Enhancement of Vehicle Lateral Stability by Non-linear Optimal Control of Yaw Dynamics

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ABSTRACT

Non-linear characteristic of tire forces is the main cause of vehicle lateral dynamics instability, while direct yaw moment control is an effective method to recover the vehicle stability. In this work, an optimal non-linear controller for yaw dynamics to track a linear model, limited by road friction coefficient, is developed. The yaw rate response of an extended 2DOF non-linear model is first predicted by Taylor series expansion and then a control law is introduced by minimizing the local differences between the predicted and the desired responses. Here, the main properties of the proposed control law and its advantages over the other conventional control methods are discussed. The derived control law has an analytical form which is easy to apply and handles the control input saturation explicitly. The effectiveness of the designed controller is investigated through simulations of severe lane change maneuvers, using a developed non-linear full vehicle dynamic model. The simulation results show that the vehicle stability can be remarkably improved when the optimal non-linear controller is applied.

Key Words: Vehicle Dynamics, Non-linear Control, Optimization, Lateral Stability, Direct Yaw Moment Control

بهبود پایداری جانبی خودرو از طریق کنترل بهینهٔ غیر خطی دینامیک چرخشی

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چکیده

مشخصهٔ غیر خطی نیروهای تایر اصلی ترین عامل ناپایداری دینامیک جانبی خودرو بوده و کنترل مستقیم گشتاور چرخشی روشی مؤثر برای پایدارسازی خودرو است. در این مقاله برای کنترل دینامیک چرخشی خودرو یک کنترل کنندهٔ بهینهٔ غیرخطی با هدف تعقیب رفتار خطی محدود شده با ضریب اصطکاک جاده طراحی می شود. برای این منظور، ابتدا با استفاده از بسط سری تیلور، پاسخ چرخشی خودرو از روی یک مدل غیر خطی دو درجهٔ آزادی توسعه داده شده، پیش بینی می شود و سپس با حداقل کردن پاسخ های مطلوب و پیش بینی شده، قانون کنترل مورد نظر به دست می آید. در ادامه، خواص عمدهٔ کنترل کنندهٔ پیشنهاد شده، به همراه مزایای آن نسبت به سایر روش های متداول کنترلی، مورد بحث قرار می گیرد. قانون کنترلی بدست آمده بصورت تحلیلی بوده و پیاده سازی آن آسان است. ضمن این که با این روش می توان محدودیت های ورودی های کنترلی را نیز در نظر گرفت. کارآیی بیشتر سیستم کنترلی با شبیه سازی مانورهای تغییر باند شدید به وسیله یک مدل کامل خودرو مورد بررسی قرار گرفته است. نتایج نشان می دهد که با اعمال کنترل کنندهٔ بهینهٔ غیر خطی، پایداری خودرو به میزان قابل توجهی افزایش می یابد.

واژه های کلیدی: دینامیک خودرو، کنترل غیر خطی، بهینه سازی، پایداری جانبی، کنترل مستقیم گشتاور چرخشی

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Nomenclatures

m	Mass of the vehicle	C_{α}	Cornering stiffness of the tire
I_z	Moment of inertia about z-axis	$C_{\alpha f}$	Cornering stiffness of the front tire
a	Distance of the mass center to the front axle	$C_{\alpha r}$	Cornering stiffness of the rear tire
b	Distance of the mass center to the rear axle	F_z	Tire vertical force
l	Distance of the wheelbase	F_y	Tire lateral force
T_w	Distance of the wheel track	C_i	Tire longitudinal stiffness
r	Yaw rate	ε_r	Road adhesion reduction factor
v	Lateral velocity	i_s	Longitudinal slip
u	Forward velocity	h	Predictive period
δ_f	Front steering angle	λ	Weighting ratio
β	Body slip angle		
α	Tire slip angle	**Subscripts**	
M_z	External yaw moment	*fr*	Front-Right tire
a_y	Lateral acceleration	*fl*	Front-Left tire
h_{cg}	Height of the mass c.g.	*rr*	Rear-Right tire
μ	Road coefficient of friction	*rl*	Rear-Left tire

1. Introduction

Vehicle dynamics control (VDC) system is the latest active safety technology introduced to control vehicle lateral stability under emergency situations. Direct yaw moment control (DYC) can be considered as the main and upper layer of VDC system [1]. A practical approach to generate a required external yaw moment, independent of lateral forces and steering angle, is the transverse distribution of the vehicle braking force between the left and right wheels. This strategy known as differential braking can be achieved using the main parts of common anti-lock braking system comprising the lower layer of VDC system.

