Mech. & Aerospace Eng. J. Vol. 1, No. 1, Aug. 2005

Regulated Sliding Mode Control of Satellite Rotation: Trade-off Between Tracking Precision and Energy Consumption

S.A.A. Moosavian[1], S.H. Sadati[2], M.R. Homaeinejad[3]
Mech. Eng. Department, Khajeh Nasir Toosi Univ. of Tech.

ABSTRACT
Although the sliding mode controller usually results in an acceptable performance, chattering phenomenon may cause practical difficulties for actuators, particularly for on-off types as is the case in space applications. In this paper, a regulated sliding mode control law to fulfill stability requirements, robustness properties, and chattering elimination is proposed. Due to the activity of the regulating routine, proper positive values for the coefficient of sliding condition are determined. To this end, first the rotation dynamics of a typical satellite described in body coordinates is derived in terms of Euler quasi coordinate. Next, focusing on the chattering phenomenon, a new approach is proposed to alleviate the chattering trend. In order to set free the actuators from permanent activity, an Error Tolerance Margin (ETM) is defined. In this method, a passive time interval is defined, during which the actuator is turned off. Also, it is assumed that only On-Off actuators are available. To evaluate the energy consumed by the actuators, a ratio named Actuator Activity Factor (AAF) is introduced. The developed control law is applied to a given satellite during a rotational maneuver in presence of parametric uncertainties and noisy feedback signals to show its ability and merits.

Key Words: Sliding Mode Control, Quasi Coordinate, Uncertainty, On-Off Actuator, Simulation

کنترل مود لغزشی تنظیم شده چرخش ماهواره: بالانس بین دقت دنبال‌کردن و مصرف انرژی

سیدعلی‌اکبر موسویان　　سیدحسین ساداتی　　محمدرضا همایی‌نژاد
دانشکده مهندسی مکانیک، دانشگاه صنعتی خواجه نصیرالدین طوسی

چکیده
اگرچه سیستم‌های کنترل مود لغزشی در اکثر مواقع دارای عملکرد مطلوبی هستند، اما پدیده نوسانات شدید سیستم کنترل می‌تواند باعث بروز برخی مشکلات برای عملگرها، به خصوص از انواع خاموش - روشن گردد. به منظور تأمین نیازهای پایداری، ویژگی‌های مقاومت و کاهش پدیده نوسانات شدید سیستم کنترل، در این مقاله، کنترل مود لغزشی اصلاح شدهٔ جدیدی پیشنهاد می‌گردد. به واسطه عملکرد یک تابع ریاضی تنظیم‌کننده شرط لغزش، مقادیر متناسب مثبتی برای این شرط، در زمان عملکرد سیستم کنترل انتخاب می‌گردد. به منظور نشان دادن قابلیت‌های الگوریتم جدید، دینامیک چرخشی یک ماهواره با توجه به دستگاه مختصات بدنی به وسیله متغیرهای شبه مختصاتی اولر بیان می‌گردد. سپس، با تمرکز بر پدیده نوسانات شدید سیستم کنترل، الگوریتم جدیدی در کنترل مود لغزشی پیشنهاد می‌گردد. به منظور جلوگیری از فعالیت همیشگی عملگرها، یک حاشیه آزادی خطا تعریف می‌شود. عملگرهای سیستم از نوع خاموش - روشن در نظر گرفته می‌شوند. برای مطالعهٔ مصرف انرژی سیستم‌های عملگری، شاخص فعالیت عملگرها تعریف شده است. الگوریتم توسعه یافتهٔ جدید در کنترل مود لغزشی، بر یک ماهواره در طول یک مانور چرخشی اعمال می‌گردد. همچنین، به منظور نشان دادن قابلیت‌ها و مزایای الگوریتم کنترلی جدید، عدم قطعیت‌های پارامتری و سیگنال‌های قرائت شده به همراه اغتشاشات در طول شبیه‌سازی در نظر گرفته می‌شوند.

واژه‌های کلیدی: مود کنترلی لغزشی، شبه محور مختصات، عدم اطمینان، عملگر خاموش - روشن، شبیه‌سازی

