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VARIABLE FORM FORMING INVESTIGATION FOR FLEXIBLE SHALLOW SHELLS ON CIRCULAR BASE

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ABSTRACT

This paper contains geometric nonlinear shallow shell of revolution analysis under static loading. Consideration is given to the action of evenly distributed vertical force for various support types. Critical force and stress are found considering geometric nonlinearity (flexibility) of steel and ferro-cements shell. Critical force coefficient and stress of shells are found by Bubnov-Galerkin method. Stresses, critical force - shape parameter dependence are presented.

Keywords: Shallow shells; nonlinearity; shells on circular base; critical force; strength; variable form.

1. INTRODUCTION

Shallow shell of revolution is promising structures. They can be used as the foundations of buildings and structures, tanks, roof structures. Therefore, the development of methods of their analysis is an urgent task. Currently, most of these structures are calculated with software based on the finite element method. They allow you to get a quick result with minimum efforts. However, the result is highly dependent on the type, numbers of finite element, and so on. Increasing the number of finite elements does not lead to accuracy. Even greater difficulties arise when we need to design geometric nonlinear constructions. In addressing some of the challenges of designing, analyzing and optimizing designs more accurate results are obtained by using direct variational methods. In some problems, the direct variational methods provides greater accuracy and adaptability compared with the finite element method.

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2. CRITICAL FORCE COEFFICIENT OF FLEXIBLE SHALLOW SHELLS ON CIRCULAR BASE SEARCHING

Critical force coefficient for geometric nonlinear shallow shells of revolution was found by Bubnov-Galerkin method [1-3]. Equations have been obtained for various types of boundary conditions. Beam functions are used for the Bubnov-Galerkin approximation and boundary conditions formulations.

Middle surface forming a variable form for shallow shells of revolution can be described by the equation

$$F(\rho) = f_0 \rho^{\xi} \qquad (1)$$

where f_0 - is the rise of arch in the center of the shell, $f_0 \le \frac{a}{5}$.

a - radius of the shell base,

ρ - dimensionless radius of the shell, range changing is $[0,1]$,

ξ - shape forming parameter, range changing is [0,4].

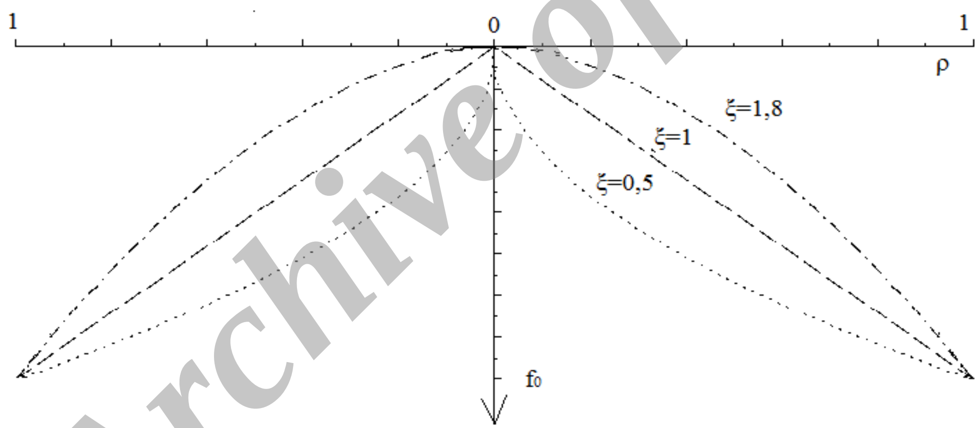

Figure 1. Dependence of the middle surface of shallow shell of rotation on the form parameter

Critical force coefficient can be described by the equation:

$$p = \frac{a^4}{E f_0^{\,4}} p_{cr} \qquad (2)$$

where

$$p=\frac{1}{t^4}k_1((k_2t^2\psi^2_{(\xi)}-k_3(k_0+t^2F_{(\xi)}))^{\frac{3}{2}}+ \\ +k_4t\psi_{(\xi)}(k_2t^2\psi^2_{(\xi)}-1{,}5k_3(k_0+t^2F_{(\xi)})));\qquad(3)$$

$$t=f_0/h\qquad(4)$$

$$k_0=\frac{\frac{4}{3}-2\alpha}{12(1-\nu^2)},\ k_1=\frac{2}{27}\left(\frac{S_4^*}{S_1}\right)^2,\ k_2=\left(\frac{1}{S_4^*}\right),\ k_3=\frac{3S_1}{(S_4^*)^2},\ \ k_4=\frac{1}{S_4^*},\qquad(5)$$

