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TRUSS SHAPE AND SIZE OPTIMIZATION WITH FREQUENCY CONSTRAINTS USING TUG OF WAR OPTIMIZATION

A. Kaveh* and A. Zolghadr
Centre of Excellence for Fundamental Studies in Structural Engineering, School of Civil Engineering, Iran University of Science and Technology, Narmak, Tehran-16, Iran

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ABSTRACT

In this paper shape and size optimization of truss structures subjected to frequency constraints is addressed utilizing a newly developed multi-agent meta-heuristic algorithm called Tug of War Optimization (TWO). The algorithm considers each candidate solution as a team participating in a series of rope pulling competitions. Frequency constraint structural optimization corresponds to highly non-linear, discontinuous, and non-convex search spaces including several local optima. Such problems call for properly balanced competent optimization algorithms. Here, viability of TWO is demonstrated using four numerical examples.

Keywords: Tug of war optimization, metaheuristic, optimal design, frequency constraints, truss structures

1. INTRODUCTION

Natural frequencies of a structural system provide useful information for its dynamic behavior. In fact, in most of the low frequency vibration problems, the response of the structure is primarily a function of its fundamental frequencies and mode shapes [1]. The problem has been introduced in 1980s by Bellagamba and Yang [2] and has received considerable attention since then. Several researchers have explored the problem using a wide variety of optimization techniques. Lin et al. [3] proposed a bi-factor algorithm based on the Kuhn–Tucker criteria for minimum weight design of structures under static and dynamic constraints. Grandhi and Venkayya [1] utilized an optimality criterion based on uniform Lagrangian density to address the problem. Sedaghati et al. [4] used a mathematical programming technique to optimize truss and frame structures subject to frequency constraints, where an integrated finite element force method was utilized to frequency analysis. Wang et al. [5] formed an optimality criterion using the differentiation of the

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*E-mail address of the corresponding author: alikaveh@iust.ac.ir (A. Kaveh)

Lagrangian function. Starting from an infeasible point, minimum weight increment was utilized for simultaneous shape and size optimization of three-dimensional truss structures. Lingyun et al. [6] addressed the problem using a hybridization of the simplex method and genetic algorithms. Gomes [7] investigated simultaneous shape and size optimization of truss structures utilizing the standard Particle Swarm Optimization algorithm. Kaveh and Zolghadr employed charged system search (CSS) and its enhanced varient [8], hybridized CSS-BBBC with trap recognition capability [9], democratic particle swarm optimization (DPSO) [10], and a hybridized PSRO [11] algorithm to investigate the problem.

Weight minimization of a structure with frequency constraints, especially when the frequencies are lower bounded, is believed to be a demanding problem [7]. Frequency constraints are highly nonlinear, non-convex and implicit with respect to the design variables [12] and thus, the problem includes several local optima.

Meta-heuristic algorithms are powerful tools for solving optimization problems. These methods do not require gradient information of the objective function and are independent on the starting point. Thanks to their global search capabilities, these algorithms are suitable for complex, nonlinear and non-convex search spaces, especially when near-global optimum solutions are sought after using limited computational effort. Some of the examples of meta-heuristic algorithms include Genetic Algorithms (GA) [13], Particle Swarm Optimization (PSO) [14], Ant Colony Optimization (ACO) [15], Big Bang-Big Crunch (BB-BC) [16], Charged System Search (CSS) [17], Ray Optimization (RO) [18], Democratic PSO (DPSO) [10], Dolphin Echolocation (DE) [19], Colliding Bodies Optimization (CBO) [20], Water Cycle, Mine Blast and Improved Mine Blast algorithms (WC-MB-IMB) [21], Search Group Algorithm (SGA) [22]. More detailed explanation of meta-heuristic algorithms can be found in [23].

In this paper the newly developed Tug of War Optimization (TWO) algorithm introduced by Kaveh and Zolghadr [24] is utilized for truss shape and size optimization with frequency constraints. The remainder of this paper is organized as follows: Weight minimization of truss structures subject to frequency constraints is stated in Section 2. Tug of war optimization algorithm is briefly presented in Section 3. Four numerical examples are studied in Section 4 in order to show the viability of the algorithm. In section 5, effect of different parameters of TWO on the performance of the algorithm is studied. Finally, some concluding remarks are provided in Section 6.

2. PROBLEM STATEMENT

In a frequency constraint shape and size optimization of truss structures, the aim is to minimize the weight of the structure while satisfying some constraints on natural frequencies. Cross-sectional areas of the members and/or the coordinates of some nodes are considered as design variables. However, topology of the structure is assumed to be unaltered during the optimization process. Each design variable should be chosen from a permissible range. The optimization problem can be mathematically stated as:

$$\begin{gathered}\text{Find } X = [x_1, x_2, x_3, \ldots, x_n] \\ \text{to minimizes } P(X) = f(X) \times f_{penalty}(X) \\ \text{subject to} \\ \omega_j \le \omega_j^* \quad \text{for some natural frequencies } j \\ \omega_k \ge \omega_k^* \quad \text{for some natural frequencies } k \\ x_{imin} \le x_i \le x_{imax}\end{gathered} \tag{1}$$

in which X is the vector of the design variables; n is the number of design variables dictated by the element grouping scheme which in turn is chosen with respect to the symmetry and practice requirements; P(X) is the penalized cost function or the objective function to be minimized; f(X) is the cost function, which is taken as the weight of the structure in a weight minimization problem; $f_{penalty}(X)$ is the penalty function which is used to make the problem unconstrained; the value for the penalty function is taken as zero for feasible solutions while non-zero values are associated to infeasible solutions; ω_j is the jth natural frequency of the structure and ω_j^* is its upper bound; ω_k is the kth natural frequency of the structure and ω_k^* is its lower bound; x_{imin} and x_{imax} are the lower and upper bounds of the design variable x_i, respectively.

The cost function is expressed as:

$$f(X) = \sum_{i=1}^{nm} \rho_i L_i A_i \tag{2}$$

where ρ_i , L_i, and A_i are the material density, length, and the cross-sectional area of member i.

The penalty function is defined as:

$$f_{penalty}(X) = (1 + \varepsilon_1 . v)^{\varepsilon_2}, \; v = \sum_{i=1}^{q} v_i \tag{3}$$

where q is the number of frequency constraints.

$$v_i = \begin{cases} 0 & \text{if the ith constraint is satisfied} \\ \left| 1 - \frac{\omega_i}{\omega^*_i} \right| & \text{else} \end{cases} \tag{4}$$

The parameters ε_1 and ε_2 are selected considering the exploration and the exploitation rate of the search space. In this study ε_1 is taken as unity, and ε_2 starts from 1.5 linearly increasing to 6 in all test examples. Such choices of parameters allow the agents to explore the search space more freely at the early stages of the optimization process. On the other hand, as the optimization proceeds, penalty values grow bigger and the agents tend to prefer feasible solutions.

