , increases when λ increases from 0.25 to 2.5. It is concluded that global average reliability index for strengthened beams with FRP is 2.78. So, HA analytical model seems to be unconservative to some extent.

3. A framework is proposed for the application of LRFD to torsional capacity of analytical models for concrete beams strengthened with externally-bonded of FRP. Resistance factors are determined based on target reliability levels. Two different target reliability levels, $\beta_T = 3.0$ and 3.5, have been considered. Calculations show that ϕ for beam should be approximately 0.915 and 0.8525 for target $\beta_T = 3.0$ and 3.5, respectively. For simplicity, in design practice, $\phi = 0.90$ and 0.85 are suggested to be used.

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