1- Assistant Prof.
2- Assistant Prof.: sadatih@yahoo.com
3- MSc Student

1- Introduction

In order to develop control systems for space assemblies, it is essential to develop proper kinematics/dynamics model for the system[1]. This has been studied under the assumption of rigid elements, [2-4], and also elastic elements, [5-6]. Due to complicated nonlinearities in space systems, and maneuver time limitations, besides restrictions of energy consumption in space, there have been various studies on the nonlinear control problem of such systems with both rigid and flexible elements [7-10]. Systems with nonlinear dynamics are controlled in a different way compared to linear systems. Systems which include uncertainties such as parametric or structural uncertainties, also need appropriate strategies to be controlled. Two important approaches used for tackling against nonlinearities and uncertainties are Robust control and Adaptive control. One of the main approaches in robust control is Sliding Mode control, [11]. Sliding Mode control is usually accompanied by a phenomenon called "chattering", [12-13]. Chattering should be avoided to reduce the energy consumption of the control system, and prevent any potential damages on actuators, especially in case of on-off type. In addition, due to high frequency content of chattering, it can easily stimulate the flexible modes which in turn may cause instability. To alleviate the chattering phenomenon, one can use saturation functions instead of switching operators, which degrade the control precision. Another approach is to choose the controller coefficients by an intelligent method such as adding an adaptive, network subsystem,or a fuzzy or neural one or even a combination of them to the main control strategy, [14]. Reaction jet actuators can be used instead of servos. However, the excitation law must be determined to provide acceptable performance and stability [15]. In this paper, a regulated sliding mode control law for space platforms during a rotational maneuver is developed, which results in considerable performance improvements. To fulfill stability requirements, a regulation procedure is proposed to determine the proper positive values for the coefficient of sliding condition, which satisfies robustness properties, and chattering elimination characteristics. To this end, the rotation dynamics of a typical satellite described in body coordinates is derived in terms of its angular velocity. Using Euler quasi-coordinates, explicit relationship between actuator torques and satellite orientation is obtained for integration purposes. Then, the developed control law is applied to a given satellite during a rotational maneuver. To examine the new controller merits, and to consider practical limitations, it is assumed that only micro reaction jet actuators are available, which can generate a constant positive or negative (on-off) force. In order to decrease the energy consumption of reaction jet actuators, an Error Tolerance Margin is defined. When the system's tracking error enters this margin, actuators will turn off until the error gets out of the margin again. Therefore, the exact demanded torque can not be applied to the system. Also, parametric uncertainties and noisy feedback signals are taken into considerations. The simulation results reveal the merits of the new regulated sliding mode control law.

2- System Dynamics

Considering typical space systems such as satellites, as shown in Figure 1, it can be assumed that the main structure consists of rigid elements. For a flexible system, an appropriate dynamics model can be substituted for the rigid model. Therefore, rotational dynamics of the main rigid body of a given satellite can be obtained based on Euler equations. To do so, the angular momentum ($\vec{H}$) about center of mass (C) can be expressed in the body attached coordinate (C_{xyz}) as:

1- For a thorough review of the dynamics and control of space multi-body systems (space free-flyers) one could see ref. [1]

$$\{H\}_{xyz}=[I]\begin{Bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{Bmatrix}, \qquad (1)$$

where, $[I]$ describes the mass moment of inertia matrix. Based on Euler equations described as:

$$\sum\overrightarrow{M_c}=\frac{d}{dt}\overrightarrow{H_c}, \qquad (2)$$

where $\sum\overrightarrow{M_c}$ defines the resultant moment of all external forces about satellite center of mass. Assuming the body attached coordinate as principal axes, Eq. (2) yields:

$$\begin{cases}M_1=I_1\dot{\omega}_1+(I_3-I_2)\omega_2\omega_3\\ M_2=I_2\dot{\omega}_2+(I_1-I_3)\omega_1\omega_3\\ M_3=I_3\dot{\omega}_3+(I_2-I_1)\omega_1\omega_2\end{cases}, \qquad (3)$$

where, M_i's are the resultant external moment about principal axes. To find a direct relationship between the control inputs (M_i's) and the satellite orientation in terms of a set of Euler angles, one can substitute the angular velocity components by Euler angle rates, see [16], as:

$$\begin{cases}\omega_1=\dot{\Phi}\sin\theta\sin\psi+\dot{\theta}\cos\psi\\ \omega_2=\dot{\Phi}\sin\theta\cos\psi-\dot{\theta}\sin\psi\\ \omega_3=\dot{\Phi}\cos\theta+\dot{\psi}\end{cases}, \qquad (4)$$

where $\phi, \theta, and\ \psi$ describe yaw, pitch, and spin angle, respectively. Substituting Eqs. (4) with Eqs. (3), and simplifying the computations, we have

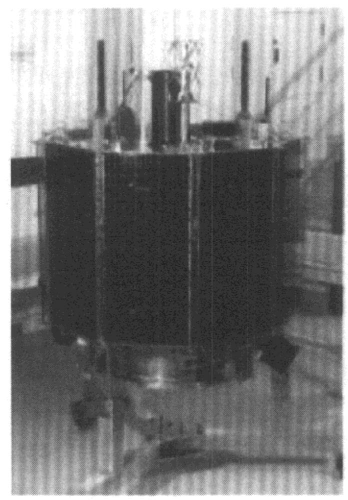

Fig. 1. The main body of a typical satellite (Surrey Space Centre UoSat-12).