3. TUG OF WAR OPTIMIZATION

3.1 Idealized tug of war framework

Tug of war or rope pulling is a strength contest in which two competing teams pull on the opposite ends of a rope in an attempt to bring the rope in their direction against the pulling force of the opposing team. The activity dates back to ancient times and has continued to exist in different forms ever since. There has been a wide variety of rules and regulations for the game but the essential part has remained almost unaltered. Naturally, as far as both teams sustain their grips of the rope, movement of the rope corresponds to the displacement of the losing team. Fig. 1 shows two teams competing in a tug of war contest.

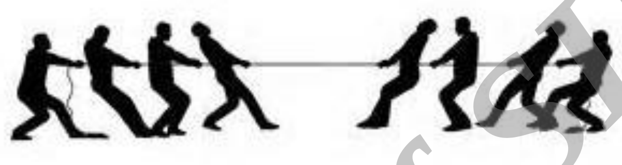


Figure 1. A competing team in a tug of war

Triumph in a real game of tug of war generally depends on many factors and could be difficult to analyze. However, an idealized framework is utilized in this paper where the two teams having weights W_i and W_j are considered as two objects lying on a smooth surface as shown in Fig. 2.

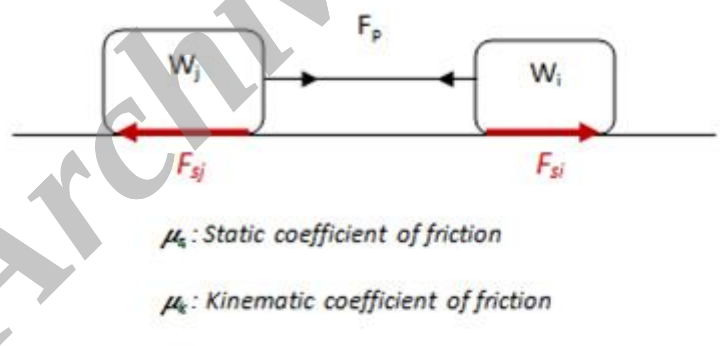


Figure 2. An idealized tug of war framework

As a result of pulling the rope, the teams experience two equal and opposite forces (F_p) according to Newton's third law. For object i, as far as the pulling force is smaller than the maximum static friction force $(W_i \mu_s)$ the object rests in its place. Otherwise the non-zero resultant force can be calculated as:

$$F_r = F_p - W_i \mu_k \tag{5}$$

As a result, the object i accelerates towards the object j according to the Newton's second law:

$$a = \frac{F_r}{\left(W_i / g \right)} \tag{6}$$

Since the object i starts from zero velocity, its new position can be determined as:

$$X_i^{new} = \frac{1}{2} a t^2 + X_i^{old} \tag{7}$$

3.2 Tug of war optimization algorithm

TWO is a population-based meta-heuristic algorithm, which considers each candidate solution $X_i = \{x_{i,j}\}$ as a team engaged in a series of tug of war competitions. The weight of the teams is determined based on the quality of the corresponding solutions, and the amount of pulling force that a team can exert on the rope is assumed to be proportional to its weight. Naturally, the opposing team will have to maintain at least the same amount of force in order to sustain its grip of the rope. The lighter team accelerates toward the heavier team and this forms the convergence operator of the TWO. The algorithm improves the quality of the solutions iteratively by maintaining a proper exploration/exploitation balance using the described convergence operator. The steps of TWO can be stated as follows:

Step 1: Initialization

A population of N initial solutions is generated randomly:

$$x_{ij}^0 = x_{j,\min} + rand(x_{j,\max} - x_{j,\min}) \qquad j = 1,2,\ldots,N \tag{8}$$

where x_{ij}^0 is the initial value of the jth variable of the ith candidate solution; $x_{j,max}$ and $x_{j,min}$ are the maximum and minimum permissible values for the jth variable, respectively; rand is a random number from a uniform distribution in the interval [0, 1]; n is the number of optimization variables.

Step 2: Evaluation of candidate designs and weight assignment

The objective function values for the candidate solutions are evaluated. All of the initial solutions are sorted and recorded in a memory denoted as the league. Each solution is considered as a team with the following weight:

$$W_i = (\frac{fit(i) - fit_{worst}}{fit_{best} - fit_{worst}}) + 1 \qquad i = 1,2,\ldots,N \tag{9}$$

where fit(i) is the fitness value for the ith particle; The fitness value of the ith team, evaluated as as the penalized objective function value for constrained problems; fit_{best} and fit_{worst} are the fitness values for the best and worst candidate solutions of the current iteration. According to Eq. (5) the weights of the teams range between 1 and 2.

Step 3: Competition and displacement

In TWO each of the teams of the league competes against all the others one at a time to move to its new position in each iteration. The pulling force exerted by a team is assumed to be equal to its static friction force $(W\mu_s)$. Hence the pulling force between teams i and j ($F_{p,ij}$) can be determined as $\max\{W_i\mu_s, W_j\mu_s\}$. Such a definition keeps the position of the heavier team unaltered.

The resultant force affecting team i due to its interaction with heavier team j in the kth iteration can then be calculated as follows:

$$F_{r,ij}^k = F_{p,ij}^k - W_i^k \mu_k \tag{10}$$

where $F_{p,ij}^k$ is the pulling force between teams i and j in the kth iteration and μ_k is coefficient of kinematic friction. Consequently, team i accelerates towards team j:

$$a_{ij}^k = (\frac{F_{r,ij}^k}{W_i^k \mu_k}) g_{ij}^k \tag{11}$$

where a_{ij}^k is the acceleration of team i towards team j in the kth iteration; g_{ij}^k is the gravitational acceleration constant defined as:

$$g_{ij}^k = X_j^k - X_i^k \tag{12}$$

where X_j^k and X_i^k are the position vectors for candidate solutions j and i in the kth iteration. Finally, the displacement of the team i after competing with team j can be derived as:

$$\Delta X_{ij}^k = \frac{1}{2} a_{ij}^k \Delta t^2 + \alpha^k \beta (X_{\max} - X_{\min}) \circ randn(1,n) \tag{13}$$

The second term of Eq. (9) introduces randomness into the algorithm. This term can be interpreted as the random portion of the search space traveled by team i before it stops after the applied force is removed. The role of α^k is to gradually decrease the random portion of the team's movement. For most of the applications α could be considered as a constant chosen from the interval [0.9, 0.99]; bigger values of α decrease the convergence speed of the algorithm and help the candidate solutions explore the search space more thoroughly. β is a scaling factor which can be chosen from the interval (0,1]. This parameter controls the steps of the candidate solutions when moving in the search space. When the search space is supposed to be searched more accurately with smaller steps, smaller values should be chosen for this parameter. For our numerical examples values between 0.01 and 0.05 seem to be appropriate for this parameter; X_{max} and X_{min} are the vectors containing the upper and lower bounds of the permissible ranges of the design variables, respectively; $\circ$ denotes element by element multiplication; $randn\,(1,n)$ is a vector of random numbers drawn from a standard normal distribution.