$$\alpha=\frac{(3+\nu)c_1+1}{(1+\nu)c_1+1}\qquad(6)$$

$$S_1=\frac{1}{96}\left(\frac{1}{14}-\frac{\alpha}{2}+\frac{3}{2}\alpha^2-2\alpha^3+\alpha^4- \right. \\ \left. -\left(\frac{(c_2-\nu)(1-4\alpha+6\alpha^2)}{(c_2-\nu)+1}+\frac{1-20\alpha+18\alpha^2}{(c_2-\nu)+1}\right)\times\left(\frac{1}{8}-\frac{\alpha}{3}+\frac{\alpha^4}{4}\right)\right),\qquad(7)$$

$$S_4^*=\frac{\frac{4}{3}-2\alpha}{16},\qquad(8)$$

E - elasticity module, ν - Poisson's ratio; c_1, c_2 – characteristics depending on support type; $\psi(\xi)$, $F(\xi)$ - functions depending on the shape forming parameter and support type of the shell, h - shell thickness.

The results obtained by this method were compared with other techniques. Table 1 shows the value for critical loads of metal shell with radius of the shell base a = 1000 mm, the rise of arch in the shell centre f_0 =100 mm and thickness h = 10 mm. Shape forming parameter ξ = 1,05.

Table 1: Comparison of critical loads

	Stupisin L.	Geniev G., Chausov N. [4]	SCAD-office
The value of the critical load, kg/cm²	328,92	303,00	216,05

Good agreements between the results in columns 1 and 2 are explained by similar methods of calculating nonlinear shells (Bubnov - Galerkin method, type of support, etc.). Calculation of the finite element method in a program complex «SCAD - office» is executed (1,600 triangular finite elements).

Finite - element numerical procedure [5] based on an approximation of a triangular area of a fourth-order polynomial. Geometrically nonlinear solution of the problem is carried out the method of successive loadings.

3. STRESS COEFFICIENT OF FLEXIBLE SHALLOW SHELLS ON CIRCULAR BASE SEARCHING

Stress coefficient for geometric nonlinear shallow shells of revolution was found by Bubnov-Galerkin method [1-3].

Shell equivalent stresses are found by use fourth stress hypothesis. Stress coefficient can be described by the equation:

$$\overline{\sigma} = \frac{a^4}{Ef_0^4}\sigma, \tag{9}$$

where,

$$\overline{\sigma} = \sqrt{\frac{1}{2}\left[\left(\overline{\sigma_r} + \overline{\sigma_v}\right)^2\right]}; \tag{10}$$

$$\overline{\sigma_r} = \overline{\sigma_r^u} + \overline{\sigma_r^m},\ \overline{\sigma_v} = \overline{\sigma_v^u} + \overline{\sigma_v^m}, \tag{11}$$

$$\overline{\sigma_r^m} = N_r,\ \overline{\sigma_v^m} = N_v, \tag{12}$$

$$\rho = \frac{r}{a}, \tag{13}$$

$$\overline{\sigma_r^u} = \left(-\frac{1}{2(1-\nu^2)}\left(\frac{dw}{d\rho} + \nu\frac{w}{\rho}\right)\right),\ \overline{\sigma_v^u} = \left(-\frac{1}{2(1-\nu^2)}\left(\nu\frac{dw}{d\rho} + \frac{w}{\rho}\right)\right), \tag{14}$$

r – distance from an arbitrary point on the middle surface of a shell to its rotation axis, changing in the range (0,a); N_r and N_v - dimensionless parameters of radial and circumferential forces, w - deflection of a median surface of shell.

The results of stresses obtained by this method were compared with other techniques. Table 2 shows the value for stress of metal shell with radius of the shell base a = 100 mm, the rise of arch in the center of the shell f_0 =2 mm and thickness h = 1 mm. Shape forming parameter ξ = 2.

Table 2: Comparison of stresses

	Stupisin L.	Kornisnin M. [6]	SCAD-office
σ (Н/м²)	2,6391 x10^7	3,1899 x10^7	1,0478 x10^8

Solution [6] obtained by the method of finite differences. The results are satisfactory agreement in columns 1, 2.

4. ANALYSIS OF CRITICAL FORCE COEFFICIENT AND STRESS

The calculation method was based on the numerical software package. The changes of the parameters of shallow shells of revolution depending on the shape forming parameter, support type and material were studied earlier [7-8].

Fig. 2 shows how the critical force parameter changes under the action of various vertical forces: distributed, centred, circular and ring. Types of support: 1– fixed; 2 – guided; 3 – simple support.

