Surge Avoidance in Gas Turbines

H. Khaleghi¹, M. Boromand², A. M. Tousi³
Aerospace Eng. Dep't.
Amirkabir Univ. of Tech.

ABSTRACT

In this work, the transient process of surge has been investigated numerically in a gas turbine engine. A one-dimensional stage-by-stage mathematical model has been developed which can describe the system behavior during aerodynamic instabilities. It is demonstrated that, these instabilities can be stabilized by the use of active control strategies, such as air bleeding and air injection. Both steady and unsteady active control systems were considered. In the steady case, mass is removed at a fixed rate from the diffuser, or mass is injected at a fixed rate into the first stage of the compressor. In unsteady control, the rate of bleeding or injection is linked with the amplitude and the frequency of the upstream pressure disturbances. Results show that both steady and unsteady strategies eliminate surge disturbances and suppress the instabilities. Therefore, they extend the stable operating range of compressor. It is also shown that smaller amount of compressed air needs to be removed in the unsteady control case. Also, a variable area diffuser is shown to be able of suppressing surge instabilities. Active control of instabilities, caused by sinusoidal perturbations of inlet total pressure, was also investigated, which showed to destabilize the stable operating condition the design point.

Key words: Surge Avoidance, Gas Turbine, Active Control

جلوگیری از واماندگی در توربین های گازی

محمد رضا خالقی، مسعود برومند و ابوالقاسم مسگر پورطوسی دانشکده مهندسی هوافضا، دانشگاه صنعتی امیرکبیر

چکیده

در این تحقیق، فرآیند گذرای واماندگی یک توربین گازی بصورت عددی بررسی شده است. یک مدل یک بعدی ریاضی مرحله به مرحله توسعه یافته که می تواند رفتار سیستم را در خلال ناپایداری های آیرو دینامیکی توصیف نماید. این ناپایداری ها می تواند با استفاده از استراتژی های کنترل فعال مثل تنفس و تزریق هوا از بین برود. دو نوع کنترل فعال به صورت های دائمی وغیر دائمی در نظر گرفته شده است. در حالت دائمی، جرم با یک نرخ ثابتی ازدیفیوزرخارج و یا با نرخ ثابتی به اولین مرحله کمپرسور تزریق می شود. در کنترل غیر دائمی، نرخ تنفس یا تزریق هوا به دامنه و فرکانس نوسان اغتشاشات فشاری بالادست جریان مربوط می شود. نتایج نشان می دهد که درهردو استراتژی تزاحمات واماندگی حذف شده و ناپایداری های مربوطه محدود می شود و بنابراین دامنه پایدار عملکردی کمپرسور افزایش می یابد. همچنین،نتایج این تحقیق نشان داد که مقدار کمتری هوای کمپرسور بایستی با داشتن کنترل غیردائمی خارج شود. یک سطح مقطع متغیر دیفیوزر قادراست کارمحدود کردن ناپایداری های واماندگی را انجام دهد. کنترل فعال ناپایداری ها که توسط تزاحمات سینوسی فشار کل ورودی به وجود می آید نیز دراین کاربررسی شده که نتیجه اش ناپایدار کردن شرایط پایدار سیستم در نقطه طراحی بود.

واژههای کلیدی: جلوگیری ازواماندگی، توربین گازی، کنترل فعال

¹⁻ PhD Student (Corresponding Author): khaleghi@aut.ac.ir

²⁻ Assistant Prof.

³⁻ Assistant Prof.