$$\begin{aligned}Q_1&=\dot{\phi}\dot{\theta}(I_1\cos\theta\sin\psi)+\dot{\phi}\dot{\psi}(I_1\sin\theta\cos\psi)-\dot{\theta}\dot{\psi}(I_1\sin\psi)+(I_3-I_2)\ (\frac{\dot{\Phi}^2}{2}\sin2\theta\cos\psi\\ &\quad+\dot{\phi}\dot{\psi}\sin\theta\cos\psi-\dot{\theta}\dot{\phi}\sin\psi\cos\theta-\dot{\theta}\dot{\psi}\sin\psi)\\ Q_2&=\dot{\phi}\dot{\theta}(I_2\cos\theta\cos\psi)-\dot{\phi}\dot{\psi}(I_2\sin\theta\sin\psi)-\dot{\theta}\dot{\psi}(I_2\cos\psi)+(I_1-I_3)\ (\frac{\dot{\Phi}^2}{2}\sin2\theta\sin\psi\\ &\quad+\dot{\phi}\dot{\psi}\sin\theta\sin\psi+\dot{\theta}\dot{\phi}\cos\psi\cos\theta+\dot{\theta}\dot{\psi}\cos\psi)\\ Q_3&=-\dot{\phi}\dot{\theta}(I_3\sin\theta)+(I_2-I_1)(\frac{\dot{\Phi}^2}{2}\sin^2\theta\sin2\psi-\dot{\theta}\dot{\phi}\sin\theta\cos2\psi-\frac{\dot{\theta}^2}{2}\sin\psi)\end{aligned}, \qquad (5a)$$

where,

$$\begin{aligned}&A_1=I_1\sin\theta\sin\psi,\ A_2=I_1\cos\psi,\\ &B_1=I_2\sin\theta\cos\psi,\ B_2=-I_2\sin\psi\\ &C_1=I_3\cos\theta,\ C_2=I_3\\ &\Delta=A_1B_2-A_2B_1\ .\end{aligned} \qquad (5b)$$

Eqs. (5) can be written in the following form:

$$\begin{cases} \ddot{\phi} = f_1 + R_1 \\ \ddot{\theta} = f_2 + R_2 \\ \ddot{\psi} = f_3 + R_3 \end{cases} \quad (6)$$

$$\begin{cases} f_1 = \dfrac{A_2Q_2 - B_2Q_1}{\Delta}, \quad R_1 = \dfrac{B_2M_1 - A_2M_2}{\Delta} \\ f_2 = \dfrac{B_1Q_1 - A_1Q_2}{\Delta}, \quad R_2 = \dfrac{A_1M_2 - B_1M_1}{\Delta} \\ f_3 = \dfrac{-Q_3}{C_2} - \dfrac{C_1}{C_2}\dfrac{1}{\Delta}(-B_2Q_1 + A_2Q_2), \\ R_3 = \dfrac{M_3}{C_2} - \dfrac{C_1}{C_2}\dfrac{1}{\Delta}(B_2M_1 - A_2M_2) \end{cases} , \quad (7)$$

where, f_i describes the dynamics for each angle and R_i are the inputs that must be determined appropriately. It should be noted that Eqs. (6) describe the system dynamics in a suitable format for developing sliding mode nonlinear controllers as will be discussed in the next section.

3- Control Law Synthesis

Suppose a multi input nonlinear system is defined by:

$$x_i^{(n_i)} = f_i(\vec{x}) + \sum_{j=1}^{m} b_{ij}(\vec{x}) \; u_j \begin{cases} i = 1 ... m \\ j = 1, ... m \end{cases} . \quad (8)$$

where $\vec{u}$ describes control input array, $\vec{x}$ the state matrix composed of $\vec{x}_i$'s as state arrays

$$\vec{x}_i = [x_i \; \dot{x}_i .... x_i^{n_i - 1}]^T , \quad (9)$$

and f_i as the dynamics of the i^{th} state as a function of state vector $\vec{x}$ (not essentially a linear function), b_{ij} is the corresponding element of input matrix "B", which describes the gain function of the j^{th} input on subsystem "i", n_i is the order of corresponding differential equation, and "m" is the number of independent inputs. The control aim can be expressed as making the state vector $\vec{x}$ following the desired time dependent vector $\vec{x}_d$. In the presence of modeling uncertainties, it is assumed that all parametric uncertainties appear in the input matrix B, and it is nonsingular in the state space domain. If $\hat{B}$ describes the estimated value of B, it is assumed to be nonsingular, too. Therefore, it is defined that,

$$\begin{aligned} & B = (I + \Delta)\hat{B} \\ & |\Delta_{ij}| \le D_{ij} \qquad i = 1, ..., m \qquad j = 1, ..., m \\ & |\hat{f}_i - f_i| \le F_i \end{aligned} \quad (10)$$

where, $\hat{f}_i$ is the estimated value of f_i, which can be obtained from dynamics model, and Δ_{ij}'s can yield the error values of the input matrix estimation procedure. In fact, the exact value of Δ_{ij} is not known but the upper bound limitation (i.e. D_{ij}) can be substituted. Therefore, the distance from a sliding surface is defined as:

$$\begin{aligned} & s_i = \left(\frac{d}{dt} + \lambda_i\right)^{n_i - 1} \tilde{x}_i \\ & \tilde{x}_i = x_i - x_{id} \end{aligned} , \quad (11)$$

