Journal of Applied Fluid Mechanics, Vol. 7, No. 3, pp. 503-512, 2014.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.

Radiation Effect on MHD Fully Developed Mixed Convection in a Vertical Channel with Asymmetric Heating

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(Received January 27, 2013; accepted April 3, 2013)

ABSTRACT

Effects of radiative heat transfer on an MHD fully developed mixed convective flow of a viscous incompressible electrically conducting fluid through a vertical channel with asymmetric heating of walls in the presence of a uniform transverse magnetic field has been studied. An exact solution of the governing equations has been obtained in closed form. It is observed that the velocity field is greatly influenced by the radiative heat transfer as well as bouyancy forces. The induced magnetic field decreases near the cool wall and it increases near the hot wall of the channel with an increase in radiation parameter. Further, an increase in radiation parameter leads to a decrease in the fluid temperature in the channel. A limiting consideration of the solutions of the governing equations of the flow are analyzed for $Ra \ll 1$.

Keywords: MHD mixed convective flow, Grashof number, radiation parameter, Prandtl number and asymmetric heating.

NOMENCLATURE

B_x	induced magnetic field component along x -direction	Pr	Prandtl number
b_x	non-dimensional induced magnetic field component	q_r	radiative heat flux
$\vec{B}$	the magnetic field vector	r_T	temperature difference ratio
e_{λ_p}	planck function	Ra	radiation parameter
g	acceleration due to gravity	T	fluid temperature
Gr	Grashof number	T_0	temperature at entrance of channel
Gr_0, Gr_1	Critical Grashof numbers	T_1	temperature of hot wall
K_{λ_0}	absorption coefficient	T_2	temperature of cool wall
k	thermal conductivity	u	velocity component in x -direction
M^2	magnetic parameter	u_1	non-dimensional fluid velocity
p	fluid pressure	(x, y)	cartesian co-ordinates

Greek symbols

α	non-dimensional pressure gradient	θ	dimensionless temperature
β	coefficient of thermal expansion	ρ	fluid density
μ_e	magnetic permeability	ρ_0	fluid density at entrance of channel
ν	kinematic viscosity	σ	conductivity of fluid
λ	wave length	τ_0, τ_1	shear stresses at cool and hot walls
η	non-dimensional width of the channel		

1. INTRODUCTION

Heat transfer in free and force convection in vertical channels occurs in any industrial processes and natural phenomena. Most of the interest in this subject is due to its applications, for instance, in the design of cooling systems or electronic devices, chemical processing equipment, microelectronic cooling and in the field of solar energy collections. Since some fluids can also emit and absorb thermal radiation, it is of interest to study the effects of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing radiation. Hence, heat transfer by thermal radiation is becoming of greater importance in space applications and higher operating temperatures. Ozisik(1989) has mentioned in an excellent review article that heat transfer by simultaneous radiation and convection has applications in numerous technological problems, including combustion, furnace design, the design of high-temperature gas-cooled nuclear reactors, nuclear-reactor safety, fluidized-bed heat exchagers, fire spreads, advanced enegy conservation devices such as open-cycle coal and natural-gas-fired MHD, solar ponds,solar collectors, natural convection in cavities and many others. On the other hand, it is worth mentionaing that heat transfer by simultaneous radiation and convection is very important in the context of space technology and processes involving high temperature. An excellent description of the fundamentals of thermal radiation has been presented in the book by Modest (2003). For a comprehensive treatment of the radiation transfer and the interractions with convection the interested reader can consult also the books by Sprrow and Cess 1970; Ozisik 1973 ; Siegel and Howell 1992). (Aung 1972; Aung *et al.*1972; Aung and Worku 1986; Barletta 2002 ; Boulama and Galanis 2004) deal with the evaluation of the temperature and velocity profiles for the vertical parallel-flow fully develoved regime. As is well known, heat exchangers technology involves convective flows in vertical channels. In most cases, these flows imply conditions of uniform heating of a channel, which can be modelled either by uniform wall temperature or uniform heat flux thermal boundary conditions. Many process in new engineering areas occur at high temperatures and knowledge of radiative heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles satellites and space vehicles are examples of such engineering areas. Cogley *et al.*(1968) have studied the differential approximation for radiative heat transfer in a non-grey gas near equilibrium. The hydrodyamic fully developed laminar convective flow through a vertical channel in the optically thin limit has been studied by Greif *et al*. (1971) whereas Gupta and Gupta (1974) have studied the same problem in the presence of a transverse magnetic field. The effects of radiative heat transfer on MHD flows in vertical channel have been studied a number of researches. Datta and Jana (1976) have discussed the effect of wall conductances on hydromagnetic convection of a radiating gas in a vertical channel. Ogulu and Motsa (2005) have studied the radiative heat transfer to magnetohydrodynamic Couette flow with variable wall temperature. Effect of radiation on unsteady free convection flow bounded by an oscillating plate with variable wall temperature have been described by Pathak *et al*.(2006). Sharma *et al.* (2007) have discussed the radiation effect on temperature distribution in three-dimensional Couette flow with suction / injection. The radiation effect on MHD free convection flow of a gas past a semi-infinite vertical plate have been studied by Takhar *et al*. (1996). The effects of wall conductance on MHD fully developed flow with asymmetric heating of the walls has been studied by Guria *et al*. (2007). Ghosh and Nandi (2000) have discussed magnetohydrodynamic fully developed combined convection flow between vertical plates heated asymmetrically. MHD fully developed mixed convection flow with asymmetric heating of the walls have been described by Ghosh *et al.* (2002). Pantokratoras (2006) has presented his results for a steady free convection flow between vertical parallel plates by considering different conditions on the wall temperature. The thermal radiation effect on fully developed mixed convection flow in a vertical channel have been examined by Grosan and Pop (2007). Suneetha *et al*. (2011) have presented the radiation and mass transfer effects on MHD free convective dissipative fluid in the presence of heat source/sink. Effects of thermal radiation on hydromagnetic flow due to a porous rotating disk with Hall effect have been studied by Anjali Devi and Uma Devi (2012). Baoku *et al.* (2012) have analyzed the influence of thermal radiation on a transient MHD Couette flow through a porous medium.