In general, the motivation for development of DYC is best understood by examining the driver's disabilities to control vehicle lateral dynamics under critical conditions. In a turning maneuver with high lateral acceleration where tire forces are approaching to or at the limit of road adhesion, the vehicle side slip angle grows and the effectiveness of vehicle steering angle in generating yaw moment becomes significantly reduced because of tire force saturation. This fact is first illustrated by the so-called β-method proposed in [2]. The decrease of restoring yaw moment generated by tire lateral force when the side slip angle increases is the basic cause of vehicle unstable motion called spin motion and adding yaw moment will recover the vehicle stability.

In this paper, according to system requirements, an optimal nonlinear approach [3-5] is applied to design a yaw moment controller. The control law is developed by minimizing the difference between the predicted and desired yaw rate responses. The proposed controller has two distinguished features: firstly, it is based on continuous nonlinear model and can handle the model nonlinearity successfully. Secondly, the optimal control law provides the possibility of using lower control energy for achievement of the specified performance and also some physical limits of control input can be satisfied.

Alternative control methods for DYC have been investigated in the literature. These methods can be mainly divided in two groups: the linear model-based methods that use optimization as a main procedure in finding the control law, and those methods that apply the nonlinear vehicle models but the optimization is not used or doesn't have a prominent role.

Several researches have been developed the well-known LQR theory to improve vehicle handling and stability [6-8]. A predictive optimal yaw stability controller based on a linearized vehicle model which was discretized via a bilinear transformation has been presented by Anwar [9]. The sliding control methods have been frequently applied to stability control because of their potential to cope with nonlinearities and intrinsic robustness [10-12]. Tahami et al. have introduced a fuzzy logic stability control based on yaw reference DYC [13]. There are insufficient studies being conducted on the optimal control of nonlinear vehicle handling dynamics and it is believed that the present paper can compensate this scarcity.

The rest of the paper is organized as follows. The dynamic models of vehicle system and tire forces are initially presented and the objectives of the control system are determined by comparing the responses of linear and nonlinear models. Then an optimization-based nonlinear control law is developed. The main properties of the proposed controller and its advantages over the other conventional control methods are discussed. Finally, the dynamic performance of the controller is investigated through the numerical simulations using a full vehicle dynamic model.

2. Vehicle System Model

In this section, vehicle dynamics model, tire model, and model responses are discussed.

2.1. Vehicle Dynamics Model

A 2DOF model for vehicle handling, shown in Fig. **1**, is used as a controller model. The basic equations of motion for this model can be derived as follows:

$$m(\dot{v}+ur)=(F_{yfl}+F_{yfr})+F_{yrl}+F_{yrr} \quad (1)$$

$$I_z\dot{r}=a(F_{yfl}+F_{yfr})-b(F_{yrl}+F_{yrr})+M_z \quad (2)$$

For the above model, the lateral velocity v and the yaw rate r are the two state variables, while M_z is the external compensating yaw moment which must be determined from the control law. It is assumed that for small steering angles, $\cos\delta_f \approx 0$. Moreover, forward speed u is assumed to be constant thus no longitudinal force is needed. The subscripts of tire lateral force F_y together with other symbols are specified in notation.

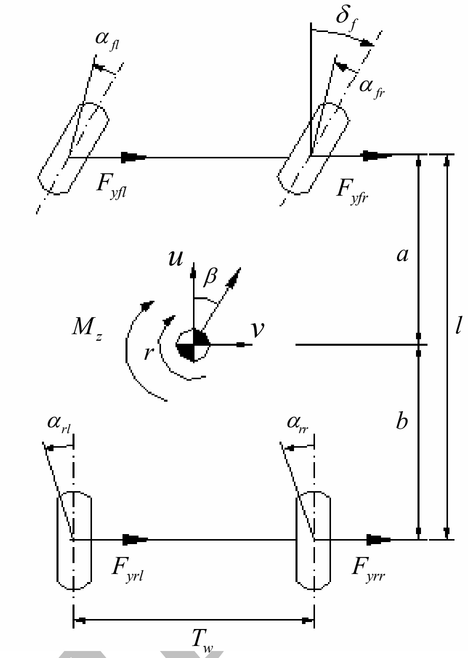

Fig. 1. 2-DOF vehicle handling model.

