A FAST HYBRID ALGORITHM FOR NONLINEAR ANALYSIS OF STRUCTURES

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ABSTRACT

The efficiency of the Newton-Raphson iteration method for solving nonlinear equations has made it popular, although the time required to achieve convergence inspires aspirations to find a more efficient alternative. In the current study a hybrid iterative algorithm is employed for solving nonlinear problems. To that effect, an alternative to the Newton-Raphson method, and related classical methods in numerical computing based on a Homotopy Perturbation Method (HPM) is introduced. In perturbation methods, perturbation quantities are used to replace a nonlinear problem by a number of manageable linear sub-problems. Then, an approximate solution is reached by summing up the results of these sub-problems. In this paper three global methods belonging to this family are discussed and then it is shown how to combine a global method with Newton-Raphson method into a hybrid algorithm as a possible way to reduce computational cost. Several well-known and difficult applications are considered for testing the performance of the new approach. The results reveal that using 2nd HPM coupled with two-point method requires less time to achieve convergence and reduces the total number of iterations.

1. INTRODUCTION

Second-order nonlinear analysis of structures has been studied extensively over the past few decades and is referred in modern design codes of practice such as the American Load and Resistance Factor Design (LRFD) specification [1] and the British Standard 5950 [2]. The numerical solution algorithms constitute one of the most important aspects in the nonlinear

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analysis of structures. Among these methods, the Newton-Raphson method is most widely used. Against the advantage of efficiency offered by this technique must be mentioned the relatively long time taken to achieve the necessary convergence Baltoz and Dhatt [3].

In recent decades, several applications of nonlinear structural analysis have been presented ([4,5]). Greco et al. [6] proposed a new formulation for geometrically nonlinear analysis of space trusses. Kassimali and Abbasnia [7] proposed a method for large deformation and stability analysis of elastic space frames based on Eulerian formulation. Based on a path-following approach, Saffari et al. [8] used a normal flow algorithm to pass limit points in nonlinear analysis of trusses.

A range of nonlinear approaches to space truss analysis can be found in the literature: some use an iterative method, some an incremental method and others a simple incremental-iterative method. Recently, a new approach to accelerate the nonlinear analysis of structures with low computational cost has been proposed ([9,10,11]). A concept to accelerate the trend in nonlinear analysis and aimed to gain the ability for analysis of complex structures has been introduced by Saffari et al. [12]. A new approach for nonlinear analysis of structures, which accelerates the convergence rate, has been introduced in references [13,14]. They employed a mathematical method, namely two–point method, to achieve the convergence state. Another mathematical technique namely homotopy perturbation method (HPM) has been applied to plane frames by Saffari et al. [15]; due to the complexity of the homotopy perturbation series they used only the first term of this series.

In the current paper, the homotopy perturbation series is first simplified in order to include also higher-order terms. Then, the number of most efficient terms of the homotopy series is found. Finally, the effective terms are included in the algorithm and are combined with two-point method. As can be seen in the numerical example section, the present method can be very effective in increasing the speed of convergence and in reducing the number of iterations.

2. NONLINEAR ANALYSIS OF STRUCTURES

In the following, large deflection inelastic analyses of structures including both geometric and material nonlinearities are briefly discussed. This is then is followed by a detailed description of the concept developed in this study.

2.1 Member behavior

2.1.1 Truss element

The accuracy in the prediction of the inelastic response of structures depends on the accuracy of the member's load–displacement relationship used in the analysis. A number of models have been introduced in the literature to predict the nonlinear behavior of space trusses. A stress–strain relationship proposed by Hill et al. [16] is adopted here to predict the inelastic post–buckling behavior of trusses, as follows. A force–strain curve (Q–u/L) assumed applicable for steel material both in tension and compression states is shown in Figure 1.
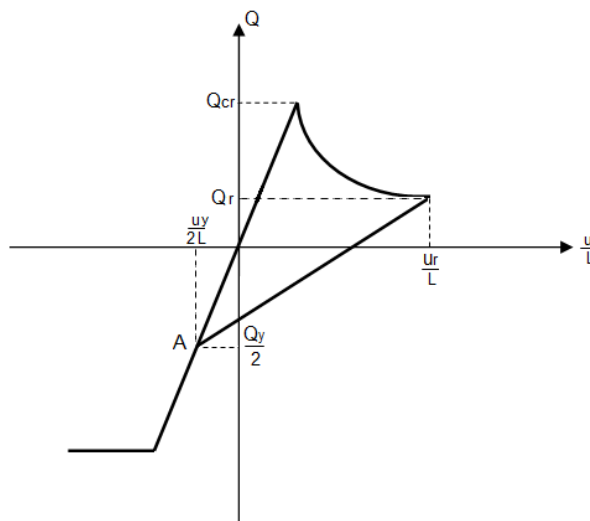
Figure 1. Assumed force-deformation curve for loading and unloading path