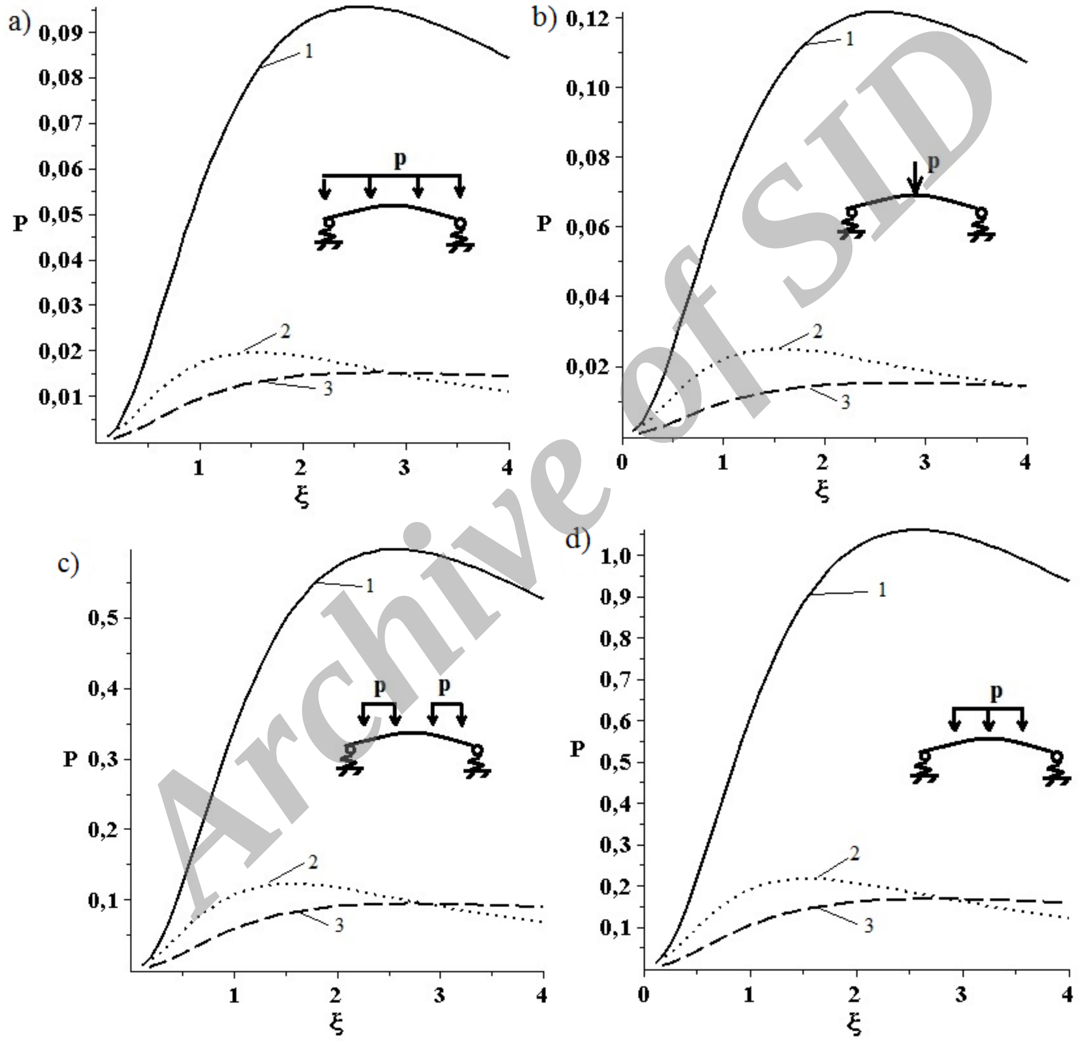

Figure 2. The dependence of the critical force on the shape forming parameter for fixed (line 1), guided (line 2) and simple support (line 3): a – distributed vertical force, b – centred force; c - ring force, d – circular force;

The graphs demonstrate a similar dependence of the critical force parameter for different force forms. This suggests that in further calculations we can use evenly distributed vertical force.

Fig. 3 shows the dependence of critical force on the middle surface form parameter.