It is clear that vehicle handling dynamics are dominated by tire lateral force response which strongly depends on tire normal load and slip angles. During a turning maneuver, lateral acceleration causes normal load to shift from the inside to the outside tires. The distribution of lateral load transfer between the front and rear axles which influences the balance of the front and rear tire forces, is crucial to directional stability. Therefore, the normal force on each tire including the static weight and dynamic lateral load transfer is given by:

$$\left.\begin{aligned} F_{zfl} &= \frac{mgb}{2l} - K_{rsf}\frac{ma_y h_{cg}}{T_w} \\ F_{zfr} &= \frac{mgb}{2l} + K_{rsf}\frac{ma_y h_{cg}}{T_w} \\ F_{zrl} &= \frac{mga}{2l} - (1-K_{rsf})\frac{ma_y h_{cg}}{T_w} \\ F_{zrr} &= \frac{mgb}{2l} + (1-K_{rsf})\frac{ma_y h_{cg}}{T_w} \end{aligned}\right\}, \quad (3)$$

where, K_{rsf} is the front roll stiffness ratio. As it is seen, although the roll motion is not included in the vehicle model, but its effect is considered on the tire forces.

In addition to normal force, the tire slip angle is another main quantity for the calculation of tire lateral force. The following equations define the slip angles of the front and rear tires:

$$\left.\begin{aligned} \alpha_{fl} &= \delta_f - \tan^{-1}\left(\frac{v+ar}{u-0.5T_w r}\right) \\ \alpha_{fr} &= \delta_f - \tan^{-1}\left(\frac{v+ar}{u+0.5T_w r}\right) \\ \alpha_{rl} &= \tan^{-1}\left(\frac{br-v}{u-0.5T_w r}\right) \\ \alpha_{rr} &= \tan^{-1}\left(\frac{br-v}{u+0.5T_w r}\right) \end{aligned}\right\} \quad (4)$$

2.2. Tire Model

The linear tire model used to calculate lateral tire force is given by:

$$F_y = C_\alpha \alpha \, . \quad (5)$$

This model can well describe vehicle lateral dynamics under normal driving conditions with low lateral acceleration. In emergency situations when lateral acceleration is high, the vehicle slip angle becomes large and tire forces saturate at the road friction limit. In this case, the dynamic behavior of vehicle is nonlinear and using the linear tire model will be inadequate. The Dugoff's tire model based on the friction ellipse idea has been widely used for nonlinear simulations [14]. In this model, the relation for lateral force of each tire is as follows:

$$F_y = \frac{C_\alpha \tan\alpha}{1-i_s} f(S) \, , \quad (6)$$

where,

$$f(S) = \begin{cases} S(2-S) & if \;\; S<1 \\ 1 & if \;\; S>1 \end{cases} \, , \quad (7)$$

and,

$$S = \frac{\mu F_z \left(1-\varepsilon_r u \sqrt{i_s^2 + \tan^2\alpha}\right)(1-i_s)}{2\sqrt{C_i^2 i_s^2 + C_\alpha^2 \tan^2\alpha}} \, . \quad (8)$$

2.3 Model Responses

Figure **2** compares the responses of linear and nonlinear vehicle models during a turning maneuver for a vehicle specified in table 1. The vehicle maneuvers on a level road ($\mu = 0.85$) at a constant speed of 30 m/s and the steering angle input $\delta_f = 0.03$ rad.

It is seen that the nonlinearity of tire force characteristics, unlike the linear model, causes vehicle to show unstable motion during high-g maneuvers. In fact, the saturation of tire force at high slip angles, lateral load transfer effects, low coefficient of friction and other characteristics which affect vehicle stability are not predicted by the linear vehicle/tire model.

Therefore, in order to compensate the loss of vehicle stability due to nonlinearity effects, the linear 2DOF vehicle plane model (bicycle model) is adopted as a desired model to be followed by the controller. In this way, since the increased rate of yaw velocities at the beginning of the maneuvers for linear and nonlinear models are almost similar, the tracking error will be the lowest at the beginning of motion.

Using the linear 2DOF vehicle equations [17], the desired yaw rate response to the driver's steering input is expressed by a second order equation. When the lateral velocity converges to zero, the yaw rate response can be reduced to a first-order lag [18], as:

$$\frac{r_d}{\delta} = G_r \frac{1}{1+T_r s} \, , \quad (9)$$

where,

$$G_r = \frac{1}{1+Nu^2}\frac{u}{l}; \quad N = \frac{m}{2l^2}\frac{bC_{\alpha r} - aC_{\alpha f}}{C_{\alpha f} C_{\alpha r}} \, ,$$

$$T_r = \frac{1}{\sqrt{P}}; \quad P = \frac{4l^2 C_{\alpha f} C_{\alpha r}}{m I_z u^2}(1+Nu^2) \; .$$

This intentional modification of yaw rate response from second to first order model increases the vehicle stability limit and prevents oscillation in relation to steering input.