The relationship between load-displacement can be expressed by the following relations:
For elastic material:

$$Q = \frac{AE}{L} u \tag{1}$$

For inelastic material:
members in tension

$$Q = \begin{cases} \dfrac{AE}{L} u & for \ |u| < u_y \\ AF_y & for \ |u| \geq u_y \end{cases} \tag{2}$$

where F_y denotes yield stress and $u_y = F_y L / E$.
b) members in compression:

$$Q = \begin{cases} \dfrac{AE}{L} u & for \ u < u_{cr} \\ Q_l + (Q_{cr} - Q_l) e^{[-(X_1 + X_2 \sqrt{u'/L})u'/L]} & for \ u \geq u_{cr} \end{cases} \tag{3}$$

Here $Q_{cr} = \pi^2 EI / L^2$ (I=weak axis moment of inertia) and Q_r is the asymptotic lower stress limit and is defined as $Q_r = rQ_{cr}$. The corresponding critical buckling displacement is $u_{cr} = Q_{cr}L / (AE)$ while u' is defined as $u' = u - u_{cr}$. Parameters X_1 and X_2 are constants depending on the slenderness ratio of the compressive members.

It should be noted that when a member is in compression state and $u \geq u_{cr}$, the tangent modulus, E_t, has to be used instead of E. The tangent modulus is obtained as [12]:

$$E_t = -\frac{1}{A}(Q_{cr} - Q_l).e^{[-(X_1 + X_2\sqrt{u'/L})u'/L]} \cdot (X_1 + \frac{3}{2}X_2\sqrt{u'/L}) \tag{4}$$

As the objective here is to evaluate numerical solution algorithms, the effects of imperfections are not considered. Moreover, sufficiently slender members are assumed, so that and material nonlinearity can be neglected in the pre-buckling stage.

2.1.2 Frame element

A perfectly plastic material associated with plastic hinge concept is used in this study to consider material non-linearity effect. In an elastic perfectly-plastic material, the effects of strain hardening are disregarded. This further implies that once the yield moment M_p is reached, the material yields and cannot withstand further stress.

It is noted that the yield moment is commonly defined by a yield criterion. A variety of yield criteria defining the yield moment have been introduced in structural engineering. In this paper, the AISC-LRFD criterion considering bending moment and axial force interaction is used for steel elements, according to which:

$$M_{pc} = \begin{cases} M_p\left(1 - \frac{|Q|}{2Q_y}\right) & for \quad \frac{|Q|}{Q_y} < 0.2 \\ \frac{9}{8}M_p\left(1 - \frac{|Q|}{Q_y}\right) & for \quad \frac{|Q|}{Q_y} \geq 0.2 \end{cases} \tag{5}$$

where M_p is the full plastic moment capacity of the cross-section in the absence of axial force, equal to ZF_y, Z stands for plastic modulus, M_{pc} represents reduced plastic moment capacity in the presence of axial force Q and $Q_y = AF_y$ where F_y denotes yield stress.
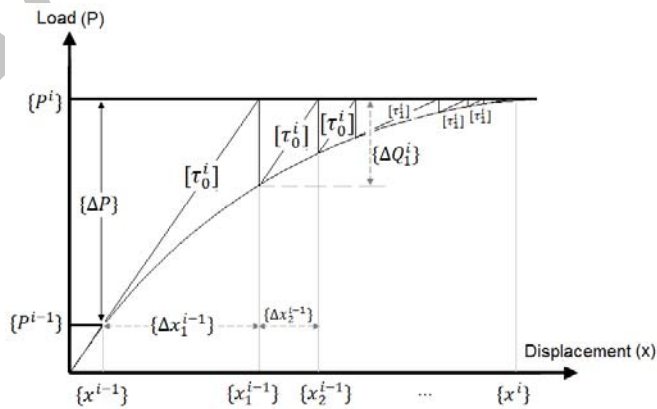
3. NONLINEAR ANALYSIS ALGORITHM

