



*Technical Note*

## IMPROVED METHOD OF ESTIMATING DEFLECTION IN PRESTRESSED STEEL I-BEAMS

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**Received:** 16 December 2012; **Accepted:** 5 June 2013/

### ABSTRACT

Prestressing steel has been popular in the recent past, due to the developments in the field of anti-corrosive coatings. The literature substantiates the application of technique of prestressing to steel structures both in safety and economy point of view. However, for all design calculations, the maximum allowable span for a given load carrying capacity is based on maximum deflection which is calculated by principle of superposition (considering the effect of prestress and total load individually). This paper proposes a method of arriving at expression for deflection of simply supported, prestressed homogenous steel I-beams calculated by considering the combined effect of prestressing and total load. A straight tendon configuration with an eccentric prestressing force is considered for study.

**Keywords:** Deflection; prestressed steel; superposition; pure bending; load carrying capacity; simply supported; homogenous.

### 1. INTRODUCTION

#### 1.1 GENERAL

The necessity for economy in steel in view of large quantities needed for construction and rehabilitation of various steel structures prompted the requirement of saving steel. This is possible by using more reinforced concrete and prestressed concrete than steel. But for long spans where prestressed concrete is not viable, prestressed steel is the ideal solution. The concept of prestressing steel is not a new frontier in the field of Civil Engineering. But it is being widely considered only in the recent past, despite a long and successful history of

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prestressing concrete members thanks to the advancements made in the field of anti corrosive coatings.

### 1.2 BASIC CONCEPT OF PRESTRESSING

The term 'prestressed steel' means application of a pre-determined concentric or eccentric force to a steel member so that the state of stress in the member resulting from this force and from any other anticipated external loading will be restricted to certain specified limit. This is done by inducing opposite stresses in a structure before it is put to its actual use by application of external forces. These forces are controlled in magnitude and direction to counter act the developed stresses in beam due to working loads. The changes occurring in the prestressing tendon due to changes in external loading can be neglected, as they are negligible.

### 1.3 TYPES OF PRESTRESSING

Based on the nature of prestressing, prestressing techniques can be broadly classified as follows.

#### 1.3.1 Internal Prestressing

In this technique, the tendons are placed inside the physical cross-section of the member to be prestressed. The transfer of prestressing force from tendon to the member is through the bond formed between them and can be done by either Pre-tensioning or Post-tensioning.

#### 1.3.2 External Prestressing

In this technique, the tendons are placed outside the physical cross-section of the member to be prestressed. The transfer of prestressing force from tendon to the member is through anchorages only. It is very difficult to establish a bond between the prestressing tendon and a steel beam. Hence external prestressing technique is adopted for prestressing of steel beams. The Transfer of prestress can be done by placing the tendon beyond the bottom most fiber or between neutral axis and bottom most fiber depending on the nature of cross-section and depending on whether it is symmetrical or unsymmetrical. Figure 1 shows the position of tendons placed beyond the bottom most fiber.

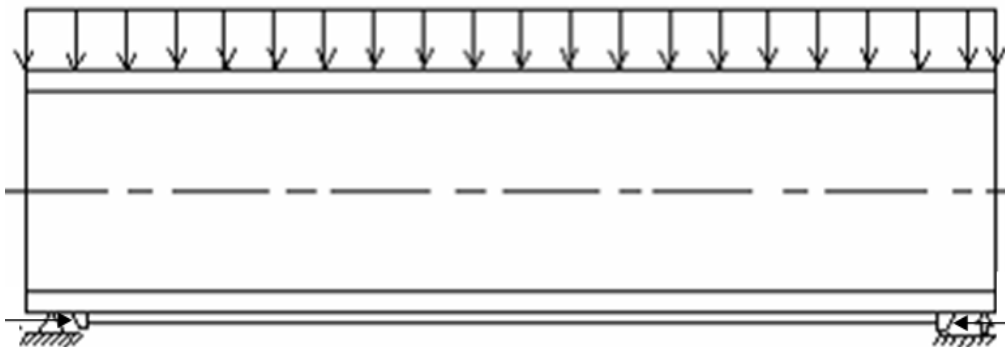


Figure 1. Tendon placed beyond the cross-section of the Beam

#### 1.4 METHODS OF PRESTRESSING

The method of prestressing often depends on the nature of structure and stresses developed by it under working loads.

