

## Structural Damage Detection, Using Fuzzy Classification and ARMA Parametric Modeling

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### ABSTRACT

In this paper, a new method is introduced for recognition, location, and severity of damage in engineering stochastic structures, based on Autoregressive Moving Average (ARMA) parametric model and fuzzy classification. The important aspect of the proposed method is the fuzzy viewpoint on stochastic structural damage diagnosis, which uses estimated ARMA parameters as feature vector. Moreover, the proposed method eliminates the optimization stage in finding membership functions parameters of fuzzy system by substituting the variances of estimated ARMA parameters directly as tuning parameters in membership functions. Another important aspect of the proposed method is the inessentiality to measure the excitation input force applied to the structure. A finite element model of a frame for diagnosing damage, wherein the damage is modeled by different stiffness reduction and location, is considered as a case study. After obtaining satisfactory results from numerical simulations, the proposed method is applied to a simply supported beam as an experimental laboratory structure, where the spring connected to the structure in different locations with different stiffness is considered as the damaged object. The results are considerably satisfactory.

**Key Words:** Damage Diagnosis, ARMA Model, Fuzzy Classification, Monte Carlo Simulation

### عیب یابی سازه ای با استفاده از مدل سازی ARMA و کلاس بندی فازی

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### چکیده

در این مقاله، یک روش جدید برای تشخیص وجود، محل و نوع عیب در سازه های با تحریک اتفاقی بر اساس مدل پارامتریک ARMA و کلاس بندی فازی ارائه شده است. یکی از مزیت های برجسته روش پیشنهادی عبارت است از وارد نمودن دیدگاه فازی برای عیب یابی سازه ها با استفاده از پارامتر های تخمین زده شده که به عنوان بردار مشخصه ها در نظر گرفته می شود. به علاوه، در روش پیشنهادی، با وارد کردن مستقیم واریانس پارامتر های مدل ARMA به عنوان پارامتر های تنظیم کننده توابع عضویت سیستم فازی، مرحله بهینه سازی که برای پیدا کردن پارامتر های بهینه آن توابع به کار می رود، حذف می شود. همچنین، عدم لزوم اندازه گیری نیروی تحریک اعمالی به سازه از مزیت های مهم دیگر روش مذکور می باشد. برای مطالعه قابلیت های روش پیشنهادی، یک مدل اجزای محدود قاب طراحی شده که در مدل مذکور عیوب به صورت کاهش سختی اعضای قاب در نظر گرفته شده اند. بعد از به دست آوردن نتایج رضایت بخش از شبیه سازی عددی مدل قاب، روش پیشنهادی برای عیب یابی روی یک مدل آزمایشگاهی تیر دو سر مفصلی به کار برده شده است. عیوب به صورت وارد نمودن فنرهایی با ضرایب متفاوت در محل های مختلف تیر مذکور به وجود آمده اند. نتایج حاصل از اعمال روش پیشنهادی روی مدل آزمایشگاهی مذکور نیز نشان دهنده قابلیت های بالای روش برای عیب یابی سازه های با تحریک اتفاقی است.

**واژه های کلیدی:** عیب یابی، مدل ARMA، کلاس بندی فازی، شبیه سازی مونت-کارلو

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## 1- Introduction

There is a vast research field with different approaches on damage detection of structures. In aerospace and modern structures, the fault diagnosing of these structures, before occurring catastrophic events, is of great importance. One of the approaches is using a parametric model, such as ARMA, in diagnosing the changes in physical parameters of structures. These researches can be grouped in time domain category of structural health monitoring (SHM) [1] where measuring the input of structures is often necessary for damage detection, in other words, the exogenous part of ARMAX model must be used for SHM [2-5]. Sohn et al. [2] have proposed a two-stage time series analysis combining ARX and AR prediction models as pattern recognition technique for SHM where main object was to extract features and to construct a statistical model that distinguishes the signals recorded under different structural conditions of a boat. In the recent work of Sohn et al. [3], the ARX model parameters are fed to an auto associative neural network which is trained to characterize the underlying dependency of extracted features (parameter of ARX model) on the unmeasured environmental and operational variety by treating these environmental effects and conditions as hidden intrinsic variables in the neural network. When a new time signal is recorded from an unknown state of the system, the parameters of the time prediction model are computed for the new data set and are fed to the trained neural network. When the structure undergoes structural degradation, it is expected that the prediction error of the neural network will increase because of damage. Based on this premise, damage classifier is constructed using a new damage detection method, proposed in [3]. In [4] and [5] a geometric approach is proposed based on ARMAX modeling of a stochastic structure excited by white Gaussian noise and is measured on a predefined point of structure for fault diagnosis. In [6] a genetic fuzzy system is proposed for crack detection in beams and helicopter rotor blades. In [7] a generalized methodology for structural fault detection using FE and fuzzy logic is presented. In [8] a fuzzy rule-based system is used for damage detection of blade in a helicopter rotor which is modeled as a cantilever beam.

In this work, a new method of damage detection and locating is proposed based on combining ARMA modeling of structure's response and fuzzy logic reasoning. The important aspect of this paper is using only the output data (response) of structure excited by white Gaussian noise and modeling the response

data by ARMA model and system identification algorithm proposed in [9] for damage diagnosing. Another very important aspect of this work is eliminating the optimization or tuning process which is a necessary stage in fuzzy rule based systems. The optimization stage is very time consuming and sometimes leads to suboptimal results [10-14]. Generally the values of obtained parameters are related to membership functions in different applications of fuzzy systems. In this paper Gaussian membership function is used for classification of ARMA parameters. The parameters of Gaussian membership function are related to mean and covariance matrix of ARMA parameters, therefore the optimization process in fuzzy rule-based system is substitute by estimating the mean and covariance of ARMA model parameters. In this work the Monte-Carlo simulation is used for covariance matrix construction of ARMA parameters [15-16]. The paper is organized as follows:

In section two, parametric modeling for feature extraction (ARMA parameters) for damage detection and locating is described. In section three, the fuzzy rule-based classification method by using ARMA parameters as the features and their covariance as the parameters of the membership function is described. The numerical simulation of frame with definition of possible fault and the proposed damage diagnosing method are applied to a frame structure in section four. In section five, a laboratory structure is used for experimental validation of the proposed method. Finally the discussion and conclusion is presented in the last section.