It should be noted that although the linear vehicle model shows a stable motion, it can't predict the road coefficient of friction. Basically, the lateral acceleration of the vehicle in terms of g unit can not exceed the maximum coefficient of friction. However, as it is illustrated in Fig. **2**, the lateral acceleration for the linear model is over 1g, whereas the friction coefficient is only 0.85. This means that the yaw rate and lateral acceleration responses of linear model lack compatibility with the maximum coefficient of friction. To eliminate this shortcoming, the desired yaw rate response described by Eq. (9) must be also limited by the following value [19]:

$$\left|r_d\right| \le \left|\mu g / u\right| . \quad (10)$$

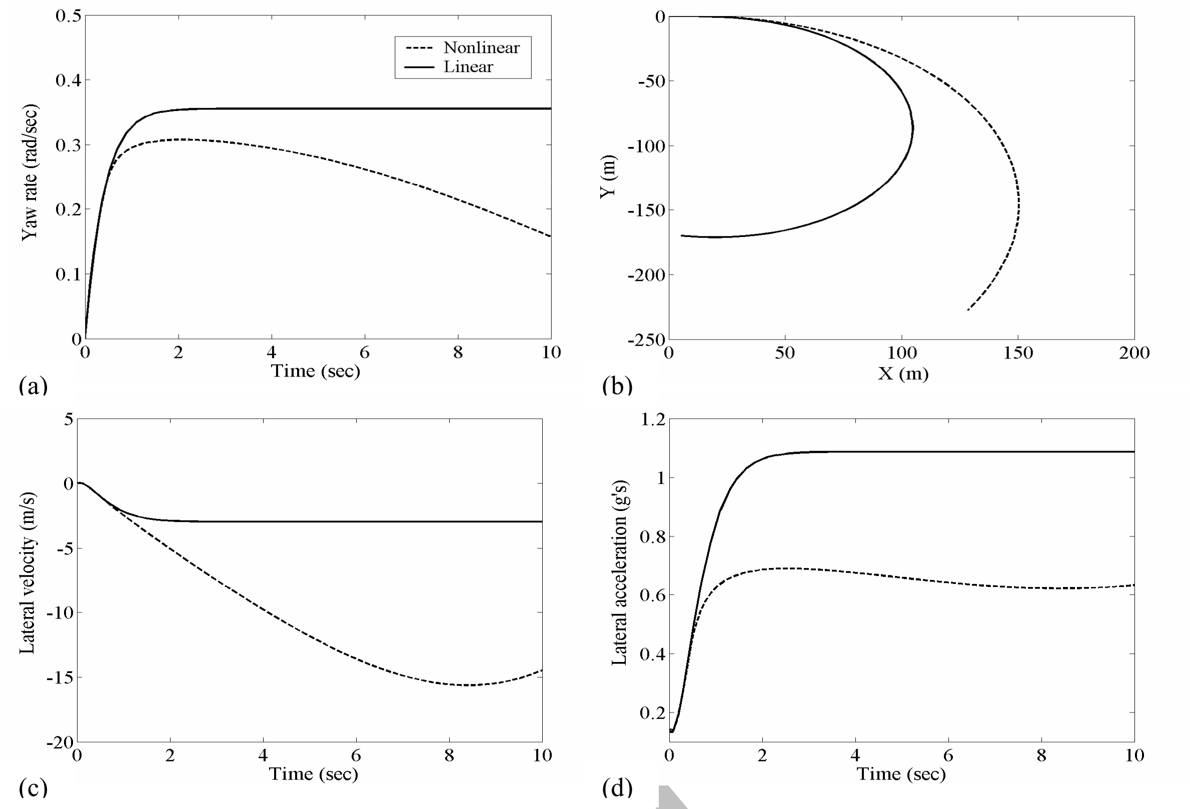

Fig. 2. Responses of linear and nonlinear 2DOF vehicle models during a turning maneuver: (a) yaw rate (b) vehicle path (c) lateral velocity (d) lateral acceleration.

The measured lateral acceleration a_y can be taken instead of μg. Combination of the limitation (10) with the desired yaw rate (9) can also broaden the vehicle stability range by decreasing the lateral velocity.

