



INELASTIC DYNAMIC ANALYSIS OF STRUCTURES UNDER BLAST LOADS USING GENERALIZED B-SPLINE METHOD

M. Amini¹, S. Shojaee^{2*} and S. Rostami²

¹Department of Civil Engineering, Imam Hossein University, Tehran, Iran

²Department of Civil Engineering, Shahid Bahonar University of Kerman, Kerman, Iran

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ABSTRACT

Quartic B-spline time integration method has been recently proposed for solving linear problems in structural dynamics. This paper developed this method to nonlinear dynamic analysis of single-degree-of-freedom (SDOF) systems under exploding loads. Using quartic B-spline basis function, this method gained second order of acceleration at each time-step. Thus it benefits from high accuracy compared to the methods in the literature. In this research, in order to applying the iterative process in the procedure, firstly, a series of standard formulas were derived from previous formulation. Then the Newton-Raphson iterative method used to develop new formulation for solving nonlinear dynamic problems. Finally, for the new scheme, a simple step-by-step algorithm is implemented and presented to calculate dynamic response of SDOF systems. The validity and effectiveness of the proposed method is demonstrated with two examples. The results were compared with those from the famous numerical method. The comparison shows that the proposed method is a fast and simple procedure with trivial computational effort.

Keywords: Quartic B-spline; time integration; nonlinear dynamic analysis; SDOF; blast load; explosion.

1. INTRODUCTION

Due to the threat from such extreme loading conditions, efforts have been made during the past three decades to develop methods of structural analysis and design to resist blast loads. Research has been undertaken over the past half a century on the modeling of blast pressure on objects and structures [1-4]. The analysis and design of structures subjected to blast loads require a detailed understanding of blast phenomena and the dynamic response of various structural elements. Some paper presents a comprehensive overview of the effects of explosion on structures. In Reference [5], an explanation of the nature of explosions and the

*E-mail address of the corresponding author: saeed.shojaee@uk.ac.ir (S. Shojaee)

mechanism of blast waves in free air is given. This paper also introduces different methods to estimate blast loads and structural response.

In structural dynamics, direct time integration algorithms are often used to obtain the solution of temporally discretized equations of motion at selected time-steps. Various time integration algorithms have been developed in the time domain using different methods. Integration algorithms are widely used for solving equation of motion so that dynamic behaviors of the systems under a specific loading can be obtained. Numerous time integration algorithms have been proposed, including the Newmark family of integration algorithms [6], Wilson method [7], and Hilber-Hughes-Taylor α -method [8].

Integration algorithms can be classified as either explicit or implicit. An integration algorithm is explicit if the displacements for the next time-step can be determined from the accelerations, velocities, and displacements at the current and previous time-steps, otherwise it is implicit. [9–14]. Meanwhile, integration algorithms are classified into two categories; conditionally and unconditionally stable. Conditionally stable algorithms require that a time-step be taken which is less than a constant time less than the smallest period of the structure. In unconditionally stable methods, instability never happens, even if a long time-step is chosen. Generally explicit algorithms are conditionally stable while most implicit methods are unconditionally stable. This method is subject to both phase and amplitude errors depending on the time-step used.

The most significant advantage of explicit methods is that it is unnecessary to solve a system of equations or to involve any iterative procedure in each time step and less storage is required than for implicit methods [11]. This also leads to an easy implementation of explicit methods. Almost all of the explicit time integration schemes are conditionally stable and for a few of them with unconditional stability, the consistency is conditional, it is the major disadvantage of explicit methods. Consequently, a very small time step and thus a very large number of time steps may be required in a time-history analysis. This may not be a disadvantage since the use of a very small time step can easily overcome the difficulty caused by the linearization errors for nonlinear systems. In addition, explicit algorithms are very efficient for shock response and wave propagation problems in which the contribution of intermediate and high frequency structural modes to the response is important. The central difference method and explicit Runge-Kutta method are the very commonly used explicit methods.

Application of cubic and quartic B-spline functions for the numerical solution of linear dynamic systems has been presented by Rostami and Shojaee in a series of papers [15-17]. Implementation of quartic B-spline for the numerical solution of dynamic systems has been done in Reference [17]. The proposed method has appropriate convergence, accuracy and low time consumption. Accuracy and stability analysis has been done profoundly in that paper. This time integration method benefits from a high order accuracy compared to the methods in the literatures. Application of cubic spline on large deformation analysis of structures has been present in research paper [18].

This paper is organized as follows. In the next section, we have a review of explosion and blast phenomenon. Section 3 is allocated to a brief review of quartic B-spline method in structural dynamic analysis. Section 4 is allotted to derivation of standard form of formulas from previous formulation. Section 5 is assigned to develop algorithm for nonlinear dynamic analysis. Finally a step-by-step algorithm of the proposed method has been introduced at the

end of this section. In section 6 the validity of this proposed method is illustrated with two examples.

2. REVIEW OF EXPLOSIONS AND BLAST PHENOMENON

An explosion is defined as a large-scale, rapid and sudden release of energy. Explosions can be categorized on the basis of their nature as physical, nuclear or chemical events. In physical explosions, energy may be released from the catastrophic failure of a cylinder of compressed gas, volcanic eruptions or even mixing of two liquids at different temperatures.

Explosive materials can be classified according to their physical state as solids, liquids or gases. Solid explosives are mainly high explosives for which blast effects are best known. They can also be classified on the basis of their sensitivity to ignition as secondary or primary explosive. The latter is one that can be easily detonated by simple ignition from a spark, flame or impact. Materials such as mercury fulminate and lead azide are primary explosives. Secondary explosives when detonated create blast (shock) waves which can result in widespread damage to the surroundings. Examples include trinitrotoluene (TNT) and ANFO [5,19].

The detonation of a condensed high explosive generates hot gases under pressure up to 300 kilo bar and a temperature of about 3000-4000°C. The hot gas expands forcing out the volume it occupies. As a consequence, a layer of compressed air (blast wave) forms in front of this gas volume containing most of the energy released by the explosion. Blast wave instantaneously increases to a value of pressure above the ambient atmospheric pressure. This is referred to as the side-on overpressure that decays as the shock wave expands outward from the explosion source. After a short time, the pressure behind the front may drop below the ambient pressure (see Fig. 1). During such a negative phase, a partial vacuum is created and air is sucked in. This is also accompanied by high suction winds that carry the debris for long distances away from the explosion source [5].

