

One-for-One Period Policy and its Optimal Solution

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ABSTRACT

In this paper we introduce the optimal solution for a simple and yet practical inventory policy with the important characteristic which eliminates the uncertainty in demand for suppliers. In this new policy which is different from the classical inventory policies, the time interval between any two consecutive orders is fixed and the quantity of each order is one. Assuming the fixed ordering costs are negligible, lead times are constant, and demand forms a Poisson process, we use queuing theory concepts to derive the long-run average total inventory costs, consisting of holding and shortage costs in terms of the average inventory. We show that the total cost rate has the important property of being entirely free of the lead time. We prove that the average total cost rate is a convex function and thus has a unique solution. We, then derive the relation for the optimal value of the time interval between any two consecutive orders. Finally we present a numerical example to compare the performance of this new policy with the classical one-for-one ordering policy. The provided example intends to re-examine the optimality of (s, S) policy in continuous review inventory models as well to establish the fact that even for the case where demand forms a Poisson process the optimality does not hold.

Keywords: Inventory control, One for one period policy, (s, S) Policy, Queuing system, Poisson demand

1. INTRODUCTION

Two fundamental questions that must be answered in controlling the inventory of any item are *when* and *how much* should be ordered for replenishment. The answers of these questions determine the *inventory control policy*. In the face of uncertainty it is more complicated to determine the inventory control policy. The most frequent uncertain factor is the stochastic nature of customer demand.

Most of the classical policies for controlling the inventory are derived from the (s, S) policy and are classified as periodic or continuous review policies. The distinction involves the frequency with which the inventory level must be observed (reviewed) in order to implement the policy. A periodic-policy is one in which the inventory level is observed only at equally spaced points in time (The time interval between such two points is called a "period." If the length of the period is T , then (s, S) policy is substituted by the (s, S, T) policy. If the maximum stock level in (s, S, T) policy equals to the reorder point, then another classical policy called periodic review (R, T) is formed, where $R=S$ is the maximum inventory level.

A continuous-review policy requires knowledge of the inventory level at all times. If the customer demand is one unit, the ordering size (Q) is always equal $S-s$. Therefore, the (s, S) policy changes into the well known continuous review (r, Q) policy, where $r=s$ is the reorder point. If the ordering

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cost is negligible in the (r, Q) policy, then the policy is called one-for-one, $(S-1, S)$, or base stock policy (Hadley and Whitin, 1963, and Love, 1979).

Hadley and Whitin (1963), Johnson and Montgomery (1974), Love (1979), Zipkin (2000), and Nahmias (2005), among others, present the methods to find the optimal or near optimal solution to minimize the inventory costs at a single stocking point with stochastic demand, based on the continuous review (r, Q) , periodic review (R, T) , and one-for-one policies.

Due to its widespread acceptance, one can find an abundance of work dealing with (s, S) policy in the literature (See, for example, Beckman, 1961; Sherbrooke, 1968; Nahmias, 1976; Sahin, 1979, 1982, 1990; Kruse, 1980, 1981; Archibald, 1981; Grave, 1985; Moinzadeh and Lee, 1986; Sivlazier, 1974; Zipkin, 1986a, 1986b; Axsäter, 1990, 1993, 2003; Lee and Moinzadeh, 1987a, 1987b; Zheng, 1992; Federgruen and Zheng, 1992; Axsäter et al., 1994; Nahmias and Smith, 1994; Matta and Sinha, 1995; Forsberg, 1995, 1996, 1997; Axsäter and Zhang, 1999; Ganeshan, 1999; Andersson and Melchior, 2001; Moinzadeh, 2002; Seo et al, 2002; Marklund, 2002; Ghalebsaz et al., 2004).

While it is well known that for the case of stochastic demand, there is no inventory control policy in which both the order size and the order interval are constant with an optimal solution, in this paper, we introduce the optimal solution for a new policy which is different from the classical inventory policies used in the literature of inventory and production control systems. This new policy is to order 1 unit at each fixed time period T . We call this new policy $(1, T)$ or *one for one period ordering policy*. In this policy *the time interval between two consecutive orders and the value of the order size are both constant*.

In what follows we discuss the advantages of this policy. Then, assuming that the fixed ordering cost is zero or negligible, demand process is a Poisson process, and the replenishment lead time is constant, we derive the long-run average total cost, the total holding and shortage costs, per unit time in terms of the average inventory for the lost sales case. To obtain the total cost rate we use some concepts from queueing theory. We prove that this total cost is a convex function in terms of the average inventory and has a unique optimal solution.

Further, we obtain the optimal value of T , the time interval between two consecutive orders, which minimizes the long-run average total cost. Finally, by a numerical example we compare the long-run average total cost of this policy with the total cost rate of the base stock, one-for-one ordering, policy for the lost sales case. The provided example intends to re-examine the optimality of (s, S) policy in continuous review inventory models in order to establish the fact that even for the case where demand forms a Poisson process the optimality does not hold.

The optimality of (s, S) policy has been addressed in the literature of inventory control by a number of scholars (Scarf, 1960, Iglehart, 1963, and Sahin 1990). The optimality of the (s, S) policy was first investigated by Schultz (1989) (Moinzadeh, 2001). He considered an $(S-1, S)$ inventory system with an arbitrarily chosen distribution for the inter-demand times and fixed lead time. In order to simplify the analysis, he assumes that orders can also be expedited and received immediately at a premium cost. Furthermore, he only considers a special case of $(S-1, S)$ inventory policies with $S=1$; that is, a maximum of one unit of inventory is allowed at any point in time. Schultz (1989) shows that by delaying the placing of orders until after demand occurs, one can achieve a lower expected total cost rate in such systems. Although Schultz (1989) examines the concept of order delays as a policy parameter, he fails to explicitly model the relationship between the delay and stocking policy parameters and only optimizes the delay in this somewhat simplified model (recall

that the maximum inventory level, S , is pre-set at unity). It is believed that in many situations the assumption that orders can be expedited and received instantaneously does not hold and setting the value of $S=1$ will lead to inferior policies (Moinzadeh, 2001). Thus the joint determination of the optimal inventory policy and the delay becomes critical.

Later, Moinzadeh (2001) revisited the optimality of (s, S) policies in continuous review inventory models where demand forms a renewal process. He explains why when orders are placed at demand epochs, (s, S) policies are not optimal in general. By introducing a delay in order placement as a policy parameter, he proposes a new policy to establish this fact. He investigates the optimality of the (s, S) policy for Gaussian distribution for the inter-demand times to show the merits of his proposed policy. When the inter-demand times follow a general distribution he develops an efficient heuristic to compute the policy parameters. He shows that under his proposed policy when inter-demand times are exponential the optimal delay time becomes zero. That is, for Poisson demand his proposed policy reduces to the (s, S) policy. Based on the results contained in his work one may conclude that if the demand follows a Poisson process then the optimality of (s, S) policy holds true.

The example presented in this paper re-examines the optimality of (s, S) policy in hope of achieving the goal that the optimality of (s, S) policy in continuous review inventory models does not hold even for the case where demand follows a Poisson process.