In the present paper, we study the effects of radiative heat transfer on MHD fully developed mixed convective flow in a vertical channel with the asymmetric heating of walls in the presence of a transverse magnetic field. We assume that the radiative heat flux follows a relation in an optically thin limit for a non-gray gas near equilibrium. The closed form solutions for velocity, temperature, shear stresses, rate of heat transfer and critical Grashof number are presented. Flow and heat transfer results for a range of values of the pertinent parameters have been reported. It is observed that the velocity field is greatly influenced by the radiative heat transfer as well as bouyancy forces. The induced magnetic field decreases near the cool wall and it increases near the hot wall of the vertical channel with an increase in radiation parameter. Further, an increase in radiation parameter leads to decrease the fluid temperature in the channel.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider a steady MHD fully developed mixed convective flow of a viscous incompressible electrically conducting fluid confined between vertical walls. The channel walls are at a distance d apart. Choose a Cartesian co-ordinates system with x-axis in the upward direction along the cool wall in the direction of flow and the axis of y is perpendicular to it. The wall at $y=0$ has a uniform temperature T_2 while the wall at $y=d$ is subjected to a uniform temperature T_1, where $T_1 > T_2$. A uniform magnetic field of strength B_0 is imposed perpendicular to the channel walls. The flow is due to buoyancy force, difference in temperature and in the presence of pressure gradient. Since the channel walls are infinitely long along the x-direction, all physical quantities, are functions of y only. The velocity components are (u, v) relative to the Cartesian frame of reference.

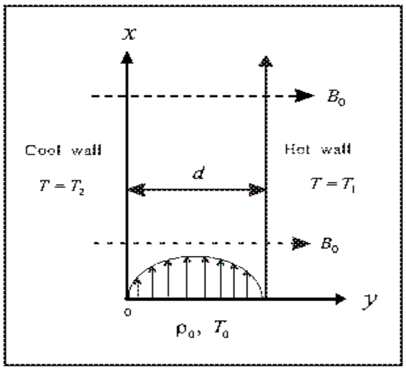

Fig. 1. Geometry of the problem

The Boussinesq approximation is assumed to hold and for the evaluation of the gravitational body force, the density is assumed to depend on the temperature according to the equation of state

$$\rho = \rho_0[1-\beta(T-T_0)], \tag{1}$$

where T is the fluid temperature, ρ the fluid density, β the coefficient of thermal expansion and T_0 and ρ_0 being the temperature and the density at the entrance of the channel.

The solenoidal equation $\nabla\cdot\vec{B}=0$ gives $B_y =$ constant $= B_0$ everywhere in the flow where $\vec{B}\equiv(B_x, B_0, 0)$. The flow being fully developed the relations $v=0$, $\frac{\partial v}{\partial y}=0$ and $\frac{\partial p}{\partial y}=0$ apply here, where p is the fluid pressure. Therefore, the continuity equation gives $\frac{\partial u}{\partial x}=0$ and hence $u=u(y)$. Using the Boussinesq approximation (1), the momentum equation along x-axis and the magnetic induction equation are

$$\nu\frac{d^2u}{dy^2}+\mu_e B_0\frac{dB_x}{dy}+\rho_0 g\beta(T-T_0)=\frac{dp}{dx}, \tag{2}$$

$$\frac{d^2B_x}{dy^2}+\sigma\mu_e B_0\frac{du}{dy}=0, \tag{3}$$

and the energy equation is

$$0 = k\frac{d^2T}{dy^2}-\frac{\partial q_r}{\partial y}, \tag{4}$$

where ν is the kinematic viscosity, μ_e the magnetic permeability, σ the conductivity of the fluid, g the acceleration due to gravity, k the thermal conductivity and q_r the radiative heat flux. It has been shown by Cogley *et al*.(1968) that in the optically thin limit for a non-gray gas near equilibrium, the following relation holds

$$\frac{\partial q_r}{\partial y}=4(T-T_0)\int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda, \tag{5}$$

where K_{λ_0} is the absorption coefficient, λ is the wave length, $e_{\lambda p}$ is the Planck's function and subscript $'0'$ indicates that all quantities have been evaluated at the temperature T_0 which is the temperature at entrance of the channel.

On the use of (5), the Eq.(4) becomes

$$0 = k\frac{d^2T}{dy^2}-4I(T-T_0), \tag{6}$$

where

$$I=\int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda$$

In asymmetric heating of the walls, the velocity, magnetic and temperature boundary conditions are respectively,

$$u=0,\ T=T_2 \text{ and } B_x=0 \text{ at } y=0,$$
$$u=0,\ T=T_1 \text{ and } B_x=0 \text{ at } y=d, \tag{7}$$

assuming the walls are electrically non-conducting.

