



Technical Report

گزارش فنی

### Parameter Estimation of Nash Conceptual Model Using Genetic Algorithm and Ordinary Least Square Methods

### تخمین پارامترهای مدل مفهومی ناش با استفاده از روش‌های الگوریتم ژنتیک و حداقل مربعات معمولی

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#### Abstract

This study is focused on two parameter estimation methods of Ordinary Least Square (OLS) and binary code Genetic Algorithm (GA) for estimating the  $k$ ,  $n$  Nash conceptual model parameters. The efficiency of these methods is compared by applying the calibrated models in simulating seven rainfall-runoff events in Heng-Chi watershed in north of Taiwan. The results of the goodness of fit criteria indicate that GA method has better performance in terms of coefficient of efficiency and has reduced the coefficient of variation of error in simulated discharge and error in peak discharge.

**Keywords:** Parameter estimation, Nash conceptual model, Ordinary Least Square, Genetic Algorithm.

#### چکیده

در این تحقیق، دو روش حداقل مربعات معمولی و الگوریتم ژنتیک دو-دویی به منظور تخمین پارامترهای مدل مفهومی ناش (n, k) مورد استفاده قرار گرفته است. کارایی این دو روش با بکارگیری پارامترهای تخمینی در شبیه سازی هفت واقعه بارندگی-رواناب واقع در حوضه Heng-Chi در شمال تایوان مورد ارزیابی قرار گرفت. نتایج حاصل از معیارهای نکوئی برازش در مورد نتایج هر دو مدل نشان داد که مدل الگوریتم ژنتیک قادر به بهبود معیار ضریب کارایی و کاهش ضریب تغییرات و خطای دبی اوج مدل نسبت به روش حداقل مربعات می باشد.

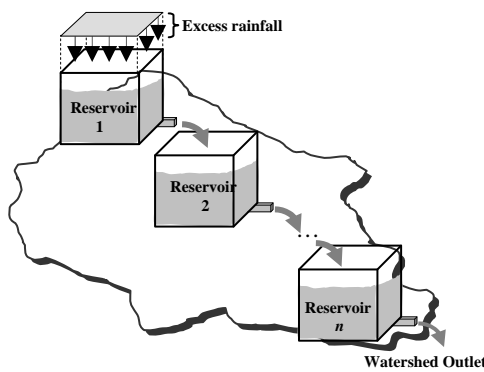
**کلمات کلیدی:** تخمین پارامتر، مدل مفهومی ناش، حداقل مربعات معمولی، الگوریتم ژنتیک.

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**Introduction**

Lumped conceptual types of models (LCM) such as the Nash conceptual model, as shown in Fig. 1, usually have few parameters which cannot be determined directly from physical catchment characteristics and therefore should be estimated by parameter estimation methods. Mathematical parameter estimation methods such as Ordinary Least Square (OLS) rely on mathematical properties of the error response surface. In OLS method, one often encounters a situation in which the derived Unit Hydrograph (UH) exhibit noise fluctuation among its ordinates. This is mainly caused by nonlinearity, time variance, distributed space, sampling intervals for time, and measurement error (Dooge et al., 1989). In recent years, evolutionary methods such as the Genetic Algorithm (GA) have been successfully used in the field of hydrology and water resources to directly map nonlinear complex relations. Researches indicate that GA may be an efficient and robust means of calibrating for a variety of models where measured time-series data are available (Wang, 1991; Mohan and Loucks, 1995).



**Figure 1- Schematic of cascade of reservoirs in Nash conceptual model**

**Nash Conceptual Model**

The Nash conceptual model as a two-parameter gamma distribution has the following structure:

$$u(t) = \frac{1}{k\Gamma(n)} \left(\frac{t}{k}\right)^{(n-1)} e^{-t/k} \quad (1)$$

$u(t)$ : ordinate of Instantaneous Unit Hydrograph (IUH) at time  $t$ ,

$t$ : time since commence of rainfall and runoff,  
 $n$ : number of linear reservoirs of equal storage coefficient,  
 $k$ : storage coefficient of reservoirs (Nash, 1957).

The merit of this model for UH derivation is that it provides all possible shapes depending on the magnitude of its parameters.

**Parameter Estimation Methods**

In the GA model, the Root Mean Square of Error (RMSE) of the simulated values obtained from the

Nash model is considered as the fitness function. The GA model, therefore, searches for the optimal solution which minimizes the simulation errors. MATLAB software has been used for formulating and running the GA model.