NOMENCLATURE

A = Area

Am = Amplitude of Surge Disturbance

 C_f = Friction Factor D_H = Hydraulic Diameter DA = Diffuser Area

E = Internal Energy Per Unit mass

Fx = Axial Force

f = Frequency of Surge Disturbance

 f_{dis} = Frequency of Inlet Total Pressure
Perturbation

 K_1 - K_5 = Constants

m_B = Mass Flow Rate of Bleed or Injected air

p = Static Pressure Q = Heat Production

t = Time

U = Axial Velocity

 U_{BX} = Axial Velocity of Bleed or injected air

W = Work Density

) =

Introduction

Turbo machines are used in a wide variety of engineering applications for power generation and propulsion. There are two major fluid dynamic instabilities in compression systems, known as rotating stall and surge. Surge is a large amplitude oscillation of the total annulus averaged flow through the compressor; whereas in rotating stall, one can finds from one to several cells of severely stalled flow rotating around the circumference, although the annulus averaged mass flow remains constant in time once the pattern is fully developed. Therefore. rotating stall is the two-dimensional or threedimensional disturbance localized to the compressor and characterized by regions of reduced or reversed flow that rotate around the annulus of the compressor [1-4]. To avoid these dangers, compressors have been designed to operate away from the peak operating

A compression system mathematical model was developed using lumped-volume techniques which make certain assumptions about compressibility within the system.

The lumped volume model uses an isentropic relationship to relate the time-dependant change in density to a time-dependant change in total pressure, and uses a steady-state form of the energy equation [5]. A stage-by-stage mathematical model was presented by Davis [6] which removed assumptions inherent in lumped-volume models. A one dimensional model developed by Garrard and Davis [7-10] was found to predict the flow oscillations of surge cycles due to perturbations of fuel flow rate. A one dimensional model was developed to predict surge disturbance propagation and engine response during surge and surge recovery, due to perturbation of total pressure and temperature, and exit nozzle area [11]. Moore and Greitzer [12] developed a 2-D model for rotating stall and surge. Their analysis was extended to the compressible flow regime by Bonnaure [13] and Hendricks [14]. It also was further modified to include actuation by Feulner [15], who also converted the model to a form compatible with control theory. Paduano used controllable inlet guide vanes for elimination of rotating stall [16,17]. Pinsley [18] studied centrifugal surge control using throttle valves as actuators. The effect of bleeding on the control of instabilities was studied by Eveker [19], Yeung [20] and Murray [21]. The reported amounted of bleeding by Yeung to achieve operating enhancements ranges from 1 to 10 percent based on the mean flow. Niazi and Stein [22] developed a three-dimensional viscous flow solver and studied the fluid dynamic phenomena that lead to the onset of instabilities in centrifugal and axial compressors and the effect of bleeding on the control of instabilities.

The next section of this study contains the model description. In the third section, the results of using steady and unsteady control for uniform inlet flow are presented. Air bleeding, air injection and variable area diffuser are used as control systems. Although one-dimensional models are not able to simulate rotating stall, they are shown to properly enable simulation of surge instability and study of active control. In the fourth section, the results of using steady and unsteady control for inlet flow with total pressure perturbation are presented.

Modeling

Figure 1 shows the engine geometry consisting of compressor, a diffuser, a combustion chamber, a two stage turbine and an exhaust duct with a convergent nozzle. Dimensions are given in mm. The compressor geometry and characteristics are taken from Rolls-Royce C-141, which its geometry and experimental data were available in house.

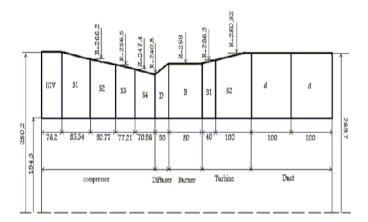


Fig. 1Geometry of the engine.