**1. Members in Axial Tension:** Prestressed steel bars are generally used as members in axial tension. They can be prestressed by applying axial tension.

**2. Flexural Members:**

a) Symmetrical I-Beam:

A symmetrical I-beam is generally prestressed by tendons placed under the bottom flange.

b) Prestressing of an Asymmetrical I-Beam:

A beam with its top flange larger in cross-section than its bottom flange is generally prestressed by tendons placed below the neutral axis and above the bottom most fiber of the beam.

**3. Pre-deflection Technique:** This technique enables the use of concrete encased high strength steel beams in cases where deflections due to cracking of concrete, or both, would otherwise be excessive. This system is generally used when shallow construction depths are required. The bending stresses and their properties of the cross-section are evaluated at four separate stages of fabrication and erection and this is the important aspect of the system. These stages are as follows.

STAGE 1: The steel beam is under the jacking forces.

STAGE 2: The concrete is placed while the jacking forces are maintained. The reinforcement in the casting may be included. The bending moment due to self weight may be deduced from the free deflection moment provided the beam is supported at its ends.

STAGE 3: Stresses occur after casting of the concrete slab forming the top flange of the beam, including concrete encasement of the web. Total loss of prestress will occur between the time of release and casting of the top slab, due to creep and shrinkage of the concrete only.

STAGE 4: Superimposed dead and live loads are applied. Separate calculations should be carried out for these two types of loads using appropriate modular ratios.

**4. Redistribution of Bending Moments:** By changing the support levels of continuous girders, one can redistribute the bending moments and consequently redistribute the stresses in the cross-section of the girder.

**5. Shortening and Lengthening of Truss Members:** Truss girders can be prestressed as a whole by judiciously choosing a member of it and by intentionally shortening or lengthening it depending on the stress induced in it due to working load.

#### 1.5 ADVANTAGES OF PRESTRESSED STEEL BEAMS

Lighter and slender members are possible by use of prestressed steel when compared with un-prestressed steel beams. The whole cross-section is effective in prestressed steel beams. But in un-prestressed steel beams, a large cross-section of beam is under stressed and hence un-utilised. High strength steel is permitted in prestressed steel members, which is more durable under aggressive environmental conditions. Prestressing of steel beams enhances the elastic range of the material and hence they deflect appreciably before ultimate failure when compared with un-prestressed steel members thereby giving ample warning before collapse.

In other words prestressed steel beams are more ductile when compared with un-prestressed steel beams. The economic advantage of any prestressed steel structure when compared with the corresponding un-prestressed steel structure is about 15% taking into account the cost of cables and the technology of prestressing. Prestressing of steel beams improves fatigue life of a structure. Hence it can be safely recommended for dynamically loaded structures. The external prestressing technique can be applied to improve the load carrying capacity of existing un-prestressed steel beams. It is not always possible in case of R.C.C (Reinforced Cement Concrete) beams. We can temporarily improve the load carrying capacity of a beam steadily by stage-by-stage prestressing to acquire the required load carrying capacity. Prestressed steel beams are ideally suitable against prestressed concrete beams for temporary bridge construction because the former can be reused. Standard components required for prestressed steel beams can be manufactured in factories and hence a lot of time is saved. Prestressing remarkably reduces ultimate deflection and hence high span depth ratios can be achieved. Prestressing steel girders increase the level of the stress at which the beam starts to buckle. The loss of tensile prestress in cables is small in case of prestressed steel structures when compared with prestressed concrete structures. Reductions in the frictional losses, because of unintentional angular changes (known as wobble) particularly get eliminated. Further as polythene sheathing is used, the friction coefficient is drastically reduced compared to the standard internal and bonded prestressing using corrugated sheet metal ducts. Tendons of External prestressed beams can easily be designed to be replaceable and restressable without major cost implications. Corresponding structural detailing provides sufficient unobstructed space requirements. Generally the webs can be made thinner resulting in a comparatively lighter structure thereby facilitating execution.