## 2- Extracting ARMA Model Parameters as Feature Vector for Fault Diagnosing

Because of stochastic nature of applied forces, the responses of these structures are also stochastic. In this paper the response of structure in time domain is used directly for damage detection. For this purpose the fuzzy classification method is applied. In fuzzy classification or clustering method, one of the most important factors, is feature selection. One of the very informative features is ARMA parameters in modeling of structure's response [17-19]. One of the most important advantages of these methods which are based on ARMA parameter estimation is that the covariance matrix of parameters are obtained during the estimation process, in other words it is assumed that the parameters are random variables asymptotic with Gaussian distribution [15]. By this way the most difficult stage of fuzzy classification method, which is finding the uncertainty bound, can be eliminated or

transferred into ARMA model estimation stage directly. In this work, a polynomial-algebraic method is used for estimating the ARMA model parameters. This method was introduced by R. Benmrad et al. [9]. Here, the method is limited to estimation of ARMA model's parameters in stationary systems (structures). By the method proposed in this paper the extension of damage detection procedure to non-stationary systems is straightforward. For estimation of ARMA model parameters, consider the output (response of structure) signal modeled by  $ARMA(m, n)$  as:

$$\begin{aligned} x_t + a_1 x_{t-1} + a_2 x_{t-2} + \dots + a_n x_{t-n} = \\ w_t + c_1 w_{t-1} + \dots + c_m w_{t-m} \quad t > t_0 \end{aligned} \quad (1)$$

where,  $t$ ,  $t_0$ ,  $x_t$ , and  $w_t$  are discrete time, starting time, signal for modeling, and white noise with variance  $\sigma_w^2$ , respectively.  $a_i$  and  $c_i$  are parameters of autoregressive and moving average part of ARMA model. Let  $B$  represents the backshift operator:

$$B[x(t)] = x(t-1). \quad (2)$$

Consider the ARMA model as:

$$\begin{cases} A[B] = 1 + a_1 B + \dots + a_n B^n \\ C[B] = 1 + c_1 B + \dots + c_m B^m \end{cases} \quad (3)$$

The polynomial defined in (3) can be manipulated by defining the  $\otimes$  as:

$$\begin{cases} B^i \otimes B^j := B^{i+j} \\ B^i \otimes f[t] := f[t-i]B^i \end{cases} \quad (4)$$

So the model in Eq. (1) can be written as:

$$I[B]x_t = w_t, \quad (5)$$

where,  $I[B]$  is defined by:

$$I[B] = [C[B]]^{-1} \otimes A[B], \quad (6)$$

with,

$$C[B] \otimes I[B] = A[B]. \quad (7)$$

The algorithm for estimation of model parameters is summarized as follows [9]:

- Step 1. Estimate parameters of  $I[B]$  by appropriate truncation in Eq. (5) by linear least square method,
- Step 2. Extract the initial estimate of MA parameters by using Eq. (6),
- Step 3. Determine AR parameters corresponding to initial MA parameters by Eq. (7),
- Step 4. Determine polynomial operator  $\beta(B)$  from:

$$I[B] = C[B]^{-1} \otimes A(B) = A(B) \otimes \beta(B), \quad (8)$$

- Step 5. Filter the signal  $x_t$  through  $\beta(B)$ ,
- Step 6. Determine the new AR coefficient by solving the linear least square equation:

$$A(B)\bar{x}_t = \varepsilon_t, \quad (9)$$

- Where,  $\bar{x}_t$  is the filtered signal. Considering that the estimation process is carrying out correctly, the residual ( $\varepsilon_t$ ) would be an accurate approximate of  $w_t$  in Eq. (1). Hence  $\varepsilon_t$  can be used as a validation tool for estimation process, and
- Step 7. Update MA parameters using Eq. (6).

Steps 4-7 may be repeated to reach an acceptable convergence to estimate model's parameters. In validation stage, two validating methods can be applied [15]. The first one is one step ahead prediction method and the second one, as described previously, is correlation validating of  $\varepsilon_t$  in Eq. (9). Covariance matrix estimation of ARMA parameters is accomplished by using a direct method or an indirect one such as Monte-Carlo simulation which needs more data. The direct method is more difficult, because it needs some analytical equations which are usually unsolvable. In the case of using PE method for ARMA parameters' estimation, there are some methods to solve the analytical equations under different assumptions and constraints on signal. In this case the Gaussian process of parameters can also be proved for PE estimation in ARMA modeling [15-16]. Because of the mentioned difficulties, in this paper the Monte-Carlo simulation method is used for finding mean value and covariance matrix of parameters with Gaussian pdf:

$$\left\{ \begin{array}{l} \bar{\theta} = \sum_{i=1}^N \hat{\theta}_i \\ \bar{p} = \sum_{i=1}^N (\hat{\theta}_i - \bar{\theta})(\hat{\theta}_i - \bar{\theta}_x)^T \end{array} \right. \quad (10)$$

where,  $\hat{\theta}_i$  is the estimated parameter in  $i$ th run (no overlapping window) and  $N$  is the number of windows. It is noted that the assumption of Gaussian distribution of parameters is admissible in PE estimation method of ARMA model [15, 16]. In next section by using the mean value of  $\hat{\theta}_i$  ( $\bar{\theta}$ ) and variance of  $\hat{\theta}_i$  derived from covariance matrix ( $\bar{p}$ ), the parameters of membership function of each ARMA model's parameters (used as features in fuzzy classifier) would be determined. Obviously the selection of parameters of the membership function must be carried in training stage and inside the predetermined different classes of structure.