It should be noted that when team j is lighter than team i, the corresponding displacement of team i will be equal to zero (i.e. ΔX_{ij}^k). Finally, the total displacement of team i in iteration k is equal to (i not equal j):

$$\Delta X_i^k = \sum_{j=1}^{N} \Delta X_{ij}^k \tag{14}$$

The new position of the team i at the end of the kth iteration is then calculated as:

$$X_i^{k+1} = X_i^k + \Delta X_i^k \tag{15}$$

Step 4: Updating the league

Once the teams of the league compete against each other for a complete round, the league should be updated. This is done by comparing the new candidate solutions (the new positions of the teams) to the current teams of the league. That is to say, if the new candidate solution i is better than the Nth team of the league in terms of objective function value, the Nth team is removed from the league and the new solution takes its place.

Step 5: Handling the side constraints

It is possible for the candidate solutions to leave the search space and it is important to deal with such solutions properly. This is especially the case for the solutions corresponding to lighter teams for which the values of ΔX is usually bigger. Different strategies might be used in order to solve this problem. For example, such candidate solutions can be simply brought back to their previous feasible position (flyback strategy) or they can be regenerated randomly. In this paper a new strategy is introduced and incorporated using the global best solution. The new value of the jth optimization variable of the ith team that violated side constraints in the kth iteration is defined as:

$$x_{ij}^{k} = GB_j + (\frac{randn}{k})(GB_j - x_{ij}^{k-1}) \tag{16}$$

where GB_j is the jth variable of the global best solution (i.e. the best solution so far); randn is a random number drawn form a standard normal distribution. There is a very slight possibility for the newly generated variable to be still outside the search space. In such cases a flyback strategy is used.

The abovementioned strategy is utilized with a certain probability (0.5 in this paper). For the rest of cases the violated limit is taken as the new value of the jth optimization variable.

Step 6: Termination

Steps 2 through 5 are repeated until a termination criterion is satisfied. The pseudo code of TWO is presented in Table 1.

Table 1: Pseudo-code of the TWO algorithm developed in this study

```
procedure Tug of War Optimization
begin
    Initialize parameters;
    Initialize a population of N random candidate solutions;
    Initialize the league by recording all random candidate solution;
    while (termination condition not met) do
        Evaluate the objective function values for the candidate solutions
        Sort the new solutions and update the league
        Define the weights of the teams of the league Wi based on fit(Xi)
            for each team i
              for each team j
                if (Wi < Wj)
                    Move team i towards team j using Eq. (13);
                end if
              end for
            Determine the total displacement of team i using Eq. (14)
            Determine the final position of team i using Eq.(15)
            Use the side constraint handling technique to regenerate violating variables
            end for
    end while
end
```

4. NUMERICAL EXAMPLES

Four numerical examples are provided in this section in order to examine the performance of TWO on frequency constraint weight minimization of truss structures. The results are compared to those of some other optimization techniques reported in the literature. A total population of 20 particles is considered for all of the examples except for the second one, where 30 agents are used. The termination criterion is taken as the number of iterations in all the examples. All numerical examples are run 50 times independently in order to provide statistical results.

4.1 A 10-bar truss

Size optimization of a 10-bar truss structure shown in Fig. 3 is considered as the first example. This is a well-known benchmark problem in the field of structural optimization subjected to frequency constraints. Cross-sectional areas of all ten members are assumed to be independent variables. A non-structural mass of 454.0 kg is attached to all free nodes. Table 2 summarizes the material properties, variable bounds, and frequency constraints for this example.

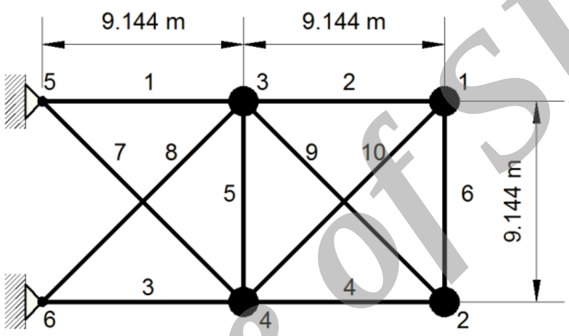


Figure 3. Schematic of a 10-bar planar truss structure

Table 2: Material properties, variable bounds and frequency constraints for the 10-bar truss structure

Property/unit	Value
E (Modulus of elasticity)/ N/m^2	6.89×10^{10}
ρ (Material density)/ kg/m^3	2770.0
Added mass/kg	454.0
Design variable lower bound/m^2	0.645×10^{-4}
Design variable upper bound/m^2	50×10^{-4}
L (Main bar's dimension)/m	9.144
Constraints on first three frequencies/Hz	$\omega_1 \geq 7$, $\omega_2 \geq 15$, $\omega_3 \geq 20$

This problem is addressed by different researchers using a wide variety of methods: Grandhi and Venkayya [1] using an optimality algorithm, Sedaghati et al. [4] utilizing a sequential quadratic programming and finite element force method, Wang et al. [5] using an evolutionary node shift method, Lingyun et al. [6] utilizing a niche hybrid genetic algorithm, Gomes employing particle swarm optimization algorithm [7] and Kaveh and Zolghadr employing standard and enhanced CSS [8], hybridized CSS-BBBC with trap recognition capability [9], democratic particle swarm optimization (DPSO) [10], and a hybridized PSRO [11].

Optimal structures found by different methods and the corresponding masses are summarized in Table 3. The optimal structure found by TWO is better than other methods. It should be noted that the structures found by standard PSO [7] and CSS [8] are obtained using a modulus of elasticity of E = 6.98 × 10^{10} N/m^2, which generally results in lighter structures. Table 4 presents the natural frequencies of the optimized structures obtained by different methods. It can be seen that all constraints are satisfied. The mean value and the standard deviation of 50 independent runs of TWO are 535.55 kg and 3.27, respectively.

Table 3: Optimal structures (cm^2) found by different methods for the planar 10-bar planar truss problem (the optimized weight does not include the added masses)

Element number	Grandhi and Venkayya [1]	Sedaghati et al. [4]	Wang et al. [5]	Lingyun et al. [6]	Gomes [7]	Kaveh and Zolghadr			
						Standard CSS [8]	DPSO [10]	PSRO [11]	Present work
1	36.584	38.245	32.456	42.23	37.712	38.811	35.944	37.075	34.544
2	24.658	9.916	16.577	18.555	9.959	9.0307	15.530	15.334	15.148
3	36.584	38.619	32.456	38.851	40.265	37.099	35.285	33.665	37.088
4	24.658	18.232	16.577	11.222	16.788	18.479	15.385	14.849	14.813
5	4.167	4.419	2.115	4.783	11.576	4.479	0.648	0.645	0.646
6	2.070	4.419	4.467	4.451	3.955	4.205	4.583	4.643	4.613
7	27.032	20.097	22.810	21.049	25.308	20.842	23.610	24.528	24.373
8	27.032	24.097	22.810	20.949	21.613	23.023	23.599	23.188	23.720
9	10.346	13.890	17.490	10.257	11.576	13.763	13.135	12.436	12.318
10	10.346	11.452	17.490	14.342	11.186	11.414	12.357	13.500	12.618
Weight (kg)	594.0	537.01	553.8	542.75	537.98	531.95	532.39	532.85	532.23

Table 4: Natural frequencies (Hz) of the optimized designs for the 10-bar planar truss.