In OLS method, Nash model is linearized by applying the logarithm operator as follows:

$$\ln Q = -\ln k^n \Gamma(n) - \frac{t}{k} + (n-1) \ln t \quad (2)$$

With replacing the  $Y=\ln Q$ ,  $X=\ln t$ ,  $C=n-1$ ,  $B=-1/k$ ,  $A = -\ln k^n \Gamma(n)$ , and  $Z=t$ , equation 2 will have a linear form as follows:

$$Y=A+BZ+CX \quad (3)$$

Where  $B$  and  $C$  must be estimated by OLS method.

To evaluate the goodness of fit for the models, three criteria are selected (Khu, 1998):

1. Nash-Sutcliffe coefficient ( $R^2$ ):

$$R^2 = 1 - \frac{\sum_{i=1}^N [Q_{obs,t} - Q_{sim,t}]^2}{\sum_{i=1}^N [Q_{obs,t} - \bar{Q}_{obs}]^2} \quad (4)$$

2. Coefficient of Variation (CV):

$$CV = \frac{1}{\bar{Q}} \left( \frac{\sum_{i=1}^N [Q_{obs,t} - Q_{sim,t}]^2}{N} \right)^{1/2} \quad (5)$$

3. Error in Peak Discharge (EPD):

$$EP = \frac{Q_{PEAK,sim} - Q_{PEAK,obs}}{Q_{PEAK,obs}} \quad (6)$$

where  $Q_{obs,i}$ = observed flow in time  $t$  ( $m^3/s$ );  $Q_{sim,i}$ = simulated flow in time  $t$  ( $m^3/s$ );  $\bar{Q}_{obs}$ = average flow in each event ( $m^3/s$ );  $N$  = number of time steps in an event;  $Q_{peak,sim}$ = simulated peak flow ( $m^3/s$ ); and  $Q_{peak,obs}$ = observed peak flow ( $m^3/s$ ).

**Case Study**

Seven rainfall-runoff events in Heng-Chi watershed (Fig. 2) which is located in northern Taiwan was selected to investigate the applicability of the two parameter estimation approaches. The characteristic of these events are shown in Table 1. In this study, constant slope method has been used for base flow separation and direct runoff data has been used for conducting the analysis.

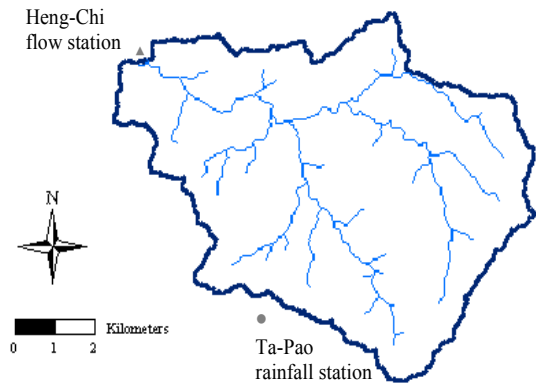


Figure 2- The Heng-Chi watershed and Location of gauging stations

Table 1- Characteristics of rainfall-runoff events

Date	Direct runoff			Excess rain	
	Peak (m <sup>3</sup> /s)	Time to Peak (hr)	Duration (hr)	Depth (mm)	Duration (hr)
10/31/2000	308.21	18	59	329.53	27
07/30/1996	199.00	30	40	133.00	36
07/10/1994	49.90	12	46	27.95	10
06/05/1993	156.94	11	20	72.20	13
08/18/1990	468.00	32	61	235.20	39
07/27/1987	140.00	7	33	54.35	9
09/17/1984	158.40	67	37	246.00	49

**Results and Discussion**

Four rainfall-runoff events shown in Table 1 were selected for calibration and parameter estimation and three other events were used for model verification. The optimum values of cross-over and mutation probability, initial population, and the number of generations for the GA model are estimated as 0.7, 0.014, 40, and 150, respectively. The estimated values of *k* and *n* for the four rainfall-runoff events using the GA model are 5.43 and 4.46, and for the OLS method are 5.53 and 4.37, respectively. The remaining three events were also simulated to check the validity of the models. For instance Fig. 3 shows two of the simulated events. As it can be seen in this figure, both models have been successfully able to simulate the overall shape and the amount of peak and time-to-peak of the hydrographs.

The three criteria shown in equations 4, 5, and 6 were applied to all three events which considered for validation and the results are presented in Table 2.