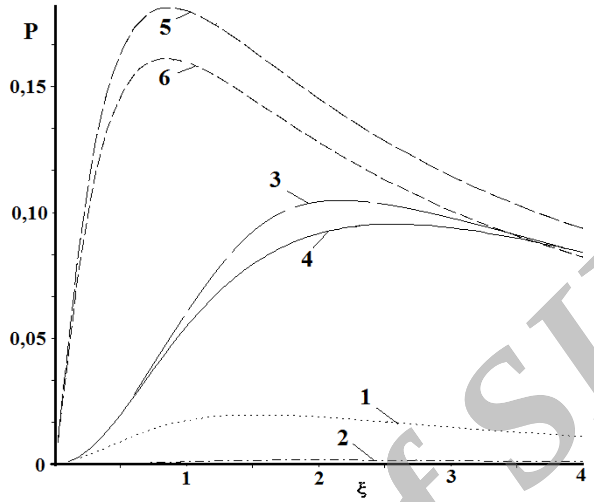

Figure 3. The dependence of the critical force and the shape forming parameter for metals (lines 1, 3 and 5) and ferro-cements (lines 2, 4 and 6): 1, 2 – fixed; 3, 4 – guided; 5, 6 – simple support

It is obvious that when rigidity decreases in the radial direction, the most optimal shape of the shell approaches the cone. With increasing rigidity this shape approximates to sphere. Critical force increases when the rigidity of circumferential fixing decreases.

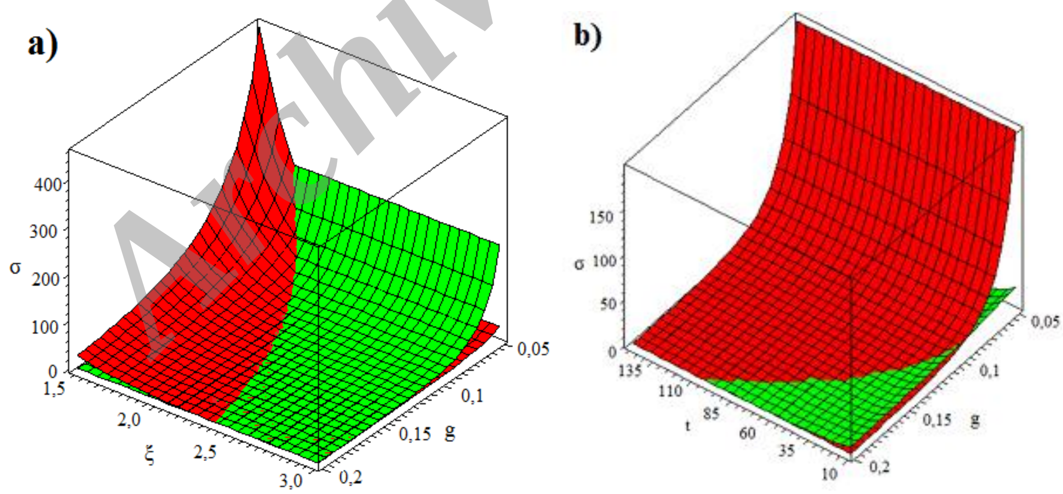

Figure 4. Dependence of the stresses on the following parameters of a shell: a) shape forming parameter ξ and parameter $g = f_0 / a$; b) relative thickness parameter $t = f_0 / h$ and parameter $g = f_0 / a$

Fig. 4 shows design parameter spaces g, t and ξ for which we have built surfaces of limit stresses in the centre of the shell (light area) and the stresses arising in the same section under a force that will cause a critical stability condition in the shell with the same shape (dark area).

These graphs demonstrate that shallow shells should be analysed for stability only when g, t and ξ parameters are such that the green area is located above the red area. When parameters g, t and ξ correspond to the positioning of the red area above the green one, the shell will be losing its strength faster, which means that such shells shall be analyzed for strength. If the parameters of a shell match the curve (track) at the intersection of these two areas, the shell shall be analysed both for the strength and stability.

5. ANALYSIS OF OPTIMUM FORMS OF GEOMETRIC NONLINEAR SHALLOW SHELLS ON CIRCULAR BASE

Analysis of optimum forms of geometric nonlinear shallow shells on circular base

Optimization problems for shells of revolution can be written as [9-10]:

$$\begin{cases} p(\xi,t) \to p_{\max}; \\ V(\xi,t) - V_0 \le 0, \xi \in G, t \in G. \end{cases} \tag{22}$$

$$\begin{cases} \sigma(\xi,t) \to \sigma_{\min}; \\ V(\xi,t) - V_0 \le 0, \xi \in G, t \in G. \end{cases} \tag{23}$$

$$\begin{cases} V(\xi,t) \to V_{\min}; \\ p(\xi,t) - p_0 \ge 0, \xi \in G, t \in G. \end{cases} \tag{24}$$