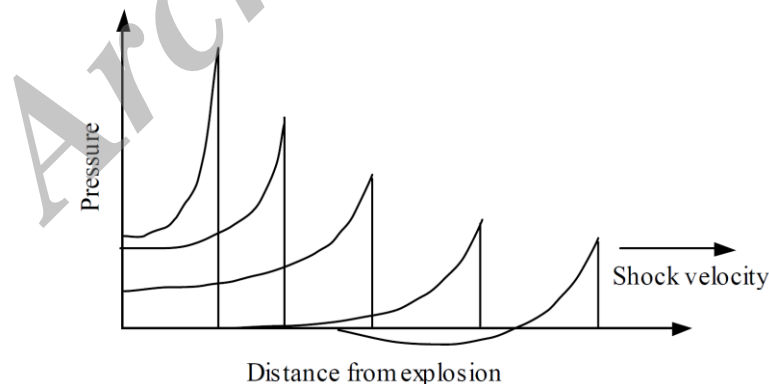


Figure 1. Blast wave propagation

2.1 Explosive air blast loading

The threat for a conventional bomb is defined by two equally important elements, the bomb size, or charge weight W , and the stand-off distance R between the blast source and the

target. The observed characteristics of air blast waves are found to be affected by the physical properties of the explosion source. Fig. 2 shows a typical blast pressure profile. At the arrival time t_A , following the explosion, pressure at that position suddenly increases to a peak value of over pressure, P_{so} , over the ambient pressure, P_o . The pressure then decays to ambient level at time, then decays further to an under pressure P_{so}^- (creating a partial vacuum) before eventually returning to ambient conditions at time $t_d + t_d^-$ the quantity P_{so} is usually referred to as the peak side-on overpressure, incident peak overpressure or merely peak overpressure [20]. The incident peak over pressures P_{so} are amplified by a reflection factor as the shock wave encounters an object or structure in its path. Throughout the pressure-time profile, two main phases can be observed; portion above ambient is called positive phase of duration t_d , while that below ambient is called negative phase of duration, t_d^- . The negative phase is of a longer duration and a lower intensity than the positive duration. As the stand-off distance increases, the duration of the positive-phase blast wave increases resulting in a lower-amplitude, longer-duration shock pulse. Charges situated extremely close to a target structure impose a highly impulsive, high intensity pressure load over a localized region of the structure; charges situated further away produce a lower-intensity, longer-duration uniform pressure distribution over the entire structure. Eventually, the entire structure is engulfed in the shock wave, with reflection and diffraction effects creating focusing and shadow zones in a complex pattern around the structure. During the negative phase, the weakened structure may be subjected to impact by debris that may cause additional damage [5].

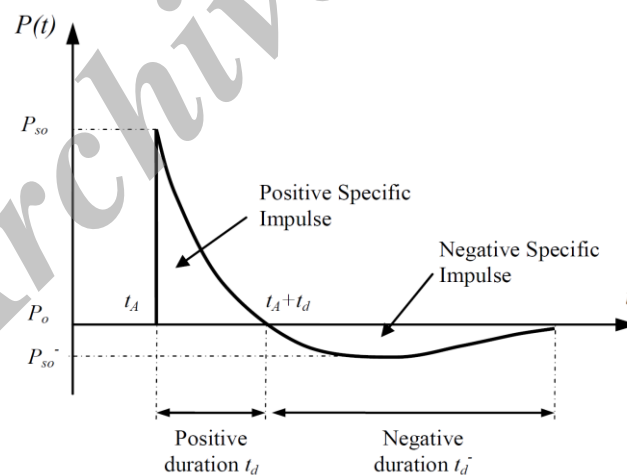


Figure 2. Blast wave pressure – Time history

2.2 Blast wave scaling laws

All blast parameters are primarily dependent on the amount of energy released by a detonation in the form of a blast wave and the distance from the explosion. A universal normalized description of the blast effects can be given by scaling distance relative to $(E / P_0)^{1/3}$ and scaling pressure relative to P_0 , where E is the energy release (kJ) and P_0 the

ambient pressure (typically $100 \text{ KN} / \text{m}^2$). For convenience, however, it is general practice to express the basic explosive input or charge weight W as an equivalent mass of TNT. Results are then given as a function of the dimensional distance parameter (scaled distance) $Z = R / W^{1/3}$, where R is the actual effective distance from the explosion. W is generally expressed in kilograms. Scaling laws provide parametric correlations between a particular explosion and a standard charge of the same substance.

2.3 Prediction of blast pressure

Blast wave parameters for conventional high explosive materials have been the focus of a number of studies during the 1950's and 1960's. Estimations of peak overpressure due to spherical blast based on scaled distance $Z = R / W^{1/3}$ were introduced by Brode in 1995 [1] as:

$$P_{so} = \frac{0.975}{Z} + \frac{1.455}{Z^2} + \frac{5.85}{Z^3} - 0.019 \quad (0.1 \leq P_{so} \leq 10 \text{ bar}) \quad (1)$$

$$P_{so} = \frac{6.7}{Z^3} + 1 \quad (P_{so} > 10 \text{ bar})$$

Henrych in 1979 [2] introduced the relations same as Brode formulas as:

$$P_{so} = \frac{14.072}{Z} + \frac{5.54}{Z^2} + \frac{0.375}{Z^3} + \frac{0.00625}{Z^4} \quad (0.05 \leq Z \leq 0.3)$$

$$P_{so} = \frac{6.194}{Z} + \frac{0.326}{Z^2} + \frac{2.132}{Z^3} \quad (0.3 \leq Z \leq 1)$$

$$P_{so} = \frac{0.662}{Z} + \frac{4.05}{Z^2} + \frac{3.288}{Z^3} \quad (1 \leq Z \leq 10)$$

Brad relations had good adaptation with experimental results for intermediate distances away from the source of explosion. While the Henrych formulas had good adaptation with experimental results for nearby distances of the source of explosion. It is recommended that for near distance ($Z \leq 0.5$) we use Henrych relations and for far distance ($Z > 0.5$) use Brad results.

Newmark and Hansen in 1961 [21] introduced a relationship to calculate the maximum blast overpressure, P_{so} , in bars, for a high explosive charge detonates at the ground surface as:

$$P_{so} = 6784 \frac{W}{R^3} + 93 \left(\frac{W}{R^3} \right)^{\frac{1}{2}} \quad (3)$$

Another expression of the peak overpressure in kPa is introduced by Mills in 1987 [22], in which W is expressed as the equivalent charge weight in kilograms of TNT, and Z is the scaled distance:

$$P_{so} = \frac{1772}{Z^3} + \frac{114}{Z^2} + \frac{108}{Z} \quad (4)$$

3. REVIEW OF QUARTIC B-SPLINE INTEGRATION METHOD

Quartic B-spline is an explicit time integration method for structural dynamics which has been proposed by Rostami et.al in a research paper [17]. In that paper by use of periodic quartic B-spline interpolation polynomial function, the authors proceeded to solve the differential equation of motion governing structural systems. In the proposed approach, a straightforward formulation was derived in a fluent manner from the approximation of the response of the system with B-spline basis. Because of using the quartic function, the system acceleration is approximated with a parabolic function. In order to access some comprehensive and exhaustive content about this method, see Reference [17].

Here the complete step-by-step algorithm of this proposed method for dynamic analysis of SDOF systems is given in Table 1.