Introducing the non-dimensional variables

$$\eta=\frac{y}{d},\ u_1=\frac{ud}{\nu},\ b_x=\frac{B_x}{\sigma\mu_e\nu B_0},\ \theta=\frac{T-T_0}{T_1-T_0}, \tag{8}$$

Eqs. (2), (3) and (6) become

$$\frac{d^2u_1}{d\eta^2}+M^2\frac{db_x}{d\eta}+Gr\,\theta+\alpha=0, \tag{9}$$

$$\frac{d^2b_x}{d\eta^2}+\frac{du_1}{d\eta}=0, \tag{10}$$

$$\frac{d^2\theta}{d\eta^2}-Ra\,\theta=0, \tag{11}$$

where $M=B_0\mu_0 d\left(\frac{\sigma}{\rho_0\nu}\right)^{\frac{1}{2}}$ is the Hartmann number, $Gr=\frac{g\beta(T_2-T_1)d^3}{\nu^2}$ the Grashof number and $Ra=\frac{4Id^2}{k}$ the radiation parameter and $\alpha=\frac{d^3}{\rho\nu^2}\left(-\frac{\partial p}{\partial x}\right)$ the non-dimensional pressure gradient.

The boundary conditions given by (7) become

$$u_1=0,\ \theta=r_T \text{ and } b_x=0 \text{ at } \eta=0$$
$$u_1=0,\ \theta=1 \text{ and } b_x=0 \text{ at } \eta=1, \tag{12}$$

where $r_T=\frac{T_2-T_0}{T_1-T_0}$ is the temperature difference ratio.

Solution of Eqs. (9) to (11) subject to the boundary conditions (12) are

$$\theta(\eta)=\begin{cases}\dfrac{\sinh\sqrt{Ra}\,\eta}{\sinh\sqrt{Ra}}+r_T\dfrac{\sinh\sqrt{Ra}\,(1-\eta)}{\sinh\sqrt{Ra}}, & Ra\neq M^2\\[2ex] \dfrac{\sinh M\eta}{\sinh M}+r_T\dfrac{\sinh M(1-\eta)}{\sinh M}, & Ra=M^2,\end{cases} \tag{13}$$

$$u_1(\eta)=\begin{cases}\left(\dfrac{\alpha}{M^2}+c\right)\left\{1-\dfrac{\sinh M\eta}{\sinh M}-\dfrac{\sinh M(1-\eta)}{\sinh M}\right\}\\ +\dfrac{Gr}{Ra-M^2}\left[\left\{\dfrac{\sinh M\eta}{\sinh M}+r_T\dfrac{\sinh M(1-\eta)}{\sinh M}\right\}\right.\\ \left.-\left\{\dfrac{\sinh\sqrt{Ra}\,\eta}{\sinh\sqrt{Ra}}+r_T\dfrac{\sinh\sqrt{Ra}\,(1-\eta)}{\sinh\sqrt{Ra}}\right\}\right], & Ra\neq M^2\\[2ex] \left(\dfrac{\alpha}{M^2}+c\right)\left\{1-\dfrac{\sinh M\eta}{\sinh M}-\dfrac{\sinh M(1-\eta)}{\sinh M}\right\}\\ -\dfrac{Gr}{2M\sinh M}\left[(r_T-\cosh M)\dfrac{\sinh M\eta}{\sinh M}\right.\\ \left.+\eta\left\{\cosh M\eta-r_T\cosh M(1-\eta)\right\}\right], & Ra=M^2,\end{cases} \tag{14}$$

$$b_x(\eta)=\begin{cases}c_1+c\eta-\left(\dfrac{\alpha}{M^2}+c\right)\left\{\eta-\dfrac{\cosh M\eta}{M\sinh M}+\dfrac{\cosh M(1-\eta)}{M\sinh M}\right\}\\ -\dfrac{Gr}{Ra-M^2}\left[\left\{\dfrac{\cosh M\eta}{M\sinh M}-r_T\dfrac{\cosh M(1-\eta)}{M\sinh M}\right\}\right.\\ \left.-\left\{\dfrac{\cosh\sqrt{Ra}\,\eta}{\sqrt{Ra}\sinh\sqrt{Ra}}-r_T\dfrac{\cosh\sqrt{Ra}\,(1-\eta)}{\sqrt{Ra}\sinh\sqrt{Ra}}\right\}\right], & Ra\neq M^2\\[2ex] c_1+c\eta-\left(\dfrac{\alpha}{M^2}+c\right)\left\{\eta-\dfrac{\cosh M\eta}{M\sinh M}+\dfrac{\cosh M(1-\eta)}{M\sinh M}\right\}\\ +\dfrac{Gr}{2M\sinh M}\left[(r_T-\cosh M)\dfrac{\cosh M\eta}{M\sinh M}\right.\\ +\dfrac{\eta}{M}\left\{\sinh M\eta+r_T\sinh M(1-\eta)\right\}\\ \left.-\dfrac{1}{M^2}\left\{\cosh M\eta-r_T\cosh M(1-\eta)\right\}\right], & Ra=M^2\end{cases} \tag{15}$$