2. ADVANTAGES OF (I, T) POLICY

When the demand is uncertain, in the classical inventory policies at least one of the two values, the order quantity and the time interval between any two consecutive orders, has an uncertain nature. For example, consider the continuous review (r, Q) and the periodic review (R, T) policies which are two usual policies employed by practitioners when demand is uncertain. When (r, Q) policy is used, the order quantity, Q , is fixed but the time interval between any two consecutive orders has an uncertain nature. For the case of (R, T) policy, the time interval between any two consecutive orders, T , is fixed whereas the value of the order quantity is uncertain. Therefore, the orders which constitute the demands for the supplier will have an uncertain nature. Hence, the uncertainty spreads into the supply chain.

For the new *one for one period* policy, discussed in this paper an order for one unit of item is placed in a pre-determined time interval. Hence, both the order size and the order interval are constant. Therefore, this policy prevents expanding the demand uncertainty for the supplier. That is, the demand for the supplier is deterministic, one unit every T units of time.

The advantages of this policy, which eliminates uncertainty in demand and leads to a uniform and deterministic demand for the supplier, are:

- 1- The safety stock in supplier is eliminated. (cost reduction)
- 2- Shortage cost in supplier due to uncertainty in demand is eliminated.
- 3- Information exchange cost for supplier due to the elimination of uncertainty of its demand is eliminated.
- 4- Inventory control and production planning in supplier are simplified.
- 5- This policy is very easy to apply.

3. COST EVALUATION

Using this policy in which an order constantly is placed for one unit of product in a pre-determined time interval, we derive the total inventory cost rate, consisting of holding and shortage costs in

terms of the average on hand inventory. The objective is to determine the optimal time interval between any two consecutive orders.

Assumptions:

- 1- Unsatisfied demand will be lost.
- 2- The lead time for an order is constant.
- 3- The fixed ordering cost is zero or negligible.

Notation:

μ	The demand rate.
π :	Cost of a lost sale.
h	Rate of holding cost.
T	Time interval between any two consecutive orders.
I	Average inventory on hand for the $(1, T)$ policy.
H	Average holding cost per unit time.
Π	Average shortage cost per unit time.
K_T	Average total cost rate, for the $(1, T)$ policy.
K_S	Average total cost rate, for the $(S-1, S)$ policy.
L	Lead time.

3.1. Methodology

To obtain the average inventory level, one can resort to some concepts of queuing theory. To do this, consider the arrival units of product to the system as the arrival process to a queuing system, the inter-demand times to the system as the service times of these units, and inventory on hand as the number of units in the queuing system. Hence, the inventory problem can be interpreted as a $D/M/1$ (Haji et al, 2006), a single channel queuing system in which the inter-arrival times are constant, equal to T , and the service times have exponential distribution with mean $1/\mu$. Thus, the arrival rate of units to the system is $\lambda=1/T$ and the service rate is μ .

Let P_0 denote the proportion of time that the number in system is zero, then for a single channel queue

$$P_0 = 1 - \rho$$

Where ρ is the ratio of the arrival rate $\lambda=1/T$ to the service rate μ , i. e.,

$$\rho = \frac{\lambda}{\mu} = \frac{1}{T\mu} \quad (1)$$

Also, let I denote the long-run average number of units in system, then for the queuing system $G/M/1$: we can write (Ross, 1993):

$$I = \frac{\rho}{1-\beta} \quad (2)$$

Where β is the solution of the following equation:

$$\beta = \int_0^{\infty} e^{-\mu t(1-\beta)} dG(t) \quad (3)$$

It can be shown that if the mean of G is greater than the mean service time $1/\mu$, then there is a unique value of β satisfying Equation (3) which is between 0 and 1. The exact value of β usually can be obtained by numerical methods.

The system under consideration is a $D/M/1$ queuing system in which the inter-arrival time of units of product to the queuing system is constant and is equal to T . Thus $G(T) = 1$, and we can write (3) as

$$\beta = e^{-T\mu(1-\beta)} = e^{-\frac{\mu}{\lambda}(1-\beta)} \quad (4)$$

Or

$$\beta = e^{-\frac{(1-\beta)}{\rho}} \quad (5)$$

Since ρ is independent of the lead time, L , so is β as defined in (4) and (5) which in turn leads to independence of I in (2) from lead time.

From (2) we can write

$$\beta = e^{-\frac{1}{I}} \quad (6)$$

The average holding cost per unit time is:

$$H = hI \quad (7)$$

Since P_0 is the proportion of time that the system is out of stock-. Thus the proportion of demand that is lost is μP_0 and the average lost sale cost per unit time is:

$$II = \pi\mu P_0 = \pi\mu(1-\rho) \quad (8)$$

Or from (2) and (6) we can write (8) as:

$$II = \pi\mu \left[1 - I \left(1 - e^{-\frac{1}{I}} \right) \right] \quad (9)$$

Thus, the total cost rate is:

$$K_T = H + II \quad (10)$$

Or from (7) and (9)

$$K_T = hI + \pi\mu \left[1 - I \left(1 - e^{-\frac{1}{I}} \right) \right] \quad (11)$$

As mentioned before, since I is independent of the value of the lead time, one can easily conclude that the total cost rate for the $(1, T)$ policy is independent of the lead time.

We now prove the following theorem for function K_T as specified in (11).

Theorem K_T is a convex function

Proof: The first derivative of K with respect to I is:

$$\frac{dK_T}{dI} = h + \pi\mu \left[-1 \left(1 - e^{-\frac{1}{I}} \right) - I \left(-\frac{1}{I^2} e^{-\frac{1}{I}} \right) \right]$$

Or

$$\frac{dK_T}{dI} = h - \pi\mu + \pi\mu \left[e^{-\frac{1}{I}} + \frac{1}{I} e^{-\frac{1}{I}} \right] \quad (12)$$

The second derivative of K with respect to I is:

$$\frac{d^2K_T}{dI^2} = \pi\mu \left[\frac{1}{I^2} e^{-\frac{1}{I}} - \frac{1}{I^2} e^{-\frac{1}{I}} + \frac{1}{I} \left(\frac{1}{I^2} e^{-\frac{1}{I}} \right) \right]$$

Or equivalently

$$\frac{d^2K_T}{dI^2} = \pi\mu \frac{1}{I^3} e^{-\frac{1}{I}} \quad (13)$$

It is clear from (13) that the second derivative of K with respect to I is positive for all values of $I > 0$. This means that K is a convex function and thus it has a unique solution which proves the theorem.

The optimal I, I^* , is the solution of

$$\frac{dK_T}{dI} = h - \pi\mu + \pi\mu \left[e^{-\frac{1}{I^*}} + \frac{1}{I^*} e^{-\frac{1}{I^*}} \right] = 0$$

Or

$$e^{-\frac{1}{I^*}} + \frac{1}{I^*} e^{-\frac{1}{I^*}} = (\pi\mu - h) / \pi\mu$$

To find the optimal value of the time interval between two consecutive orders, T^* , note that from (1), (2), and (6) we can write

$$\frac{1}{\mu T^*} = I^* \left(1 - e^{-\frac{1}{I^*}} \right)$$

Thus, the optimal T, T^* , can be obtained from the following relation

$$T^* = \frac{1}{\mu I^* (1 - e^{-\frac{1}{I^*}})} \quad (14)$$

4. NUMERICAL RESULTS

This section is devoted to comparison between the one-for-one period that is, the $(1, T)$ policy with the one-for-one policy in terms of total cost rate.