The Newton-Raphson method is one of the most popular iterative methods for solving nonlinear equations. Via this method, an approximate solution is estimated, and then an unknown value is added as a corrector value to improve the initial solution. This procedure is illustrated in Figure 2 (a).
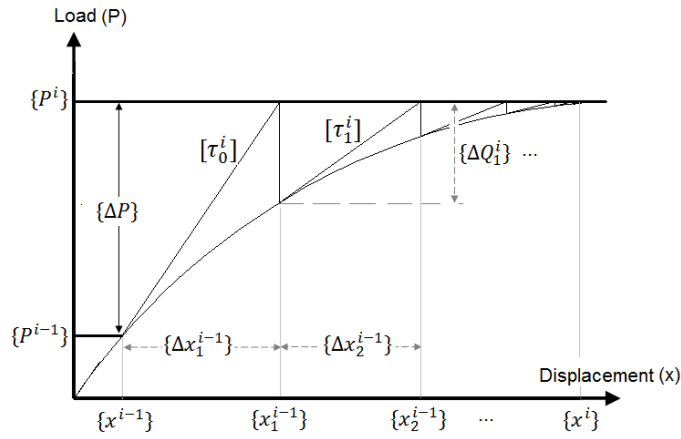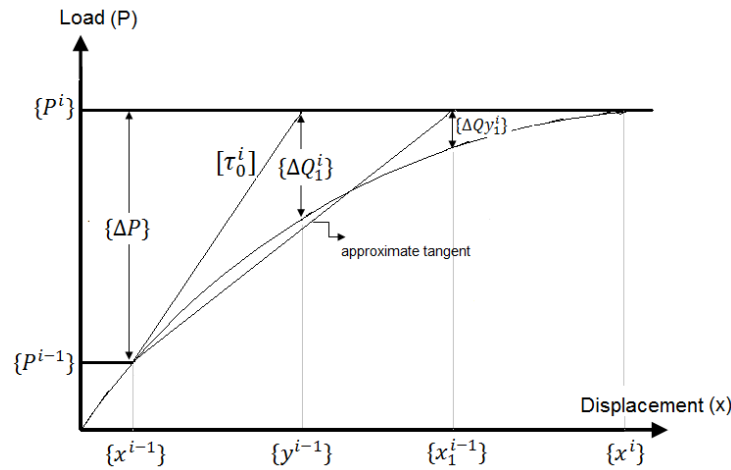
(a) Newton-Raphson method
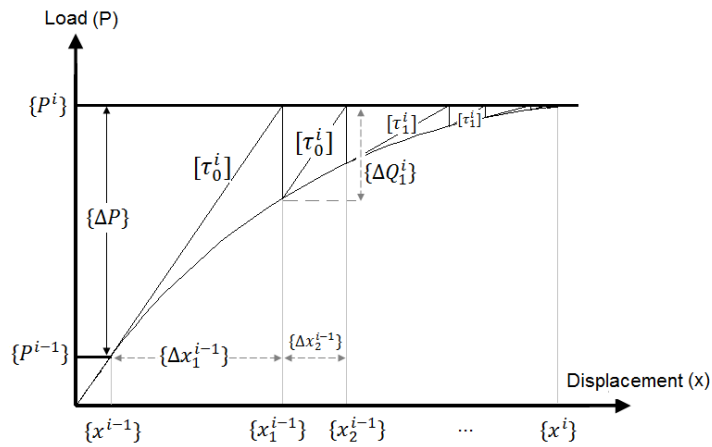
(b) 1st HPM

(c) 2nd HPM

(d) Two-point method
Figure 2. Iterative methods

Since formation of the tangent stiffness matrix is a time-consuming process, some algorithms are combined together to accelerate the convergence rate. Such combinations are described in the following.

3.1 Modified Homotopy Perturbation Method (HPM) for nonlinear problems

In the current paper, the homotopy perturbation series is first simplified in order to include also higher-order terms. Then, the number of most efficient terms of the homotopy series is found. These sets of equations are then solved iteratively.

This paper develops and improves the methodology which has been presented in reference Saffari et al. [15] for application to structural engineering problems. A schematic representation of this method is illustrated in Figure 2 (b). In the current paper, the homotopy perturbation series is first simplified in order to include also higher-order terms. Then, the number of most efficient terms of the homotopy series is found.

Consider a series of n nonlinear equations $\Delta Q_{1(x)} = 0, \Delta Q_{2(x)} = 0,....., \Delta Q_{n(x)} = 0$ with n unknown variables $x_1, x_2,......, x_n$ as follows:

$$\{\Delta Q_{(x)}\} = \{P\} - \{f_{(x)}\} = 0 \tag{6}$$

in which $\{P\}$ and $\{f_{(x)}\}$ external and resultant internal forces respectively. If, respectively, x_n and q are the exact solution of the system of equations and an initial guess, then using Taylor series expansion, Eqn 6 can be written as:

$$\{\Delta Q_{(x)}\} = \{\Delta Q_{(q)}\} - [\tau](\{x\} - \{q\}) + \{R_{(x)}\} = 0 \tag{7}$$

where $[\tau]$ is the system tangent stiffness matrix or $-[\tau]$ is Jacobin matrix of $\{\Delta Q_{(x)}\}$:

$$\tau_{ij} = -\frac{\partial \Delta Q_i}{\partial x_j} = \frac{\partial f_i}{\partial x_j} \tag{8}$$