where

$$c_1=\begin{cases}\dfrac{\alpha}{2M^2}+\dfrac{Gr(1-r_T)}{2\left(Ra-M^2\right)}\left[\dfrac{1+\cosh M}{M\sinh M}-\dfrac{1+\cosh\sqrt{Ra}}{\sqrt{Ra}\sinh\sqrt{Ra}}\right], & Ra\neq M^2\\[2ex] \dfrac{\alpha}{2M^2}+\dfrac{Gr(1-r_T)}{4M^3\sinh^2 M}(1+\cosh M)(M+\sinh M), & Ra=M^2,\end{cases} \tag{16}$$

$$c=\begin{cases}-\dfrac{\alpha}{M^2}\left\{1+\dfrac{M\sinh M}{2(1-\cosh M)}\right\}\\ +\dfrac{1}{2}\cdot\dfrac{Gr(1+r_T)}{Ra-M^2}\cdot\dfrac{M\sinh M}{1-\cosh M}\left[\dfrac{1-\cosh M}{M\sinh M}-\dfrac{1-\cosh\sqrt{Ra}}{\sqrt{Ra}\sinh\sqrt{Ra}}\right], & Ra\neq M^2\\[2ex] -\dfrac{\alpha}{M^2}\left\{1+\dfrac{M\sinh M}{2(1-\cosh M)}\right\}+\dfrac{Gr(1+r_T)}{4M^2\sinh M}(\sinh M-M), & Ra=M^2.\end{cases} \tag{17}$$

and α is given by (19).

It is seen from the expressions (13)-(15) that the velocity field and induced magnetic field depend on the Grashof number Gr, whereas the temperature distribution is independent of Gr. In the absence of radiation ($Ra=0$), the velocity distribution and the induced magnetic field coincide with Guria *et al.*(2007) in the case of without wall conductance.

3. EXPRESSION FOR PRESSURE GRADIENT

It is noticed from the Eqs. (14) and (15) that the parameter α is still to be evaluated. Using the rate of mass flow

$$\int_0^1 u_1\,d\eta=1, \tag{18}$$

we have, on the substitution of the value of u_1, which is obtained from Eq. (14) as

$$\alpha=\begin{cases}\dfrac{2M^2(\cosh M-1)}{M\sinh M+2(1-\cosh M)}\\ +\dfrac{Gr(1+r_T)}{Ra-M^2}\cdot\dfrac{M^3\sinh M}{M\sinh M+2(1-\cosh M)}\\ \times\left[\dfrac{1-\cosh M}{M\sinh M}-\dfrac{1-\cosh\sqrt{Ra}}{\sqrt{Ra}\sinh\sqrt{Ra}}\right], & Ra\neq M^2\\[2ex] \dfrac{2M^2(\cosh M-1)}{M\sinh M+2(1-\cosh M)}\\ \times\left[1-\dfrac{1}{4}Gr(1+r_T)\dfrac{\sinh M-M}{M^2\sinh M}\right], & Ra=M^2.\end{cases} \tag{19}$$

4. RESULTS AND DISCUSSION

To study the effects of radiative heat transfer and the magnetic field on the MHD fully developed flow with asymmetric heating of the walls, the dimensionless velocity u_1, the induced magnetic filed b_x and temperature distribution θ are depicted graphically against η for several values of radiation parameter Ra, magnetic parameter M^2, bounyancy parameter Gr and temperature difference ratio r_T in Figs.2 to 13. Figures.2 and 3 depicts the effects of the radiative heat transfer and magnetic field on the velocity field. It is seen from Fig.2 that the fluid velocity u_1 increases in the range $0\le\eta\le 0.38$ while it decreases in the range $0.38<\eta\le 0.86$ and again it increases in the range $0.86<\eta\le 1$ with an increase in radiation parameter Ra. It is manifested that there is a closeness of the curves near the hot wall. Fig.3 reveals that the fluid velocity u_1 increases in the range $0\le\eta\le 0.42$ while it decreases in the range $0.42<\eta\le 1$ with an increase in magnetic parameter M^2. It means that the Lorentz force imposed by a transverse magnetic field to an electrically conducting fluid, which slows down the fluid motion near the hot wall and enhances the fluid motion near cool wall. It is also manifested that there is a closeness of the curves near the hot wall. It is noticed from Fig.4 that the fluid velocity u_1 decreases in the region $0\le\eta\le 0.53$ and it increases in the region $0.53<\eta\le 1$ with increase in Grashof number Gr. Fig.5 shows that with an increase in r_T, the fluid velocity u_1 increases in the region $0\le\eta\le 0.53$ and it decreases in the region $0.53<\eta\le 1$. It is observed from Fig.6 that the induced magnetic field b_x decreases at any point near the cool wall and it increases near the hot wall with an increase in Ra. Fig.7 shows that the induced magnetic field b_x decreases with an increase in M^2. It is seen from Fig.8 that the induced magnetic field b_x increases with an increase in Gr. Fig.9 reveals that the induced magnetic field b_x decreases with an increase in r_T. The temperature profiles have been drawn for different values of Ra and r_T in Figs.10 and 11. It is seen from Figs.10 and 11 that the fluid temperature θ decreases with an increase in radiation parameter Ra while it increases with an increase in temperature difference ratio parameter r_T.