where, λ_i's are controller parameters and in fact are time constants in a low pass filter sequence, [12], and $\tilde{x}_i$ describes the tracking error of x_i. Eq. (11) can be written as:

$$s_i = x_i^{(n_i - 1)} - x_{r_i}^{(n_i - 1)} , \quad (12)$$

where, $x_{r_i}^{(n_i - 1)}$ is computed based on the error between $\vec{x}_i$ and $\vec{x}_{i_d}$. For instance, considering a system with two state arrays and two independent inputs, this can be obtained:

$$s_i = (\frac{d}{dt} + \lambda_i).\tilde{x}_i = \dot{x}_i - (\dot{x}_{id} - \lambda_i \tilde{x}_i) \quad i = 1,2 \quad , (13a)$$

which yields:

$$x_{r_i} = \dot{x}_{i_d} - \lambda_i \tilde{x}_i \quad i = 1,2 \quad . \qquad (13b)$$

Therefore, in a general case, the control inputs must be determined such that they satisfy the following *sliding condition:*

$$\frac{1}{2}\frac{d}{dt}s_i^2 \le -\eta_i |s_i| \qquad \eta_i > 0 \ , \qquad (14)$$

where, η_i s are controller parameters chosen as positive values which reflect how the states are converged to their sliding surfaces. Higher values of η_i show that the corresponding state reaches its sliding surface faster. Assuming that K_i s are positive values that must be determined so that the sliding condition (14) is satisfied,. then the control law is obtained as:

$$\vec{u} = \hat{B}^{-1}\left(\vec{x}_r^n - \vec{\hat{f}} - \vec{K}Sgn(s)\right) \ . \qquad (15)$$

where K_i s can be calculated using *Filippov's Construction of Equivalent Dynamics*, [12] by $\dot{s}_i = 0$ (for i=1,...,n), which yields:

$$(1 + D_{ii})K_i + \sum_{j \ne i} D_{ij} K_j = F_i + \sum_{j=1}^{n} D_{ij} \left| x_{rj}^{n_j} - \hat{f}_j \right| + \eta_i \qquad i = 1,\dots,n \quad . \qquad (16)$$

In fact, Eqs. (16) define a set of "n" equations with "n" unknowns (i.e. K_i's). To solve these equations, one should choose η_i s. As explained before, η_i is a factor that indicates the speed of the corresponding state in approaching its sliding surface. Therefore, if one can determine η_i in such a way that the speed of the corresponding state becomes lower based on the absolute value of distance from the sliding surface, and becomes zero on the surface, then the performance will be as desired and chattering will be alleviated if not vanishing. Therefore, rather than the conventional heuristic method to choose η_i, here it is proposed to select that:

$$\eta_i(t) = (\eta_{0_i} \left|1 - e^{|s_i|}\right|) \times \frac{1}{2}[1 - \text{sgn}(|s_i| - s_{acti}^*)] + \eta_{0i} \times \frac{1}{2}[1 + \text{sgn}(|s_i| - s_{acti}^*)] \quad . \qquad (17)$$

In Eq. (17) the parameter s_{acti}^* is a positive constant value that indicates the activity margin of ERP (Etta Regulating Process) mechanism. For instance, if a fast transient response is desired, s_{acti}^* should be selected tight (a small value), while the value of η_{0i} should be chosen large enough. On the other hand, if the smoothness of the response is more important, s_{acti}^* can be chosen loose (large value), while η_{0i} can have any value. In practice, one can follow the procedure shown in Figure 2, to use Eq. (17) for selection of η_i, which will be referred to as Etta Regulating Process (ERP). To select the initial value, η_{0i}, any large value can be chosen. In fact, in the absence of any uncertainties, there are two aspects in the conventional heuristic method of choosing η_i. First, regarding the noise effects, the controller must be capable enough to tackle against separation of the state trend from the corresponding sliding surface. Therefore, with a reasonable high value of η_i, the noise effect will be decreased. Second, after reaching the state trend to the sliding surface, chattering may happen. The amplitude of this phenomenon increases with a high value of η_i. Therefore, the noise rejection characteristics conflict the chattering effect reduction. However, following the proposed procedure even with high initial values for η_{0i}, it will compensate between the two effects, and alleviates the chattering soon, which will be shown in simulations in the next section. In figure 2, the switch, threshold can be chosen with respect to the initial value of η_i's and the required approaching speed of the systems state vector toward the sliding surface. If the initial value of η_i's are large enough to compete the control activity, the ERP activation margin, i.e., threshold of switch can be chosen a tight extent. But if the initial values are not large enough, the threshold must be chosen wide enough to activate the ERP to add enough values to the parameters to afford the control activity. In the next part, more details about the switch threshold will be discussed. So if the initial values to be chosen are large enough heuristically, the quality of approaching the sliding surface can be tuned as desired.