Thus, $\{R_{(x)}\}$ is defined as follows:

$$\{R_{(x)}\} = \{\Delta Q_{(x)}\} - \{\Delta Q_{(q)}\} + [\tau](\{x\} - \{q\}) \tag{9}$$

Then $\{x\}$ can be given by solving Eqn 7:

$$\{x\} = \{q\} + [\tau]^{-1}\left(\{\Delta Q_{(q)}\} + \{R_{(x)}\}\right) \tag{10}$$

To reach approximate solution of $\{x\}$ at first, homotopy is shaped:

$$H(\bar{x}, p) = \{\bar{x}\} - \{q\} + [\tau_{(q)}]^{-1}\left(\{\Delta Q_{(q)}\} + p\{R(\bar{x})\}\right) = 0 \tag{11}$$

In recent situation $\{\bar{x}\}$ will be the same $\{x_n\}$. In reference He [17] had been shown that:

$$\{\bar{x}\} = \{x_0\} + p\{x_1\} + p^2\{x_2\} + \dots \tag{12}$$

Substituting Eqn 12 into Eqn. 11 then using Taylor series of $\{G(x)\}$ around $\{x_0\}$:

$$\begin{aligned}
\{x_0\} + p\{x_1\} + p^2\{x_2\} + p^3\{x_3\}\dots = \\
= \{q\} - [\tau_{(q)}]^{-1}\{\Delta Q(q)\} - p[\tau_{(q)}]^{-1}\left(R(x_0) + p[\tau_{(x_0)}]\{x_1\} + p^2[\tau_{(x_0)}]\{x_2\} + p^3[\tau_{(x_0)}]\{x_3\} + \dots\right) = 0
\end{aligned} \tag{13}$$

By equating the terms with power p, we have:

$$\{x_0\} = q + [\tau_{(q)}]^{-1}\{\Delta Q_{(q)}\} \tag{14}$$

$$\{x_1\} = [\tau_{(q)}]^{-1}\{\Delta Q_{(x_0)}\} \tag{15}$$

Neglecting high derivatives terms of Taylor series of $\{R(x)\}$ around $\{x_0\}$ then $\{x_2\}$, $\{x_3\}$, $\{x_4\}$, … can be estimated as follows:

$$\{x_2\} \approx [\tau_{(q)}]^{-1}\left([\tau_{(x_0)}]\{x_1\}\right) \tag{16}$$

$$\{x_3\} \approx [\tau_{(q)}]^{-1} ([\tau_{(x_0)}]\{x_2\}) \tag{17}$$

$$\{x_4\} \approx [\tau_{(q)}]^{-1} ([\tau_{(x_0)}]\{x_3\}) \tag{18}$$

Using above approximation higher order of homotopy series can be available and then it can be applied to nonlinear analysis process. Graphically, the proposed method is shown in Figure 2 (c).

3.2 Two-point technique

It is possible to accelerate the convergence rate of analysis using an approach which has been recently proposed by Saffari and Mansouri [14]. Briefly, in this method, displacements are updated as follows:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f\left(x_n - \frac{f(x_n)}{f'(x_n)}\right)}{f'(x_n)} g(t_n) \tag{19}$$

in which:

$$g(t) = \frac{t^3 + 2t + 1}{1 - t^2} \tag{20}$$

and parameter t is explained in detail in reference Saffari and Mansouri [14]. The graphical representation of the method is indicated in Figure 2 (d).

3.3 Implementation of hybrid methods in structural engineering

Combining the simplified homotopy perturbation method and two-point method produces the following formulation to estimate the displacement vector $\{x_n\}$:

$$\{y_n\} \approx [\tau_{(x_n)}]^{-1} ([\tau_{(x_0)}]\{x_n\}) \tag{21}$$

$$\{x_{n+1}\} \approx \{y_n\} + [\tau_{(x_n)}]^{-1} ([\tau_{(x_0)}]\{y_n\}) g(t) \tag{22}$$

4. NORMAL FLOW ALGORITHM

The Newton–Raphson method is one powerful approach to evaluate the response of a structure to a set of successive loads. However, this method diverges when the solution is close to limit point [18]. As mentioned earlier, in this research normal flow algorithm is used to trace the equilibrium path. If i is the number of the step, j is the number of the iteration, and the total load on the structure is $\{P\}_i^j$, or equivalently, the product of a total ratio λ_i^j and a given reference external load $\{P_{ref}\}$, applied through a series of load increments.