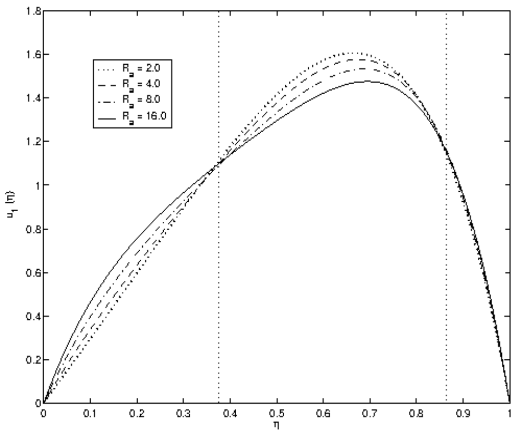

Fig. 2. Variation of velocity for different Ra **when** $M^2 = 5$, $r_T = 0.4$ **and** $Gr = 100$

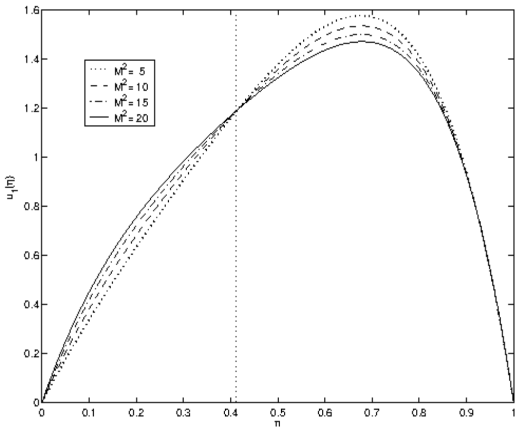

Fig. 3. Variation of velocity for different M^2 **when** $Ra = 4$, $r_T = 0.4$ **and** $Gr = 100$

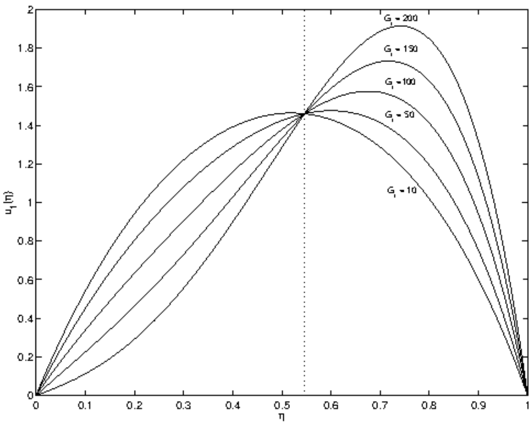

Fig. 4. Variation of velocity for different Gr **when** $M^2 = 5$, $Ra = 4$ **and** $r_T = 0.4$

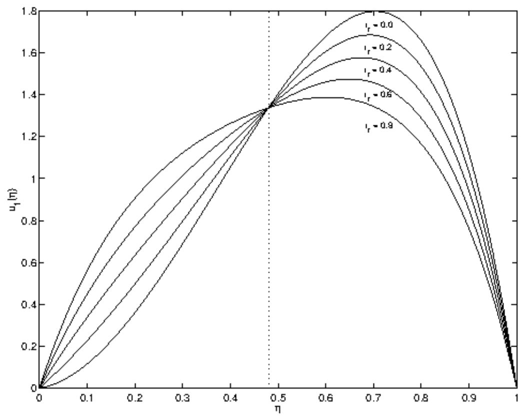

Fig. 5. Variation of velocity for different r_T **when** $M^2 = 5$, $Ra = 4$ **and** $Gr = 100$

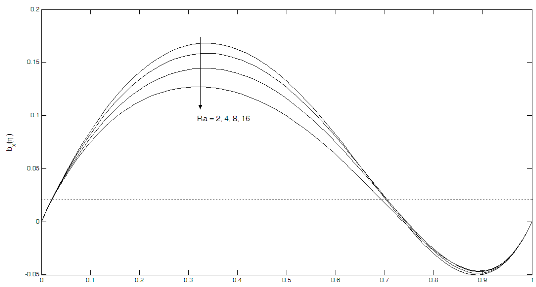

Fig. 6. Variation of induced magnetic field for different Ra **when** $M^2 = 5$, $r_T = 0.4$ **and** $Gr = 100$

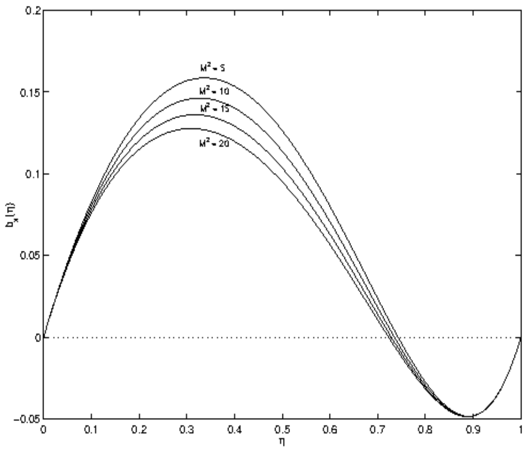

Fig. 7. Variation of induced magnetic field for different M^2 when $Ra = 4$, $r_T = 0.4$ and $Gr = 100$