Frequency number	Grandhi and Venkayya [1]	Sedaghati et al. [4]	Wang et al. [5]	Lingyun et al. [6]	Gomes [7]	Kaveh and Zolghadr			
						Standard CSS [8]	DPSO [10]	PSRO [11]	Present work
1	7.059	6.992	7.011	7.008	7.000	7.000	7.000	7.000	7.000
2	15.895	17.599	17.302	18.148	17.786	17.442	16.187	16.143	16.194
3	20.425	19.973	20.001	20.000	20.000	20.031	20.000	20.000	20.000
4	21.528	19.977	20.100	20.508	20.063	20.208	20.021	20.032	20.002
5	28.978	28. 173	30.869	27.797	27.776	28.261	28.470	28.469	28.478
6	30.189	31.029	32.666	31.281	30.939	31.139	29.243	29.485	28.894
7	54.286	47.628	48.282	48.304	47.297	47.704	48.769	48.440	48.603
8	56.546	52.292	52.306	53.306	52.286	52.420	51.389	51.157	51.148

Fig. 4 presents the convergence curve of the best run of TWO for the 10-bar planar truss.

4.2 A 72-bar spatial truss

A 72-bar spatial truss as depicted in Fig. 5 is presented as the second example. Four non-structural masses of 2270 kg are attached to the uppermost four nodes. The shape of the structure is kept unchanged during the optimization process and the design variables only include cross-sectional areas of the members, which are grouped into 16 groups. Material properties, variable bounds, frequency constrains and added masses are listed in Table 5.

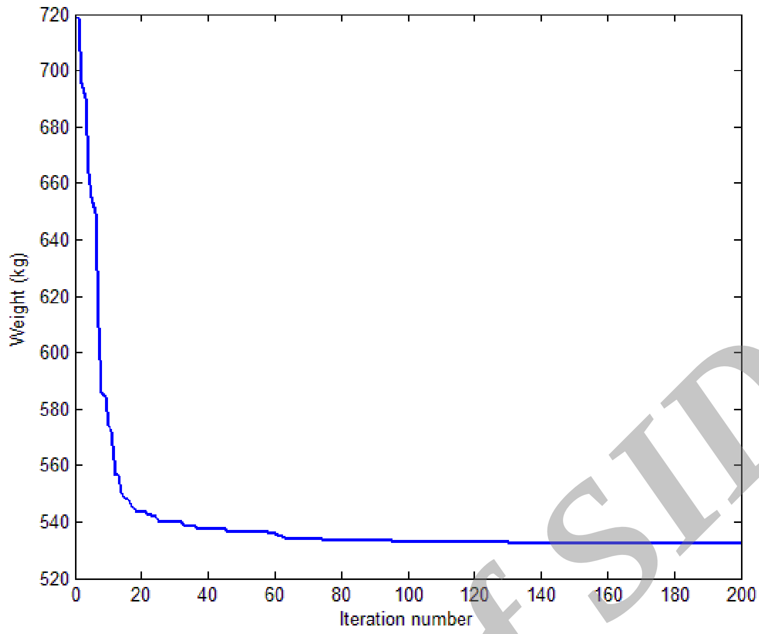


Figure 4. Convergence curve of the best run of TWO for the 10-bar planar truss

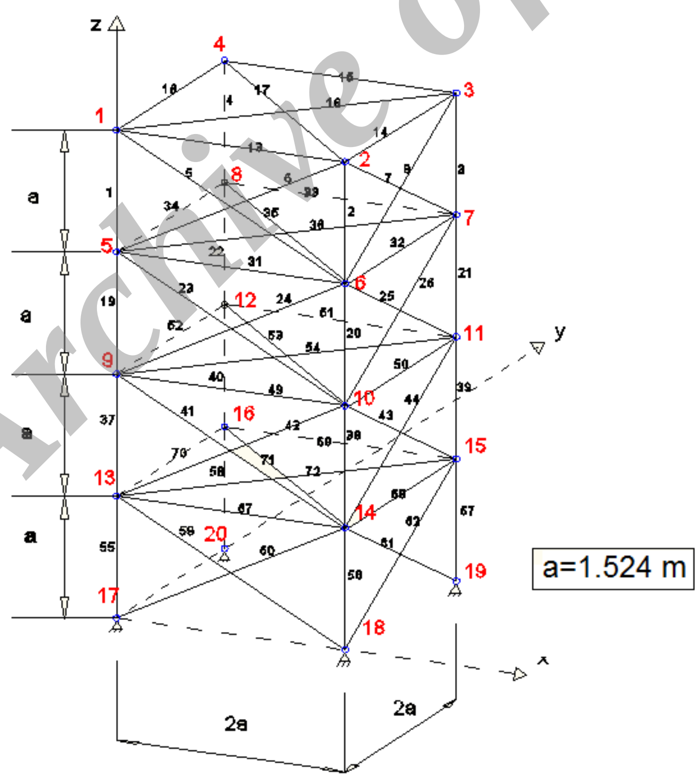


Figure 5. The 72-bar spatial truss

Table 5: Material properties and frequency constraints for the 72-bar spatial truss

Property/unite	Value
E (Modulus of elasticity)/ N/m^2	6.89×10^{10}
ρ (Material density)/ kg/m^3	2770.0
Added mass/kg	2270
Design variable lower bound/m^2	0.645×10^{-4}
Constraints on first three frequencies/Hz	ω_1=4.0 , $\omega_3 \geq 6$

Optimized designs obtained by different optimization methods are summarized in Table 6. It should be mentioned that a modulus of elasticity of $E=6.98\times10^{10}$ N/m^2 is used by Gomes [7] and Kaveh and Zolghadr [8, 9]. This generally results in lighter structures. The mean value and the standard deviation of 50 independent runs of TWO are 336.1 kg and 5.8, respectively. Table 7 presents the first five natural frequencies for the optimized structures found by different methods. It could be seen that all of the frequency constraints are satisfied. Fig. 6 presents the convergence curve for the best run of TWO.