#### *1.6 DISADVANTAGES OF PRESTRESSED STEEL BEAMS*

Longer spans are not preferred to avoid possible bulking. Externally prestressed members can easily be damaged with minor equipment. For certain cross-sections and construction procedures, the handling of the tensioning devices is difficult. The availability of builders and engineers experienced with the technique of external prestressing of steel is scanty. Initial equipment cost is very high. Prestressing tendons are brittle as high strength steel is used. The tendons are more exposed to environmental influences, fire, aggressive chemicals etc. The deviators and anchor plates which are delicate components, have to be placed very accurately. As the tendons are not bonded to the concrete (or bonded only at particular points) the ultimate strength is developed in the ultimate design resulting in a higher prestressing steel consumption. Usually the height of the cross-section cannot be fully utilised. Therefore a greater depth or additional prestressing is required.

#### *1.7 APPLICATIONS OF TECHNIQUE OF PRESTRESSING STEEL*

The technique of prestressing is applied to steel structures to

- (1) obtain economy in steel by utilising high strength steel materials.
- (2) increase the area of the elastic range of steel.
- (3) reduce the deformability of the structure.
- (4) increase the stability of the structure.

Depending on the type of structure and its future working conditions, prestress may be

applied during the erection or at the manufacturing plant. Prestressing can be applied in single or multiple stages. In utilising the material, the greater effect may be obtained in multistep prestressing. This prestressing is possible only if the load is constant. The sequence of steps used to create prestressing depends on the type of structure or loading and is subject to its effect on the economies being sought from prestressing. Prestressing is used in the design of new structures as well as for the existing ones. Reinforcing of an existing structure by prestressing results in an increase in its load carrying capacity and stiffness with minimal consumption of additional material. Considering its structural use, prestressing has been successfully applied in the design of new structures such as girders, frames, arches, trusses, buildings, towers, masts and bridges as well as to strengthen old bridges.

Figure 2 shows the World's first prestressed steel bridge, in Minneapolis – St. Paul that was designed by T.Y.Lin. Some of the typical applications of technique of prestressed steel where external tendons are feasible, practical and economical are as follows.

Trusses: Prestressed trusses are used in industrial buildings and in the roofs of the boilers.

Bridges: Many large metal bridges have been built recently with the application of prestressing (bridge girders prestressed by tendons).