We study the effect of varying the values of L and π/h on the total cost rates for both policies. The adopted values for the parameters are:

$$h = 1; \mu = 1, 2, 3;$$

$$\pi / h = 0.1, 0.5, 1(1)10;$$

$$L = 1(1)10 (5) 20 (10) 60.$$

The total cost rate for, $(S-1, S)$, is as follows (Tijms, 1986):

$$K_S = h \left[S - L\mu \left(1 - \frac{(L\mu)^j / S!}{\sum_{x=0}^S (L\mu)^x / x!} \right) \right] + \pi\mu \frac{(L\mu)^j / S!}{\sum_{x=0}^S (L\mu)^x / x!} \quad (15)$$

Where L stands for the lead time.

It is obvious from (15) that K_S depends on the lead time but K_T is entirely independent from the lead time as mentioned before.

Table 1 presents the outcome of comparing the total cost rates for the $(1, T)$ and $(S-1, S)$ policies. In this table we consider the values 0.1, 0.5, 1, 2, 3, and 4 for π/h . Additional values for π/h are used in Table 2. The value of μ in Table 1 and Table 2 is assumed to be 1 while in Tables 3 and 4 μ is 2.

The upper section of Table 1 presents the optimal values of T (cycle times), T^* , the optimal values of inventory on hand, I^* , and the optimal values of total cost rates, K_T^* , for the $(1, T)$ policy for different values of π/h . The lower section of this table displays the optimal values of S as well as the optimal values of the total cost rates, K_S^* , for the $(S-1, S)$ policy for different values of L and π/h . As the contents of Table 1 indicate the performance of $(1, T)$ and $(S-1, S)$ policies are exactly the same for all values of L as long as π/h does not exceed 1 (obviously $S \geq 0$).

When $\pi/h \geq 2$, application of $(S-1, S)$ policy amounts to a lower total cost rate provided that, in the worst case considered, $L \leq 8$. Each row of this table also provides the total cost rate differentials, $\Delta K = K_S - K_T$, for various values of L . As can be seen the $Max \Delta K$ occurs at the largest values of L and the largest values of π/h , i.e., $Max \Delta K = 5.0302 - 4.1246 = 0.9056$ corresponding to $L = 60$ and $\pi/h = 10$. For the values of parameters used in this example, the maximum percent change in total cost increase due to application of $(S-1, S)$ policy instead of $(1, T)$ policy is more than 21 percent.

Table 1. Cost comparisons between (1, T) and (S-1, S) policies for $\mu = 1$

T^*	∞	∞	∞	2.0636	1.7095	1.5565						
I^*	0.0000	0.0000	0.0000	0.5958	0.8412	1.0403						
$K(T^*)$	0.1000	0.5000	1.0000	1.6266	2.0863	2.4704						
π/h	0.1	0.5	1	2	3	4						
L	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*
1	0	0.1000	0	0.5000	0,1	1.0000	1	1.5000	2	1.8000	2	2.0000
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = -0.1266$		$\Delta K = -0.2863$		$\Delta K = -0.4704$	
2	0	0.1000	0	0.5000	0,1	1.0000	2	1.6000	2	2.0000	3	2.2632
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = -0.0266$		$\Delta K = -0.0863$		$\Delta K = -0.2072$	
3	0	0.1000	0	0.5000	0,1	1.0000	2	1.6471	3	2.0769	3	2.4231
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0204$		$\Delta K = -0.0093$		$\Delta K = -0.0473$	
4	0	0.1000	0	0.5000	0,1	1.0000	2	1.6923	3	2.1549	4	2.4854
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0657$		$\Delta K = 0.0687$		$\Delta K = 0.0151$	
5	0	0.1000	0	0.5000	0,1	1.0000	3	1.7076	4	2.1867	5	2.5638
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0810$		$\Delta K = 0.1005$		$\Delta K = 0.0934$	
6	0	0.1000	0	0.5000	0,1	1.0000	3	1.7213	4	2.2261	5	2.6040
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0947$		$\Delta K = 0.1398$		$\Delta K = 0.1336$	
7	0	0.1000	0	0.5000	0,1	1.0000	3	1.7379	5	2.2472	6	2.6446
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1113$		$\Delta K = 0.1609$		$\Delta K = 0.1742$	
8	0	0.1000	0	0.5000	0,1	1.0000	4	1.7464	5	2.2691	6	2.6770
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1197$		$\Delta K = 0.1828$		$\Delta K = 0.2066$	
9	0	0.1000	0	0.5000	0,1	1.0000	4	1.7519	6	2.2862	7	2.7006
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1253$		$\Delta K = 0.1999$		$\Delta K = 0.2302$	
10	0	0.1000	0	0.5000	0,1	1.0000	4	1.7600	6	2.2987	7	2.7266
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1333$		$\Delta K = 0.2124$		$\Delta K = 0.2562$	
15	0	0.1000	0	0.5000	0,1	1.0000	6	1.7799	8	2.3466	10	2.7965
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1532$		$\Delta K = 0.2603$		$\Delta K = 0.3261$	
20	0	0.1000	0	0.5000	0,1	1.0000	7	1.7912	11	2.3728	13	2.8412
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1646$		$\Delta K = 0.2865$		$\Delta K = 0.3708$	
30	0	0.1000	0	0.5000	0,1	1.0000	10	1.8028	15	2.3991	18	2.8866
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1761$		$\Delta K = 0.3128$		$\Delta K = 0.4163$	
40	0	0.1000	0	0.5000	0,1	1.0000	13	1.8088	19	2.4140	23	2.9118
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1822$		$\Delta K = 0.3277$		$\Delta K = 0.4415$	
50	0	0.1000	0	0.5000	0,1	1.0000	16	1.8126	23	2.4235	28	2.9279
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1859$		$\Delta K = 0.3372$		$\Delta K = 0.4575$	
60	0	0.1000	0	0.5000	0,1	1.0000	19	1.8151	28	2.4300	33	2.9389
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1885$		$\Delta K = 0.3437$		$\Delta K = 0.4685$	

Table 1. (continued) Cost comparisons between (1, T) and (S-1, S) policies for $\mu = 1$

T^*	1.4682	1.4097	1.3675	1.3354	1.3101	1.2894
I^*	1.2130	1.3679	1.5096	1.6410	1.7641	1.8804
$K(T^*)$	2.8075	3.1116	3.3908	3.6505	3.8942	4.1246