Table 1: Traditional Quartic B-spline step-by-step time integration algorithm; linear systems

1. Initial calculation

1.1. Determine stiffness \mathbf{k} , mass \mathbf{m} , and damping ratio ξ of the system.

1.2. Specify the force values applied to the system in each time instant F_i .

1.3. Determine initial value of displacement u_0 and velocity \dot{u}_0 . Then determine the initial acceleration using relation below:

$$\ddot{u}_0 = \left(\frac{F_0}{m} - 2\xi\omega\dot{u}_0 - \omega^2 u_0 \right)$$

1.4. Select appropriate time-step ($\Delta t < \Delta t_{critical}$) and calculate constant parameters α , β , θ and γ as

$$\alpha = \left(\frac{1}{2\Delta t^2} - \frac{\xi\omega}{3\Delta t} + \frac{\omega^2}{24} \right) \quad \beta = \left(\frac{-1}{2\Delta t^2} - \frac{\xi\omega^2}{\Delta t} + \frac{11\omega^2}{24} \right)$$

$$\theta = \left(\frac{-1}{2\Delta t^2} + \frac{\xi\omega}{\Delta t} + \frac{11\omega^2}{24} \right) \quad \gamma = \left(\frac{1}{2\Delta t^2} + \frac{\xi\omega}{3\Delta t} + \frac{\omega^2}{24} \right) \quad \text{where } \omega = \sqrt{\frac{k}{m}}$$

1.5. Using the below relations, first determine F'_0 unknown value and then obtain \ddot{u}_0 .

$$F'(t_0) = \frac{-F(t_2) + 4F(t_1) - 3F(t_0)}{2\Delta t} \Rightarrow \ddot{u}_0 = \left(\frac{F'(t_0)}{m} - 2\xi\omega\dot{u}_0 - \omega^2 u_0 \right)$$

1.6. Using the below terms determine the four unknown coefficients (C_{-4} , C_{-3} , C_{-2} and C_{-1}).

$$C_{-4} = \left(1 - \frac{11\omega^2\Delta t^2}{12} \right) u_0 - \left(\frac{3\Delta t}{2} - \frac{11\xi\omega\Delta t^2}{6} \right) \dot{u}_0 - \frac{\Delta t^3 \ddot{u}_0}{4} + \frac{11\Delta t^2 F(t_0)}{12m}$$

$$C_{-3} = \left(1 + \frac{\omega^2\Delta t^2}{12} \right) u_0 + \left(\frac{-\Delta t}{2} + \frac{\xi\omega\Delta t^2}{6} \right) \dot{u}_0 + \frac{\Delta t^3 \ddot{u}_0}{12} - \frac{\Delta t^2 F(t_0)}{12m}$$

$$C_{-2} = \left(1 + \frac{\omega^2\Delta t^2}{12} \right) u_0 + \left(\frac{\Delta t}{2} + \frac{\xi\omega\Delta t^2}{6} \right) \dot{u}_0 - \frac{\Delta t^3 \ddot{u}_0}{12} - \frac{\Delta t^2 F(t_0)}{12m}$$

$$C_{-1} = \left(1 - \frac{11\omega^2\Delta t^2}{12}\right)u_0 - \left(\frac{3\Delta t}{2} - \frac{11\xi\omega\Delta t^2}{6}\right)\dot{u}_0 + \frac{\Delta t^3\ddot{u}_0}{4} + \frac{11\Delta t^2 F(t_0)}{12m}$$

2. For each time-step ($i = 0, 1, \dots, n$)

2.1. Calculate displacement, velocity and acceleration simultaneously, by

$$u(t_i) = \frac{1}{24}(C_{i-4} + C_{i-1}) + \frac{11}{24}(C_{i-3} + C_{i-2})$$

$$\dot{u}(t_i) = \frac{1}{6\Delta t}(C_{i-1} - C_{i-4}) + \frac{1}{2\Delta t}(C_{i-2} - C_{i-3})$$

$$\ddot{u}(t_i) = \frac{1}{2\Delta t^2}(C_{i-4} - C_{i-3} - C_{i-2} + C_{i-1})$$

$$\dddot{u}(t_i) = \frac{1}{\Delta t^3}(C_{i-1} - C_{i-4}) + \frac{3}{\Delta t^3}(C_{i-3} - C_{i-2})$$

2.2. Calculate unknown coefficients C_j from $j = 0$ to $(n-1)$ by

$$C_i = \frac{1}{\gamma} \left(\frac{F(t_{i+1})}{m} - \alpha C_{i-3} - \beta C_{i-2} - \theta C_{i-1} \right)$$

4. ALGORITHM IN STANDARD FORM

In order to enable the algorithm to solve the nonlinear problems it is necessary to, first, convert previous formulation to a standard form. For this purpose, it is necessary to solve equations below in order to obtain the C_{i-4} to C_i . These equations are derived from Table 1.

$$u(t_i) = \frac{1}{24}(C_{i-4} + C_{i-1}) + \frac{11}{24}(C_{i-3} + C_{i-2}) \quad (5.a)$$

$$\dot{u}(t_i) = \frac{1}{6\Delta t}(C_{i-1} - C_{i-4}) + \frac{1}{2\Delta t}(C_{i-2} - C_{i-3}) \quad (5.b)$$

$$\ddot{u}(t_i) = \frac{1}{2\Delta t^2}(C_{i-4} - C_{i-3} - C_{i-2} + C_{i-1}) \quad (5.c)$$

$$\dddot{u}(t_i) = \frac{1}{\Delta t^3}(C_{i-1} - C_{i-4}) + \frac{3}{\Delta t^3}(C_{i-3} - C_{i-2}) \quad (5.d)$$

Now, having Eq. (5) in hand, it is possible to solve the system equations and write this four unknowns in terms of displacement, velocity, acceleration and variation of acceleration as

$$C_{i-1} = u_i + \frac{3\Delta t}{2}\dot{u}_i + \frac{11\Delta t^2}{12}\ddot{u}_i + \frac{\Delta t^3}{4}\dddot{u}_i \quad (6.a)$$

$$C_{i-2} = u_i + \frac{\Delta t}{2}\dot{u}_i - \frac{\Delta t^2}{12}\ddot{u}_i - \frac{\Delta t^3}{12}\dddot{u}_i \quad (6.b)$$

$$C_{i-3} = u_i - \frac{\Delta t}{2} \dot{u}_i - \frac{\Delta t^2}{12} \ddot{u}_i + \frac{\Delta t^3}{12} \dddot{u}_i \quad (6.c)$$

$$C_{i-4} = u_i - \frac{3\Delta t}{2} \dot{u}_i + \frac{11\Delta t^2}{12} \ddot{u}_i - \frac{\Delta t^3}{4} \dddot{u}_i \quad (6.d)$$

Setting Eq. (6.a) at the current time (t) equal to the Eq. (6.b) at the next time ($t + \Delta t$), we will get to

$$C_{i-1} = u_i + \frac{3\Delta t}{2} \dot{u}_i + \frac{11\Delta t^2}{12} \ddot{u}_i + \frac{\Delta t^3}{4} \dddot{u}_i = u_{i+1} + \frac{\Delta t}{2} \dot{u}_{i+1} - \frac{\Delta t^2}{12} \ddot{u}_{i+1} - \frac{\Delta t^3}{12} \dddot{u}_{i+1} \quad (7)$$