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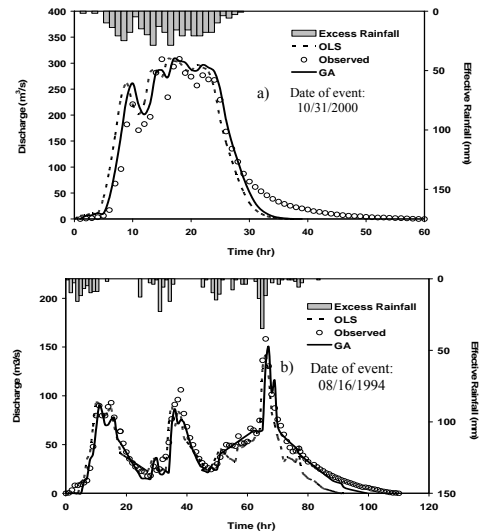


Figure 3- Comparison between simulated and observed hydrographs of the events used for model validation

Table 2- Results of goodness of fit of the models

Event	R <sup>2</sup>		EP		CV	
	OLS	GA	OLS	GA	OLS	GA
10/31/2000	0.75	0.79	-4.31	-3.80	0.63	0.58
07/30/1996	0.93	0.96	-0.76	-0.23	0.29	0.23
07/10/1994	0.74	0.79	-3.06	-2.85	0.52	0.46

The values in Table 2 indicate that the simulated events using the GA method have the higher coefficient of efficiency and lower variation and error in peak discharge compared to the OLS method.

**References**

Dooge, J. C. I., and Bruen, M. (1989). "Unit hydrograph stability and linear algebra." *J. Hydrol.*, Vol. 111, 377-390.

Khu, S. T., (1998). "Automatic calibration of NAM model with multi objective consideration." National University of Singapore/ Danish Hydraulic Institute, D2K Technical Report 1298-1, 18<sup>th</sup> December.

Mohan, S., and Loucks, D. P., (1995). "Genetic algorithms for estimating model parameters." *Proc. 22 Annu. Conf. Integr. Water Res. Plan 21 Century*, p 460-463.

Nash, J. E. (1957). "The form of instantaneous unit hydrograph." *Int. Assn. Sci. Hydro. Publ. No. 51*, 546-557, LAHS, Gentbrugge, Belgium.

Wang, Q. J., (1991). "The genetic algorithm and its application to calibrating conceptual rainfall-runoff models." *Water Resources Researches*, Vol. 27, No. 9, 2467-2471.



## Extended Abstract

## Application of Finite Elements Concept in Water Resources Management: Introduction of Model and FEWREM Software

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### Introduction

For planning and management of water resources systems, different models have been developed. Some of these models have been widely accepted due to their dynamic approaches in modeling many types of problems. However, most of them are used just to simulate hydro-systems. Models which use optimization in a complete form are rare. That is because of computational burden imposed by present algorithms. Some of the more famous models applied in water resources systems planning and analysis, are WEAP21, MIKEBASIN, RIBASIM, ARSP, SOCRATES, MODSIM, and CRIM (Close *et al.*, 2003; McKinney *et al.*, 1999). As mentioned, the main reason that a model does not make use of efficient optimization, is the lack of an appropriate structure for modeling and solving large systems justified in time and precision. In this research a framework is proposed for modeling and solving such large systems with efficient precision, and in a desired time span.

### Objectives

The main objective of this research is the introduction of a framework for systematic modeling of large hydro-systems in a way, so that most solution algorithms could be done with desired precision and within reasonable time. A model called, FEWREM (Finite Elements Water Resources Management Model) is also developed to show application of this framework in a sample waterbasin.

### Methodology

To attain this objective, the finite elements concept is applied for provision of a structure, which is decomposable and aggregatable simultaneously. A system approach is applied in this structure for intra-element behavior modeling. Elements have interactions with their neighbor elements. However, a system approach could deal with a simple network flow model up to a more sophisticated model inside an element. By these two concepts combined, the basic element is formulated. This basic element is used for generating different parts of a hydro-system. Elements are then assembled based on the desired solution method. For a direct solving method, all elements may be entirely assembled. Elements may partially be assembled if a decomposition-coordination algorithm is selected. They may be assembled and aggregated if an aggregation-disaggregation algorithm is desired. By each of these three methodologies, the model is solved and results are obtained for decision making.