π/h	5	6	7	8	9	10
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L	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*
1	2	2.2000	2	2.4000	3	2.5000	3	2.5625	3	2.6250	3	2.6875
	$\Delta K = -0.6075$		$\Delta K = -0.7116$		$\Delta K = -0.8908$		$\Delta K = -1.0880$		$\Delta K = -1.2692$		$\Delta K = -1.4371$	
2	3	2.4737	3	2.6842	4	2.8571	4	2.9524	4	3.0476	4	3.1429
	$\Delta K = -0.3338$		$\Delta K = -0.4274$		$\Delta K = -0.5337$		$\Delta K = -0.6981$		$\Delta K = -0.8466$		$\Delta K = -0.9817$	
3	4	2.6489	4	2.8550	4	3.0611	5	3.2106	5	3.3207	5	3.4307
	$\Delta K = -0.1586$		$\Delta K = -0.2566$		$\Delta K = -0.3298$		$\Delta K = -0.4399$		$\Delta K = -0.5736$		$\Delta K = -0.6939$	
4	5	2.7916	5	2.9907	5	3.1897	5	3.3888	6	3.5231	6	3.6403
	$\Delta K = -0.0159$		$\Delta K = -0.1209$		$\Delta K = -0.2011$		$\Delta K = -0.2617$		$\Delta K = -0.3711$		$\Delta K = -0.4843$	
5	5	2.8487	6	3.1103	6	3.3022	6	3.4940	6	3.6859	7	3.8078
	$\Delta K = 0.0412$		$\Delta K = -0.0013$		$\Delta K = -0.0887$		$\Delta K = -0.1565$		$\Delta K = -0.2084$		$\Delta K = -0.3168$	
6	6	2.9141	6	3.1791	7	3.4057	7	3.5908	7	3.7758	8	3.9500
	$\Delta K = 0.1067$		$\Delta K = 0.0675$		$\Delta K = 0.0149$		$\Delta K = -0.0597$		$\Delta K = -0.1184$		$\Delta K = -0.1746$	
7	6	2.9760	7	3.2353	7	3.4842	8	3.6823	8	3.8611	8	4.0400
	$\Delta K = 0.1685$		$\Delta K = 0.1238$		$\Delta K = 0.0934$		$\Delta K = 0.0318$		$\Delta K = -0.0331$		$\Delta K = -0.0846$	
8	7	3.0061	8	3.2980	8	3.5336	8	3.7691	9	3.9434	9	4.1165
	$\Delta K = 0.1987$		$\Delta K = 0.1864$		$\Delta K = 0.1427$		$\Delta K = 0.1186$		$\Delta K = 0.0492$		$\Delta K = -0.0081$	
9	8	3.0482	8	3.3374	9	3.5888	9	3.8131	10	4.0233	10	4.1913
	$\Delta K = 0.2407$		$\Delta K = 0.2258$		$\Delta K = 0.1980$		$\Delta K = 0.1626$		$\Delta K = 0.1291$		$\Delta K = 0.0667$	
10	8	3.0748	9	3.3713	9	3.6445	10	3.8625	10	4.0771	11	4.2646
	$\Delta K = 0.2673$		$\Delta K = 0.2597$		$\Delta K = 0.2537$		$\Delta K = 0.2120$		$\Delta K = 0.1828$		$\Delta K = 0.1400$	
15	11	3.1758	12	3.5021	13	3.7909	13	4.0541	14	4.2796	14	4.4996
	$\Delta K = 0.3683$		$\Delta K = 0.3906$		$\Delta K = 0.4000$		$\Delta K = 0.4036$		$\Delta K = 0.3854$		$\Delta K = 0.3750$	
20	14	3.2349	15	3.5799	16	3.8849	17	4.1600	17	4.4157	18	4.6378
	$\Delta K = 0.4275$		$\Delta K = 0.4683$		$\Delta K = 0.4941$		$\Delta K = 0.5095$		$\Delta K = 0.5215$		$\Delta K = 0.5132$	
30	20	3.3030	21	3.6685	23	3.9977	24	4.2940	24	4.5649	25	4.8130
	$\Delta K = 0.4955$		$\Delta K = 0.5569$		$\Delta K = 0.6069$		$\Delta K = 0.6435$		$\Delta K = 0.6707$		$\Delta K = 0.6884$	
40	25	3.3400	27	3.7179	29	4.0573	30	4.3667	31	4.6516	32	4.9150
	$\Delta K = 0.5325$		$\Delta K = 0.6063$		$\Delta K = 0.6665$		$\Delta K = 0.7162$		$\Delta K = 0.7574$		$\Delta K = 0.7904$	
50	31	3.3633	33	3.7496	35	4.0977	37	4.4163	38	4.7093	39	4.9823
	$\Delta K = 0.5558$		$\Delta K = 0.6380$		$\Delta K = 0.7069$		0.7658		$\Delta K = 0.8151$		$\Delta K = 0.8577$	
60	36	3.3802	39	3.7717	41	4.1267	43	4.4509	45	4.7509	46	5.0302
	$\Delta K = 0.5728$		$\Delta K = 0.6602$		$\Delta K = 0.7359$		$\Delta K = 0.8004$		$\Delta K = 0.8566$		$\Delta K = 0.9056$	

Table 2. Cost comparisons between (1, T) and (S-1, S) policies for $\mu = 2$

T^*	∞	∞	1.0318	0.7782	0.7048	0.6677						
I^*	0.0000	0.0000	0.5958	1.0403	1.3679	1.6410						
$K(T^*)$	0.2000	1.0000	1.6266	2.4704	3.1116	3.6505						
π/h	0.1	0.5	1	2	3	4						
L	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*
1	0	0.2000	0,1	1.0000	2	1.6000	3	2.2632	4	2.6842	4	2.9524
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = -0.0266$		$\Delta K = -0.2072$		$\Delta K = -0.4274$		$\Delta K = -0.6981$	
2	0	0.2000	0,1	1.0000	2	1.6923	4	2.4854	5	2.9907	5	3.3888
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0657$		$\Delta K = 0.0151$		$\Delta K = -0.1209$		$\Delta K = -0.2617$	
3	0	0.2000	0,1	1.0000	3	1.7213	5	2.6040	6	3.1791	7	3.5908
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.0947$		$\Delta K = 0.1336$		$\Delta K = 0.0675$		$\Delta K = -0.0597$	
4	0	0.2000	0,1	1.0000	4	1.7464	6	2.6770	8	3.2980	8	3.7691
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1197$		$\Delta K = 0.2066$		$\Delta K = 0.1864$		$\Delta K = 0.1186$	
5	0	0.2000	0,1	1.0000	4	1.7600	7	2.7266	9	3.3713	10	3.8625
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1333$		$\Delta K = 0.2562$		$\Delta K = 0.2597$		$\Delta K = 0.2120$	
6	0	0.2000	0,1	1.0000	5	1.7689	8	2.7625	10	3.4347	11	3.9553
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1423$		$\Delta K = 0.2921$		$\Delta K = 0.3231$		$\Delta K = 0.3048$	
7	0	0.2000	0,1	1.0000	5	1.7788	9	2.7898	11	3.4881	13	4.0205
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1522$		$\Delta K = 0.3194$		$\Delta K = 0.3766$		$\Delta K = 0.3700$	
8	0	0.2000	0,1	1.0000	8	1.7824	11	2.8109	13	3.5228	14	4.0741
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1557$		$\Delta K = 0.3406$		$\Delta K = 0.4112$		$\Delta K = 0.4236$	
9	0	0.2000	0,1	1.0000	7	1.7876	12	2.8274	14	3.5525	15	4.1270
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1609$		$\Delta K = 0.3570$		$\Delta K = 0.4410$		$\Delta K = 0.4765$	
10	0	0.2000	0,1	1.0000	7	1.7912	13	2.8412	15	3.5799	17	4.1600
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1646$		$\Delta K = 0.3708$		$\Delta K = 0.4683$		$\Delta K = 0.5095$	
15	0	0.2000	0,1	1.0000	10	1.8028	18	2.8866	21	3.6685	24	4.2940
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1761$		$\Delta K = 0.4163$		$\Delta K = 0.5569$		$\Delta K = 0.6435$	
20	0	0.2000	0,1	1.0000	13	1.8088	23	2.9118	27	3.7179	30	4.3667
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1822$		$\Delta K = 0.4415$		$\Delta K = 0.6063$		$\Delta K = 0.7162$	
30	0	0.2000	0,1	1.0000	19	1.8151	33	2.9389	39	3.7717	43	4.4509
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1885$		$\Delta K = 0.4685$		$\Delta K = 0.6602$		$\Delta K = 0.8004$	
40	0	0.2000	0,1	1.0000	25	1.8183	43	2.9533	51	3.8007	56	4.4968
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1917$		$\Delta K = 0.4829$		$\Delta K = 0.6891$		$\Delta K = 0.8463$	
50	0	0.2000	0,1	1.0000	31	1.8203	53	2.9621	63	3.8189	69	4.5258
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1937$		$\Delta K = 0.4918$		$\Delta K = 0.7073$		$\Delta K = 0.8753$	
60	0	0.2000	0,1	1.0000	37	1.8217	63	2.9682	75	3.8314	82	4.5459
	$\Delta K = 0.0000$		$\Delta K = 0.0000$		$\Delta K = 0.1950$		$\Delta K = 0.4978$		$\Delta K = 0.7198$		$\Delta K = 0.8954$	