3. Control System Design

In this section, the controller's development and performance are discussed.

3.1. Controller development

The main goal of the control system is to make the actual yaw rate r to follow the desired yaw rate r_d. This handling performance criterion, i.e. the yaw rate tracking, is more suitable for DYC than the zero lateral velocity [16].

The nonlinear vehicle system dynamics described by equations (1) and (2), can be written in the state-space form by considering yaw rate as the output of the system:

$$\dot{x}_1 = f_1(x, \delta_f) \tag{11}$$

$$\dot{x}_2 = f_2(x, \delta_f) + \frac{1}{I_z} M_z \tag{12}$$

$$y = x_2 \tag{13}$$

where, $x = [v\ r]^T$ is the state vector and y is the output. M_z represents the control input and the vehicle steering angle $\delta_f(t)$ is considered as the external disturbance. The nonlinear tire model given in Eq. (6), has been incorporated in f_1 and f_2.

Now, an optimal predictive control law is developed for the design of yaw rate tracking controller. Briefly, the nonlinear response of the yaw rate for the next time interval, $r(t+h)$, is first predicted by Taylor series expansion and then the current control $M_z(t)$ will be found based on continuous minimization of predicted tracking error. Note that h denotes to the predictive period and is a real positive number.

Let us first approximate $r(t+h)$ by a *k*th-order Taylor series at *t*:

$$r(t+h) = r(t) + h\dot{r}(t) + \frac{h^2}{2!}\ddot{r}(t) + \cdots + \frac{h^k}{k!} r^{(k)}(t) \quad . \tag{14}$$

Now, the key issue is to choose the order k in a way which is suitable for the purposes of controller design on the basis of predictions. Following Lu [4], the expansion order k is determined by the relative degree of nonlinear system. According to equations (7)-(9), our system has a well-defined relative degree 1 determined as the lowest order of the derivative of yaw rate r in which the input M_z first appears explicitly [20]. Therefore, the first order Taylor series is sufficient for the expansion:

$$r(t+h) = r(t) + h\dot{r}(t) \, . \tag{15}$$

Substituting (12) into (15) yields:

$$r(t+h) = r(t) + h[f_2(x,\delta_f) + \frac{1}{I_z} M_z] \, . \tag{16}$$

Note that the arguments of functions may be frequently dropped through the rest of paper for simplicity of notations.

Now, we consider a performance index that penalizes the next instant tracking error and the current control expenditure in the following form:

$$J[M_z(t)] = \frac{1}{2} w_1 [r(t+h) - r_d(t+h)]^2 + \frac{1}{2} w_2 [M_z^2(t)], \tag{17}$$

where $w_1 > 0$ and $w_2 \geq 0$ are weighting factors indicating the relative importance of the corresponding terms. Minimization of the performance index must be sought in order to improve the yaw tracking accuracy at the next instant and consequently obtain the optimum handling behavior of the vehicle.

We can expand the desired yaw rate in the same manner as we did before:

$$r_d(t+h) = r_d(t) + h\dot{r}_d(t) \, . \tag{18}$$

Now, the expanded performance index can be obtained as a function of control input by substituting equations (16) and (18) into (17) as:

$$J[M_z(t)] = \frac{1}{2} w_1 [(r_d - r) + h(\dot{r}_d - f_2) - \frac{h}{I_z} M_z]^2 + \frac{1}{2} w_2 M_z^2 \quad . \tag{19}$$

The necessary condition for optimality is

$$\frac{\partial J}{\partial M_z} = 0 \quad , \tag{20}$$

which, leads to

$$M_z(t) = \frac{1}{h} \frac{I_z}{1 + \lambda I_z^2 h^{-2}} [(r_d - r) + h(\dot{r}_d - f_2)] \, , \tag{21}$$

where, λ is the weighting ratio:

$$\lambda = \frac{w_2}{w_1} \, . \tag{22}$$

It is considered that the analytically defined predictive control law, Eq. (21), is a closed form which depends on the states of the system and steering angle.

In the derived control law, the predictive period h is treated as a controller parameter rather than the integration step size. The roles of this parameter will be demonstrated later in this paper.

3.2. Controller performance

In this section, the important properties of the derived optimal nonlinear control law (21) are discussed and some relation and distinctions between this controller and other conventional control approaches are investigated.