Table 6: Optimal cross-sectional areas for the 72-bar space truss (cm^2)

Group number	Elements	Sedaghati et al. [4]	Gomes [7]	Kaveh and Zolghadr: Standard CSS [8]	Kaveh and Zolghadr: Enhanced CSS [8]	Kaveh and Zolghadr: CSS-BBBC [9]	PSRO [11]	Present method
1	1–4	3.499	2.987	2.528	2.252	2.854	3.840	3.380
2	5–12	7.932	7.849	8.704	9.109	8.301	8.360	8.086
3	13–16	0.645	0.645	0.645	0.648	0.645	0.645	0.647
4	17–18	0.645	0.645	0.645	0.645	0.645	0.699	0.646
5	19–22	8.056	8.765	8.283	7.946	8.202	8.817	8.890
6	23–30	8.011	8.153	7.888	7.703	7.043	7.697	8.136
7	31–34	0.645	0.645	0.645	0.647	0.645	0.645	0.654
8	35–36	0.645	0.645	0.645	0.646	0.645	0.651	0.647
9	37–40	12.812	13.450	14.666	13.465	16.328	12.136	13.097
10	41–48	8.061	8.073	6.793	8.250	8.299	8.839	8.101
11	49–52	0.645	0.645	0.645	0.645	0.645	0.645	0.663
12	53-54	0.645	0.645	0.645	0.646	0.645	0.645	0.646
13	55–58	17.279	16.684	16.464	18.368	15.048	17.059	16.483
14	59–66	8.088	8.159	8.809	7.053	8.268	7.427	7.873
15	67-70	0.645	0.645	0.645	0.645	0.645	0.646	0.651
16	71-72	0.645	0.645	0.645	0.646	0.645	0.645	0.657
	Weight (kg)	327.605	328.823	328.814	328.393	327.507	329.80	328.83

Table 7: Natural frequencies (Hz) obtained by different methods for the 72-bar space truss

Frequency number	Sedaghati et al. [4]	Gomes [7]	Kaveh and Zolghadr				
			Standard CSS [8]	Enhanced CSS [8]	CSS-BBBC [9]	PSRO [11]	Present method
1	4.000	4.000	4.000	4.000	4.000	4.000	4.000
2	4.000	4.000	4.000	4.000	4.000	4.000	4.000
3	6.000	6.000	6.006	6.004	6.004	6.000	6.000
4	6.247	6.219	6.210	6.155	6.2491	6.418	6.259
5	9.074	8.976	8.684	8.390	8.9726	9.143	9.082

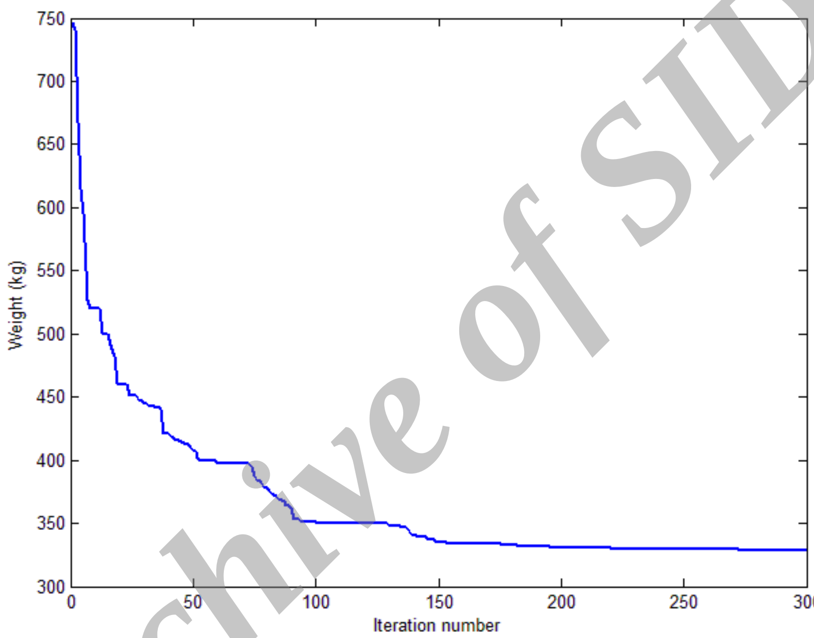


Figure 6 convergence curve of the best run of TWO for the 72-bar planar truss

4.3 A Simply supported 37-bar planar truss

Shape and size optimization of a simply supported 37-bar planar truss as shown in Fig. 7 is studied as the third example. The elements of the lower chord are modeled as bar elements with constant rectangular cross-sectional areas of 4×10^{-3} m^2. The rest of the members, which for the sizing variables of the problem are modeled as bar elements and are grouped with respect to symmetry. The y-coordinate of all the nodes on the upper chord can vary in a symmetrical manner to form the shape variables. A non-structural mass of 10 kg is attached to all free nodes of the lower chord. Constraints are imposed on the first three natural frequencies of the structure.

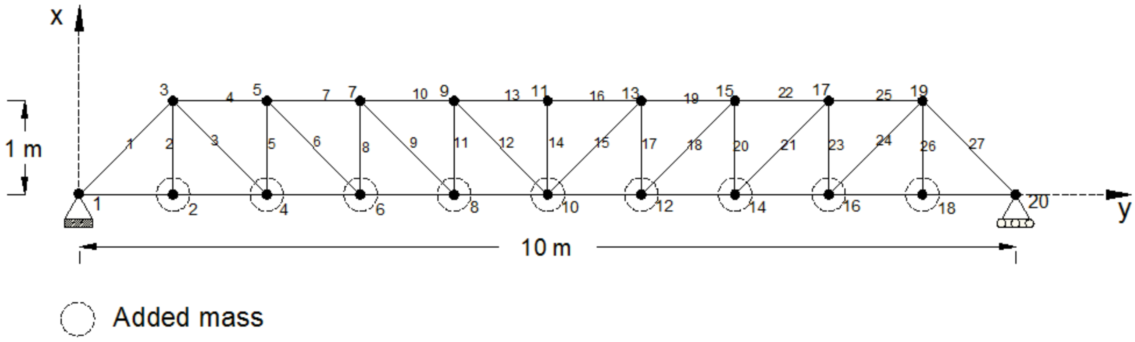


Figure 7. A simply-supported planar 37-bar truss

This example has been investigated by different researchers including Wang et al. [5] using an evolutionary node shift method, Lingyun et al. [6] using a niche hybrid genetic algorithm, Gomes [7] utilizing particle swarm algorithm, and Kaveh and Zolghadr using CSS [8], democratic PSO [10], and a hybridized PSRO algorithm [11]. Material properties, frequency constrains, added masses and variable bounds for this example are listed in Table 8. Final cross-sectional areas and node coordinates obtained by different methods together with the corresponding weight are shown in Table 9.