Mathematically, this is written as:

$$\{P\}_i^j = \lambda_i^j \{P_{ref}\} \tag{23}$$

The method of normal flow algorithm is schematically presented in Figure 3 and a detailed discussion is provided in reference Saffari et al. [8].
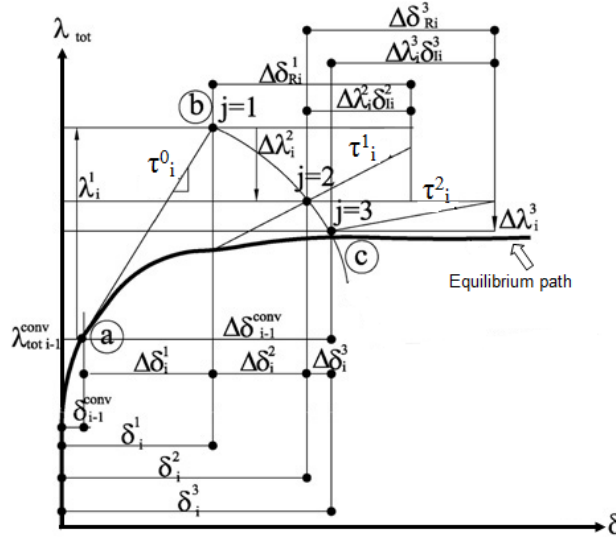

Figure 3. The modifying iterative steps in the normal flow algorithm

In this paper, a modified Euclidian convergence criterion is adopted for displacement control which is defined Saffari et. [8].

A direct method of updating is adopted, such that the load increment is related to the number of iterations. The sign of the determinant of the tangential stiffness matrix of the previous step can be computed through the following relationship:

$$\lambda_{i+1}^j = \pm \lambda_i^j \left(\frac{J_D}{J_M} \right)^\gamma \tag{24}$$

where the exponent γ is a certain number, J_D is the number of iterations assumed at the beginning of the calculations and J_M is the number of iterations performed in the previous step.

5. NUMERICAL EXAMPLES

A program implementing HPM and two-point method has been written in MATLAB and representative results are provided. Three numerical examples were solved in a

microcomputer environment (32 bit Pentium 1.66 GHz processor: 2CPUs) such that the efficiency of the proposed procedure (developed as above) and the Newton-Raphson method of predicting the nonlinear behavior of space trusses and frames could be compared. Using a tolerance ε=10⁻⁵, the nonlinear equations were subject to successive iterations until the convergence criteria were satisfied. Newton–Raphson method as well as other hybrid algorithms has been applied to two cases of elastic and inelastic post–buckling (IPB) analyses of structures.

5.1 Example 1
The geometric dimensions of the geodesic dome truss shown in Figure 4 are taken from Ramesh and Krishnamoorthy [19].
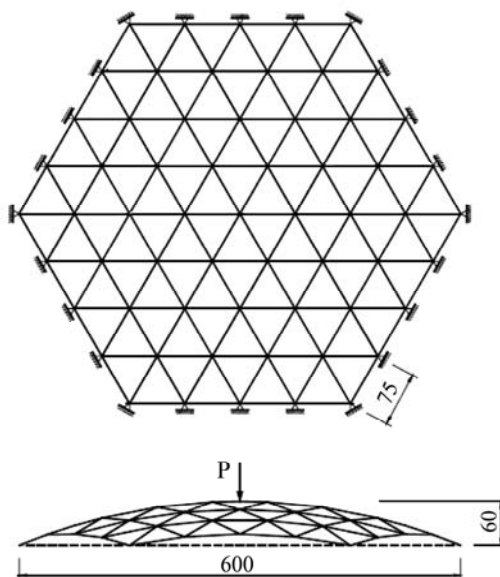

Figure 4. Geodesic dome truss, dimensions are given in *cm*

This truss has 156 members and 61 nodes with pin supports at the outer nodes and one vertical load $P = 8$ kN at the center, $\Delta\lambda_1^1 = 0.01$, $\lambda_{max} = 0.5$, $\gamma = 0.1$, $J_D = 10$, $J_{max} = 100$. The elevation of the truss is defined by the following equation:

$$x^2 + y^2 + (z + 7.2)^2 = 60.84 \tag{25}$$

All members have identical cross sections, i.e., with $E = 6895$ kN/cm², $A = 6.5$ cm², $F_y = 400$ kN/cm², $I = 1$ cm⁴. The load-displacement curve for this structure is shown in Figure 5. A comparison between obtained curves and those available in reference [19], demonstrates the accuracy of proposed algorithm.
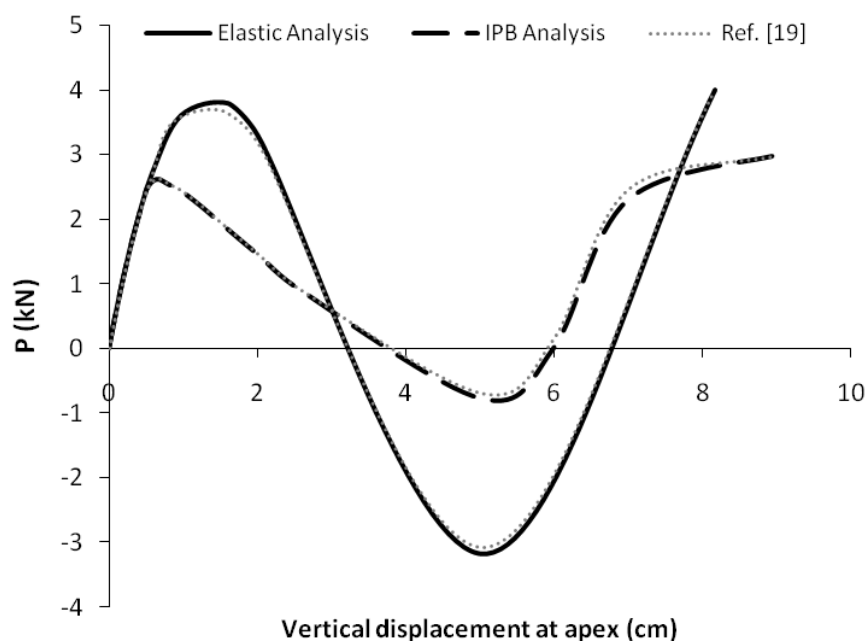
Figure 5. Load-displacement curves of geodesic dome truss at apex

The comparison between the results of applying the four approaches is listed in Table 1.
It is observed that in all cases the N-R $+2^{nd}$ HPM coupled with two-point algorithm has better performance than the other methods.

Table 1: Comparison of CPU time and num. of iteration for example 1

method	Time (sec)		Number of iterations	
	Elastic analysis	IPB	Elastic analysis	IPB
N-R	27.5306	48.0764	111	201
N-R $+1^{st}$ HPM	21.0562	36.6521	89	184
N-R $+2^{nd}$ HPM	11.0312	19.2401	62	121
N-R +2nd HPM + Two-point	10.7811	18.5207	55	94

5.2 Example 2
The circular dome truss taken from [20] is shown in Figure 6. This structure is subjected to a vertical load $P = 500$ kN at the apex and has 168 elements with 73 nodes with a total of 147 degrees-of-freedom. There are pin supports around the truss.
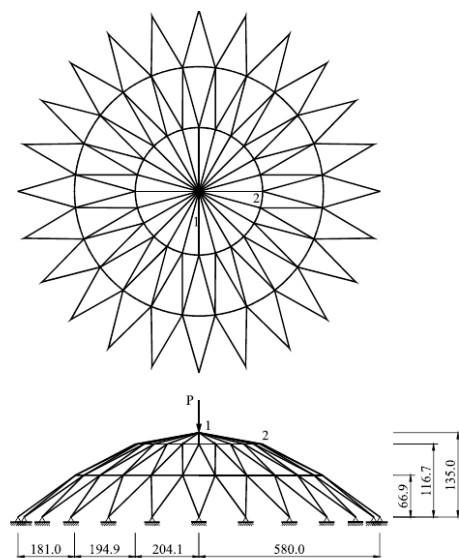
Figure 6. Circular dome truss, dimensions are given in *cm*

The cross-sectional area A is equal to $50.431\,\text{cm}^2$ for all the members. The elastic modulus of the members E is 2.04×10^4 kN/cm^2, $F_y = 25$ kN/cm^2, $I = 52.94$ cm^4, $\Delta\lambda_1^1 = 0.01$, $\lambda_{\max} = 2$, $J_D = 5$, $J_{\max} = 100$, $\gamma = 0.1$.

Figure 7 illustrates the numerical responses obtained from the proposed formulation for the three analyses. Obtained curves are the same in reference [20].
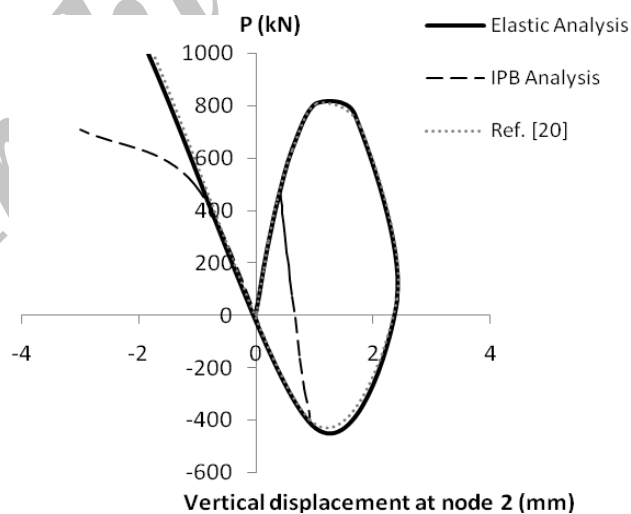

Figure 7. Load-displacement curves of circular dome truss at node 2

The computational results in Table 2 show that N-R + 2nd HPM coupled with two-point algorithm requires less CPU time among other methods. Therefore, they are of practical interest and can compete with Newton-Raphson method and work better than it.