### 3- Damage Detection Using Fuzzy Classification with ARMA Parameters as Feature Vectors

Fuzzy classification is one of the most important applications of fuzzy logic in engineering. In this section, the location of damage and its severity is diagnosed by the fuzzy classification method based on structures' response and using of ARMA model parameters. In fact the ARMA model parameters of response signal, in any predetermined point of the structure, are considered as fuzzy classification features.

Typical fuzzy classifiers consist of interpretable if-then rules with fuzzy antecedents and class labels in the consequent part. The antecedents (if-parts) of the rules partition the input space into a number of fuzzy regions by fuzzy sets, while the consequents (then-parts) describe the output of the classifier in these regions. Fuzzy logic improves rule-based classifiers by allowing the use of overlapping class definitions and improves the interpretability of the results by providing more insight into the decision making process. Each of the applied fuzzy classification rules describes one of the  $N_c$  classes in the data set. The rule antecedent is a fuzzy description in the  $n$ -dimensional feature space and the rule consequent is a crisp (non-fuzzy) class label from  $\{1, 2, \dots, N_c\}$  [20]:

$$\begin{array}{l} R_i: \\ \text{if } x_1 \text{ is } A_{i1} \text{ and } x_2 \text{ is } A_{i2} \text{ and } \dots \text{ and } x_n \text{ is } A_{in} \\ \text{then} \\ \text{output is class}_i, \quad i=1,2,\dots,N_c \end{array} \quad (11)$$

Here  $n$  denotes the number of features,  $X = \{x_1, x_2, \dots, x_n\}^T$  is the input vector (features),  $\text{class}_i$  is the output of the  $i$ th rule and  $A_{i1}, A_{i2}, \dots, A_{in}$  are the antecedent fuzzy sets. The "and" connective is modeled by the product operator (max operator), allowing for interaction between the propositions in the antecedent. The degree of activation of the  $i$ th rule is calculated as:

$$D_i = \prod_{j=1}^n A_{ij} \quad i=1,2,\dots,N_c. \quad (12)$$

The output of the fuzzy classifier ( $y$ ) is determined by the rule that has the highest degree of activation:

$$y = \text{class}_{i^*}, \quad i^* = \arg \max_{1 \leq i \leq N_c} D_i. \quad (13)$$

In this paper the inputs (features) of fuzzy classification system are ARMA model's parameters and its outputs of it is the damaged element number and the degree of its severity. For every feature the antecedent fuzzy sets (membership function) may be defined as:

$$\mu(f_i^j) = \exp \left[ -0.5 \times \left( \frac{f_i^j - m_i^j}{\sigma_i^j} \right)^2 \right], \quad (14)$$

where,  $\mu(f_i^j)$  is  $i$ th membership function of  $j$ th feature,  $f_i^j$  is the absolute value,  $m_i^j$  is the related mean value (midpoint) and  $\sigma_i^j$  is the variance of the  $j$ th feature in  $i$ th class. Features are random (stochastic) variables, with Gaussian pdfs as discussed in previous section. It must be noted that,  $m_i^j$  and  $\sigma_i^j$  are the mean value and variance of ARMA parameters for every damage location and

severity (every classes) which was calculated by the method explained in previous section by using Eq. (10). Obviously the stage of calculating of  $m_i^j$  and  $\sigma_i^j$  may be referred as training stage in the proposed method of this paper. Now,  $D_i$  (degree of activation for the  $i$ th rule) is defined as:

$$D_i = \prod_{j=1}^n \left[ \exp \left( -0.5 \times \left( \frac{f^j - m_i^j}{\sigma_i^j} \right)^2 \right) \right], \quad (15)$$

where,  $n$  is the number of features,  $f^j$  is the  $j$ th feature (input into fuzzy system) and this feature is one of the response's features (ARMA model parameters) of a structure,  $m_i^j$  is the mean value (midpoint of Gaussian membership function) and  $\sigma_i^j$  is the variance of  $j$ th feature in  $i$ th fuzzy class. After calculating  $D_i$ , the damaged element and its severity is identified by the maximum  $D_i$ :

$$i^* = \arg \max_{1 \leq i \leq N_c} D_i, \quad (16)$$

where,  $i^*$  is the rule number with highest degree of activation. The correspondent class to  $i$ th rule represents the damaged element number and its severity. The location of damage and its severity are considered as crisp and linguistic variable sets respectively such as:

Location = [element 1, element 2, ..., element n]

Severity = [undamaged, tiny damage, moderate damage, severe damage, very severe damage]

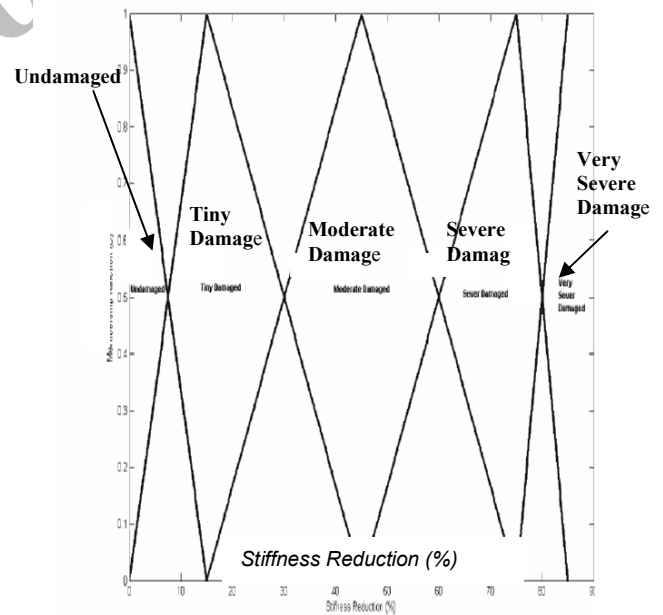
For example, when the number of elements is three, the fuzzy rules corresponding to different classes of structure may be such as shown in Table 1.