Then, if we arrange the above equation in terms of u_{i+1} , it can be expressed by

$$u_{i+1} = u_i + \frac{\Delta t}{2} (3\dot{u}_i - \dot{u}_{i+1}) + \frac{\Delta t^2}{12} (11\ddot{u}_i + \ddot{u}_{i+1}) + \frac{\Delta t^3}{12} (3\dddot{u}_i + \dddot{u}_{i+1}) \quad (8)$$

Similarly, if we do this process for Eq. (6.b) at the current time (t) and Eq. (6.c) at the next time ($t + \Delta t$), we will get to an equation. Then, if we arrange the outcome in terms of \dot{u}_{i+1} , we will have

$$\dot{u}_{i+1} = -\dot{u}_i + \frac{2}{\Delta t} (u_{i+1} - u_i) + \frac{\Delta t}{6} (\ddot{u}_i - \ddot{u}_{i+1}) + \frac{\Delta t^2}{6} (\dddot{u}_i + \dddot{u}_{i+1}) \quad (9)$$

Finally, if we follow the above process for both Eq. (6.c) and (6.d), and then write the resulted equation in terms of \ddot{u}_{i+1} , we get to

$$\ddot{u}_{i+1} = \frac{-\ddot{u}_i}{3} + \frac{1}{3\Delta t} (\dot{u}_i + 11\dot{u}_{i+1}) + \frac{2}{\Delta t^2} (u_i - 3u_{i+1}) + \frac{4}{\Delta t^3} (u_{i+1} - u_i) \quad (10)$$

The aim is to remove the term \ddot{u}_{i+1} from Eq. (8) and (9). Therefore, substituting Eq. (10) in Eq. (8) and (9), it is possible to rewrite these equations in the following forms, respectively.

$$u_{i+1} = u_i + \frac{\Delta t}{2} (5\dot{u}_i - 3\dot{u}_{i+1}) + \frac{\Delta t^2}{12} (17\ddot{u}_i + 7\ddot{u}_{i+1}) + \frac{\Delta t^3}{3} \dddot{u}_i \quad (11)$$

$$\dot{u}_{i+1} = \frac{-\dot{u}_i}{3} - \frac{4}{3\Delta t} (u_i - u_{i+1}) + \frac{\Delta t}{9} (\ddot{u}_i + 2\ddot{u}_{i+1}) + \frac{\Delta t^2}{18} \dddot{u}_i \quad (12)$$

Replacing Eq. (12) with \dot{u}_{i+1} in Eq. (13) will result to

$$u_{i+1} = u_i + \Delta t \dot{u}_i + \frac{\Delta t^2}{12} (5\ddot{u}_i + \ddot{u}_{i+1}) + \frac{\Delta t^3}{12} \ddot{\ddot{u}}_i \quad (13)$$

It can be seen that the only term with $i+1$ index is \ddot{u} in the right hand of the above equation. Now, if we use the above equation instead of u_{i+1} in Eq. (12), this equation will appear in the following form after simplification. Here, also the only term with $i+1$ index is \ddot{u} in the right hand.

$$\dot{u}_{i+1} = \dot{u}_i + \frac{\Delta t}{3} (2\ddot{u}_i + \ddot{u}_{i+1}) + \frac{\Delta t^2}{6} \ddot{\ddot{u}}_i \quad (14)$$

Replacing Eq. (13) and (14) with u_{i+1} and \dot{u}_{i+1} , in Eq. (10) respectively, this equation will appear in the following shape after simplification. In this equation also the only term with $i+1$ index is \ddot{u} .

$$\ddot{u}_{i+1} = \frac{2}{\Delta t} (\ddot{u}_{i+1} - \ddot{u}_i) - \ddot{\ddot{u}}_i \quad (15)$$

From Eqs. (13) to (15) it is understood that response values including displacement, velocity, acceleration and variation of acceleration at the end of each time-step are dependent on those values in the end of previous time-step. So this new formulation of the proposed method is an implicit scheme.

Now, having equations (13), (14) and (15) we can rewrite this equation in terms of $\Delta \ddot{u}_i$ as follows:

$$\Delta u_i = \Delta t \dot{u}_i + \frac{\Delta t^2}{2} \ddot{u}_i + \frac{\Delta t^2}{12} \Delta \ddot{u}_i + \frac{\Delta t^3}{12} \ddot{\ddot{u}}_i \quad (16)$$

$$\Delta \dot{u}_i = \Delta t \ddot{u}_i + \frac{\Delta t}{3} \Delta \ddot{u}_i + \frac{\Delta t^2}{6} \ddot{\ddot{u}}_i \quad (17)$$

$$\Delta \ddot{u}_i = \frac{2}{\Delta t} \Delta \ddot{u}_i - 2\ddot{\ddot{u}}_i \quad (18)$$

By sort of Eq. (16) in terms of $\Delta \ddot{u}_i$ we have:

$$\Delta \ddot{u}_i = \frac{12}{\Delta t^2} \Delta u_i - \frac{12}{\Delta t} \dot{u}_i - 6\ddot{u}_i - \Delta t \ddot{\ddot{u}}_i \quad (19)$$

In the above equation the only incremental term is Δu_i . Replacing Eq. (19) in Eq. (17), after simplifying we will have a new equation as below

$$\Delta \dot{u}_i = \frac{4}{\Delta t} \Delta u_i - \Delta t \ddot{u}_i - 4\dot{u}_i - \frac{\Delta t^2}{6} \ddot{\ddot{u}}_i \quad (20)$$

Substituting Eq. (19) and (20) into the incremental equation of motion;

$$m \Delta \ddot{u}_i + c \Delta \dot{u}_i + k \Delta u_i = \Delta p_i \quad (21)$$

After simplifying gives

$$\left(k + \frac{4c}{\Delta t} + \frac{12m}{\Delta t^2} \right) \Delta u_i = \Delta p_i + \left(\left(\frac{12m}{\Delta t^2} + 4c \right) \dot{u}_i + (6m + c \Delta t) \ddot{u}_i + \left(m \Delta t + \frac{c}{6} \Delta t^2 \right) \ddot{\ddot{u}}_i \right) \quad (22)$$

It is possible to rewrite Eq. (22) as,

$$\hat{k} \Delta u_i = \Delta \hat{p}_i \quad (23)$$

where

$$\hat{k} = k + \frac{4c}{\Delta t} + \frac{12m}{\Delta t^2} \quad (24)$$

and

$$\Delta \hat{p}_i = \Delta p_i + \left(\frac{12m}{\Delta t^2} + 4c \right) \dot{u}_i + (6m + c \Delta t) \ddot{u}_i + \left(m \Delta t + \frac{c}{6} \Delta t^2 \right) \ddot{\ddot{u}}_i \quad (25)$$

In order to write a computer code, the complete algorithm used in this proposed method is summarized in Table 2.

If we compare this algorithm with the same algorithm presented for quartic B-spline method in Ref. [17] it will be recognized that the new procedure is summarized and much simpler.

Table 2: Generalized Quartic B-spline step-by-step time integration algorithm; linear systems

1. Initial calculation

1.1. Determine stiffness \mathbf{k} , mass \mathbf{m} , and damping ratio ξ of the system. And select appropriate time-step Δt .