Table 2. (continued) Cost comparisons between (1, T) and (S-1, S) policies for $\mu = 2$

T^*	0.6447	0.6287	0.6169	0.6076	0.6002	0.5941
I^*	1.8804	2.0961	2.2941	2.4782	2.6508	2.8140
$K(T^*)$	4.1246	4.5529	4.9464	5.3126	5.6565	5.9816

π/h	5	6	7	8	9	10
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L	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*	S^*	K_S^*
1	4	3.1429	4	3.3333	4	3.5238	5	3.6606	5	3.7339	5	3.8073
	$\Delta K = -0.9817$		$\Delta K = -1.2195$		$\Delta K = -1.4226$		$\Delta K = -1.6521$		$\Delta K = -1.9225$		$\Delta K = -2.1743$	
2	6	3.6403	6	3.8746	6	4.1089	7	4.2550	7	4.3805	7	4.5060
	$\Delta K = -0.4843$		$\Delta K = -0.6783$		$\Delta K = -0.8375$		$\Delta K = -1.0577$		$\Delta K = -1.2760$		$\Delta K = -1.4756$	
3	8	3.9500	8	4.1938	8	4.4375	9	4.6532	9	4.8035	9	4.9538
	$\Delta K = -0.1746$		$\Delta K = -0.3591$		$\Delta K = -0.5089$		$\Delta K = -0.6594$		$\Delta K = -0.8530$		$\Delta K = -1.0278$	
4	9	4.1165	10	4.4332	10	4.6765	10	4.9199	11	5.1135	11	5.2761
	$\Delta K = -0.0081$		$\Delta K = -0.1196$		$\Delta K = -0.2699$		$\Delta K = -0.3928$		$\Delta K = -0.5430$		$\Delta K = -0.7055$	
5	11	4.2646	11	4.5911	12	4.8737	12	5.1132	12	5.3527	13	5.5302
	$\Delta K = 0.1400$		$\Delta K = 0.0383$		$\Delta K = -0.0727$		$\Delta K = -0.1994$		$\Delta K = -0.3038$		$\Delta K = -0.4514$	
6	12	4.3685	13	4.7176	13	5.0274	14	5.2819	14	5.5163	15	5.7433
	$\Delta K = 0.2439$		$\Delta K = 0.1648$		$\Delta K = 0.0810$		$\Delta K = -0.0308$		$\Delta K = -0.1402$		$\Delta K = -0.2383$	
7	14	4.4593	14	4.8309	15	5.1381	15	5.4336	16	5.6642	16	5.8932
	$\Delta K = 0.3347$		$\Delta K = 0.2780$		$\Delta K = 0.1916$		$\Delta K = 0.1210$		$\Delta K = 0.0078$		$\Delta K = -0.0884$	
8	15	4.5269	16	4.9086	17	5.2488	17	5.5321	18	5.8017	18	6.0253
	$\Delta K = 0.4023$		$\Delta K = 0.3558$		$\Delta K = 0.3024$		$\Delta K = 0.2194$		$\Delta K = 0.1452$		$\Delta K = 0.0437$	
9	17	4.5921	17	4.9915	18	5.3271	19	5.6317	19	5.9041	20	6.1501
	$\Delta K = 0.4675$		$\Delta K = 0.4387$		$\Delta K = 0.3806$		$\Delta K = 0.3190$		$\Delta K = 0.2476$		$\Delta K = 0.1685$	
10	18	4.6378	19	5.0451	20	5.4023	20	5.7201	21	5.9946	21	6.2574
	0.5132		$\Delta K = 0.4922$		$\Delta K = 0.4559$		$\Delta K = 0.4075$		$\Delta K = 0.3381$		$\Delta K = 0.2758$	
15	25	4.8130	26	5.2660	27	5.6624	28	6.0128	29	6.3288	30	6.6230
	$\Delta K = 0.6884$		$\Delta K = 0.7131$		$\Delta K = 0.7159$		$\Delta K = 0.7002$		$\Delta K = 0.6724$		$\Delta K = 0.6414$	
20	32	4.9150	34	5.3914	35	5.8124	36	6.1914	37	6.5352	38	6.8514
	$\Delta K = 0.7904$		$\Delta K = 0.8385$		$\Delta K = 0.8660$		$\Delta K = 0.8788$		$\Delta K = 0.8787$		$\Delta K = 0.8698$	
30	46	5.0302	48	5.5391	50	5.9928	51	6.4064	53	6.7813	54	7.1280
	$\Delta K = 0.9056$		$\Delta K = 0.9863$		$\Delta K = 1.0463$		$\Delta K = 1.0937$		$\Delta K = 1.1248$		$\Delta K = 1.1464$	
40	60	5.0943	63	5.6221	65	6.0955	67	6.5270	68	6.9232	70	7.2909
	$\Delta K = 0.9697$		$\Delta K = 1.0692$		$\Delta K = 1.1490$		$\Delta K = 1.2143$		$\Delta K = 1.2667$		$\Delta K = 1.3093$	
50	74	5.1354	77	5.6752	80	6.1623	82	6.6066	84	7.0166	85	7.3976
	$\Delta K = 1.0108$		$\Delta K = 1.1224$		$\Delta K = 1.2159$		$\Delta K = 1.2939$		$\Delta K = 1.3601$		$\Delta K = 1.4160$	
60	87	5.1641	91	5.7130	94	6.2095	97	6.6631	99	7.0829	101	7.4735
	$\Delta K = 1.0395$		$\Delta K = 1.1601$		$\Delta K = 1.2630$		$\Delta K = 1.3504$		$\Delta K = 1.4264$		$\Delta K = 1.4919$	

For the case $\mu = 2$, Table 2 provides a similar set of results for the same range of values for L and π/h for which the maximum percent change in total cost increase due to application of $(S-1, S)$ policy instead of $(1, T)$ policy is better than 24 percent.