**Table (1):** Fuzzy classes descriptions correspondent to Fuzzy rule number.

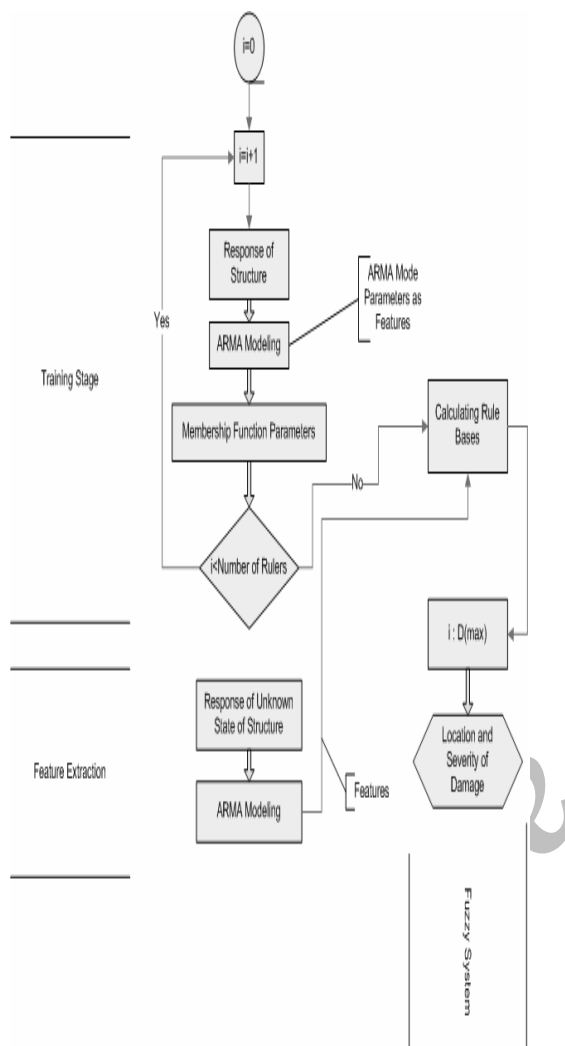
Fuzzy rule number	Fuzzy classes description
1	Undamaged
2	tiny damaged in element 1
3	tiny damaged in element 2
4	tiny damaged in element 3
5	Moderate damage in element 1
6	Moderate damage in element 2
7	Moderate damage in element 3
8	Severe damage in element 1
9	Severe damage in element 2
10	Severe damage in element 3
11	Very severe damage in element 1
12	Very severe damage in element 2
13	Very severe damage in element 3

Membership function of severity set is assumed to be triangular as shown in Fig. 1.

Membership Function (D)



**Figure ( 1):** Membership function of fuzzy set of severity.



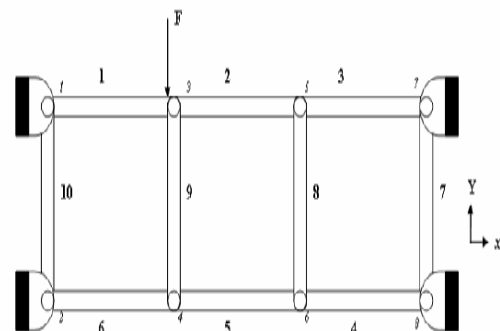
**Figure ( 2):** Flowchart of proposed damage detection method.

Arrange of the membership function are based on the definition of severity bounds. The designed fuzzy classification system generates two kinds of outputs, the fuzzy and crisp values that are related to severity and location of damage respectively. The crisp value which shows the damage location may also be interpreted as fuzzy number with considering the activation degree of  $D_i$  calculated on different classes. The fuzzy output of the fuzzy system can be considered as fuzzy linguistic phenomenon on severity of damage in structure.

In previous works on application of this kind of fuzzy classification method, some time consuming optimization method such as non-linear programming and genetic algorithms have been used for finding the  $m_i^j$  and  $\sigma_i^j$  [10-14]. In this paper the process of finding  $m_i^j$  and  $\sigma_i^j$  is cast directly into ARMA parameters' estimation process. By using this kind of features, the most difficult stage of the fuzzy classification is eliminated. Certainly the effect of variance and mean value estimation in ARMA modeling has very important role in the accuracy of classification or fault diagnosis scheme. The whole process of fault diagnosis of a structure using ARMA model parameters and fuzzy classification is depicted in Fig. 2.

**4- Numerical Simulation**

In the previous chapter, a new fault diagnosing method based on fuzzy classification method and ARMA modeling was introduced. In this section the proposed method is used in a FE model of a structural frame forced by a non-measured white Gaussian noise on one of its nodes and the response is measured on the other node, so the structure is completely stochastic. The Three scenarios of damage were designed and four degree of severity was considered as described in section three. The frame with ten elements is shown in Fig. 3.