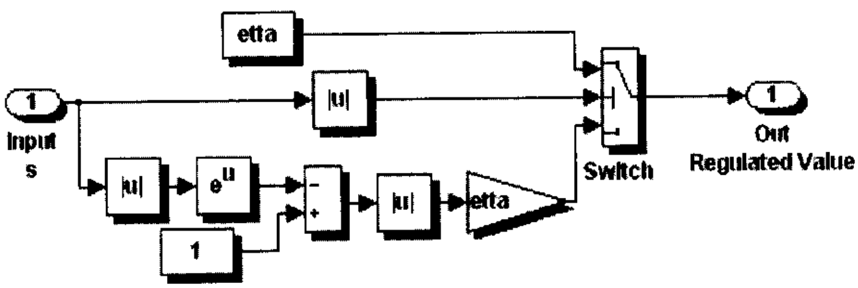

Fig. 2. The Procedure for choosing η_i.

4- Simulation Results

In this section, comparison between the performance of the two systems; one having both ERP and ETM, and the other only ERP, is done. We simulate a 3-DOF rotating satellite about a fixed position of its center of mass. The important task is that the state matrix can follow the desired one, i.e. $[\vec{\Phi}\ \vec{\theta}\ \vec{\Psi}]_d$, despite actuators limitations such as saturation, dead band, and quantization during a 3-DOF rotating maneuver. The satellite specifications are given in Table (1).

Table (1). The satellite specifications.

Specification	**Value<Unit>**
Moment of Inertia-Axis(ϕ)	$550 < kg.m^2 >$
Moment of Inertia-Axis(θ)	$550 < kg.m^2 >$
Moment of Inertia-Axis(ψ)	$450 < kg.m^2 >$
Total Mass	$1050 < kg >$

Parametric uncertainties and measurement faults for the two controllers have been considered as follows:

1- 5% uncertain variations in the dynamic parameters, i.e. mass and mass moments of inertia.

2- 5% deviations in measurements.

Obtained results show that when the state vector of the system reach the sliding surface, the effect of parametric uncertainties will become negligible as shown in Fig (3).

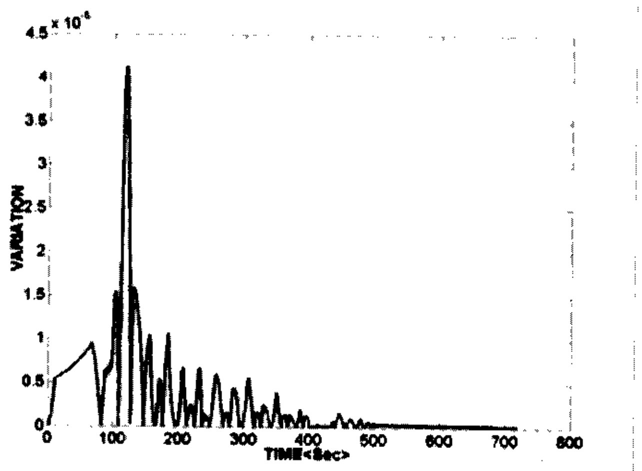

Fig. 3. Variation of ψ channel dynamics due to parametric uncertainties.

The model for the actuators has been shown in Fig. 4. In this model, the constant value can be be determined by calculating the required moment obtained from inverse dynamics. Since the maneuver takes place in a relatively long time interval, i. e. 720 s, the time constants for switching between "on" and "off" actions of the actuators can be neglected. According to this model, as shown in Fig. 4, if the demanding controlling torque is positive/negative, the actuator will deliver its constant value in positive/negative direction, respectively.

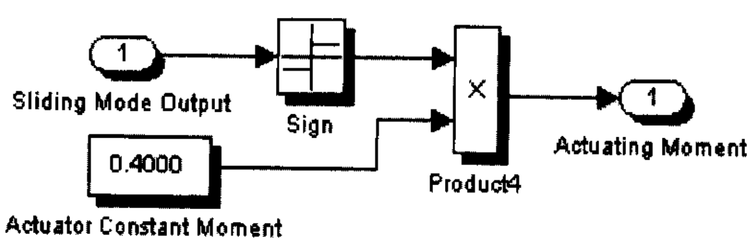

Fig. 4. The reaction jet actuators model.

In order to achieve a lower activity of actuators, in expense of losing some tracking precision, an Error Tolerance Margin (ETM) is defined as:

$$G_{etm,i} = \frac{1}{2}[1 + \mathrm{sgn}(|\tilde{x}_i| - \tilde{x}_i^*)]$$
$$M_{act,i} = M_{reaction,i} \times G_{etm,i} \quad , \qquad (18)$$
$$\tilde{x}_i = x - x_d, \quad i = 1,2,3.$$

In Eq. (18), $\tilde{x}^*$ is a positive value that defines the permissible error tolerance. In fact, if this parameter is chosen large, the passive time of actuators will increase and vice-versa. By this definition, the activity of actuators can decrease, but the tracking accuracy of the system worsens. The coefficients of conventional and chatter avoidance controllers are chosen as:

$$\lambda_1 = \lambda_2 = \lambda_3 = 15$$
$$\eta_{01} = \eta_{02} = \eta_{03} = 0.0450 \qquad (19)$$
$$s^*_{act1} = s^*_{act2} = s^*_{act3} = 0.045$$
$$\tilde{x}_1^* = \tilde{x}_2^* = \tilde{x}_3^* = 0.50^\circ$$