Governing Equations

The governing equations are the unsteady, onedimensional equations of continuity, momentum and energy. These equations for an inviscid flow are expressed in the conservative form (Equations 1 to 4). It must be noted that these 1-D equations are used for simulation of compressor and turbine flows and not for the combustion chamber. The combustion chamber is considered as a zero dimensional component which energy release is modeled by an increase in total temperature as:

$$\frac{\partial M}{\partial t} + \frac{\partial N}{\partial x} = S,$$

$$M = \begin{bmatrix} \rho \\ \rho U \\ \rho e \end{bmatrix}, \tag{2}$$

$$N = \begin{bmatrix} \rho U \\ \rho U^2 + p \\ \rho U e + p U \end{bmatrix}, \tag{3}$$

$$S = \begin{bmatrix} -\rho U \frac{d(\ln A)}{dx} - \frac{d\dot{m}_B}{Adx} \\ -\frac{\rho U^2}{A} \frac{dA}{dx} + \frac{FX}{Adx} - 2C_f \frac{\rho U|U|}{D_H} - \frac{d\dot{m}_B U_{Bx}}{Adx} \\ \frac{\partial Q/\partial t}{Adx} - \frac{\dot{W}_{shaft}}{Adx} - \frac{\rho U}{A} \left(\frac{U^2}{2} + \frac{\gamma}{\gamma - 1} \frac{p}{\rho}\right) \frac{dA}{dx} - (e_B + p/\rho) \frac{d\dot{m}_B}{Adx} \end{bmatrix}$$
(4)

Component Characteristics

To provide stage force (FX) and shaft work (W_{shaft}) input to the momentum and energy equations, a set of quasi-steady stage characteristics must be available for closure. The stage characteristics provide the pressure and temperature variation across each stage as a function of normalized corrected mass flow rate. The compressor has four stages and an inlet guide vane (IGV) with different pressure and temperature characteristics. During transition to surge, the steady stage forces derived from the steady characteristics are modified for dynamic behavior via a first-order time lag equation. Appropriate time constants must be used for each stage to provide the correct transient behavior. The pressure and temperature characteristics of the third stage are given in Fig.2. Characteristics of other stages are also modified for dynamic behavior.

Burner is considered as a zero dimensional component. The energy release from the combustion chamber is considered by an increase in total temperature. The air-fuel ratio and combustion chamber loss of a typical engine are used in the model. The ratio of exit to inlet total pressure of combustion chamber is 0.96. Turbine stages characteristics, with specified power rating, were obtained in order to take the matching condition into account.

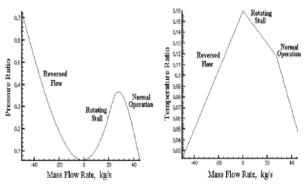


Fig. 2 Compressor characteristic performance of the third stage.

Numerical Scheme and Boundary Conditions

The method of characteristics is used as the Numerical scheme to solve the governing equations. A variable time step is used to satisfy the Courant condition. For more details of MOC one may refer to reference [23].

Specified total pressure and temperature during normal forward flow is the inlet boundary conditions. The exit boundary condition is the specification of

exit Mach number or static pressure. During reverse flow the inlet is converted to an exit boundary with the specification of the ambient static pressure. Therefore, both the inlet an exit boundaries function as exit boundaries during a surge cycle. The boundary conditions are properly applied to the combustor to take the zero dimensional modeling into account. The initial values are determined for the boundary conditions at some specified compressor operating points.

Control Strategies

In this study, two types of active control systems are considered: steady and unsteady. In steady control, including steady air bleeding and air injection, a fixed fraction of mass flow rate is removed from or injected to the compression system. In unsteady case, the mass flow rate of removed or injected air is linked to the pressure fluctuation upstream of compressor during instabilities. Figure 3 illustrates the schematic of the unsteady control system which is used in the present study.

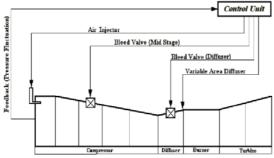


Fig. 3 Schematic of the unsteady control system.

The Validation of Results

To obtain the stable operating conditions, the equations are solved by a time marching technique. To validate the results, the predicted overall of the compressor (for characteristic conditions) is compared with the experimental data in Fig.4. Close agreement between the model steady state results and experimental data, especially near the surge point, is obtained. Table 1 shows the comparison between the surge point obtained from the model and the experimental surge point. As shown in figure 4 the experimental curve is sufficiently close to the theoretical curve near the surge point. Such results can be attributed to the fact that one-dimensional modeling is quite close to the nature of surge.