3. Sheet Structures and Wall Structures: These are prestressed so that they can take the compressive load effectively.

4. Masts and Towers: Prestressed steel is used in Masts and Towers to increase the rigidity of the structure.

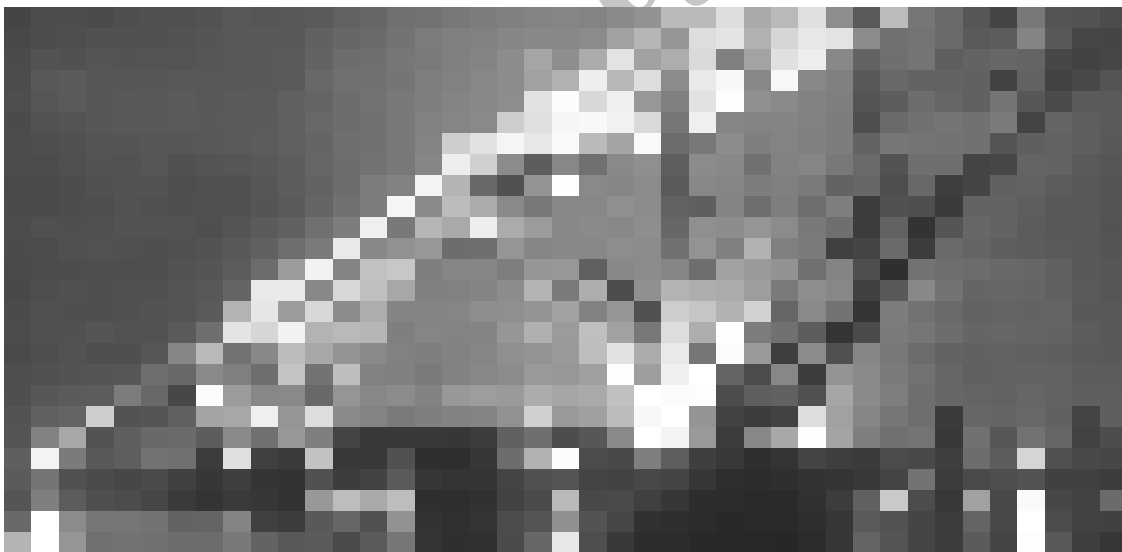


Figure 2. World's 1<sup>st</sup> Prestressed Steel Bridge designed by T.Y.Lin. (Minneapolis–St.Paul)

In general, the technique of prestress can be safely recommended for Rehabilitation of existing steel structures and is ideal for temporary bridge construction. It is economical for both simple and continuous spans and is ideal for Temporary bridge construction as the components can be reused. It is also suitable for precast segmental construction and is safe for incremental launching procedures.

## 2. LITERATURE REVIEW

Prestressed metal structures have been proposed since 1837, when [27] Whipple S (1847) in the United States learned to compensate for the poor tensile capacity of cast-iron members through prestressing in Bridges. [8] Dischinger F (1949) in Germany, beginning in 1935, began to conceive much wider applications for prestressed steel. His proposals included highway and railway bridges utilising prestressed plate girders, box girders, trusses and other structural forms. [11] Fritz.B (1955) reaffirmed the work. As a result of the European work in prestressed steel, [7] Coff L (1950) in the United States proposed a 250-foot span prestressed steel plate girder bridge. In the late 19<sup>th</sup> and early 20<sup>th</sup> centuries, [2] Barnard P.F (1960) designed many U.S. bridges with trussed floor beams. [26] Vasiliev A.A (1961) analysed the behavior of prestressed beam with a view to determine optimum geometric parameters. [10] Finn E.V and Needham F.H (1964) performed an extensive testing program for a 90-foot span prestressed steel truss. [19, 20] Petrov A.M (1965, 1967) examined the parameters of prestressed steel beams for designing with cables all along the beam length. [9] Ferjencik P (1972) and [24] Tochacek M and Amrhein F.G (1971) described progress in prestressed steel design in Czechoslovakia. Research began in 1960 and actual design specifications were adopted as a result of that research. Ferjencik developed a catalog of applications of prestressing including applying it to girders and trusses. [25] Tochacek M and Mehta (1972) pointed out that the safety factor for the portions of prestressed steel structures subjected to a range of both tension and compression can be reduced by 20 % under a working stress design. [3] Belenya E (1972) conducted tests on prestressed homogenous beam. [16] Kalburgi R.V (1975) developed the equations for finding the optimum prestressed steel I-section in non-dimensional parameters. [18] Michael S Troitsky et al. (1989) made a study on prestressed – steel continuous – span girders. [13] Hamid Saadatmanesh et al. (1989) made an experimental study on the behavior of prestressed composite beams. [14] Hamid Saadatmanesh et al. (1989) also made an analytical study on prestressed composite beams. [15] Hamid Saadatmanesh et al. (1990) formulated certain guidelines for flexural design of prestressed concrete beam in positive and negative moment regions. [4] Bilal M. Ayyub et al. (1990) made a study on the behavior of prestressed, composite steel-concrete beams under positive bending moment and compared the benefits of different types of prestressing. [17] Mark A Bradford (1991) made a study on buckling of prestressed steel girders. [5] Bilal M. Ayyub et al. (1992) experimentally examined the behavior of prestressed composite steel concrete girders subjected to negative moment. [6] Bilal M. Ayyub et al. (1992) made an analytical study of negative moment on prestressed composite girders. [22] Rao M.G.R (1993) suggested a direct method for optimum design of prestressed metal beams working with elastic and elasto-plastic states. [23] Russel D and Syder P.E (1995) made a study on prestressed steel girders for single span bridges. [1] Anjaneya Prasad V (2001) developed modified equations of flexure for design of prestressed homogenous steel beams by considering the effect of overload and under load factors. [21] Raman Singh V.S.M et al. (2002) made a case study on prestressed steel bridge. He suggested measures that can be taken in case of steel bridges for strengthening compression