Figure 1 depicts the behavior of ΔK in terms of the percent change as a function of L , π/h , and $\mu = 1$.

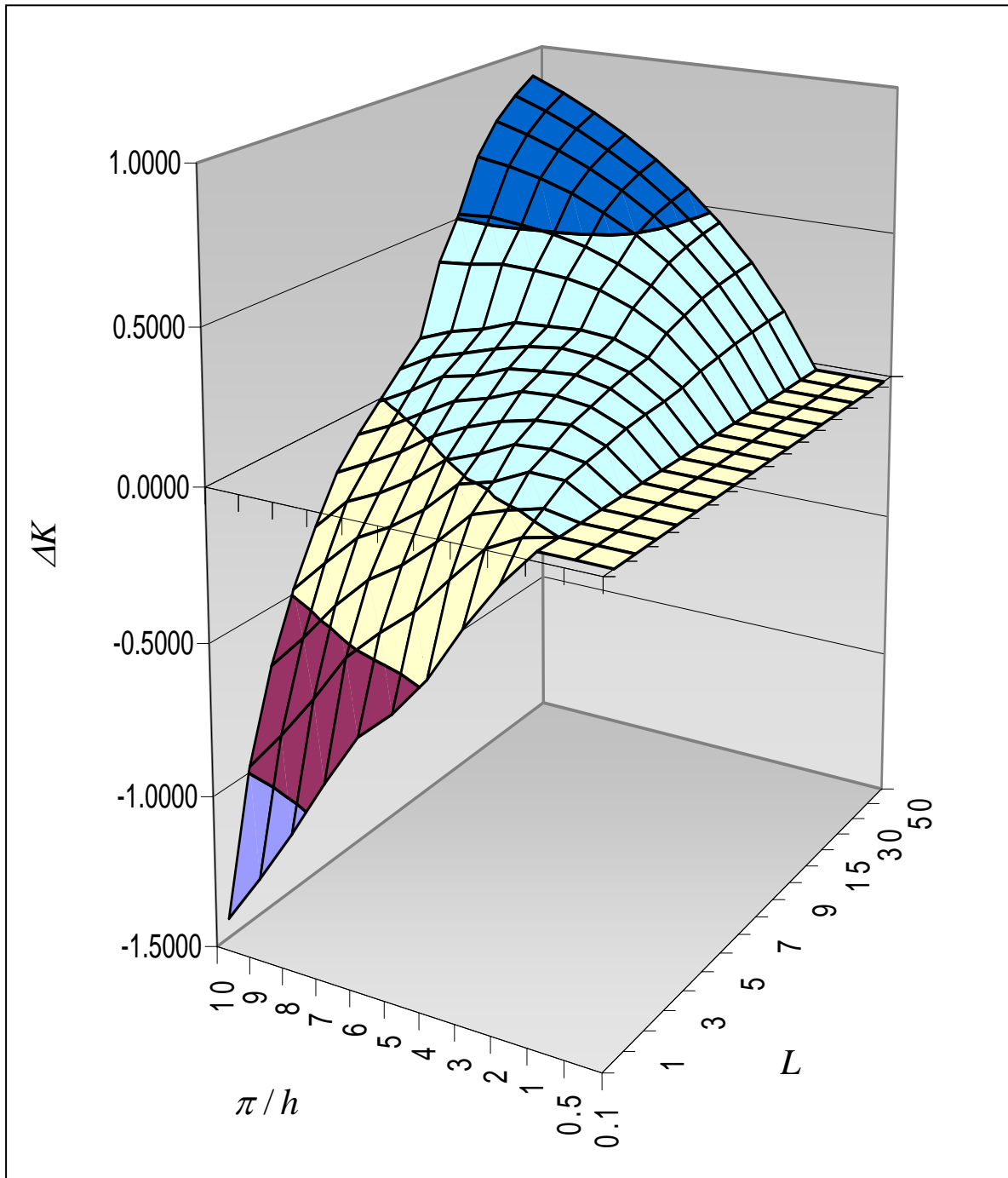


Figure 1. The value of ΔK for $\mu=1$ as a function of L and π/h

Figures 2 and 3 display the variations in percent change in ΔK for $\mu = 2$ and 3 with the same values of L , and π/h .