Tab. 1 Experimental and computational surge point of compressor.

	Mass Flow Rate, kg/s	Compressor Pressure Ratio
Experiment	16.67	2.49
Computation	16.5	2.49

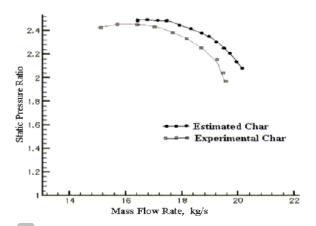


Fig. 4 Compressor overall characteristics.

Uniform Inlet Flow

In steady control, a fixed fraction of the mass flow rate is removed through a valve which can be placed at the diffuser or at the interstage of compression system, or a fixed fraction of the mass flow rate is injected into the first stage of the compressor. Figure 5 shows the static pressure at the compressor face versus time. Steady bleeding from diffuser, equal to 4.3% of mean mass flow rate was applied to the unstable operating condition at point B (shown in Fig.6). As shown, this amount of bleeding can remove surge disturbance.

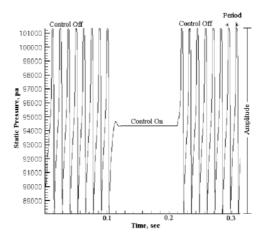


Fig. 5 Inlet static pressure fluctuation.

To study the effect of bleeding, steady bleeding from diffuser and also interstage (second stage) was considered. Bleeding equal to 3 % and 4.3 % of mean flow rate was applied to the unstable condition at point B. For 70 % and 100 % of bleeding which is equal to 3 % and 4.3 % of mean flow rate respectively, the stable controlled operating points are shown as points C and D in Fig.6. In Fig.6, the horizontal axis is the mass flow rate after bleed valve and the vertical axis is the static pressure ratio of the compressor. For 70 % of bleeding (3 % of mean flow rate), the computed mass flow rate is 15.75 kg/s and the corresponding overall static pressure ratio is 2.37. For 100 % of bleeding (4.3 % of mean flow rate), the mass flow rate is 15.65 kg/s and the static pressure ratio is 2.35. This reduction of static pressure ratio is due to removing more compressed air in 100 % bleeding.

To investigate the effect of interstage bleeding, the same amount of bleeding (4.3 % of mean flow rate) was applied to the unstable condition at point B. Mass is removed from the interstage (second stage) and the new operating point is shown as point E which has the mass flow rate of 15.6 kg/s and static pressure ratio of 2.38. As illustrated, interstage bleeding results in higher pressure ratio.

3.3 % of the mean flow rate was injected into the first stage, during the unstable condition at point B, to study the effect of injection on the performance of the compressor. The new stable operating condition is point F in Fig.6. The mass flow rate of point F is 15.85 and the corresponding pressure ratio is 2.5.

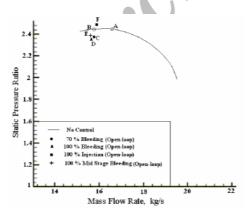


Fig. 6 Compressor characteristic performance for steady control

The steady bleeding is inefficient and must be turned off during design operation. In unsteady control, the removed mass flow rate is linked to the pressure fluctuation upstream of the compressor. Although such strategy is not possible in one-dimensional analysis, using periodic functions for bleeding mass flow rate is shown to improve the stable operating range. The amount of mass, which is removed from diffuser, is linked to the amplitude and frequency of surge disturbance. The amplitude and frequency of pressure fluctuation during surge is found to be 12 Kpa and 80 Hz from Fig.5.