member. [12] Guptha L.M (2002) made a study on prestressed steel structures. [28] Vincenzo Nunziata (2002) deals with the basic principles and the technology of prestressing steel structures. 40 m long pre-stressed steel beams were tested by loading concrete blocks of 25 kN each.

**Concluding remarks:** Even though the concept of prestressing steel beams is being adopted in the design and analysis of different bridges all over the world since long and more significantly in the recent past, there are many assumptions being considered for the same. From the existing literature, it can be observed that the maximum allowable span for a given load carrying capacity is based on the maximum deflection, which is calculated by the principle of superposition (considering the effect of prestress and total load individually). The combined effect of prestressing and total load on the deflection will be different from the effect of prestress and total load considered individually on the maximum deflection. This aspect considerably affects the economy of the structure as a whole and compels the design engineer to adopt a higher factor of safety. Hence, the objective of the present work is to formulate an expression for evaluating deflection by considering the combined effect of prestressing and total load on the deflection and by considering the effect of prestress and total load individually.

### 3. ESTIMATION OF DEFLECTION IN PRESTRESSED BEAMS

#### 3.1 Importance of control of Deflection

Deflection forms an important criterion for safety of any structure. Suitable control on deflection is very essential for the following reasons.

1. Excessive sagging of any structural members is unsightly and may sometimes render the floor unsuitable for use.
2. Large deflections under dynamic effects and under the influence of variable loads may cause discomfort to users.
3. Excessive deflections are likely to cause damage to finishes, partitions and associated structures.

#### 3.2 Effect of Tendon Profile on Deflection

In the most of the cases of prestressed beams and tendons are located with eccentricities towards the soffit of beams to counteract the sagging bending moments due to transverse loads. Consequently the prestressed beams deflect upwards (camber) on the application of prestress. Since the bending moment is the product of the prestressing force and eccentricity, the tendon profile itself will represent the shape of the bending moment diagram.

#### 3.3 Expressions for Estimation of Deflection

The method of computing deflections of simply supported eccentrically prestressed beam with straight cable in two different approaches will be discussed in the following sections.

##### 3.3.1 Combined Effect of Prestress and External Load on Deflection

Consider a simply supported beam of span 'l', Flexural rigidity  $EI_x$  subjected to a total uniformly distributed load of  $w$ /unit length and prestressed by a force 'P' at an eccentricity

'e'. Let the deflection of the beam at a distance 'x' from left support be 'y'. (Figure 3).