It should be mentioned that the conventional heuristic method of choosing controller coefficients for various trials does not result in much difference. The initial error of each Euler angle is chosen to be about 2°. Figure 5 shows the corresponding desired values of Euler angles of the satellite for the rotating maneuver. These values have been chosen such that the main performance characteristics of the controllers can be compared. Applying the new idea of regulation procedure to the selection of η_i, based on Eqs. (17,18), the obtained results will be compared. Figure 8 shows a typical result for the ERP algorithm, i.e., the regulated value of η_3. Application of the ERP to other subsystem parameters yields similar results.

Figure 6 compares tracking errors for the Euler angles (ϕ,θ,ψ channels) between the original system (ERP alone), and the new proposed algorithm (both ERP and ETM). It can be seen that the new algorithm makes the error confined within the allowable error margin of each subsystem, while the conventional ERP yields a small value of error. This is due to the fact that ETM switches off the actuators when the absolute value of the error becomes less than the error margin. However, the ERP algorithm causes a smooth approach toward the sliding surface by proper variation of η_i's, and consequently, is able to keep the kinetic energy of the rotating object close to its nominal value. These results reveal the merits of the new regulated sliding mode control law. In order to investigate the effect of ETM on actuators performance, Actuators Activity Factor is defined as follows:

$$AAF(t) = \frac{\int_0^t |M_{reaction}|.dt}{2\times|M^*_{reaction}|.t_f} \times 100 \quad , t \le t_f . \qquad (20)$$

In Eq.(20), $M_{reaction}$ indicates the moment generated by the corresponding actuator during its activity and $M^*_{reaction}$ indicates the constant value of the moment that can be generated by the flow jet of actuators. If for two system, AAF(t_f)s are the same, the corresponding actuator activities of the two systems will be the same. In figure 7, the AAF of each subsystem for two kinds of sliding mode control strategy is shown. It can be seen that the trade-off between about 1.5 degrees of error in each subsystem is about 8% reduction in AAF for the first and the third axes and more than 40% for the second axis. In figure 8, the effect of ERP mechanism on regulating one of switching gains is shown. For all subsystems the regulation schematic is the same.

Mech. & Aerospace Eng. J. Vol. 1, No. 1, Aug. 2005

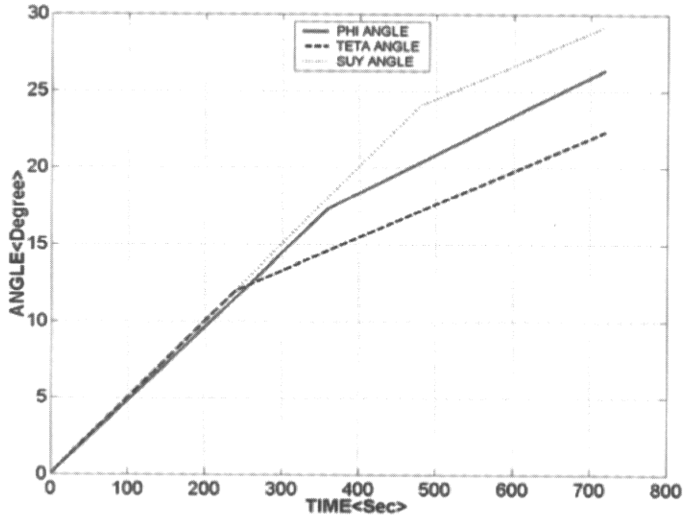

Fig. 5. Desired values for Euler Angles during the maneuver.