Three forms of periodic functions are used for bleeding control. In the following equations, K_1 , K_2 , K_3 , ϕ are chosen to be 1, 0.9, 0.5 and $\pi/4$ respectively:

$$\dot{m}_B = \overline{\dot{m}}_B + \overline{\dot{m}}_B * K_1 * \frac{Am}{P_1} * \left[\sin \left(2\pi . t. f + \varphi \right) \right]$$
 (5)

$$\dot{m}_{B} = \overline{\dot{m}}_{B} + \overline{\dot{m}}_{B} * K_{2} * \frac{Am}{P_{1}} * \left[\frac{\sin(2\pi . t. f + \varphi) +}{\cos^{2}(2\pi . t. f + \varphi)} \right]$$
(6)

$$\dot{m}_{B} = \overline{\dot{m}}_{B} + \overline{\dot{m}}_{B} * K_{3} * \frac{Am}{P_{1}} * \begin{bmatrix} \sin(2\pi . t. f + \varphi) + \\ \cos^{2}(2\pi . t. f + \varphi) + \\ \cos^{3}(2\pi . t. f + \varphi) \end{bmatrix}$$
 (7)

In the above equations, "P₁", "t", "Am", "f" and "φ" are respectively ambient pressure, time, amplitude, frequency of the fluctuations and the phase lag. The constants K1, K2, K3 are chosen to ensure the bleed rate is less than 3 % of mean flow rate. The parameter $\overline{\dot{m}}_B$ is averaged mass flow rate and is set to be 2.3 % of the mean mass flow rate. Figure 7 is given for better understanding the trend of control function. Results are shown in figure 8. Point G, H and I are new stable operating points corresponding to equation 5-7 respectively. For point G the computed mass flow rate is 15.6 kg/s and the static pressure ratio is 2.4. Point H has mass flow rate of 15.7 kg/s and static pressure ratio of 2.37. Point I has the minimum mass flow rate equal to 15.5 kg/s with the static pressure ratio of 2.37. The minimum mass flow rate obtained is related to the equation 7 and the maximum pressure ratio is related to equation 5. This behavior may be attributed to the effect of different shapes of the equations shown in figure 7. The similar shapes to the nature of surge may lead to higher pressure ratios or lower mass flow rates. As shown, smaller amount of compressed air need to be removed in unsteady control, and also it leads to operating point with

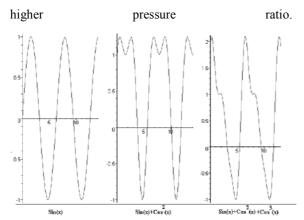


Fig. 7 Schematic shape of three control functions.

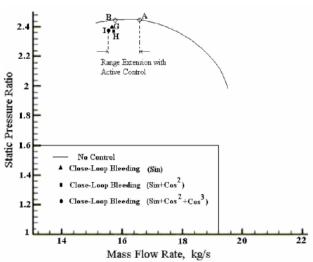


Fig. 8 Compressor characteristic performance for unsteady control

Variable Area Diffuser

To study the effect of variable area diffuser, which is located before the burner, the area of the diffuser is changed with time. The area variation is linked to the amplitude and frequency of surge disturbance. The following periodic control law was chosen in this study:

$$DA = DA_{d} - \frac{|DA_{d} * K_{4} * Am}{|1/P_{1} * \sin(2\pi . t. f. K_{5} + \varphi)|}(8)$$

 DA_d is the design area of the diffuser. Constants K_4 and K_5 are chosen to be 0.2 and 0.1 respectively so that the area variation does not exceed 2.5% of the design area.

This control law was applied to the unstable operating condition at point B shown in figure 6. Figure 9 shows the inlet mass flow rate versus time.

As illustrated, using an appropriate form of diffuser area variation eliminates the surge instability and leads to stable controlled condition.

The area of diffuser decreases periodically by using the above control law. As a result, compressor back pressure decreases periodically. Reduction of back pressure has the same effect of bleeding. Therefore, variable area diffuser is capable of eliminating compressor instabilities.