**Figure (3):** FE model of frame.

Every three possible damage position are on an element numbered by one, two or three. The input force is applied on node-3 and the response is measured at node-5. Both of force and response are applied and measured concurrently in Y-direction. Physical parameters of structure (element) are selected as follow with considering that the simulation time to be as minimum as possible:

$$E = 70 \text{ kPa}, \quad \rho = 780 \frac{\text{kg}}{\text{m}^3},$$

$$L = 1 \text{ m}, \quad A = 0.4 \times 10^{-4} \text{ and } ,$$

$$I = 0.133 \times 10^{-6}.$$

For real time simulation the Newmark Alpha-Hilbert method [21] with the following parameters is used:

$$\alpha = -0.1, \quad \beta = 0.3 \text{ and } \gamma = 0.6$$

Based on obtained natural frequencies, the frequency range of interest, and Nyquist sampling theorem [22], the time step is selected as 0.1 seconds, with the total simulation time for each window being 100 seconds. Also, it should be noted that the state of white Gaussian noise is not fixed on different simulation windows. As an example the FRF of an undamaged and a damaged case on element 1 with severity 25% stiffness reduction are shown in Fig's. 4 and 5.

As it is seen, in these figures the damage occurrence is not visible neither in time domain nor in FRF plots, but as it will be shown, the proposed method of damage detection clarifies the position and severity of damage. The ARMA (10,10) is selected based on validating process. The one-step ahead prediction, residual and residual correlation method are illustrated by Figs. 6 and 7 for the mentioned undamaged and damaged states of structure, respectively.

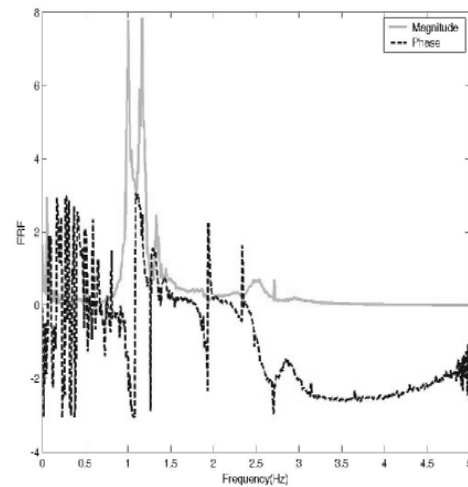
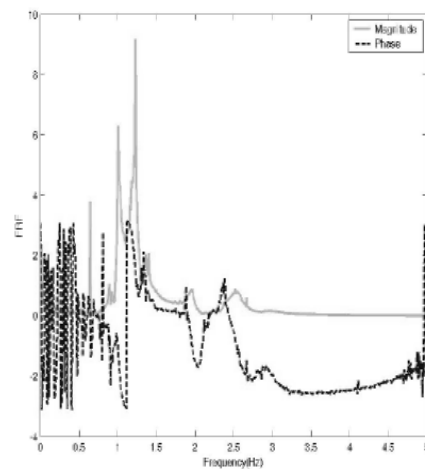
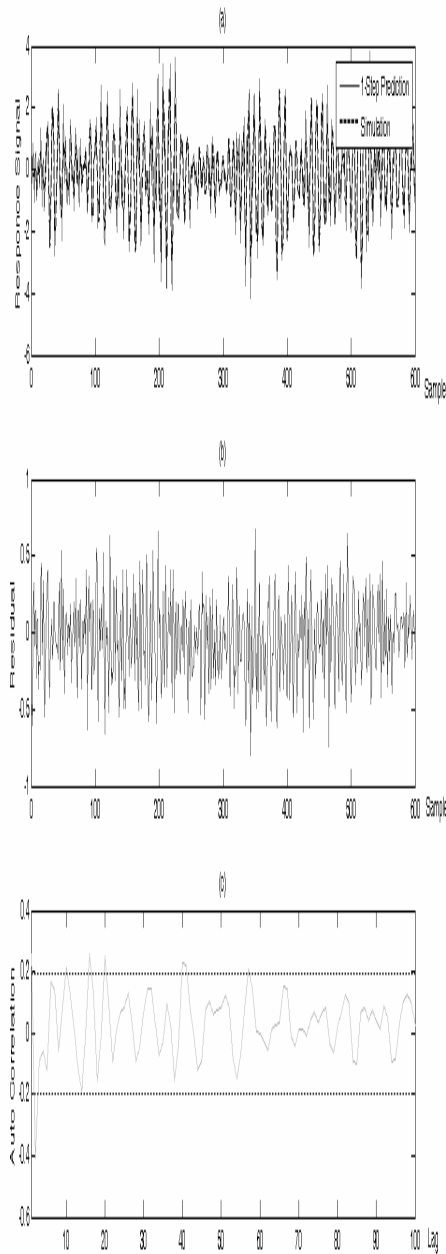