### Results and Discussion

The FEWREM model is developed and used for a case study based on the proposed framework for modeling. FEWREM uses elements which are suitable for hydro-systems without hydro-electric generation. Other components of a hydro-system like reservoirs, demand sites, groundwater, rivers and pumping are considered in its basic element. Of course, elements could be simply changed to consider more components of a hydro-system. FEWREM uses the direct solution algorithm within the GAMS solver. The case study is composed of a hydro-system with three reservoirs, two domestic demand sites, three irrigation demand sites, and the groundwater. This system is modeled with

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FEWREM and MODSIM for comparison. FEWREM is mainly superior to MODSIM according to its multi-periodicity. FEWREM has operated the system with more effectiveness. MODSIM has operated the system with 1574 MCM slack for domestic and irrigation demands. However, FEWREM has operated the system with 1104 MCM slack only on Irrigation demands. On the other hand, MODSIM has kept reservoirs full in most of the periods while FEWREM kept them full only in summer. These are the result of a single-period solution in comparison to a multi-period approach. Figure 1 shows summary of results.

FEWREM model has kept less water in the system and released more for demands satisfaction. However, MODSIM has kept reservoirs full and released less water. Therefore, FEWREM model operation led to less slacks in comparison to MODSIM. Both models have the same toolbars for system management, but their decision making structure differs. Decision making structure of FEWREM has shown more effectiveness.

**Conclusion**

In this research a framework, based on combination of finite elements and system concept was proposed for modeling and solution of large hydro-systems. Within this framework decomposition-coordination and aggregation-disaggregation algorithms could be applied more efficiently. Based on this framework, a model called FEWREM was also developed. FEWREM makes use of a multi-period optimization algorithm for model solution. This model is compared with MODSIM through a case study. In comparison to MODSIM, FEWREM has operated the system more effectively for demands due to the different approaches.

**Keywords:** Finite Elements, Systems Approach, FEWREM, Water Resources Management, and Modeling

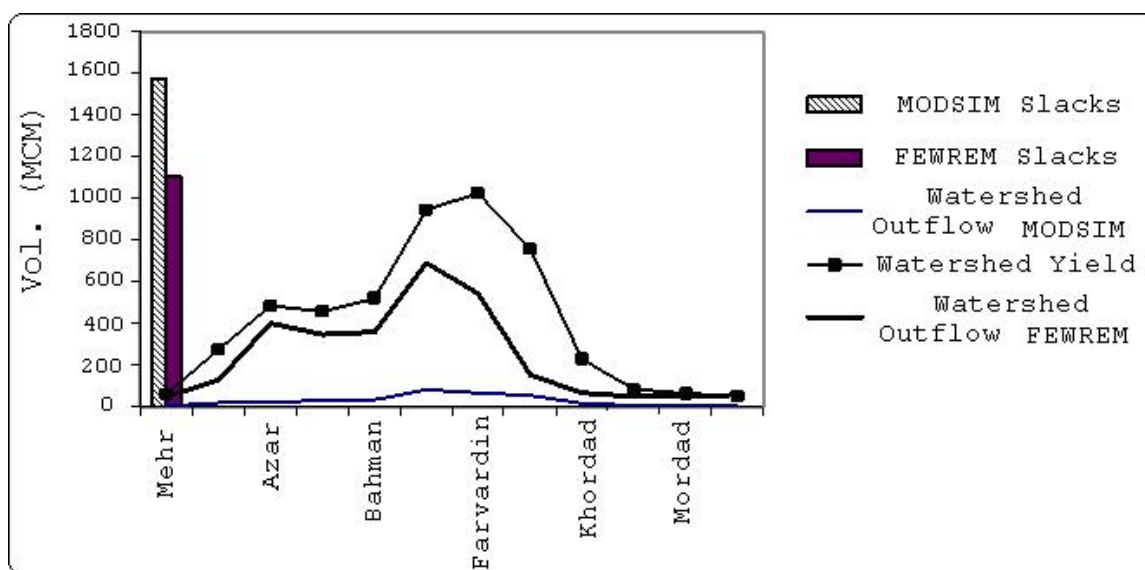


Figure 1- Effectiveness of FEWREM and MODSIM models in operating the hydro-system and their total slacks in meeting demands

**References**

Close A., Haneman W.M., Labadie J.W., Loucks D.P., Lund J.R., MacKinney D.C., and Stedinger J.R., (2003), *A Strategic Review of CALSIM II and its Use for Water Planning, Management, and Operations in Central California*, California bay Delta Authority Science Program Association of Bay Governments, Oakland, California.

McKinney D.C., Cai X., Rosegrant M.W., Ringler C. and Scott C.A., (1999), *Modeling Water Resources Management at the Basin Level: Review and Future Directions*, International Water Management Institute.