1.2. Determine the force value applied to the system in each time instant and initial value of displacement u_0 and velocity \dot{u}_0 .

$$1.3. \ddot{u}_0 = \frac{1}{m}(F_0 - c\dot{u}_0 - ku_0)$$

$$1.4. F'_0 = \frac{1}{2\Delta t}(-F_2 + 4F_1 - 3F_0)$$

$$1.5. \ddot{u}'_0 = \frac{1}{m}(F'_0 - c\ddot{u}_0 - k\dot{u}_0)$$

$$1.6. \hat{k} = k + \frac{4c}{\Delta t} + \frac{12m}{\Delta t^2}$$

$$1.7. \alpha = \frac{12m}{\Delta t} + 4c, \quad \beta = 6m + c\Delta t, \quad \gamma = m\Delta t + \frac{c}{6}\Delta t^2$$

2. For each time step ($i=0,1,\dots,n$)

$$2.1. \Delta \hat{p}_i = \Delta p_i + \alpha \dot{u}_i + \beta \ddot{u}_i + \gamma \ddot{u}'_i$$

$$2.2. \Delta u_i = \Delta \hat{p}_i / \hat{k}$$

$$2.3. \Delta \dot{u}_i = \frac{4}{\Delta t} \Delta u_i - \Delta t \ddot{u}_i - 4\dot{u}_i - \frac{\Delta t^2}{6} \ddot{u}'_i$$

$$2.4. \Delta \ddot{u}_i = \frac{12}{\Delta t^2} \Delta u_i - \frac{12}{\Delta t} \dot{u}_i - 6\ddot{u}_i - \Delta t \ddot{u}'_i$$

$$2.5. \Delta \ddot{u}'_i = \frac{2}{\Delta t} \Delta \ddot{u}_i - \ddot{u}'_i$$

$$2.6. u_{i+1} = u_i + \Delta u_i, \quad \dot{u}_{i+1} = \dot{u}_i + \Delta \dot{u}_i, \quad \ddot{u}_{i+1} = \ddot{u}_i + \Delta \ddot{u}_i, \quad \ddot{u}'_{i+1} = \ddot{u}'_i + \Delta \ddot{u}'_i$$

5. DEVELOPING THE ALGORITHM FOR NONLINEAR SYSTEMS

In this section, the standard form of quartic B-spline method described in previous section for linear systems is developed for nonlinear systems.

Incremental equilibrium equation for a nonlinear system given as

$$m\Delta \ddot{u}_i + c\Delta \dot{u}_i + \Delta f s_i = \Delta p_i \quad (26)$$

The incremental resisting force is

$$(\Delta f s)_i = (k_i)_{\text{sec}} \Delta u_i \quad (27)$$

where the secant stiffness $(k_i)_{\text{sec}}$, shown in Fig. 3, cannot be determined because u_{i+1} is not known. If we make the assumption that over a small time step Δt , the secant stiffness $(k_i)_{\text{sec}}$, could be approximated by

$$(\Delta f s)_i = (k_i)_{\text{sec}} \Delta u_i \quad (28)$$

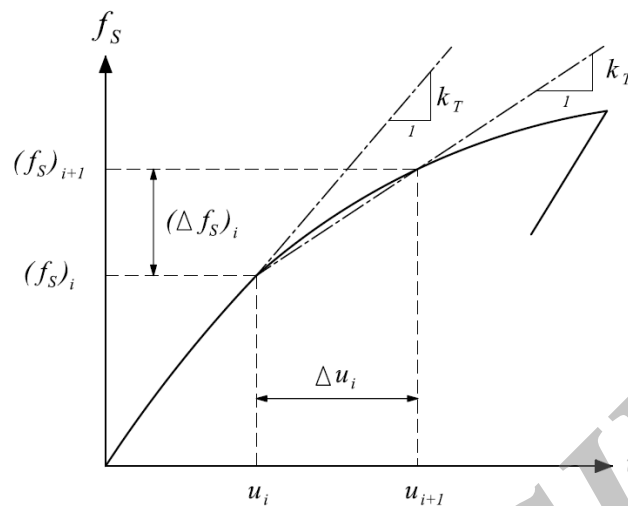


Figure 3. Secant and tangent stiffness

The similarity between the Eq. (26) and the corresponding equation for linear systems, Eq. (21), suggests that the no iterative formulation of quartic B-spline method presented earlier for linear systems may also be used in the analysis of nonlinear response. All that needs to be done is to replace k in Eq. (24) by the tangent stiffness k_i to be evaluated at the beginning of each time step. This change implies that step 6.1 of Table 2 should follow step 2.1 For nonlinear system the \ddot{u}_{i+1} term from $\ddot{u}_{i+1} = \ddot{u}_i + \Delta\ddot{u}_i$ would give different values of \ddot{u}_{i+1} and the latter value is preferable because it satisfies equilibrium at time $i + 1$.

This procedure with a constant time step Δt can lead to unacceptably inaccurate results. Significant errors arise for two reasons: (1) the tangent stiffness was used instead of the secant stiffness, and (2) use of a constant time step delays detection of the transitions in the force-deformation relationship [9].

First, we consider the second source of error, illustrated by the force-deformation relation of Fig. (4a). Suppose that the displacement at time i , the beginning of a time step, is u_i and the velocity \dot{u}_i is positive (i.e., the displacement is increasing); this is shown by point a . Application of the previously described numerical procedure for the time step results in displacement u_{i+1} and velocity \dot{u}_{i+1} at time $i + 1$; this is shown by point b . If \dot{u}_{i+1} is negative, then at some point b' during the time step, the velocity become zero, changed sign, and the displacement started decreasing. In the numerical procedure, if we do not bother to locate b' , continue with the computations by starting the next time step at point b , can use the tangent stiffness associated with the unloading branch of the force-deformation diagram, this procedure locates the point c at the end of the next time step with displacement u_{i+1} and negative velocity. On the other hand, if the time instant associated with b' (when the velocity actually became zero) could be determine, computations for the next time step would start with the state of the system at b' and determine the displacement and velocity at the end of the time step, identified as c' . Not locating b' has the effect of overshooting to b and not following the exact path on the force-deformation diagram. This departure from the

exact path would occur at each reversal of velocity, leading to errors in the numerical results. A similar problem arises at sharp corners in the force-deformation relationship, as in elasto-plastic systems [9].