One of the most important properties of the proposed controller is its tracking capability. If there is no initial yaw tracking error, i.e. $r(0) = r_d(0)$, the controller will maintain a perfect tracking for all $t \in [0, t_f]$, provided that the control weighting factor w_2 is zero. This can be seen in a straightforward fashion by substituting the control law (21) into Eq. (12). In this way, the tracking error dynamics of the yaw rate is obtained as follows:

$$\dot{e} + \frac{1}{h} e = 0 \, , \tag{23}$$

where,

$$e = r_d - r \,. \tag{24}$$

It is obvious that the closed loop system is exponentially stable for any $h > 0$.

The error dynamics (23) is linear and time invariant. We see that the proposed tracking controller technique naturally leads to a special case of feedback linearization. But the current control law (21) has some important advantages over the input/output linearization control. When the control input saturates, the feedback linearization is often unachievable, but when the control computed from Eq. (21) exceeds the control bounds, the use of the maximum control value will be still the best choice that minimizes the performance index. Moreover, it can be established that the predictive controller is robust in the presence of a class of modeling uncertainties and doesn't need the exact knowledge of the system nonlinearity unlike the feedback linearization. Optimal property of the proposed control law is another important advantage that provides the possibility of limiting the control effort by regulation of weighting factors.

Now, the important roles of parameter h in the control law can be stated as well. From one point, according to the error dynamics (23), h is the time constant of the closed loop system, but from the other, $1/h$ is seen as the controller gain in Eq.(21). Thus, h is treated as a controller parameter which can affect both the control effort and the convergence rate in tracking. It can be adjusted to improve the performance of the controller.

4. Numerical Simulation

Computer simulations are carried out to verify the effectiveness of the designed nonlinear optimal control system. In order to predict the vehicle response accurately and achieve more reliable results, simulation studies have been conducted using a full vehicle dynamic model with 8DOF. This model has been previously developed and validated by experimental results [14]. The longitudinal velocity, lateral velocity, yaw rate, roll rate and rotational speeds of four wheels constitute the degrees of freedom for this model. Therefore, the normal load transfers of tires, the roll steer, the roll camber and other model complexity effects are considered on evaluation of the controller performance. The vehicle parameters employed for computer simulations are given in table 1

Table 1. Specification data for the vehicle under study.

Parameters	Value
m	1280 kg
I_z	2500 kg m^2
a	1.203 m
b	1.217 m
l	2.42 m
$C_{\alpha f} = C_{\alpha r}$	30000 N/rad
C_i	50000 N/unit slip
K_{rsf}	0.444 m
h_{cg}	0.5 m
T_w	1.33 m
ε_r	0.015
h	0.2 sec

Fig. **3** shows the simulation results for vehicle behavior between the cases with and without control during a lane change maneuver. The vehicle runs on a level dry road ($\mu = 1$) at the constant speed of 80 km/h for 2s and the steering angle, as shown in Fig. **3a**, performs a single sine change with the amplitude of 4.5 deg and the frequency of 0.5 Hz. The control input is limited by the given value of $M_{z\max} = 1500\, N.m$.

The time responses of yaw rate, side-slip angle and lateral acceleration illustrated in Figs. **3c**, **3d** and 3e indicate that the uncontrolled vehicle becomes unstable after the negative steering sine input and consequently can't complete the lane change path as shown in Fig. **3b**. This fact is due to the saturation of nonlinear tires, so that the necessary lateral force can't be generated when the slip angle increases. In contrast, the proposed controller can stabilize the vehicle successfully. The yaw rate can follow the behavior of desired value in spite of the control input limitation. The time response of control input is seen in Fig. **3f.** As it is expected, the external yaw moment is remained below the given maximum value through a suitable value of weighting ratio, i.e. $\lambda = 1.4e-8$.