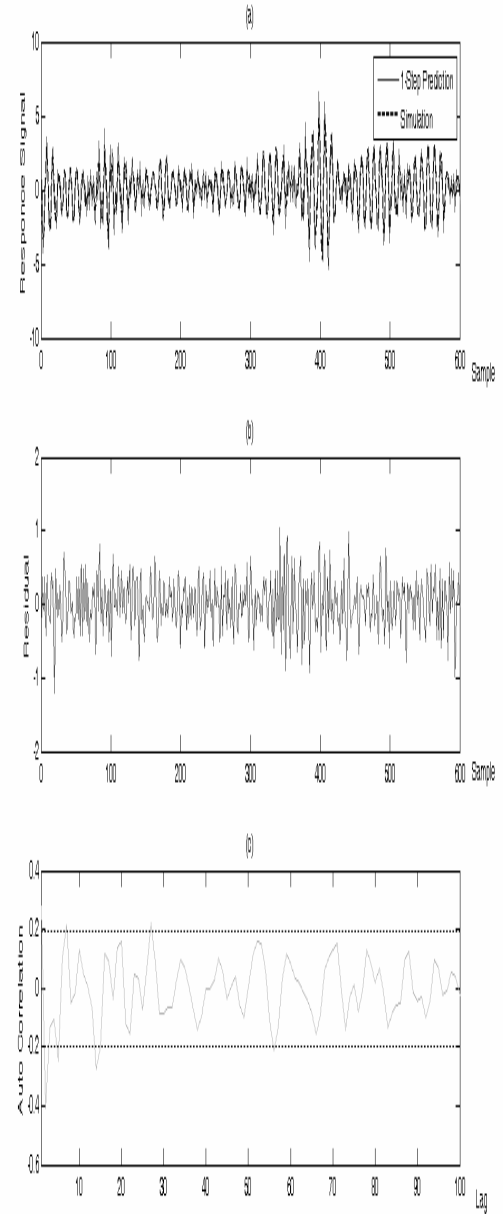
Figure (4): FRF of undamaged structure.



Figure( 5): FRF of damaged structure.



**Figure (6):** Validation results of undamaged structure, a) One-step ahead prediction method, b) Residual, c) Auto-Correlation. The dashed lines give the 95 percent confidence level.



**Figure (7):** Validation results of damaged structure, a) One-step ahead prediction method b) Residual, c) Auto-Correlation. The dashed lines give the 95 percent confidence level.



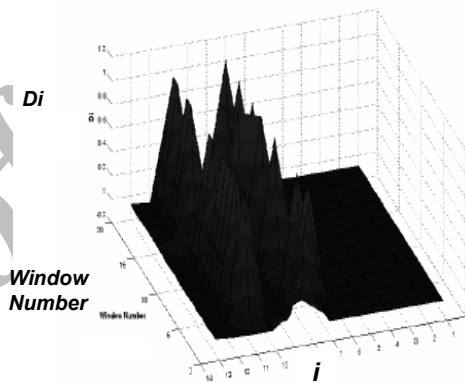
In designing of fuzzy system, it is assumed that the damage is on one of the frame elements 1, 2 or 3 with possible severity of 15% to 75% reduction in stiffness. Table 2 shows fuzzy rule number corresponding to every damage element number and its severity.

**Table (2):** Corresponding damage element number and severity for every fuzzy rule number.

Fuzzy rule number	Damaged element number	Severity of damage
1	-	Undamaged
2	1	tiny damaged in element
3	2	tiny damaged in element
4	3	tiny damaged in element
5	1	Moderate damage in element
6	2	Moderate damage in element
7	3	Moderate damage in element
8	1	Severe damage in element
9	2	Severe damage in element
10	3	Severe damage in element
11	1	Very severe damage in element
12	2	Very severe damage in element
13	3	Very severe damage in element

It must be noted that the damage was modeled as reduction of element stiffness. For the damage detection of the system, ten windows were considered for every fault case in frame. For verification of proposed method, each of the three elements of frame was damaged separately with reduction of stiffness 0%, 15%, 40%, 75% and 90%. Decision making process about damage was carried on ten no overlapping windows. The total results for ten windows are shown in Table 3.

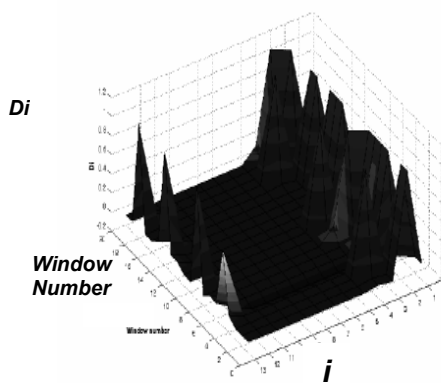
The results show that the classification of structure state is satisfactory except for the 10<sup>th</sup> fuzzy rule. As a sample the plot of  $D_i$  when the fuzzy class is 1 and 11 is shown in Figs. 8 and 9 respectively.



**Figure (8):** Plot of  $D_i$  when the state of frame is corresponding to class 11.

**Table (3):** Total results of simulation test (\*indicating the incorrect result).

Fuzzy rule number	Window →	1	2	3	4	5	6	7	8	9	10
1	↓	1	1	1	1	1	1	1	12	12	1
2		2	2	2	8	2	2	13	13	5	2
3		3	3	3	13	3	3	3	13	3	13
4		4	4	3	3	3	4	13	4	4	4
5		5	5	5	5	5	5	5	8	5	5
6		6	6	6	6	6	6	8	6	6	13
7		7	8	7	8	7	8	8	8	7	7
8		8	8	8	8	13	8	8	8	8	8
9		9	9	9	8	8	9	9	9	9	9
10*		11	11	11	11	8	13	13	11	13	11
11		11	11	9	11	11	11	11	11	11	11
12		12	12	11	12	11	12	12	12	12	12
13		13	13	13	13	13	13	13	13	11	13



**Figure (9):** Plot of  $D_i$  when the state of frame is corresponding to class 1.

As it is clear from the mentioned figures, by considering corresponding class to the highest level of  $D_i$ , the competition is between classes 8 and 11 in Fig. 8 as well as between the first four classes (1-4) in Fig. 9.