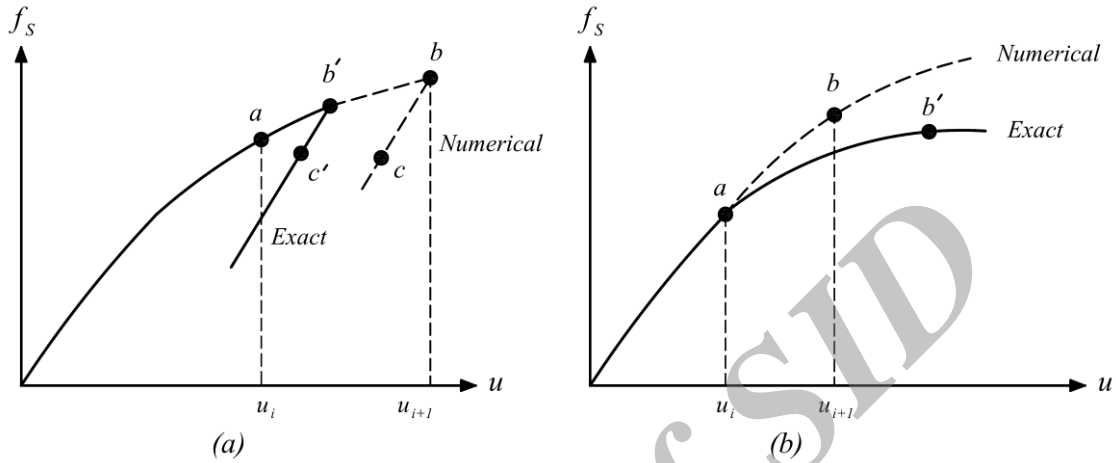


Figure 4. Probable errors in numerical method

These errors could be avoided by locating b' accurately. This could be achieved by retracing the integration over the time interval t_i to t_{i+1} with a smaller time step. Alternatively, an iterative process may be used in which integration is resumed from time i with a step smaller than the full time step, whose size is progressively adjusted so that at the end of such an adjusted time step, the velocity is close to zero.

Now, we return to the first source of error that is associated with the use of tangent stiffness instead of the unknown secant stiffness, and is illustrated by the force-deformation relation of Fig. (4b). The displacement at time i , the beginning of a time step t_i to t_{i+1} leads to the displacement u_{i+1} , identified as point b . if we were able to follow the curve exactly, the result may have been the displacement at b' . This discrepancy accumulating over a series of time steps may introduce significant errors.

These errors can be minimized by using an iterative procedure. The key equation that is solved at each time step is Eq. (23) so for nonlinear systems, the terms k replace by k_T to emphasize that this is the tangent stiffness; so the Eq. (23) and (24) become

$$\hat{k}_T \Delta u = \Delta \hat{p} \quad (29)$$

and

$$\hat{k}_T = k_T + \frac{4c}{\Delta t} + \frac{12m}{\Delta t^2} \quad (30)$$

Fig. (5) shows a schematic plot of Eq. (29). The relationship is nonlinear because the tangent

stiffness k_T depends on the displacement u and hence the slope \hat{k}_T is not constant [9].

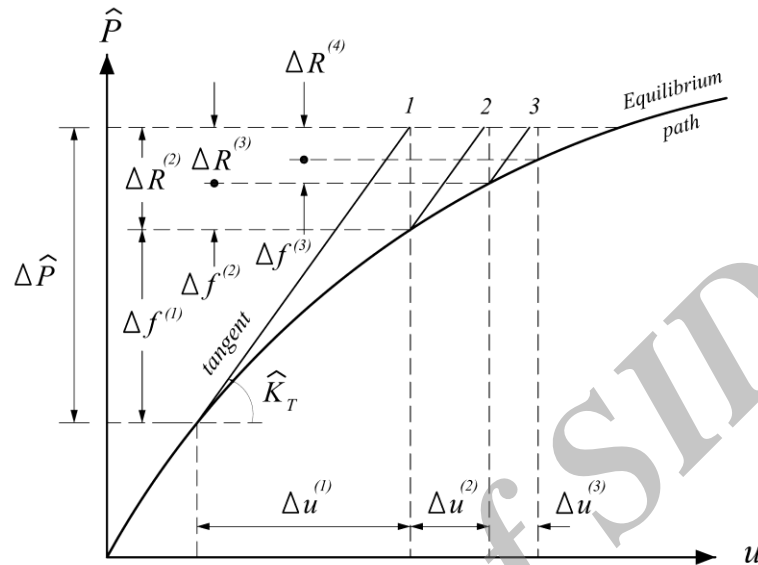


Figure 5. Modified Newton-Raphson iteration in one increment

The iterative procedure is described next with reference to Fig (5). The first iterative step is the application of Eq. (29) in the procedure described previously:

$$\hat{k}_T \Delta u^{(1)} = \Delta \hat{p} \quad (31)$$

To determine $\Delta u^{(1)}$ (corresponding to point b in Fig. (4b)), the first approximation to the final Δu (corresponding to point b' in Fig. (4b)). Associated with $\Delta u^{(1)}$ is the true force $\Delta f^{(1)}$, which is less than $\Delta \hat{p}$, and a residual force is defined: $\Delta R^{(2)} = \Delta \hat{p} - \Delta f^{(1)}$.

The additional displacement $\Delta u^{(2)}$ due to this residual force is determined from

$$\hat{k}_T \Delta u^{(2)} = \Delta R^{(2)} = \Delta \hat{p} - \Delta f^{(1)} \quad (32)$$

This additional displacement is used to find a new value of the residual force, and the process is continued until convergence is achieved. This iterative process for the time step t_i to t_{i+1} , summarized in Table 3, is known as the modified Newton-Raphson method.

Table 3: Modified Newton-Raphson iteration

1. Initialize data			
$u_{i+1}^0 = u_i$	$f_s^{(0)} = (fs)_i$	$\Delta R^{(1)} = (\Delta \hat{p})_i$	$\hat{k}_T = \hat{k}_i$
2. Calculation for each iteration, $j = 1, 2, 3, \dots$			

$$2.1. \text{ Solve: } \hat{k}_T \Delta u^{(j)} = \Delta R^{(j)} \Rightarrow \Delta u^{(j)}$$

$$2.2. u_{i+1}^{(j)} = u_{i+1}^{(j-1)} + \Delta u^{(j)}$$

$$2.3. \Delta f^{(j)} = f_s^{(j)} - f_s^{(j-1)} + (\hat{k}_T - k_T) \Delta u^{(j)}$$

$$2.4. \Delta R^{(j+1)} = \Delta R^{(j)} - \Delta f^{(j)}$$

3. Repetition for the next iteration. Replace j by $j+1$ and repeat calculation step 2.1 to 2.4.

The iteration process is terminated after n iterations when the incremental displacement $\Delta u^{(n)}$ becomes small enough compared to the current estimate of $\Delta u = \sum_{j=1}^n \Delta u^{(j)}$; that is $(\Delta u^{(n)} / \Delta u) < \varepsilon$. Then the displacement increment over the time step t_i to t_{i+1} is given by:

$$\Delta u_i = \sum_{j=1}^n \Delta u^{(j)} \quad (33)$$

This is an accurate value of Δu_i that replaces the one obtained without iteration from Eq. (23); the latter is the same as $\Delta u^{(1)}$ obtained after one iteration. With Δu_i known, the rest of the computation proceeds as before.

Table 4 summarizes the time-stepping solution as it might be implemented on the computer code.

Table 4: Generalized Quartic B-spline step-by-step time integration algorithm; nonlinear systems

1. Initial calculation

1.1. Determine stiffness \mathbf{k} , mass \mathbf{m} , and damping ratio ξ of the system. And select appropriate time-step Δt .