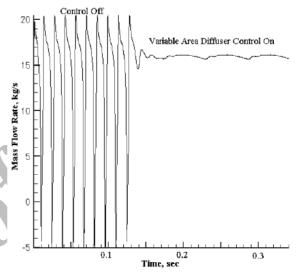


Fig. 9 Inlet mass flow rate (variable area diffuser).

Intel Total Pressure Perturbation

Instabilities Due to Inlet Total Pressure Perturbation As mentioned in the previous sections, Transient interaction of shock waves and boundary layer at the entry may lead to sinusoidal variations of pressure that can affect compressor instabilities. AS suggested by Tesch and Steenken [24], the following form for modeling of the perturbation was considered:

$$(P_T) = (P_T)_{SS} + (P_T)_{SS} *$$

$$Amplitude \quad Percent * Sin(2 \cdot \pi \cdot f_{dis} \cdot t).$$
(9)

In the above equation, $(P_T)_{SS}$ is the steady state inlet total pressure, "t" is time and " f_{dis} " is the perturbation frequency. Instead of amplitude, a parameter named amplitude-percent was used.

To study the engine response, we first applied the above sinusoidal perturbation with the amplitude-percent of 5 and frequency of 10 Hz to the stable condition at surge point. Since no surge disturbance was observed, we increased the frequency at constant amplitude-percent. The first surge disturbance, with

the frequency of 25 Hz, was captured when the frequency of perturbation was increased to 30 Hz. The inlet mass flow rate versus time is given in Fig.10.

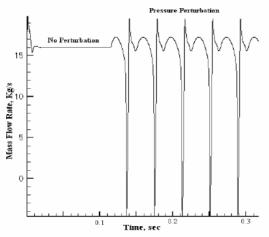


Fig. 10 Inlet mass flow rate (pressure perturbation)

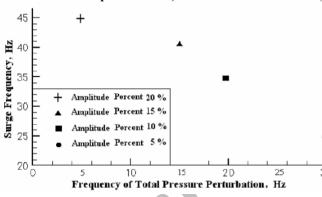


Fig.11 Minimum frequency of the perturbations that leads to surge

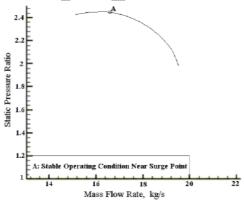


Fig. 12 Compressor overall characteristics

The same methodology was applied to the stable operating condition at surge point for other magnitudes of amplitude-percent. Figure 11 shows the minimum frequency of the perturbations that leads to surge for various amplitude-percent magnitudes. It also indicates the surge frequency.

Steady Control Results

To study the effect of steady bleeding, 4 % of the mean mass flow rate was extracted from the diffuser. This amount of bleeding was applied to the stable operating condition at point A which has been destabilized with inlet perturbation of total pressure of amplitude percent of 5 and frequency of 30 Hz. As shown in Fig.11 this perturbation is capable of destabilizing the stable condition at point A which is closed to the surge point (Fig.12). Fig.13 shows the inlet mass flow rate versus time for this case. As illustrated, this amount of bleeding can remove surge disturbance completely.

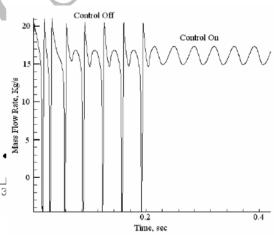


Fig. 13 Inlet mass flow rate (Steady bleeding).

Table. 2 Minimum amount of required steady bleeding for various inlet perturbations.

Amplitude-Percent	5	10	15	20
Minimum Required Steady Bleeding	2.2	3	4.2	5.5
(% of mean mass flow rate)				

To obtain the minimum mass flow rate needed to be removed for stabilizing the instabilities, we reduce the steady removed mass. Bleeding equal to 2.2 % of mean mass flow rate was found to be optimum. The same methodology was used in the case of perturbations of higher amplitude percent. The

frequency of perturbations was chosen from Fig.11 to ensure that the inlet perturbations can destabilize the compressor. Table.2 shows the minimum amount of steady bleeding needed for various amplitude percent magnitudes. As the amplitude percent increases, higher amount of mass flow rate is needed to be removed in order to stabilize the instabilities.