### 5- Experimental Test

The results in previous section showed that the proposed method can detect the damage in structure and its location. A laboratory structure was considered for experimental evaluation of the proposed method. The structure was a simply supported beam, excited by a shaker. The exciting input force was white Gaussian noise in the frequency range of 0 to 1600 Hz applied by B&K shaker type 4809. The response was measured by an accelerometer. The time length of the test and time resolution was 0.2 and 0.00012 seconds respectively. Response is acceleration of structure measured with B&K accelerometer type 4508. By connecting a spring to the structure, an artificial damage was applied on the structure as shown in Fig. 10.



Figure (10): Experimental setup.

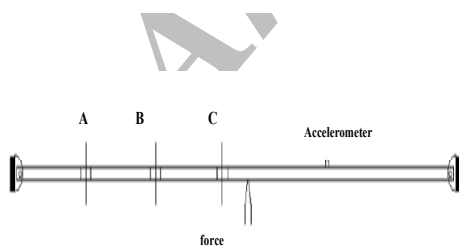


Figure (11): Schematic view of Beam.

Different spring positions and stiffness correspond to different damage positions and severities respectively. The possible damage position was assumed to be in one of the three locations of beam called A, B and C Fig. 11.

The severity of damage was considered to be related to the stiffness of spring. Springs with the same materials, equal lengths and diameters, but with different wire diameters (3.7, 4.2 and 5) are selected to simulate the damage severity. In this study three positions with three severities were used for constructing the fuzzy rules of fuzzy system in training stage. In validation stage of damage diagnosing process, the response from unknown state of structure was fed into the fuzzy system for evaluation of proposed method in diagnosing of position of spring and the severity. The fuzzy set of severity was defined as follow:

Severity = [no damage, tiny damage, moderate damage, severe damage]

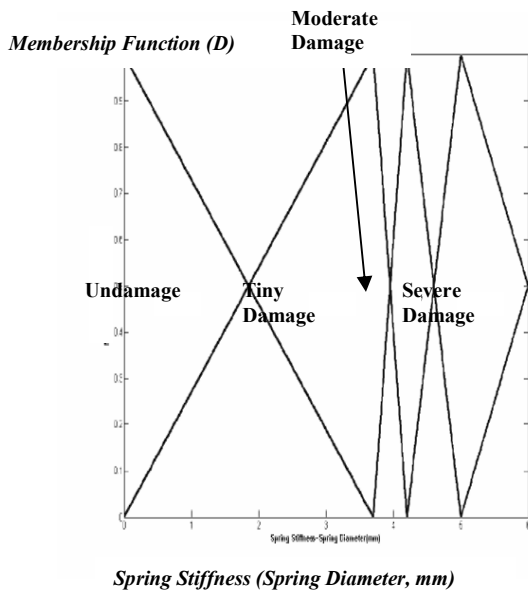
The fuzzy classes and corresponding rule numbers are shown in Table 4.

Table (4). Fuzzy classes descriptions correspondent to Fuzzy rule number

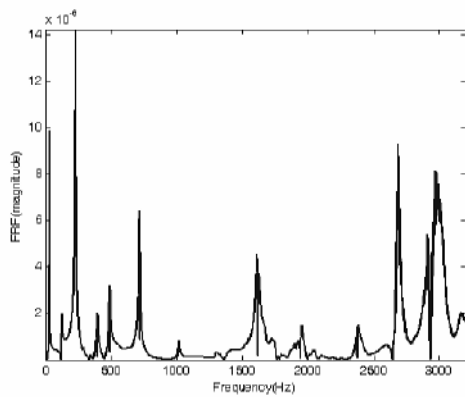
Fuzzy rule number	Fuzzy classes description
1	Undamaged
2	tiny damaged in position 1
3	tiny damaged in position 2
4	tiny damaged in position 3
5	Moderate damage in position 1
6	Moderate damage in position 2
7	Moderate damage in position 3
8	Severe damage in position 1
9	Severe damage in position 2
10	Severe damage in position 3

The applied membership functions are also shown in Fig. 12.

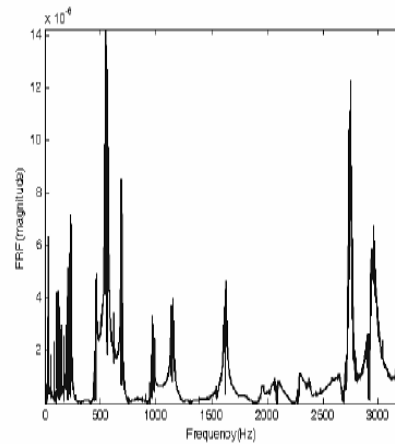
The centers of severity membership functions are related to the mentioned three stiffness of spring. In validating stage, three springs with different stiffness and positions were considered. The stiffness of the springs in validation stage was 3.5, 4.7 and 5.2. As an example the FRF of structure are shown in Figs. 13 and 14 in undamaged and damaged (sever damage in position A) state of structure respectively.