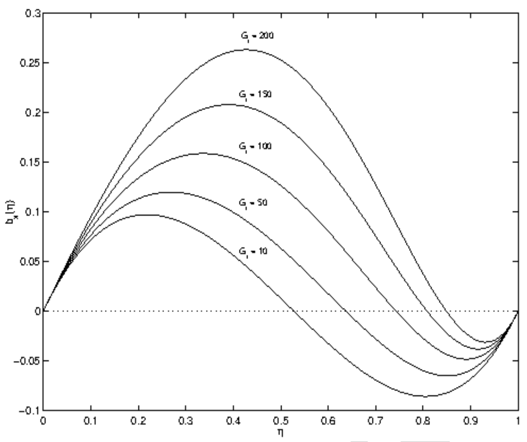

Fig. 8. Variation of induced magnetic field for different Gr when $M^2 = 5$, $Ra = 4$ and $Gr = 100$

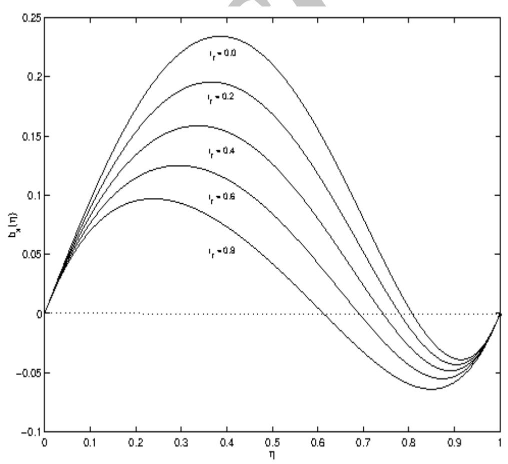

Fig. 9. Variation of induced magnetic field for different r_T when $M^2 = 5$, $Ra = 4$ and $Gr = 100$

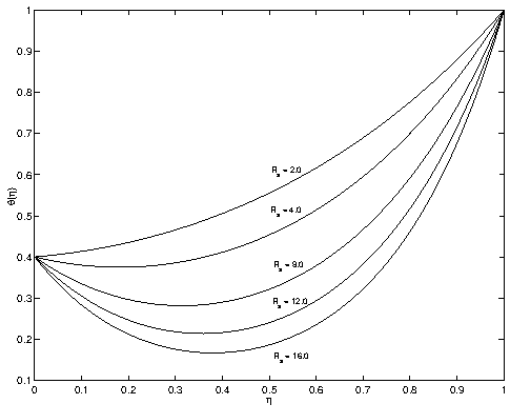

Fig. 10. Variation of temperature for different Ra when $r_T = 0.4$

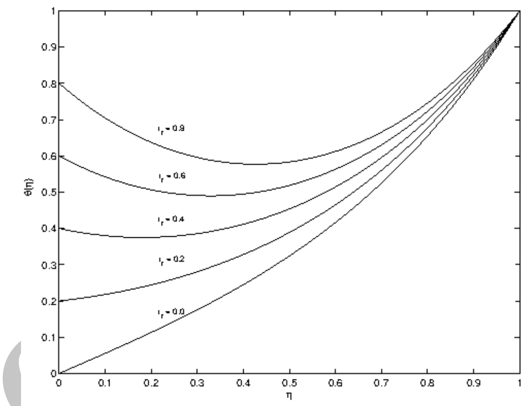

Fig. 11. Variation of temperature for different r_T when $Ra = 4$

One of the important characteristics of this problem is the shear stress at the walls. The non-dimensional shear stress at the cool wall $(\eta = 0)$ and hot wall $(\eta = 1)$ are respectively given by

$$\tau_{x_0} = \left(\frac{du_1}{d\eta}\right)_{\eta=0} \text{ and } \tau_{x_1} = \left(\frac{du_1}{d\eta}\right)_{\eta=1}, \qquad (20)$$

where

$$\left(\frac{du_1}{d\eta}\right)_{\eta=0} = \begin{cases} \dfrac{M^2(\cosh M - 1)}{M\sinh M + 2(1-\cosh M)} \\ \times\left[1 + \dfrac{Gr(1+r_T)}{Ra - M^2}\left(\dfrac{1-\cosh M}{M\sinh M} - \dfrac{1-\cosh\sqrt{Ra}}{\sqrt{Ra}\sinh\sqrt{Ra}}\right)\right] \\ + \dfrac{Gr}{Ra - M^2} \\ \times\left[\dfrac{M(1-r_T\cosh M)}{\sinh M} - \dfrac{\sqrt{Ra}(1-r_T\cosh\sqrt{Ra})}{\sinh\sqrt{Ra}}\right], Ra \neq M^2 \\ \\ \dfrac{M^2(\cosh M - 1)}{M\sinh M + 2(1-\cosh M)} \\ \times\left[1 + Gr(1+r_T)\dfrac{(1-\cosh M)(\sinh M - M)}{2M^3\sinh^2 M}\right] \\ -\dfrac{Gr}{2M\sinh M} \\ \times\left[(r_T - \cosh M)\dfrac{M}{\sinh M} + (1 - r_T\cosh M)\right], Ra = M^2 \end{cases}$$