$$\begin{cases} V(\xi,t) \to V_{\min}, \\ \sigma(\xi,t) - \sigma_0 \le 0, \xi \in G, t \in G. \end{cases} \tag{25}$$

$$\begin{cases} V(\xi,t) \to V_{\min}; \\ p(\xi,t) - p_0 \ge 0, \xi \in G, t \in G; \\ \sigma(\xi,t) - \sigma_0 \le 0, \xi \in G, t \in G. \end{cases} \tag{26}$$

$$G = \{\xi : 0{,}5 < \xi_i < 2, i = \overline{1,n}; t : 0{,}001 < t_j < 0{,}1, j = \overline{1,m}\}$$

where,

$V(x_i)$ - volume function,

$p(x_i)$ - critical force function,

$\sigma(x_i)$ - stress functions.

Fig. 5 depicts the dependence of the volume of the shells on the parameter ξ and thickness of shell.

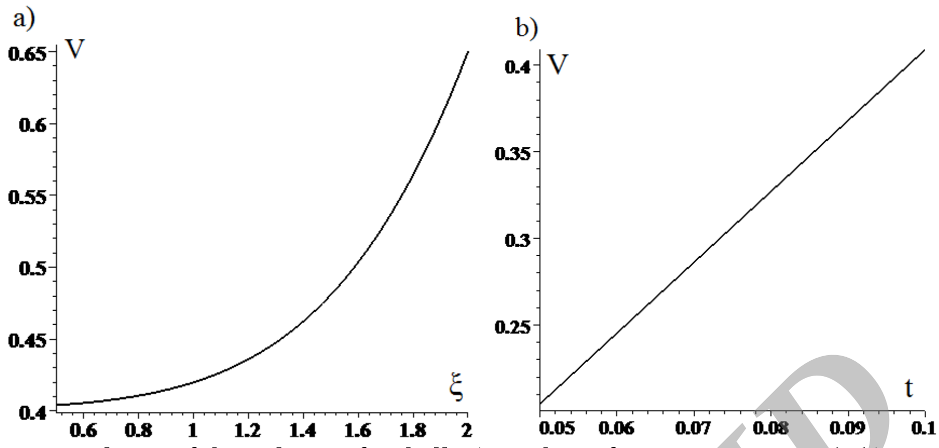

Figure 5. Dependence of the volume of a shell: a) on shape forming parameter ξ, b) on thickness of shell

The volume of the shells decreases with decreasing thickness and decreasing of the parameter ξ.

Optimization algorithms are realized in the "Maple" environment. The solutions that allow us to design rationally shaped shells depending on the changes in their form have been successfully obtained for both variants of the stated problems [11-12].

In some cases just due to a rational shape of the envelope it is possible to save up to 35% of its material with a considerable weight reduction of the structure.

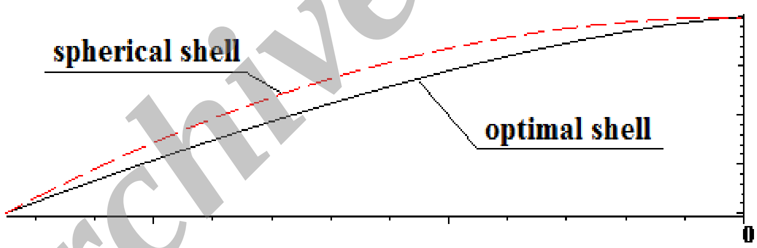

Figure 6. Middle surface of a spherical shell and the shape that is optimal to sustain critical force impact

The offered method makes it possible to optimize geometrically nonlinear shallow shells due to optimal form distribution when there is a need to:

– optimize stress in the median surface of a shell for any kinds of support with a limitation on the volume of used material;
– optimize critical force for any kinds of support;
– optimize the stress in the middle surface of the shell for any kinds of support with a pre-set critical force value and a limitation on the volume of used material;
– optimize critical force at pre-set values of stresses in the middle surface of the shell and a limitation on the volume of used material;
– optimize the volume of a used material for any kinds of support at pre-set values of the stresses and critical force.

6. CONCLUSIONS

This methodology can be applied to determine critical force and stresses for geometrically nonlinear shallow shells of revolution with variable form of the middle surface. In some problems, the proposed method provides greater accuracy and adaptability compared with the finite element method. The dependences demonstrate that it is possible to find an optimal relationship between the thickness and the form of geometrically nonlinear shallow shells of revolution by applying the criteria of maximum critical force and minimum stresses. The optimization algorithm can ensure considerable savings in the weight of building structures.

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