Table 2: Comparison of CPU time and num. of iteration for example 2

method	Time (sec)		Number of iterations	
	Elastic analysis	IPB	Elastic analysis	IPB
N-R	26.86136	87.29523	188	251
N-R +1st HPM	20.17294	64.7803	153	201
N-R +2nd HPM	11.56912	37.3771	105	174
N-R +2nd HPM + Two-point	10.02325	33.5691	100	146

5.3 Example 3

This truss, shown in Figure 8 with 264 elements and 97 nodes with pin supports at the outer nodes, gives a possibility of comparison with results in the reference Greco et al. [6]. The axial stiffness for all members is $EA = 640 \times 10^3$ kN, $F_y = 25$ kN/cm^2, $I = 30.04$ cm^4. The external loading is due to equipment self-weight, consisting of $P = 50$ kN at the crown node and $\Delta \lambda_1^1 = 0.01$, $\lambda_{max} = 1$, $\gamma = 0.1$, $J_D = 2$, $J_{max} = 100$.
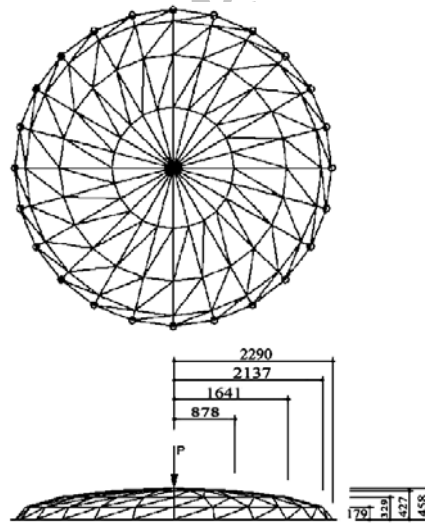

Figure 8. Schewdeler's dome truss, dimensions are given in *cm*

Figure 9 shows the variation of vertical displacement at central node with the load *P*. Achieved curves show a good accuracy between obtained and available results in reference [6].
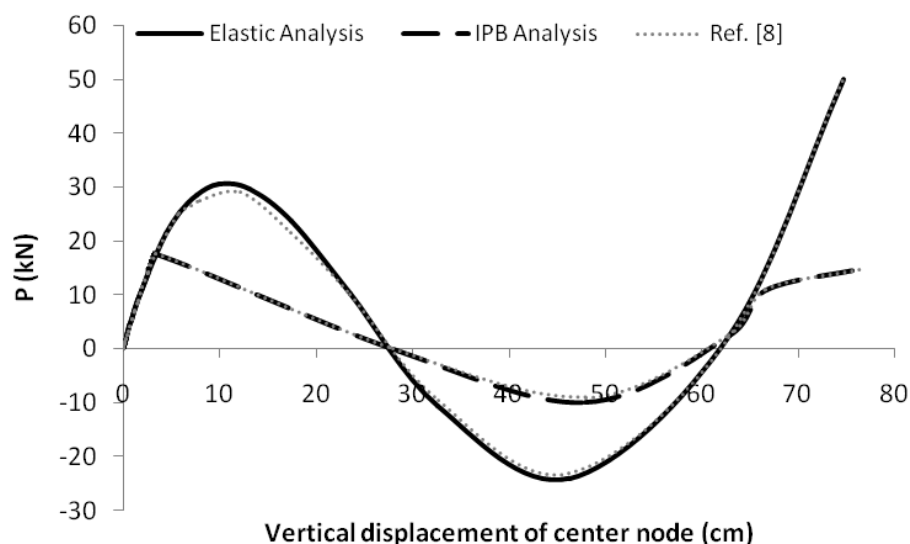
Figure 9. Central node vertical displacement-force of Schewdeler's dome

To compare the performance of the proposed method, the results of analyses are summarized in Table 3. The computations are carried out with a high precision. It can be easily seen that less computing time are used by N-R +2nd HPM coupled with Two-point algorithm rather than the others.