(21)

and

$$\left(\frac{du_1}{d\eta}\right)_{\eta=1} = \begin{cases} \dfrac{M^2(1-\cosh M)}{M\sinh M + 2(1-\cosh M)} \\ \times\left[1+\dfrac{Gr(1+r_T)}{Ra-M^2}\left(\dfrac{1-\cosh M}{M\sinh M}-\dfrac{1-\cosh\sqrt{Ra}}{\sqrt{Ra}\sinh\sqrt{Ra}}\right)\right] \\ +\dfrac{Gr}{Ra-M^2} \\ \times\left[\dfrac{M(\cosh M - r_T)}{\sinh M}-\dfrac{\sqrt{Ra}(\cosh\sqrt{Ra}-r_T)}{\sinh\sqrt{Ra}}\right], Ra \neq M^2 \\ \\ \dfrac{M^2(1-\cosh M)}{M\sinh M + 2(1-\cosh M)} \\ \times\left[1+Gr(1+r_T)\dfrac{(1-\cosh M)(\sinh M - M)}{2M^3\sinh^2 M}\right] \\ -\dfrac{Gr}{2M\sinh M} \\ \times\left[(r_T\cosh M - 1)\dfrac{M}{\sinh M}+(\cosh M - r_T)\right], Ra = M^2 \end{cases} \tag{22}$$

The values of non-dimensional shear stresses at the walls $\eta = 0$ and $\eta = 1$ are shown graphically against r_T for different values of Ra and M^2 in Figs.12 and 13. It is observed from Fig.12 that for fixed values of M^2 and Gr the shear stress τ_{x_0} at the cool wall increases while the shear stress τ_{x_1} at the hot wall decreases with an increase in radiation parameter Ra. On the other hand, Fig.13 shows that the shear stress τ_{x_0} at the cool wall increases while the shear stress τ_{x_1} at the right wall decreases with increase in M^2. It is interesting to note that the shear stress due to the flow does not vanish at the walls $\eta = 0$ and $\eta = 1$ when buoyancy forces $Gr = 0$. Thus we arrive an interesting conclusion that there is no flow reversal in the absence of the buoyancy forces.

The shear stresses at the cool and hot wall vanish if

$$Gr_0 = \begin{cases} \dfrac{(Ra-M^2)M^2(1-\cosh M)}{(A+B)[M\sinh M + 2(1-\cosh M)]}, & Ra \neq M^2 \\ \\ \dfrac{2M^2(1-\cosh M)}{(A+B)[M\sinh M + 2(1-\cosh M)]}, & Ra = M^2, \end{cases} \tag{23}$$

$$Gr_1 = \begin{cases} \dfrac{(Ra-M^2)M^2(1-\cosh M)}{(A_1+B_1)[M\sinh M + 2(1-\cosh M)]}, & Ra \neq M^2 \\ \\ \dfrac{2M^2(1-\cosh M)}{(A_1+B_1)[M\sinh M + 2(1-\cosh M)]}, & Ra = M^2, \end{cases} \tag{24}$$

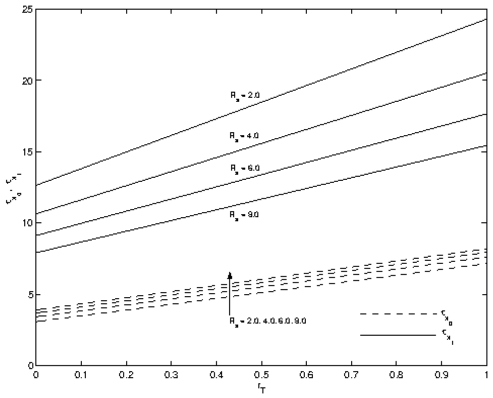

Fig. 12. Shear stress for different Ra when $M^2 = 5$, $Ra = 4$ and $Gr = 50$

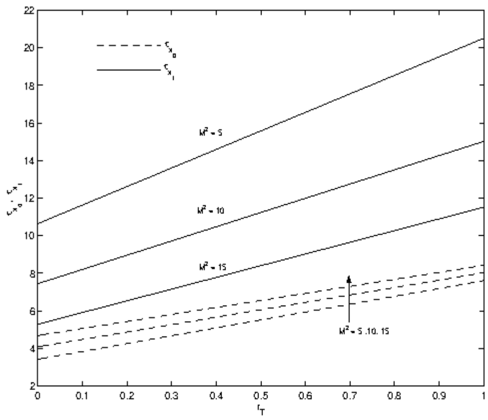

Fig. 13. Shear stress for different M^2 when $M^2 = 5$, $r_T = 0.4$ and $Gr = 50$

Table 1 The values of Critical Grashof number $Gr_0 \times 10^{-3}$ at the cool wall. Gr_0

	Ra			r_T		
M^2	2	4	6	0	0.2	0.4
5	0.1821947	0.2327067	0.2946396	0.1056287	0.1453024	0.2327067
10	0.2102455	0.2710896	0.347127	0.1214226	0.1677217	0.2710896
15	0.2385835	0.3102064	0.4012581	0.1373196	0.1903684	0.3102064
20	0.2671151	0.3498836	0.4567396	0.1532741	0.2131661	0.3498836

Table 2 The values of Critical Grashof number $Gr_1 \times 10^{-3}$ at the hot wall.