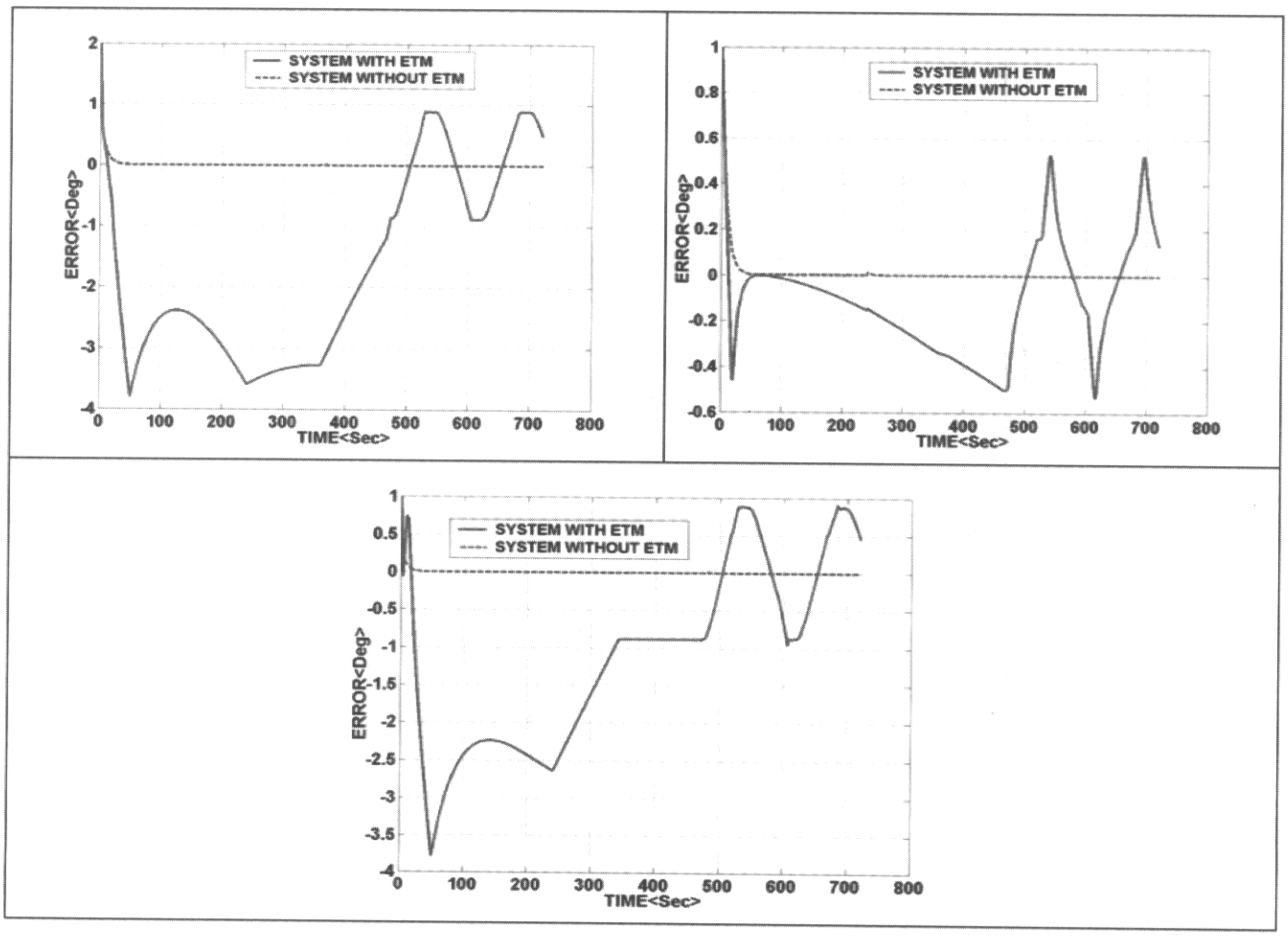

Fig. 6. Euler angles tracking error; (Dashed Line: Conventional ERP; Solid Line: The new proposed controller, Left: ϕ, Right: θ and Bottom: ψ angle).