Then bending moment at any section at a distance of 'x' from left support is given by

$$M_x = \left( \frac{w \times l \times x}{2} \right) - \left( \frac{w \times x^2}{2} \right) - P(e - y) \quad (1)$$

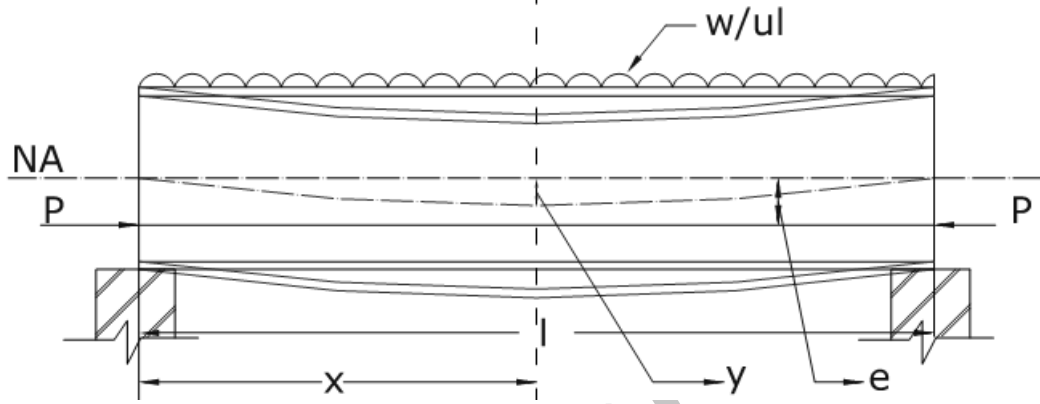


Figure 3. Deflected shape of the prestressed steel I-beam

Hence from the theory of pure bending

$$\begin{aligned} E \times I_x \times \frac{d^2 y}{dx^2} &= - \left( \frac{w \times l \times x}{2} \right) + \left( \frac{w \times x^2}{2} \right) + P(e - y) \\ \Rightarrow \frac{d^2 y}{dx^2} + \left( \frac{P \times y}{E \times I_x} \right) &= - \left( \frac{w \times l \times x}{2 \times E \times I_x} \right) + \left( \frac{w \times x^2}{2 \times E \times I_x} \right) + \left( \frac{P \times e}{E \times I_x} \right) \end{aligned} \quad (2)$$

$$\Rightarrow \left( D^2 + \left( \sqrt{\frac{P}{E \times I_x}} \right)^2 \right) y = - \left( \frac{w \times l \times x}{2 \times E \times I_x} \right) + \left( \frac{w \times x^2}{2 \times E \times I_x} \right) + \left( \frac{P \times e}{E \times I_x} \right) \quad (3)$$

The Complementary function (C.F) of this differential equation is given by

$$C.F = C_1 \times \cos \left( \sqrt{\frac{P}{E \times I_x}} \times x \right) + C_2 \times \sin \left( \sqrt{\frac{P}{E \times I_x}} \times x \right) \quad (4)$$

The Particular Integral (P.I) of the Differential Equation is given by



$$\begin{aligned}
 P.I &= \frac{1}{\left(D^2 + \left(\sqrt{\frac{P}{E \times I_x}}\right)^2\right)} \left\{ -\left(\frac{w \times l \times x}{2 \times E \times I_x}\right) + \left(\frac{w \times x^2}{2 \times E \times I_x}\right) + \left(\frac{P \times e}{E \times I_x}\right) \right\} \\
 &= \frac{1}{\left(\frac{E \times I_x \times D^2}{P} + P\right)} \left(\frac{E \times I_x}{P}\right) \left\{ -\left(\frac{w \times l \times x}{2 \times E \times I_x}\right) + \left(\frac{w \times x^2}{2 \times E \times I_x}\right) + \left(\frac{P \times e}{E \times I_x}\right) \right\} \\
 &= \left(\frac{E \times I_x \times D^2}{P} + P\right)^{-1} \left(\frac{E \times I_x}{P}\right) \left\{ -\left(\frac{w \times l \times x}{2 \times E \times I_x}\right) + \left(\frac{w \times x^2}{2 \times E \times I_x}\right) + \left(\frac{P \times e}{E \times I_x}\right) \right\}
 \end{aligned} \tag{5}$$