1.2. Determine the force value applied to the system in each time instant and initial value of displacement u_0 and velocity \dot{u}_0 .

$$1.3. \ddot{u}_0 = \frac{1}{m} (F_0 - c u_0 - k u_0)$$

$$1.4. F'_0 = \frac{1}{2\Delta t} (-F_2 + 4F_1 - 3F_0)$$

$$1.5. \ddot{u}'_0 = \frac{1}{m} (F'_0 - c \dot{u}_0 - k u_0)$$

$$1.6. \alpha = \frac{12m}{\Delta t} + 4c, \quad \beta = 6m + c\Delta t, \quad \gamma = m\Delta t + \frac{c}{6}\Delta t^2$$

2. For each time step ($i = 0, 1, \dots, n$)

$$2.1. \Delta \hat{p}_i = \Delta p_i + \alpha u_i + \beta \ddot{u}_i + \gamma \ddot{u}'_i$$

2.2. Determine the tangent stiffness k_i

$$2.3. \hat{k}_i = k_i + \frac{4c}{\Delta t} + \frac{12m}{\Delta t^2}$$

2.4. Solve for Δu_i from \hat{k}_i and $\Delta \hat{p}_i$ using the iterative procedure of Table 3.

$$2.5. \Delta \dot{u}_i = \frac{4}{\Delta t} \Delta u_i - \Delta t \ddot{u}_i - 4\dot{u}_i - \frac{\Delta t^2}{6} \ddot{u}_i$$

$$2.6. \Delta \ddot{u}_i = \frac{12}{\Delta t^2} \Delta u_i - \frac{12}{\Delta t} \dot{u}_i - 6\ddot{u}_i - \Delta t \ddot{u}_i$$

$$2.7. \Delta \ddot{u}_i = \frac{2}{\Delta t} \Delta \ddot{u}_i - \ddot{u}_i$$

$$2.8. u_{i+1} = u_i + \Delta u_i, \quad \dot{u}_{i+1} = \dot{u}_i + \Delta \dot{u}_i, \quad \ddot{u}_{i+1} = \frac{1}{m} (F_{i+1} - c\dot{u}_{i+1} - ku_{i+1}), \quad \ddot{u}_{i+1} = \ddot{u}_i + \Delta \ddot{u}_i$$

Although the modified Newton-Raphson converges by more number of iterations compared to conventional Newton-Raphson method, it reduces the number of operations to invert stiffness matrix (for multi degrees of freedom). In the previous table, modified Newton-Raphson method was used for iterative process. Any other method such as Arc-Length method [23] also can be used.

Using the proposed algorithm a computer code is written by Matlab. Using this computer code many examples have been analyzed. A couple of those examples are given in the next section.

6. NUMERICAL EVALUATION

In this section, the validity of the proposed method is confirmed with examination of several results. Two examples with nonlinear behavior are considered.

6.1 A SDOF system with nonlinear behavior under blast load

Fig. 6.a shows a SDOF portal frame with nonlinear behavior under an exploding blast load that shows in Fig. 6.b. The properties such as frame elevation, mass and modulus of elasticity are shown in the figure. In this example damping ratio $\xi = 0.078$ and the force-deformation relation (stiffness) is $12.35/\sqrt{|u|+1}$ kips/in. In this example, $\Delta T = 0.1$ sec has been selected as time increment.

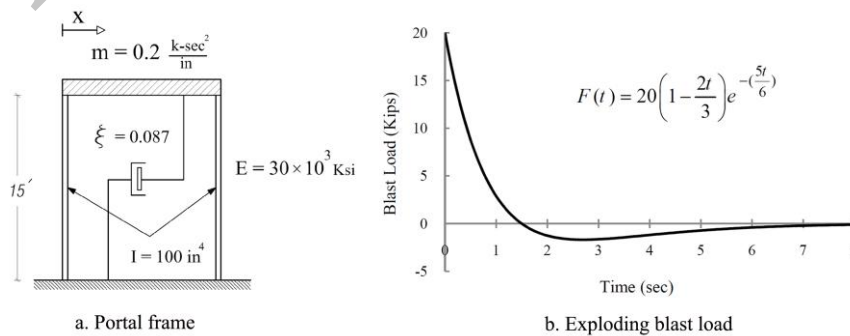


Figure 6. A portal frame under exploding blast load

Figures 7 to 9 show the analysis results including relative displacement, velocity and acceleration of the aforesaid system resulted by Newmark, Wilson, Central difference and Quartic B-spline (proposed) methods during 8-seconds after the exploding. As the graphs show the proposed generalized method provides acceptable estimate of the results.