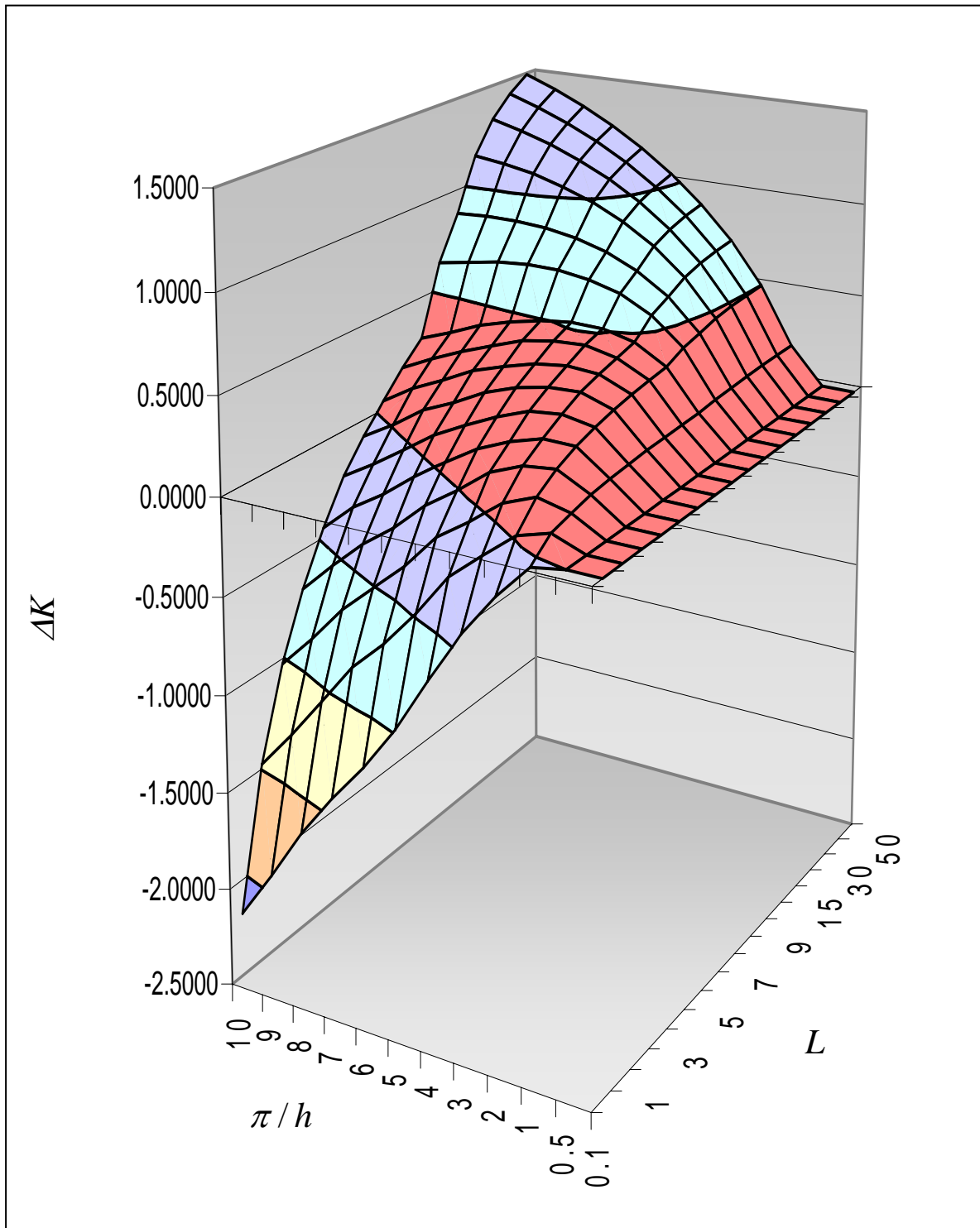


Figure 2. The value of ΔK for $\mu = 2$ as a function of L and π/h

As can be seen, the increase in percent change in ΔK becomes more pronounced as μ increases from 1 to 3.

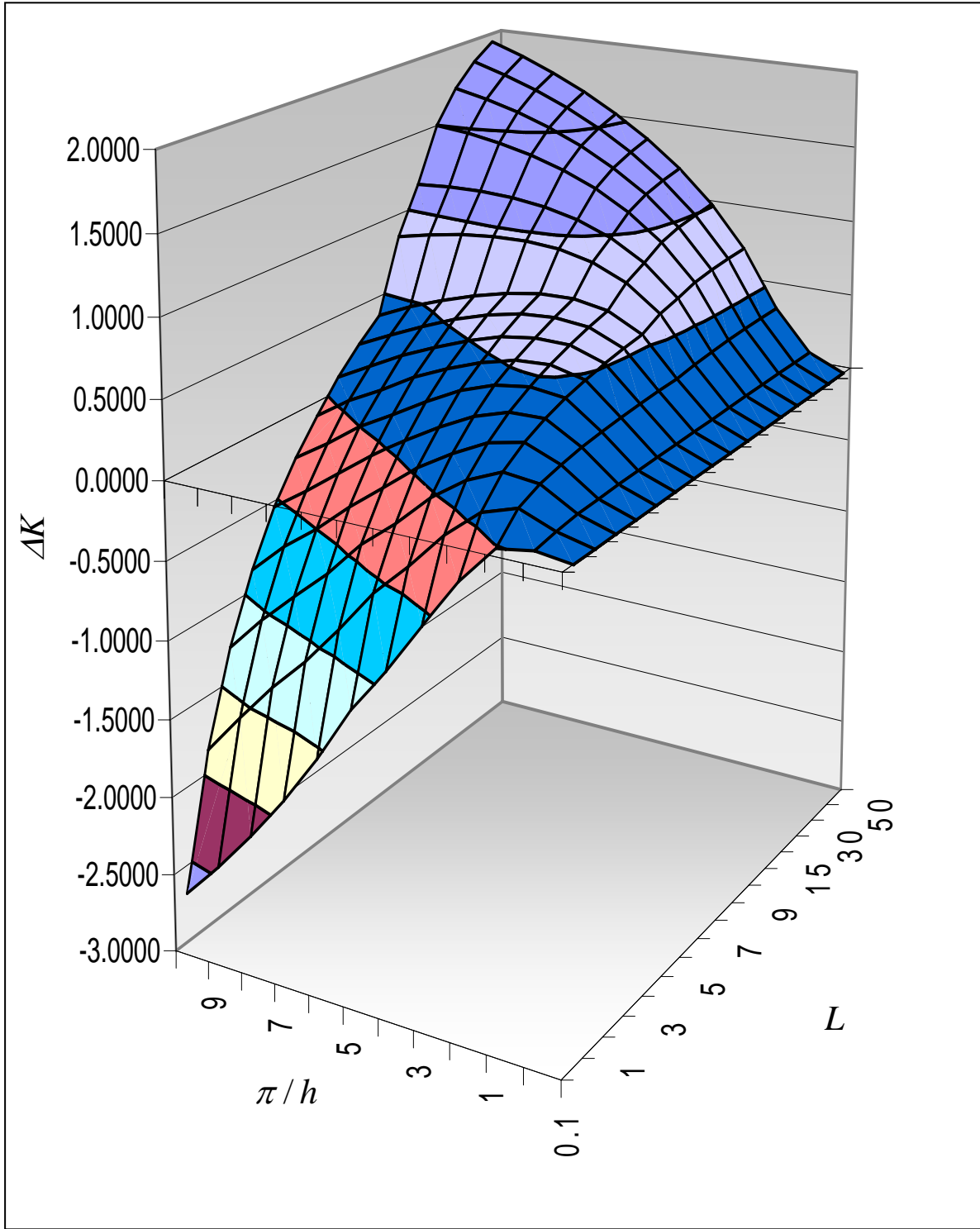


Figure 3. The value of ΔK for $\mu = 3$ as a function of L and π/h

5. CONCLUSION

While it is well known that for the case of stochastic demand, there is no inventory control policy in which both the order size and the order interval are constant with an optimal solution, in this paper, we introduced the optimal solution for a new policy which is different from the classical inventory policies. An important characteristic of this policy is that it eliminates the uncertainty in demand for suppliers. This new policy is to order 1 unit at each fixed time period T . We called this new policy *one for one period ordering* or $(1, T)$ policy. In this policy *the time interval between two consecutive orders and the value of the order size are both constant*.

We assumed that the fixed ordering cost is zero or negligible, demand process is a Poisson process, and the replenishment lead time is constant. Then we derived the long-run average total cost, consisting of holding and shortage costs, per unit time for the lost sales case in terms of the average inventory level. To obtain this total cost rate we used some concepts from queueing theory and proved that it is a convex function in terms of the average inventory on hand. The one-for-one period, or $(1, T)$ policy enjoys several advantages one of which is the very significant property that establishes the fact that its total cost rate is free of the lead time length.

Further, we obtained the optimal value of T , the time interval between two consecutive orders, which minimizes the total expected cost. Finally, by a numerical example we compared the total cost rate of this policy with the total cost rate of the one-for-one ordering or $(S-1, S)$ policy for the lost sales case. This example also served to re-examine the optimality of (s, S) policy in continuous review inventory models and established the fact that even for the case where demand forms a Poisson process the optimality of (s, S) policy does not hold.