Table 8: Material properties, frequency constraints, and variable bounds for the simply supported 37-bar planar truss

Property/unit	Value
E (Modulus of elasticity)/ N/m^2	2.1×10^{11}
ρ (Material density)/ kg/m^3	7800
Design variable lower bound/m^2	1×10^{-4}
Design variable upper bound/m^2	10×10^{-4}
Added mass/kg	10
Constraints on first three frequencies/Hz	$\omega1\geq20$, $\omega2\geq40$, $\omega3\geq60$

Table 9: Optimized designs obtained for the planar 37-bar truss problem

Variable	Wang et al. [5]	Lingyun et al. [6]	Gomes [7]	Kaveh and Zolghadr			
				Standard CSS [8]	DPSO [10]	PSRO [11]	Present method
Y3 , Y19 (m)	1.2086	1.1998	0.9637	0.8726	0.9482	1.0087	1.0039
Y5 , Y17 (m)	1.5788	1.6553	1.3978	1.2129	1.3439	1.3985	1.3531
Y7 , Y15 (m)	1.6719	1.9652	1.5929	1.3826	1.5043	1.5344	1.5339
Y9 , Y13 (m)	1.7703	2.0737	1.8812	1.4706	1.6350	1.6684	1.6768
Y11 (m)	1.8502	2.3050	2.0856	1.5683	1.7182	1.7137	1.7728
A1, A27 (cm^2)	3.2508	2.8932	2.6797	2.9082	2.6208	2.6368	2.8892
A2, A26 (cm^2)	1.2364	1.1201	1.1568	1.0212	1.0397	1.3034	1.0949
A3, A24 (cm^2)	1.0000	1.0000	2.3476	1.0363	1.0464	1.0029	1.0213
A4, A25 (cm^2)	2.5386	1.8655	1.7182	3.9147	2.7163	2.3325	2.6776
A5, A23 (cm^2)	1.3714	1.5962	1.2751	1.0025	1.0252	1.2868	1.1981

A6, A21 (cm^2)	1.3681	1.2642	1.4819	1.2167	1.5081	1.0704	1.1387
A7, A22 (cm^2)	2.4290	1.8254	4.6850	2.7146	2.3750	2.4442	2.6537
A8, A20 (cm^2)	1.6522	2.0009	1.1246	1.2663	1.4498	1.3416	1.4171
A9, A18 (cm^2)	1.8257	1.9526	2.1214	1.8006	1.4499	1.5724	1.3934
A10, A19 (cm^2)	2.3022	1.9705	3.8600	4.0274	2.5327	3.1202	2.7741
A11, A17 (cm^2)	1.3103	1.8294	2.9817	1.3364	1.2358	1.2143	1.2759
A12, A15 (cm^2)	1.4067	1.2358	1.2021	1.0548	1.3528	1.2954	1.2776
A13, A16 (cm^2)	2.1896	1.4049	1.2563	2.8116	2.9144	2.7997	2.1666
A14 (cm^2)	1.0000	1.0000	3.3276	1.1702	1.0085	1.0063	1.0099
Weight (kg)	366.50	368.84	377.20	362.84	360.40	360.97	360.27

Table 9 shows that TWO has obtained the best result among the algorithms. The mean weight and the standard deviation of 50 independent runs of TWO are 363.75 kg and 2.48 kg, respectively. Table 10 presents the first five natural frequencies of the optimized structures. Fig. 8 presents the convergence curve of the best run of TWO for the simply supported 37-bar planar truss.

Table 10: Natural frequencies (Hz) evaluated at the optimized designs for the planar 37-bar truss

Frequency number	Wang et al. [5]	Lingyun et al. [6]	Gomes [7]	Kaveh and Zolghadr			
				Standard CSS [8]	DPSO [10]	PSRO [11]	Present method
1	20.0850	20.0013	20.0001	20.0000	20.0194	20.1023	20.0279
2	42.0743	40.0305	40.0003	40.0693	40.0113	40.0804	40.0146
3	62.9383	60.0000	60.0001	60.6982	60.0082	60.0516	60.0946
4	74.4539	73.0444	73.0440	75.7339	76.9896	75.8918	76.5062
5	90.0576	89.8244	89.8240	97.6137	97.2222	97.2470	96.5840

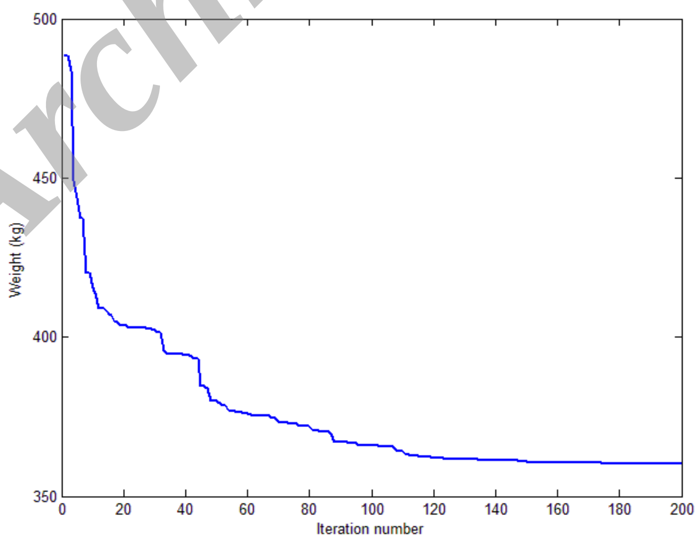


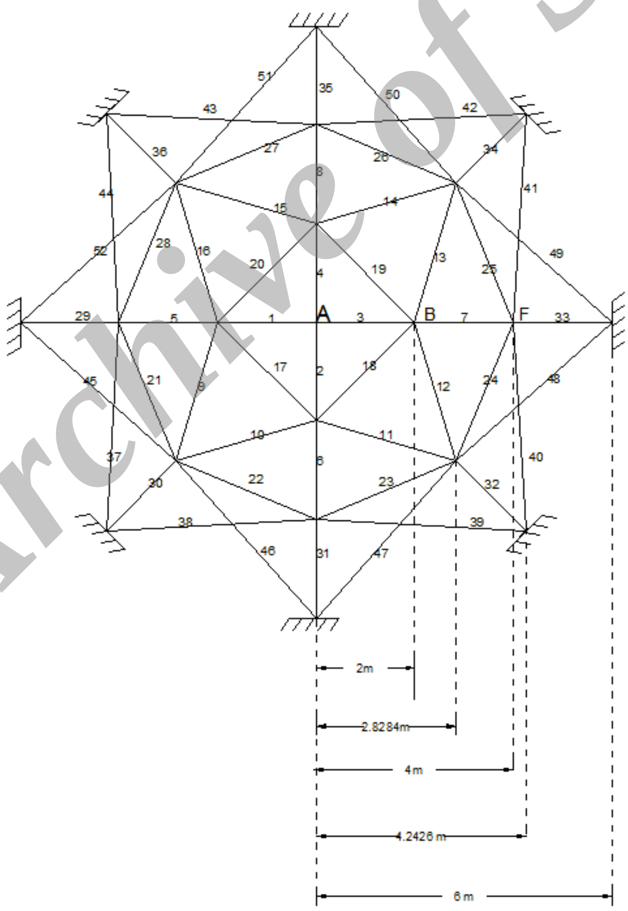
Figure 8. Convergence curve of the best run of TWO for the simply supported 37-bar planar truss