Using the first two terms of the Binomial expansion

$$\begin{aligned}
 P.I &= \left(1 - \frac{E \times I_x \times D^2}{P}\right) \left\{ -\left(\frac{w \times l \times x}{2 \times P}\right) + \left(\frac{w \times x^2}{2 \times P}\right) + e \right\} \\
 &= \left\{ -\left(\frac{w \times l \times x}{2P}\right) + \left(\frac{w \times x^2}{2P}\right) + e - \left(\frac{w \times E \times I_x}{P^2}\right) \right\}
 \end{aligned} \tag{6}$$

Hence, the solution of the differential equation i.e., the deflection equation is given by  
 $y = C.F + P.I$

$$\begin{aligned}
 y &= C_1 \times \cos\left(\sqrt{\frac{P}{E \times I_x}} \times x\right) + C_2 \times \sin\left(\sqrt{\frac{P}{E \times I_x}} \times x\right) - \left(\frac{w \times l \times x}{2 \times P}\right) + \left(\frac{w \times x^2}{2 \times P}\right) + \\
 &\quad e - \left(\frac{w \times E \times I_x}{P^2}\right)
 \end{aligned} \tag{7}$$

Boundary conditions for this beam are

1. At  $x = 0$ ,  $y = 0$

$$\text{Hence } C_1 = \left(\frac{w \times E \times I_x}{P^2}\right) - e \tag{8}$$

2. At  $x = l$ ,  $y = 0$

$$\text{Hence } C_2 = \left( \left( \frac{w \times E \times I_x}{P^2} \right) - e \right) \times \tan \left( \sqrt{\frac{P}{E \times I_x}} \times \frac{l}{2} \right) \quad (9)$$

Substituting the values of  $C_1$  and  $C_2$  in Equation 7 and rearranging the terms, we have

$$y = \left( \frac{w \times E \times I_x}{P^2} - e \right) \times \left\{ \frac{\cos \left( \sqrt{\frac{P}{E \times I_x}} \times \left( x - \frac{l}{2} \right) \right)}{\cos \left( \sqrt{\frac{P}{E \times I_x}} \times \frac{l}{2} \right)} - 1 \right\} + \left( \frac{w \times x}{2 \times P} \right) \times (x - l) \quad (10)$$

This is the generalised expression for finding the deflection of a simply supported eccentrically prestressed beam considering the combined effect of prestress and total load.

Maximum deflection by symmetry occurs at  $x = l/2$ .

Hence,

$$y_{\max} = \left( \frac{w \times E \times I_x}{P^2} - e \right) \times \left\{ \sec \left( \sqrt{\frac{P}{E \times I_x}} \times \frac{l}{2} \right) - 1 \right\} - \left( \frac{w \times l^2}{8 \times P} \right) \quad (11)$$

### 3.3.2 Deflection by Principle of Superposition

(By considering the effect of prestress and total load individually)

1. Upward deflection due to prestressing force.

Consider a simply supported beam of span 'l', prestressed by a force 'P', at an eccentricity 'e'. Then bending moment at any section on the beam is

$$M_x = -P \times e$$

$$E \times I_x \times \frac{d^2 y}{dx^2} = (P \times e) \quad (12)$$

By Integrating with respect to 'x' on both sides

$$\frac{dy}{dx} = \left( \frac{P \times e \times x}{E \times I_x} \right) + A_1 \quad (13)$$

Integrating again with respect to 'x' on both sides

$$y = \left( \frac{P \times e \times x^2}{2 \times E \times I_x} \right) + (A_1 \times x) + A_2 \quad (14)$$

Applying the boundary conditions

1. At  $x = 0, y = 0$

$$\Rightarrow A_2 = 0 \quad (15)$$

2. At  $x = l, y = 0$

$$\Rightarrow A_1 = -\left(\frac{P \times e \times l}{2 \times E \times I_x}\right) \quad (16)$$

$$\text{Hence } y = \left(\frac{P \times e \times x^2}{2 \times E \times I_x}\right) - \left(\frac{P \times e \times l \times x}{2 \times E \times I_x}\right) \quad (17)$$

This is the expression to calculate the upward deflection i.e., camber due to the eccentric prestressing force alone.