REFERENCES

- [1] Andersson J., Melchior P. (2001), A two-echelon inventory model with lost sales; *International Journal of Production Economics* 69(3); 307-315.
- [2] Archibald B.C. (1981), Continuous review (s, S) policies with lost sales; *Management Science* 27; 1171-1178.
- [3] Axsäter S. (1990), Simple solution procedures for a class of two-echelon inventory problems; *Operation Research* 38; 64-69.
- [4] Axsäter S. (1993), Exact and approximate evaluation of batch-ordering policies for two-level inventory systems; *Operations .Research* 41; 777-785.
- [5] Axsäter S. (2003), Approximate optimization of a two-level distribution inventory system; *International Journal of Production Economics* 81-82; 545-553.
- [6] Axsäter S., Forsberg R., Zhang W.F. (1994), Approximating general multi-echelon inventory systems by Poisson models; *International Journal of Production Economics* 35; 201-206.
- [7] Axsäter S., Zhang W.F. (1999), A joint replenishment policy for multi-echelon inventory control; *International Journal of Production Economics* 59; 243-250.
- [8] Beckman M. (1961), An inventory model for arbitrary interval and quantity distribution of demand; *Management Science* 8; 35-57.

- [9] Federgruen A., Zheng Y. S. (1992), An efficient algorithm for computing an optimal (r, Q) policy for continuous review stochastic inventory systems; *Operations Research* 40; 808-812.
- [10] Forsberg R. (1995), Optimization of order-up-to-S policies for two-level inventory systems with compound Poisson demand; *European Journal of Operational Research* 81; 143-153.
- [11] Forsberg, R. (1996), Exact evaluation of (R, Q) -policies for two-level inventory systems with Poisson demand; *European Journal of Operational Research*; 96; 130-138.
- [12] Forsberg R. (1997), Evaluation of (R, Q) policies for two-level inventory systems with generally distributed customer inter-arrival times; *European Journal of Operational Research* 99; 401-411.
- [13] Ganeshan R. (1999), Managing supply chain inventories: A multiple retailer, one warehouse, multiple supplier model; *International Journal of Production Economics* 59; 341-354.
- [14] Ghalebsaz-Jeddi B, Shultes B. C, Haji R. (2004), A multi-production continuous review inventory system with stochastic demand, backorders, and a budget constraint; *European Journal of Operational Research* 158; 456-469.
- [15] Graves S. (1985), A multi-echelon inventory model for a repairable item with one-for-one replenishment; *Management Science* 31; 1247-1256.
- [16] Hadley G., Whitin. T. (1963), Analysis of inventory systems. Prentice Hall. Englewood Cliffs. NJ.
- [17] Haji R., Pirayesh M., Baboli A. (2006), A new replenishment policy in a two-echelon inventory system with stochastic demand, *International Conference on Service System and Service Management-ICSSSM'06* Proceeding, 256-260 Troyes, France, 25-27 October
- [18] Iglehart. D. (1963), Optimality of (s, S) policies in the infinite horizon dynamic inventory problem; *Management Science* 9; 259-267.
- [19] Johnson L.A., Montgomery D.C. (1974), Operation research in production planning, scheduling, and inventory control, John Wiley & Sons, New York.
- [20] Kruse W. K. (1980), Waiting time in an $(S-1, S)$ inventory system with arbitrary distributed lead times; *Operations Research* 28; 348-352.
- [21] Kruse W. K. (1981), Waiting time in a continuous review (s, S) inventory system with constant lead times; *Operations Research* 29; 202-206.
- [22] Lee H.L., Moinzadeh K. (1987a), Two-parameter approximations for multi-echelon repairable inventory models with batch ordering policy; *IIE Transactions* 19; 140-149.
- [23] Lee H.L., Moinzadeh K. (1987b), Operating characteristics of a two-echelon inventory system for repairable and consumable items under batch ordering and shipment policy; *Naval Research Logistics Quarterly* 34; 356-380.
- [24] Love S. (1979), Inventory control, McGraw-Hill Book Company, New York

- [25] Marklund J., (2002), Centralized inventory control in a two-level distribution system with Poisson demand; *Naval Research Logistics* 49;798-822.
- [26] Matta K.F., Sinha D. (1995), Policy and cost approximations of two-echelon distribution systems with a procurement cost at the higher echelon; *IIE Transactions* 27(5); 646-656.
- [27] Moinzadeh K. (2001), An improved ordering policy for continuous review inventory systems with an arbitrary inter-demand time distributions; *IIE Transactions* 33; 111-118.
- [28] Moinzadeh K. (2002), A multi-echelon inventory system with information exchange, *Management Science* 48(3); 414-426.
- [29] Moinzadeh; K., Lee H.L. (1986), Batch size and stocking levels in multi-echelon repairable systems; *Management Science* 32; 1567-1581.
- [30] Nahmias S. (1976), On the equivalence of three approximate continuous review inventory model; *Naval Research Logistics Quarterly* 23; 31-38.
- [31] Nahmias S. (2005), Production and operations analysis; 5th edition, McGraw-Hill/Irwin; New York.
- [32] Nahmias S., Smith S.A. (1994), Optimizing inventory levels in a two-echelon retailer system with partial lost sales; *Management Science* 40; 582-596.
- [33] Ross S. M. (1993), Introduction to probability models Fifth Edition, Academic press, New York.
- [34] Sahin I. (1979), On the stationary analysis of continuous review (s, S) inventory systems with constant lead times; *Operations Research* 27; 717-729.
- [35] Sahin I. (1982), On the objective function behavior in (s, S) inventory models; *Operations Research* 30; 709-724.
- [36] Sahin I. (1990), Regenerative inventory systems Springer-Verlag, New York. NY.
- [37] Scarf H. E. (1960), The optimality of (s, S) policies in the dynamic inventory problem. In *Mathematical Methods in the social Sciences*. Stanford University Press. Stanford. CA.
- [38] Schultz C. R. (1989), Replenishment delays for expensive slow-moving items; *Management Science* 35; 1454-1462.
- [39] Seo Y., Jung S., Hahm J. (2002), Optimal reorder decision utilizing centralized stock information in a two-echelon distribution system; *Computers & Operations Research* 29; 171-193.
- [40] Sherbrooke C. C. (1968), METRIC: a multi-echelon technique for recoverable item control; *Operations Research* 16; 122-141.
- [41] Sivlazier B. D. (1974), A continuous review (s, S) inventory system with arbitrary interval distribution between unit demand; *Operations Research* 22; 65-71.

- [42] Tijms H.C. (1986), *Stochastic modeling and analysis-A computational approach*. John Wiley and Sons, New York.
- [43] Zheng Y. S. (1992), On properties of stochastic inventory systems; *Management Science* 38; 87-130.
- [44] Zipkin P. (1986a), Stochastic lead times in continuous review inventory systems; *Naval Research Logistics Quarterly* 35; 763-774.
- [45] Zipkin P. (1986b), Inventory service level measures: convexity and approximations; *Management Science* 32; 975-981.
- [46] Zipkin P. H. (2000), *Foundation of inventory management*. McGraw-Hill, New York.

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