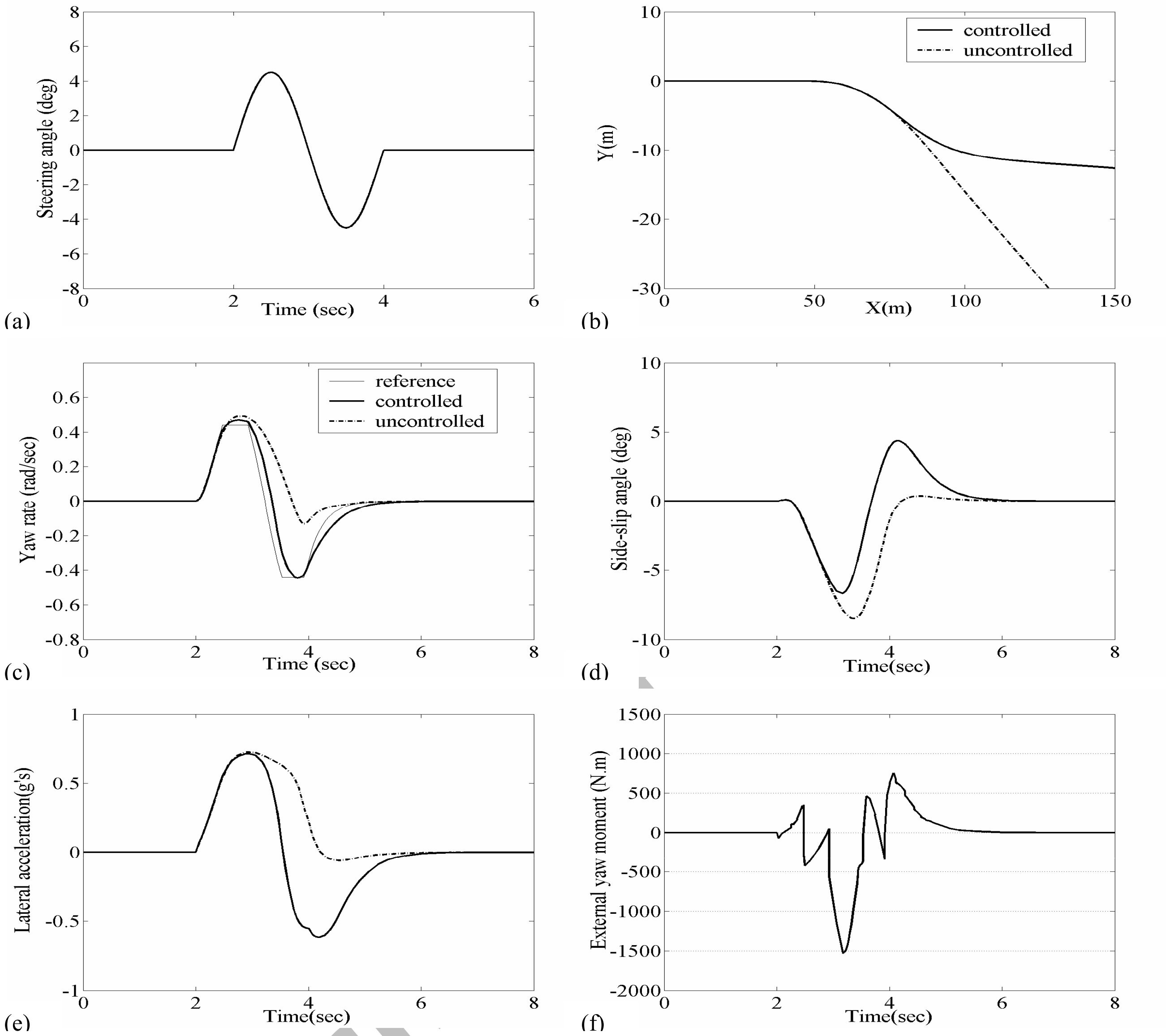

Fig. 3. Simulation results of a lane change maneuver on dry road: (a) steering angle input (b) vehicle path (c) yaw rate (d) side-slip angle (e) lateral acceleration (f) external yaw moment.

To illustrate the effectiveness of the optimal nonlinear controller and its superior performance, let's consider the former lane change test, but with low road friction coefficient ($\mu = 0.4$). The limitation of control input is applied again. Simulation results of the uncontrolled and controlled vehicles are shown in Fig. **4.**

Fig. **4a** shows that the time response of yaw rate with proposed control law follows its desired value. The existing tracking error is due to control input saturation and weighting ratio.

It should be noted that in order to decrease the control input, we can increase the value of weighting ratio to some extent, otherwise the yaw rate can't follow the behavior of reference model and the vehicle remains unstable. It is found that when the input computed from the control law with a suitable value of weighting ratio exceeds the control bounds, the use of maximum control value, shown in Fig.**4d**, can be the best choice which minimizes the performance index. The time history of the side-slip angle which is an important safety criterion is compared in fig. **4b** for two different cases of the vehicle. . This indicates that the significant reduction of side-slip angle is certainly achieved by the designed controller based on tracking the desired yaw rate limited by Eq. (10).