2. Downward deflection due to self-weight and live load.

Consider a simply supported beam of span 'l', Flexural rigidity EI, which is subjected to a total uniformly distributed load of w/unit length. Then bending moment at any section, which is at a distance 'x' from the left end is given by

$$M_x = \left(\frac{w \times l \times x}{2}\right) - \left(\frac{w \times x^2}{2}\right) \quad (18)$$

Applying the theory of pure bending

$$EI \frac{d^2 y}{dx^2} = -\left(\frac{w \times l \times x}{2}\right) + \left(\frac{w \times x^2}{2}\right) \quad (19)$$

Integrating with respect to 'x' on both sides

$$\frac{dy}{dx} = -\left(\frac{w \times l \times x^2}{4 \times E \times I_x}\right) + \left(\frac{w \times x^3}{6 \times E \times I_x}\right) + B_1 \quad (20)$$

Integrating again with respect to 'x' on both sides

$$y = -\left(\frac{w \times l \times x^3}{12 \times E \times I_x}\right) + \left(\frac{w \times x^4}{24 \times E \times I_x}\right) + (B_1 \times x) + B_2 \quad (21)$$

Applying the boundary conditions

1. At  $x = 0, y = 0$

$$\Rightarrow B_2 = 0 \quad (22)$$

2. At  $x = l, y = 0$

$$\Rightarrow B_1 = \left( \frac{w \times l^3}{24 \times E \times I} \right) \quad (23)$$

Hence

$$y = -\left( \frac{w \times l \times x^3}{12 \times E \times I_x} \right) + \left( \frac{w \times x^4}{24 \times E \times I_x} \right) + \left( \frac{w \times l^3 \times x}{24 \times E \times I_x} \right) \quad (24)$$

This is the deflection equation of a simply supported beam subjected to uniformly distributed load alone. The generalized expression for net deflection due to prestressing force and self-weight of the beam is obtained by algebraic sum of Equations (17) and (24).

$$y = -\left( \frac{w \times l \times x^3}{12 \times E \times I_x} \right) + \left( \frac{w \times x^4}{24 \times E \times I_x} \right) + \left( \frac{w \times l^3 \times x}{24 \times E \times I_x} \right) + \left( \frac{P \times e \times x^2}{2 \times E \times I_x} \right) - \left( \frac{P \times e \times l \times x}{2 \times E \times I_x} \right) \quad (25)$$

The maximum deflection by symmetry occurs at  $x = \left( \frac{l}{2} \right)$

$$\therefore y_{\max} = \left( \frac{5 \times w \times l^4}{384 \times E \times I_x} \right) - \left( \frac{P \times e \times l^2}{8 \times E \times I_x} \right) \quad (26)$$

#### 4. CONCLUSIONS

Based on the study of variables within the scope of this research, the following general and specific conclusions can be drawn.

1. For a simply supported prestressed steel I-beam, the generalised expression for deflection of the beam by considering the combined effect of prestress and total load is given by

$$y = \left( \frac{w \times E \times I_x}{P^2} - e \right) \times \left\{ \frac{\cos \left( \sqrt{\frac{P}{E \times I_x}} \times \left( x - \frac{l}{2} \right) \right)}{\cos \left( \sqrt{\frac{P}{E \times I_x}} \times \frac{l}{2} \right)} - 1 \right\} + \left( \frac{w \times x}{2 \times P} \right) \times (x - l)$$

3. Maximum deflection by symmetry occurs at center of the span is given by

$$y_{\max} = \left( \frac{w \times E \times I_x}{P^2} - e \right) \times \left\{ \sec \left( \sqrt{\frac{P}{E \times I_x}} \times \frac{l}{2} \right) - 1 \right\} - \left( \frac{w \times l^2}{8 \times P} \right)$$

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