Table 3: Comparison of CPU time and num. of iteration for example 3

method	Time (sec)		Number of iterations	
	Elastic analysis	IPB	Elastic analysis	IPB
N-R	115.6921	184.1164	195	266
N-R +1st HPM	84.6741	134.2376	162	214
N-R +2nd HPM	48.0445	76.3232	114	183
N-R +2nd HPM + Two-point	40.9501	65.5133	111	157

5.4 Example 4

Figure 10 shows a two-bay six-storey frame subjected to distributed gravity and lateral loads. The beam and column cross-sections are shown in the figure while all loading magnitudes are scaled to a predefined reference value *P* (Chan 2000). The elastic modulus for all members *E* is adopted as 20500 kN/cm^2. Incremental load (ΔP) is selected equal to 2.044 kN.
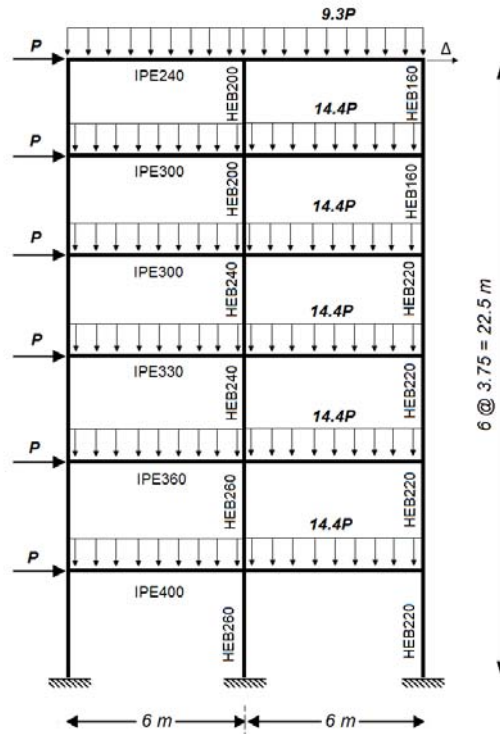
Figure 10. Two-bay six-storey frame

Obtained curve (Figure 11) is compatible with reference [21]. The efficiency and performance of the proposed method can be deduced from the results shown in Table 4.
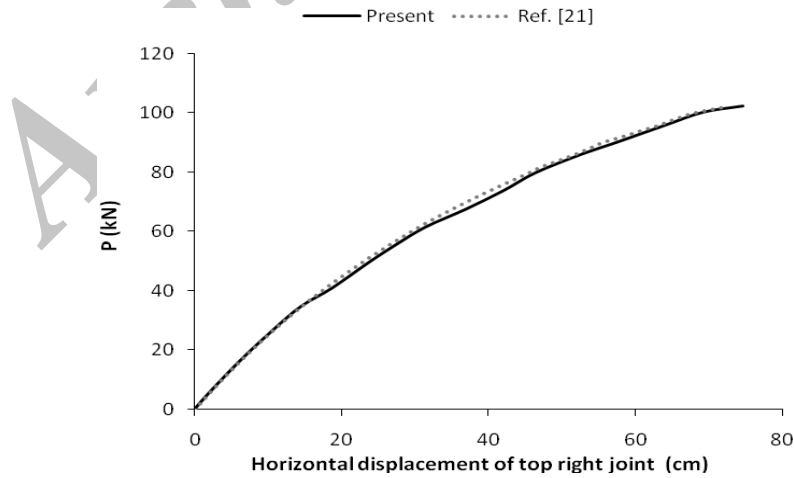

Figure 11. Load-deflection curve for example 4

Table 4: Comparison of CPU time for example 4 (sec)

method	Time (sec)		Number of iterations	
	Elastic analysis	IPB	Elastic analysis	IPB
N-R	19.62	24.35	87	101
N-R +1st HPM	9.019	11.19	71	98
N-R +2nd HPM	8.037	9.97	41	66
N-R +2nd HPM + Two-point	6.2316	7.82	30	59

As can be seen in the table, when proposed HPM is used the computational time is reduced compared with those taken by other methods. In particular, in the current example, the rate of reduction in the computing time is up to 68% when compared with the classic Newton-Raphson method. Therefore, the proposed method is of practical interest whenever the accuracy and efficiency are both concerned in the nonlinear analysis of structures.

6. CONCLUSIONS

In this paper, a new hybrid approach was developed aiming at achieving convergence acceleration compared to conventional Newton-Raphson algorithms. Both geometrical and material nonlinearity was considered. A mathematical formulation, known as Homotopy Perturbation Method, was numerically extended, applied and combined for nonlinear analysis of structures. In the examples under consideration, the proposed hybrid methods converged in less time and number of iterations than the classical Newton-Raphson method. It is thus concluded that this method can be regarded as an efficient technique for nonlinear analysis of large structures for which computational time aspects impose practical constraints.

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