Unsteady Control Results

The frequency of surge disturbance is chosen from Fig.11 and the pressure fluctuation during surge is also obtained from computations for each case. As discussed, unsteady one dimensional bleeding with periodic forms, can reduce the amount of bleeding. Therefore, it is more efficient. A sinusoidal form (equation 10) is used to consider periodic bleeding.

$$\dot{m}_B = \overline{\dot{m}}_B + \overline{\dot{m}}_B * K_4 * \frac{Am}{P_1} * \left[\sin(2\pi t \cdot f + \varphi) \right]$$
 (10)

In equation 10, P_1 , t, Am, and f are the ambient pressure, time, the amplitude and the frequency of the fluctuations, respectively. " ϕ " is the phase lag and is chosen to be $\pi/4$. m_B is the averaged mass flow rate which is needed to be removed. The constant K_4 is chosen to ensure that the bleed rate does not exceed 20 % of the averaged removed mass flow rate. The above control law was applied to the same operating conditions in steady part. Tab.3 shows the minimum averaged mass flow rate needed to be removed in order to stabilize the instabilities. As shown, smaller amount of compressed air need to be removed in unsteady control, as compared to steady ease.

CONCLUSION

A one-dimensional unsteady computer code has been developed which enables simulation of surge disturbance propagation through entire jet engines. The effect of active control on the instabilities was studied and the following observations and lower conclusions were obtained:

- 1- steady control can eliminate surge disturbance. If the amount of bleeding air increases, the new stable operating point has lower mass flow rate and pressure ratio.
- 2- using air injection, as the control system, the new operating point has higher pressure ratio and also higher mass flow rate, as compared to air bleeding.
- 3- if the bleeding air is injected into the first stage of the compressor, the required amount of bleeding reduces.

- 4- Interstage bleeding leads to a new stable operating point with higher pressure ratios compared to bleeding from the diffuser, so it is more efficient.
- 5- smaller amount of compressed air need to be removed in unsteady control. This leads to a new operating point with higher pressure ratio.
- 6- compressor back pressure reduces as the diffuser area decreases. Therefore, variable area diffuser can be used to stabilize compressor instabilities.
- 7- inlet perturbations can destabilize the stable operating condition at design point.

Acknowledgment

This research was supported by Amirkabir University of Technology, Iran, Tehran, which in greately appreciated.

References

- 1. Eveker, K.M., Gysling, D. L., Nett, C. N., and Sharma, O.P., "Integrated Control of Rotating Stall and Surge in High-Speed Multistage Compression Systems", ASME J. Turbomach., Vol. 120, , pp.440-445, 1998.
- 2. Greitzer, E.M., "Surge and Rotating Stall in Axial Flow Compressors—Part 1, Theoretical Compression System Model", ASME J. Eng. for Power, Vol. 98, No.2, pp.190-198, 1976.
- 3. Greitzer, E.M., "Review-Axial Compressor Stall Phenomena, "ASME J. Fluids Eng., Vol. 102, pp.134-151, 1980.
- 4. Greitzer, E.M., "The Stability of Pumping Systems- The 1980 Freeman Scholar Lecture", ASME J. Fluid Eng., Vol.103, pp. 193-242, 1981.
- 5. Davis, M.W. Jr., "A Stage-by-Stage Dual-Spool Compression System Modeling Technique", ASME Gas Turbine Conf., ASME Paper 82-GT-189, London, 1982.
- 6. Davis, M.W. Jr., "Stage-by-Stage Poststall Compression System Modeling Technique", J. Propulsion, Vol. 7, No. 6, 1991.
- 7. Hale, A.A., and M.W. Davis, Jr., "Dynamic Turbine Engine Compressor Code, DYNTECC Theory and Capabilities", AIAA Paper -92-3190, 1992.
- 8. Garrard, G.D. "ATEC: The Aerodynamic Turbine Engine Code For the Analysis of Transient and Dynamic Gas Turbine Engine System Operation-Part 1, Model Development", ASME pp. 96-GT-193, 1996.
- 9. Garrard, G.D "ATEC: The Aerodynamic Turbine Engine Code For the Analysis of Transient and Dynamic Gas Turbine Engine System Operations—Part 2: Numerical Simulations", ASME pp. 96-GT-194, 1996.