4.4 A 52-bar dome-like truss

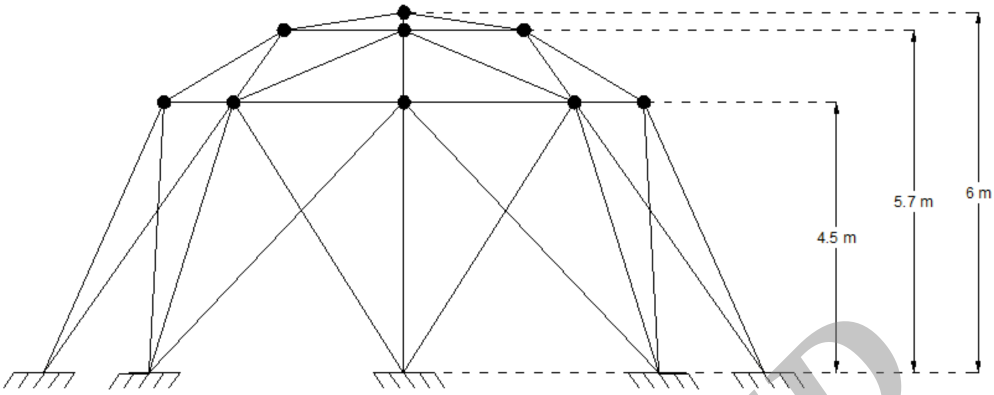
As the last example, simultaneous shape and size optimization of a 52-bar dome-like truss is considered. Initial layout of the structure is depicted in Fig. 9. Non-structural masses of 50 kg are attached to all free nodes. Material properties, frequency constraints and variable bounds for this example are summarized in Table 11. The elements of the structure are categorized in 8 groups according to Table 12. All free nodes are permitted to move ±2m from their initial position in a symmetrical manner.

Table 11: Material properties and frequency constraints and variable bounds for the 52-bar space truss

Property/unit	Value
E (Modulus of elasticity)/ N/m^2	2.1×10^{11}
ρ (Material density)/ kg/m^3	7800
Added mass/kg	50
Allowable range for cross-sections/ m^2	$0.0001\leq A\leq 0.001$
Constraints on first three frequencies/Hz	$\omega_1\leq 15.916$ $\omega_2\geq 28.648$



(a) Top view



(b) Side view

Figure 9. Initial layout of the spatial 52-bar truss

Table 12: Element grouping for the spatial 52-bar truss

Group number	Elements
1	1-4
2	5-8
3	9-16
4	17-20
5	21-28
6	29-36
7	37-44
8	45-52

This example has been solved by Lin et al. [3] using a mathematical programming technique and Lingyun et al. [6] using a niche hybrid genetic algorithm. Gomes [7] has studied the problem using particle swarm optimization algorithm. The authors have studied the problem using CSS [8], hybridized CSS-BBBC with a trap recognition capability [9], democratic PSO [10], and a hybridized PSRO algorithm [11].

Table 13 summarizes the best results obtained by different methods for this example. It can be seen the structure found by TWO is lighter than those of other methods. The mean weight and the standard deviation of 50 independent runs of TWO are 214.25 kg and 12.64 kg, respectively. Table 14 shows the first five natural frequencies of the final structures found by various methods for the 52-bar dome-like space truss. The convergence curve of the best run of TWO for this problem is shown in Fig. 10.

Table 13: Optimized designs obtained for the spatial 52-bar truss problem

Variable	Lin et al. [3]	Lingyun [6]	Gomes [7]	Kaveh and Zolghadr Standard CSS [8]	DPSO [10]	PSRO [11]	Present method
Z_A (m)	4.3201	5.8851	5.5344	5.2716	6.1123	6.252	6.012
X_B (m)	1.3153	1.7623	2.0885	1.5909	2.2343	2.456	1.598
Z_B (m)	4.1740	4.4091	3.9283	3.7093	3.8321	3.826	4.287
X_F (m)	2.9169	3.4406	4.0255	3.5595	4.0316	4.179	3.641
Z_F (m)	3.2676	3.1874	2.4575	2.5757	2.5036	2.501	2.888
A1 (cm^2)	1.00	1.0000	0.3696	1.0464	1.0001	1.0007	2.1245
A2 (cm^2)	1.33	2.1417	4.1912	1.7295	1.1397	1.0312	1.1341
A3 (cm^2)	1.58	1.4858	1.5123	1.6507	1.2263	1.2403	1.1870
A4 (cm^2)	1.00	1.4018	1.5620	1.5059	1.3335	1.3355	1.3180
A5 (cm^2)	1.71	1.911	1.9154	1.7210	1.4161	1.5713	1.3637
A6 (cm^2)	1.54	1.0109	1.1315	1.0020	1.0001	1.0021	1.0299
A7 (cm^2)	2.65	1.4693	1.8233	1.7415	1.5750	1.3267	1.3479
A8 (cm^2)	2.87	2.1411	1.0904	1.2555	1.4357	1.5653	1.4446
Weight (kg)	298.0	236.046	228.381	205.237	195.351	197.186	194.25

Table 14: Optimized designs obtained for the spatial 52-bar truss problem

Frequency number	Lin et al. [3]	Lingyun [6]	Gomes [7]	Kaveh and Zolghadr Standard CSS [8]	DPSO [10]	PSRO [11]	Present method
1	15.22	12.81	12.751	9.246	11.315	12.311	9.265
2	29.28	28.65	28.649	28.648	28.648	28.648	28.667
3	29.28	28.65	28.649	28.699	28.648	28.649	28.667
4	31.68	29.54	28.803	28.735	28.650	28.715	28.686
5	33.15	30.24	29.230	29.223	28.688	28.744	29.734