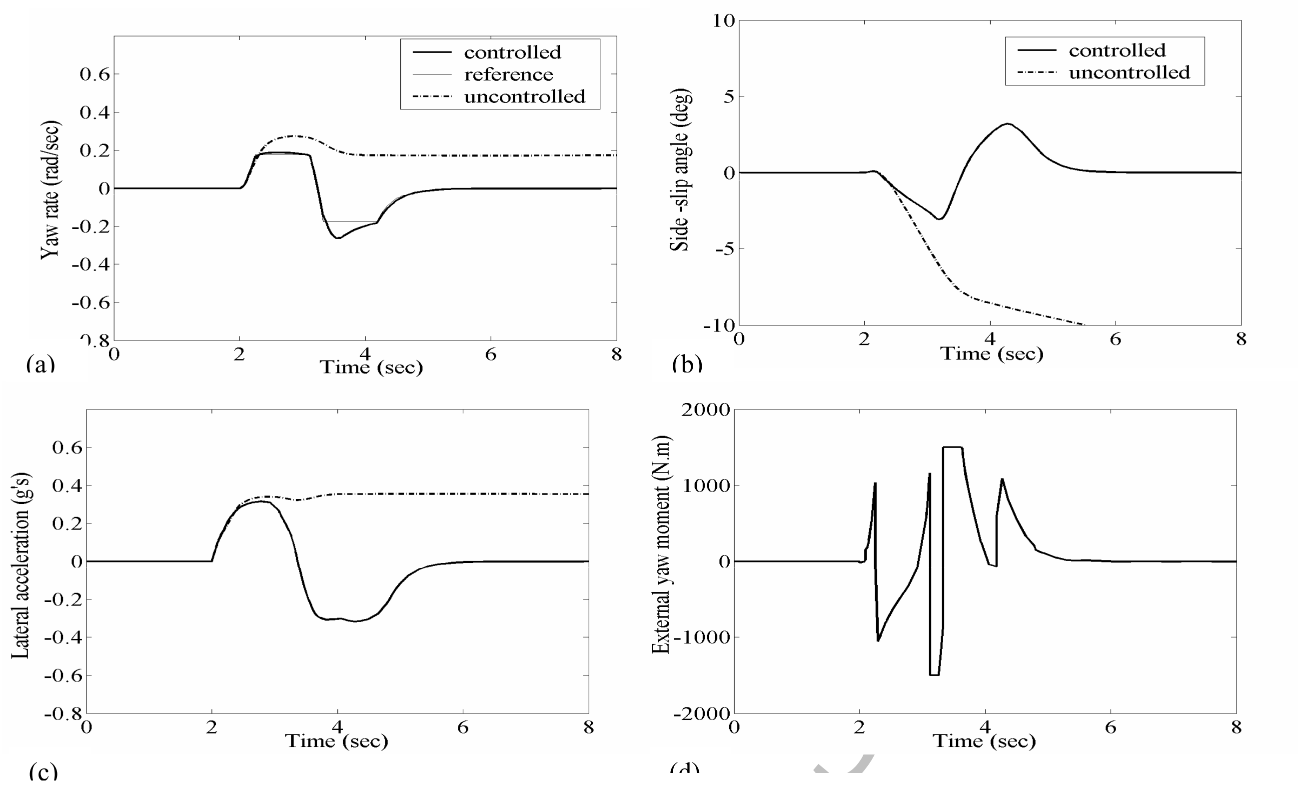

Fig. 4. Simulation results of the lane change maneuver on low-μ road: (a) yaw rate (b) side-slip angle (c) lateral acceleration (d) external yaw moment.

The lateral acceleration in g units, as shown in Fig. **4c**, is less than the limit of road adhesion.

5. Conclusions

The nonlinearity of tire force characteristics causes vehicle to show unstable motion during emergency driving conditions, whereas the behavior of linear vehicle model is stable. In this paper, according to the system requirements, an optimal yaw rate tracking law is developed for DYC based on the response prediction of an extended 2DOF nonlinear vehicle dynamics model and then high efficiency of this method for stabilizing vehicle motion is investigated through a full vehicle dynamic model with 8DOF. In order to compensate the loss of vehicle stability due to nonlinearity effects, a linear vehicle model limited by road friction coefficient is adopted as a desired model to be followed by the controller.

The simulation results show that when the proposed controller is engaged with the model, the yaw rate can follow its desired value and thus a significant decrease of lateral velocity can be achieved. The proposed optimal nonlinear control law has some significant features. It has an analytical closed form which is easy to apply; it is exponentially stable if the control weighting term is zero; and it handles control input saturation explicitly.

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