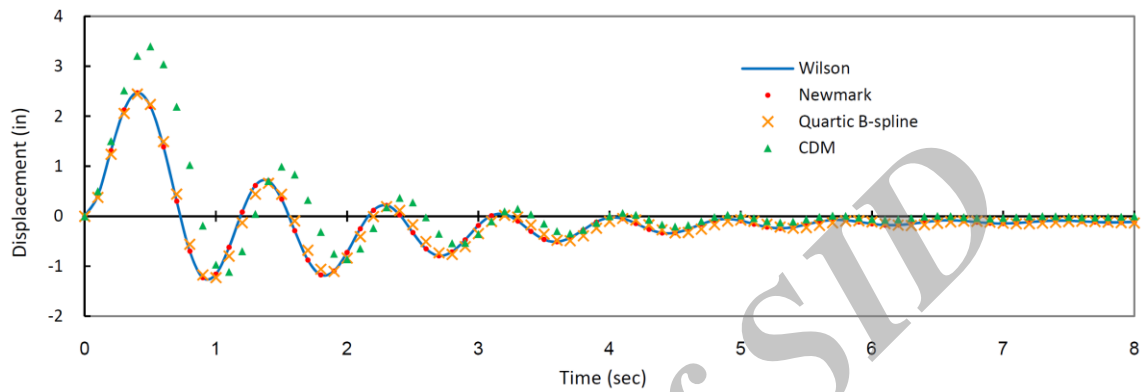


Figure 7. Displacement time-histories

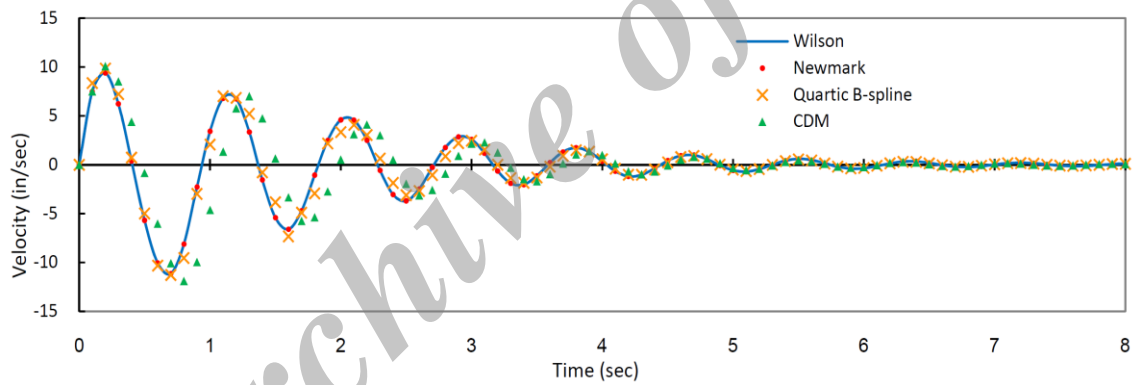


Figure 8. Velocity time-histories

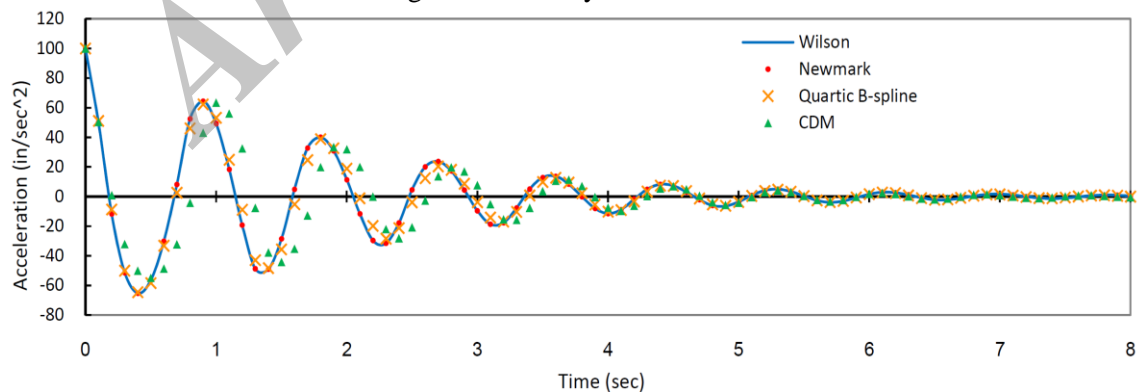


Figure 9. Acceleration time-histories

6.2 A portal frame with elasto-plastic behavior under blast load

For the portal frame in previous example a bilinear behavior (elasto-plastic), as depicted in the Fig. 10, is defined. Here also the excitation function is the blast load has been shown in Fig. 6.b.

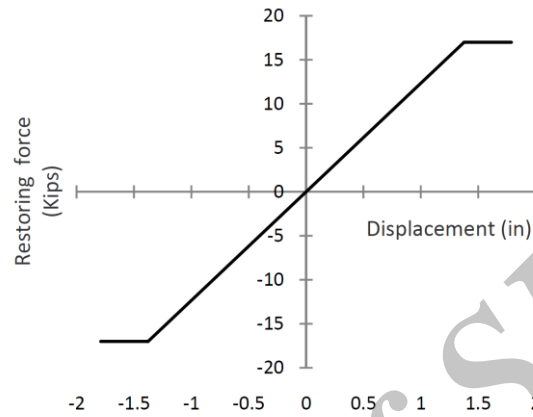


Figure 10. Elasto-plastic behavior

Fig. 11 shows the analysis result i.e. displacement time-history of the above system for three Wilson, Newmark and quartic B-spline (proposed) methods during an 8-seconds duration after explosion. Fig. 12 also shows the hysteresis diagram of bilinear behavior of the system. As the graph shows, the result of proposed method is nearly coincident with the results of two other methods.

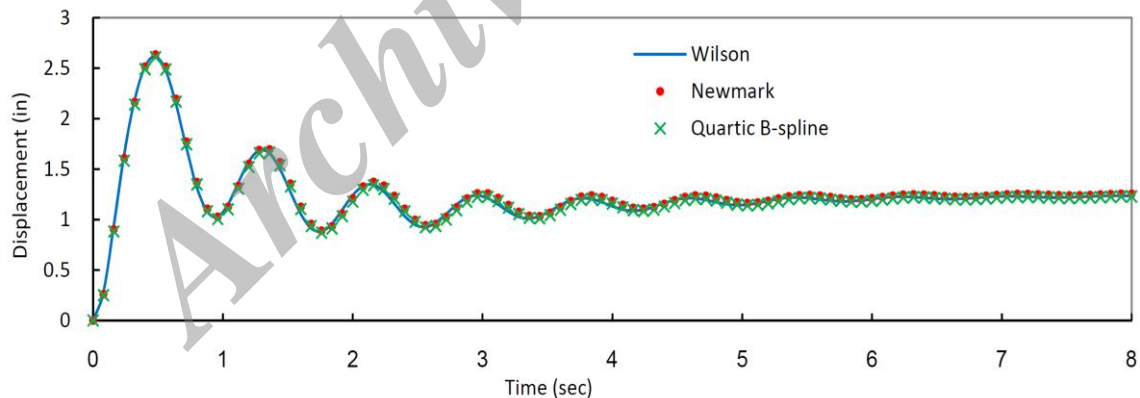


Figure 11. Displacement time-histories

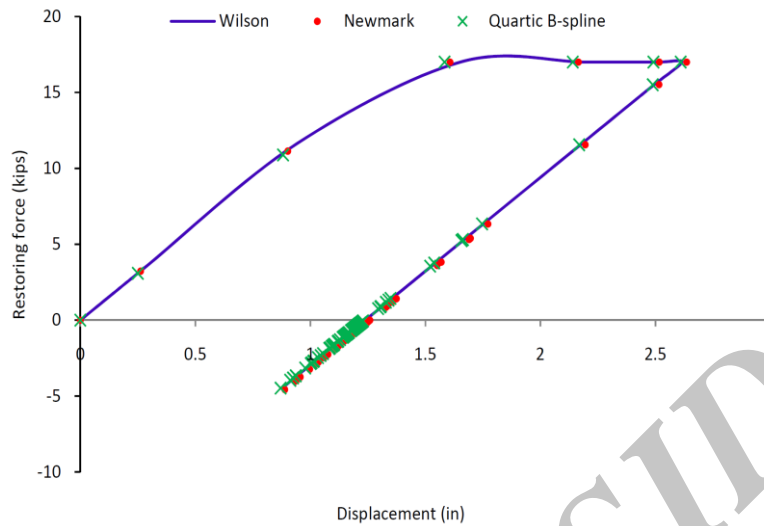


Figure 12. Hysteresis diagram

7. CONCLUSIONS

So many efforts have been made during the past three decades to develop methods of structural analysis and design to resist blast loads. This paper developed quartic B-spline time integration method for inelastic dynamic analysis of SDOF systems under blast loads. A series of standard formula were derived from previous formulation and a new simple and efficient algorithm presented for linear analysis. Newton-Raphson iterative method used to develop new algorithm for solving nonlinear dynamic problems. A simple and effective step-by-step algorithm is implemented and presented to calculate nonlinear dynamic response of SDOF systems. Quartic B-spline time integration method gains second order of acceleration at each time-step so it benefits from high order accuracy. The numerical evaluation shows that the proposed method is a fast and simple procedure with trivial computational effort; therefore, it may be an appropriate choice for inelastic time-history analysis under blast and explosive loads as a time integration method.

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