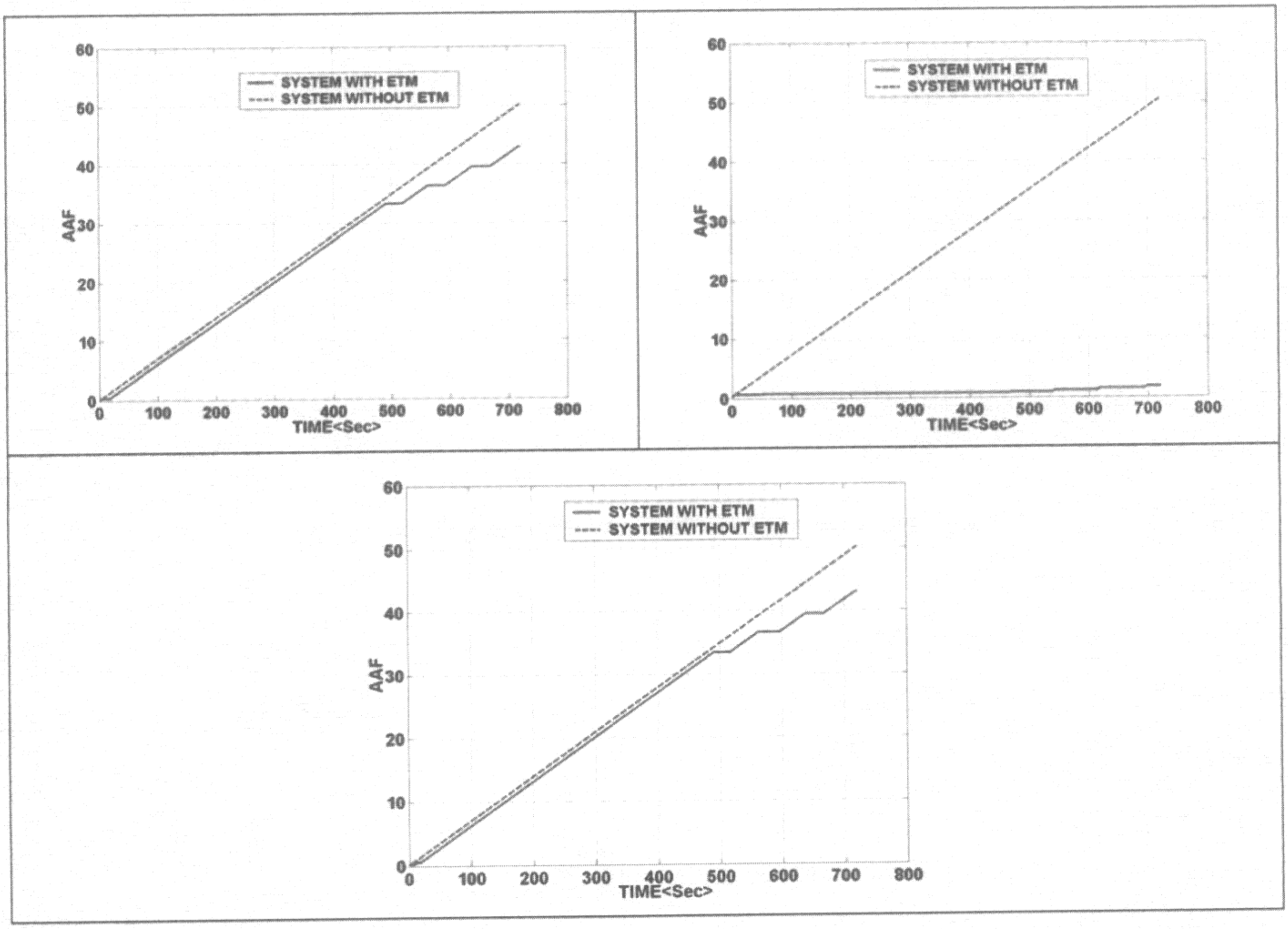

Fig. 7. AAF of each system as a function of time; (Dashed Line: Conventional ERP; Solid Line: The new proposed controller, Left: ϕ, Right: θ and Bottom: ψ angle).

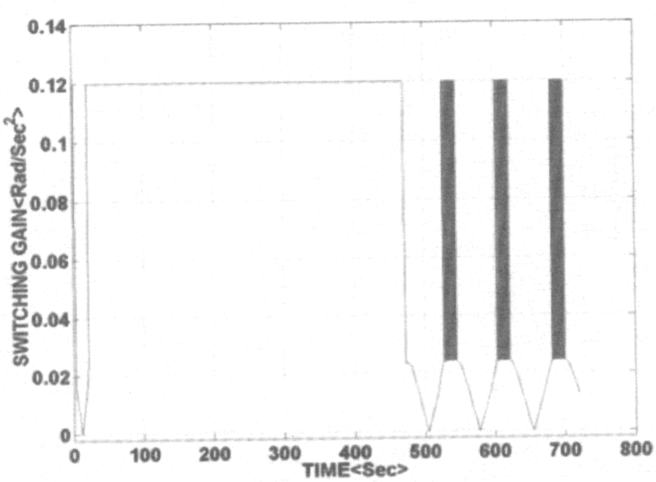

Fig . 8. The ERP result for determination of η_1 .

5- Conclusion

In this paper, a regulated sliding mode control law was developed and applied to space platforms during a rotational maneuver. Since on-off reaction jet actuators in space can not deliver the exact demanding moment to the system, the control becomes more complicated. The sliding mode controller is a reliable algorithm with acceptable performance, while the chattering frequency is often high and may excite flexible dynamics, which are of main concern for space systems due to minimum weight design as a dominant trend. Therefore, it is not easy to find proper values for the coefficient of sliding condition (η_i's) that can provide an acceptable performance during a usual maneuver. To alleviate the chattering phenomenon, a new procedure was proposed to change η_i's and smoothly converge it to the appropriate positive values. This satisfies robustness properties, and chattering elimination characteristics. After all, in order to decrease the on-off actuators activity, an Error Tolerance Margin (ETM) was defined. By this definition, when the state vector of the system entered in a region chosen by the designer, the on- off actuators were made unable. Therefore, in the output of the system, some errors were observed while the total energy consumption of the system decreased (Trade-off between tracking precision and energy consumption). To check up the performance of the system due to the insertion of the ETM into control algorithm, an Actuator Activity Factor (AAF) was introduced. To this end, the rotation dynamics of a typical satellite described in body coordinates was derived in terms of its angular velocity. Using Euler quasi-coordinates, explicit relationship between actuator torques and satellite orientation was obtained for integration purposes. Then, control input functions were obtained based on a multi-input sliding mode control law. Next, focusing on the chattering phenomenon to fulfill energy limitations in space, a new approach was proposed to alleviate (ideally eliminate) the chattering trend. Finally, the developed control law was applied to a given satellite during a rotational maneuver. The simulation results revealed that the on-off actuators switching intensity can be decreased by losing some tracking precision within the acceptable errors of the mission.

Acknowledgement

The authors of this paper wish to thank Professor Ali Ghaffari and Dr. Ali Nahvy for their detailed comments and suggestions.

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