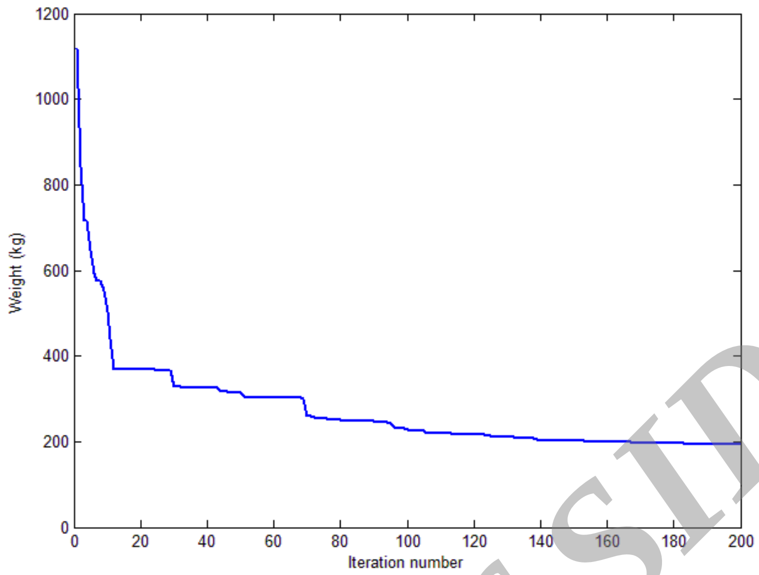
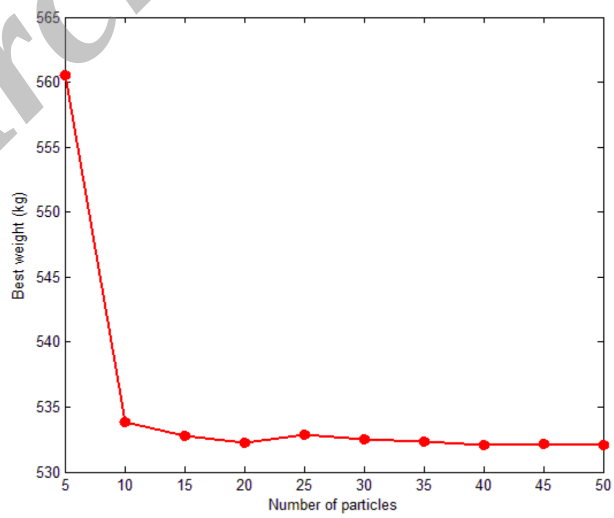


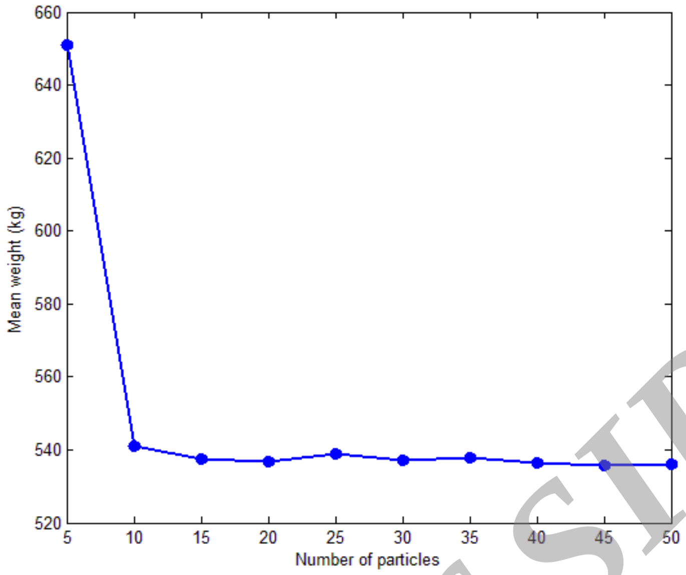
Figure 10. convergence curve of the best run of TWO for the 52-bar truss problem

5. EFFECT OF PARAMETERS

In this section effect of different parameters of TWO on the performance of the algorithm is studied. For this purpose the 10-bar truss structure is optimized using different values for population size, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and μ_k. The problem is solved 20 times using each set of parameters to eliminate the effect of randomly generated initial solutions. Fig. 11 shows the best and mean weight values for this example using different population sizes. It can be seen that a population size of 20 could be sufficient and bigger values do not improved the results significantly.



(a)



(b)

Figure 11 Effect of population size on the performance of TWO on 10-bar truss problem a) Best weight b) Mean weight

Table 15 presents the effect of parameter μ_k on the performance of the algorithm. It can be seen that the best results in terms of best and mean weight are obtained when μ_k decreases linearly from 1 to zero. The standard deviation and number of infeasible designs are also comparatively acceptable. It can also be seen that the performance of the algorithm is not affected drastically for other values of μ_k. This means that even if the user does not set the optimal values of parameters, adequate results are still attainable.

Table 15: Effect of μ_k on the performance of TWO on the 10-bar truss problem

μ_k	0	0.5	1	linear
Best weight (kg)	532.63	533.13	532.25	532.23
Mean weight (kg)	538.85	539.02	537.02	535.55
std	3.81	2.97	3.55	3.27
Number of infeasible designs	6	2	0	1

Table 16 summarizes the performance of TWO for different values of parameter $\boldsymbol{\alpha}$. It can be seen that the best performance in terms of best and mean weight values is associated with $\boldsymbol{\alpha}$=0.97. However, selecting this parameter in the range (0.9, 0.99) will also result in acceptable results according to the table.

Table 16: Effect of α on the performance of TWO on the 10-bar truss problem

α	0.8	0.85	0.9	0.95	0.97	0.99
Best weight (kg)	533.69	536.46	533.81	532.24	532.23	532.84
Mean weight (kg)	567.55	549.84	542.27	537.98	535.55	537.89
std	23.22	13.96	6.649	3.39	3.27	3.11
Number of infeasible designs	3	5	4	0	1	2

Finally, Table 17 shows the effect of parameter β on the performance of the presented algorithm. It can be seen that the algorithm performs acceptable for a relatively large range of this parameter namely (0.005, 0.1). For bigger values however, the quality of the results slightly decreases.

Table 17: Effect of parameter β on the performance of TWO

β	0.005	0.01	0.02	0.05	0.1	0.5	1
Best weight (kg)	532.21	532.23	532.25	532.48	532.76	533.75	534.97
Mean weight (kg)	537.16	535.55	535.63	535.85	5.3538	538.04	542.82
std	2.75	3.27	3.42	3.16	3.03	3.9	6.84
Number of infeasible designs	3	1	0	1	0	3	5

6. CONCLUDING REMARKS

Shape and size optimization of truss structures subject to frequency constraints is addressed in this paper using a newly developed multi-agent meta-heuristic algorithm, called tug of war optimization (TWO). The algorithm considers each of the candidate solutions as a team competing in a series of rope pulling competitions.

An idealized framework is presented in order to simplify the physical nature of a game of tug of war, in which the teams are considered as two bodies lying on a smooth surface. It is then assumed that the pulling force that a team can exert is proportional to its weight and the two teams sustain their grip of the rope during the contest. The weights of the teams are determined base on the quality of the solutions they represent.

Four numerical examples are provided in order to examine the efficiency of the algorithm. The results are compared to those of some other optimization algorithms on the same problems. The results indicate that the performance of TWO is comparable to the other documented methods. In fact for two of the examples the optimized structures obtained by TWO are lighter than those of other methods.

In view of the significant effect of the internal parameters of a meta-heuristic algorithm on its performance, the effects of the parameters of TWO on the results obtained for the problems under consideration are studied using the 10-bar truss example. The optimal parameter values for frequency constraint structural optimization problems are presented. It should be noted that the parameter values are believed to be problem dependent and these values might not be globally optimal for other cases. The results indicate that for non-optimal parameter values, the obtained results are still reasonable for the 10-bar truss problem.

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