- 10. Garrard, G.D., Davis, M.W., Jr., Wehofer, S., and Cole, G. "A One-dimensional, Time Dependent Inlet, Engine Numerical Simulation for Aircraft Propulsion Systems", ASME Paper # 97-GT-333, 1997
- 11. Khaleghi, H., Boroomand, M. and Tousi, A. M., "A Stage by Stage Simulation and Performance Prediction of Gas Turbine", The 43rd AIAA Aerospace Exhibit and Meeting, Nevada, 2005
- 12. Moore, F.K. and Greitzer, E.M., "A Theory of Post-Stall Transients in Axial Compression Systems-Part I- Development of Equations and Part II-Application," ASME J. Eng. for Gas Turbine and Power, Vol. 108, pp. 68-97, 1986.
- 13. Bonnaure, L.P., "Modeling High Speed Multistage Compressor Stability, ", M.Sc. Dep't. of Aeronautics and Astronautics, Thesis, 1991.
- 14. Hendricks, G.J., Bonnaure, L.P., Longley, J.P., Greitzer, E.M. and Epstein, A.H., "Analysis of Rotating Stall Onset in High Speed Axial Flow Compressors", AIAA Paper No. 93-2233, 1993.
- 15. Feulner, M.R., Hendricks, G.J., and Paduano, J.D., "Modeling for Control of Rotating Stall in High Speed Multistage Axial Compressors", ASME Paper No. 94-GT-200, 1994.
- 16. Paduano, J., Valvani, L., Epstein, A., Greitzer, E., and Guenette, G., "Modeling for Control of Rotating Stall", Automatica, Vol. 30, No. 9, pp. 1357-1373, 1994
- 17. Paduano, J., Epstein, A., Valvani, L., Longley, J., Greitzer, E., and Guenette, G., "Active Control of Rotating Stall in a Low-Speed Axial Compressor", J. Turbomach., Vol. 115, pp. 48-56, 1993.
- 18. Pinsley, J.E., Guenette, G.R., Epstein, A.H. and Greitzer, E.M., "Active Stabilization of Centrifugal Compressor Surge", J. Turbomach., Vol. 113, pp. 723-732, 1991.
- 19. Eveker, K.M. and Nett, C.N., "Model Development for Active Surge Rotating Stall Avoidance Aircraft Gas Turbine Engines", The 1991 American Control Conf., 1991.
- 20. Yeung, S. and Murray, R.M., "Reduction of Bleed Valve Requirements for Control of Rotating Stall Using Continues Air Injection", The 1997 IEEE Int. Conf. on Control Application, Hartford, CT, pp.683-690,1997.
- 21. Wang, Y. and Murray, R.M., "Effects of the Shape of Compressor Characteristic on Actuator Requirements for Rotating Stall, Control", The 1998 American Control Conf., 1998.
- 22. Niazi, S., Stein, A. and Sankar, L.N., "Numerical Studies of Stall and Surge Alleviation in a High-Speed Transonic Fan Rotor," AIAA Paper 2000-0226, 2000.

23. Manning, J.R., "Computerized Method of Characteristics Calculation for Unsteady Pneumatic Line Flows", J. Basic Eng., pp. 231-240, 1968.
24. Tesch W.A., Steenken W.G., "Dynamic Blade Row Compression Model for Stability Analysis", AIAA paper No. 76-203, AIAA 14th Aerospace Science Meeting, Washington D.C., pp. 26-28, 1976.