**Figure (12):** Membership function of fuzzy set of severity in experimental test.



**Figure (13):** FRF of undamaged structure in experimental test.



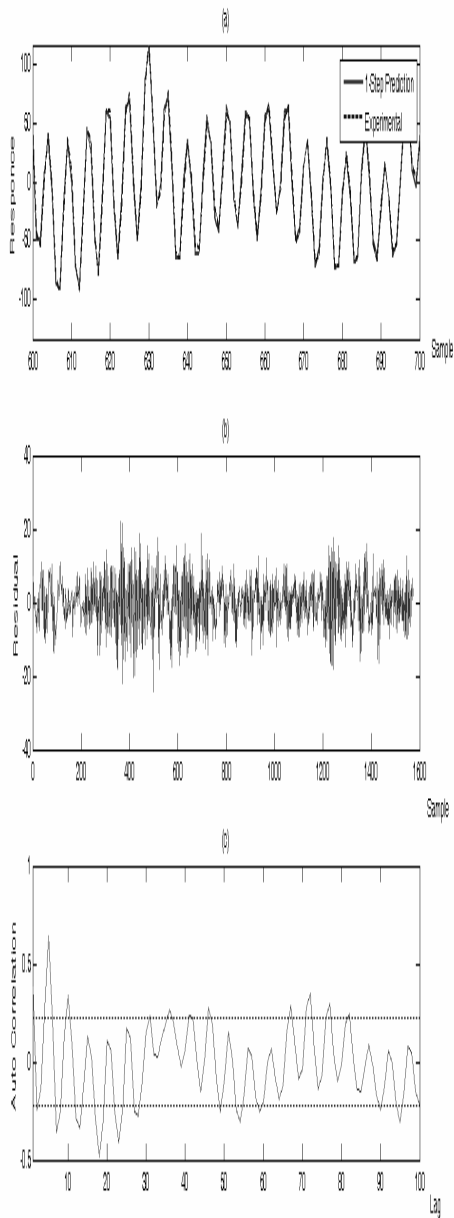
**Figure (14):** FRF of damaged structure in experimental test.

By considering over parameterization manner for capturing high frequencies of response, the order of ARMA model was selected (10, 10). The one-step ahead prediction, residual and residual correlation method are illustrated in Figs.15 and 16 for the mentioned undamaged and damaged states of structure respectively.

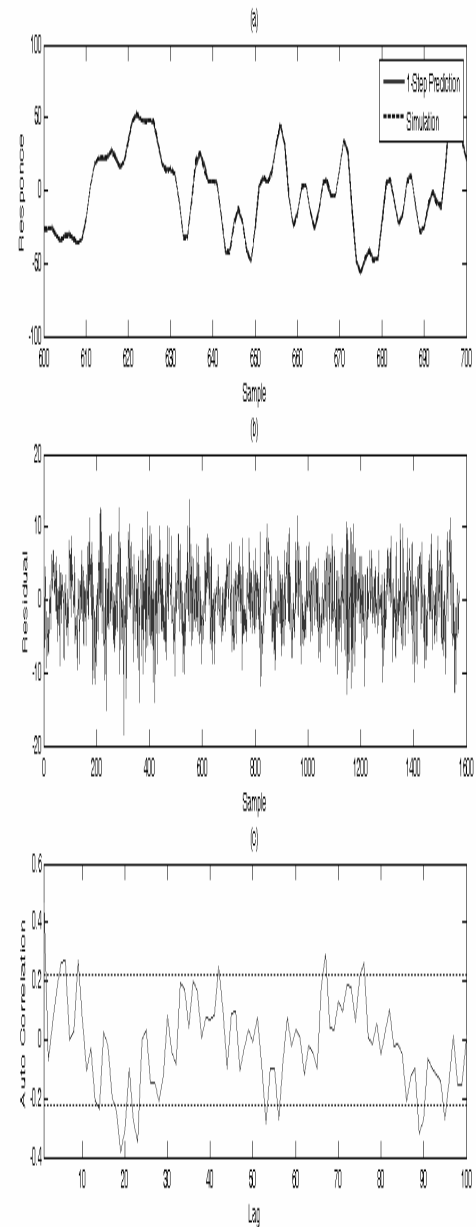
The total results are presented in Table 5. The results are satisfactory except for the fuzzy rules with number three and nine.

**6- Conclusion**

In this research, the ARMA parameters was used as the features for damage detection, locating and severity prediction by using fuzzy classification. One of the important aspects of the present research was disregarding the optimization process of fuzzy classification by using the variance of ARMA parameters directly in Gaussian shape membership function of parameters. Also, Monte-Carlo simulation method was used for finding the variance of ARMA parameters. In numerical simulation of this study a FE model of frame was used for validating the proposed method. It was assumed that the damage is in one of three selected elements in the frame and is modeled by stiffness reduction of element. The diagnosing results were satisfactory except for one case. In experimental validation of the introduced method, a laboratory structure which was a simply supported beam was used. The damage in the structure was created by connecting a spring in some



**Figure (15):** Validation results of undamaged structure in experimental test, a) One-step ahead prediction method, b) Residual, c) Auto-Correlation. The dashed lines give the 95 percent confidence level.



**Figure (16):** Validation results of damaged structure in experimental test, a) One-step ahead prediction method, b) Residual, c) Auto-Correlation. The dashed lines give the 95 percent confidence level.

**Table (5):** Total results of experimental test (\* indicating the incorrect result).

Fuzzy rule number ↓	Window →	1	2	3	4	5	6	7	8	9	10
1		1	1	1	1	10	1	1	1	1	1
2		10	2	2	2	2	2	2	2	2	2
3*		10	10	8	10	10	10	10	10	10	10
4		4	4	4	4	4	4	4	4	4	4
5		5	5	5	5	5	5	5	5	5	5
6		6	6	6	6	6	6	6	6	6	6
7		7	7	7	7	7	7	7	7	7	7
8		8	8	8	8	8	8	8	8	8	8
9*		10	7	7	10	7	7	10	10	10	10
10		10	10	10	10	10	10	10	10	10	10

locations of beam with different stiffness. The damage diagnosing result was very satisfactory except for two cases. Both of the mentioned structures were excited by the white Gaussian noise and the excitation force was not measured. One of the reasons of incorrect cases in both of numerical and experimental simulation is related into selection of membership function shape. The overlaps of different classes or fuzzy rules may be minimized by considering different shapes. Another reason is related to selection of ARMA model's order. In this paper, because of the algorithm using in estimation of ARMA parameters, there is limitation in choosing the order of model (the order of AR and MA part of ARMA model must be the same).

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