	Ra			r_T		
M^2	2	4	6	0	0.2	0.4
5	0.1197925	0.109435	0.1033845	0.07737131	0.09065144	0.109435
10	0.1361429	0.123871	0.1167035	0.08797246	0.10288010	0.123871
15	0.1524667	0.138243	0.1299393	0.09855560	0.11507350	0.138243
20	0.1687417	0.152538	0.1430853	0.10910590	0.1272174	0.1525389

The values of the critical Grashof numbers Gr_0 and Gr_1 due to the flow at the cool wall $\eta = 0$ and hot wall $\eta = 1$ are entered in the Tables 1 and 2 for different values of magnetic parameter M^2, radiation parameter Ra and the temperature difference ratio parameter r_T. It is seen from the Table 1 that the critical Grashof number Gr_0 at the cool wall $(\eta = 0)$ due to the flow increases with increase in either Ra or r_T or M^2. Table 2 shows that the the critical Grashof number Gr_1 at the hot wall $(\eta = 1)$ due to the flow increases with an increase in either M^2 or r_T. On the other hand, with increase in the radiation parameter Ra, the critical Grashof number Gr_1 decreases.

In the absence of radiative heat transfer ($Ra = 0$), the critical Grashof numbers Gr_0 and Gr_1 are given by

$$Gr_0 = \frac{2M^4(\cosh M - 1)}{(1-r_T)[M^2(1+\cosh M) + 4\cosh M - 4(1+\cosh M)]}, \tag{25}$$

$$Gr_1 = -Gr_0, \tag{26}$$

which are identical with the Eq. (39) of Guria *et al.* (2007).

4.1. Limiting Case

Now, we discuss the case when the radiation parameter is $Ra \ll 1$. In this case, Eqs. (13) to (15) become

$$\theta(\eta) = \eta + \frac{Ra}{6}(\eta^3 - \eta) + r_T\left[1-\eta) + \frac{Ra}{6}\eta(1-\eta)(\eta-2)\right], \tag{27}$$

$$u_1(\eta) = \left(\frac{\alpha}{M^2} + c\right)\left\{1 - \frac{\sinh M\eta}{\sinh M} - \frac{\sinh M(1-\eta)}{\sinh M}\right\} + \frac{Gr}{Ra - M^2}\left[\left\{\frac{\sinh M\eta}{\sinh M} + r_T\frac{\sinh M(1-\eta)}{\sinh M}\right\} - \left[\eta + \frac{Ra}{6}(\eta^3-\eta) + r_T\left\{1-\eta) + \frac{Ra}{6}\eta(1-\eta)(\eta-2)\right\}\right]\right] \tag{28}$$

$$b_x(\eta) = \frac{\alpha}{2M^2} + \frac{Gr(1-r_T)}{2(Ra-M^2)}\cdot\frac{1+\cosh M}{M\sinh M} + c\eta - \left(\frac{\alpha}{M^2} + c\right)\left\{\eta - \frac{\cosh M\eta}{M\sinh M} + \frac{\cosh M(1-\eta)}{M\sinh M}\right\} - \frac{Gr}{Ra-M^2}\left\{\frac{\cosh M\eta}{M\sinh M} - r_T\frac{\cosh M(1-\eta)}{M\sinh M}\right\} - \frac{3Gr}{(Ra-M^2)(Ra+6)}\left\{\frac{1}{2}(1-r_T) - \eta^2 + r_T(1-2\eta+\eta^2)\right\} \tag{29}$$

where

$$c = \left[-\frac{\alpha}{M^2} + \frac{Gr(1+r_T)}{2(Ra-M^2)}\right]\left[\frac{M\sinh M + 2(1-\cosh M)}{2(\cosh M - 1)}\right],$$

$$\alpha = \frac{2M^2(\cosh M - 1)}{M\sinh M + 2(1-\cosh M)} + \frac{M^2 Gr(1+r_T)}{2(Ra-M^2)} \tag{30}$$

In limit $Ra \to 0$, Eqs. (27) to (28) for the temperature, the velocity and the induced magnetic field yield respectively

$$\theta(\eta) = \eta + r_T(1-\eta), \tag{31}$$

$$u_1(\eta) = \frac{1}{M^2}\left[\left(Gr r_T + \alpha + M^2 c\right)\left\{1 - \frac{\sinh M\eta}{\sinh M} - \frac{\sinh M(1-\eta)}{\sinh M}\right\} + Gr(1-r_T)\left(\eta - \frac{\sinh M\eta}{\sinh M}\right)\right], \tag{32}$$

$$b_x(\eta) = -\frac{1}{M^2}\left[(Gr r_T + \alpha + M^2 c)\left\{\eta - \frac{\cosh M\eta}{M\sinh M} + \frac{\cosh M(1-\eta)}{M\sinh M}\right\} + Gr(1-r_T)\left(\frac{1}{2}\eta^2 - \frac{\cosh M\eta}{M\sinh M}\right)\right] + c\eta + c_1, \tag{33}$$

where

$$c = \left[r_T Gr + \alpha + \frac{1}{2}Gr(1-r_T)\right]\frac{[M\sinh M + 2(1-\cosh M)]}{2M^2(\cosh M - 1)}$$

$$c_1 = \frac{1}{2M^2}\left[(r_T Gr + \alpha) + \frac{1}{2}Gr(1-r_T)\left\{\frac{M\sinh M - 2(1+\cosh M)}{M\sinh M}\right\}\right]$$

$$\alpha = \frac{2M^2(\cosh M - 1)}{M\sinh M + 2(1-\cosh M)} - \frac{1}{2}Gr(1+r_T). \tag{34}$$

The velocity and the induced magnetic field given by (32) and (33) coincide with Eqs. (14) and (15) of Guria *et al.* (2007) in the case of without wall conductance.

5. CONCLUSION

An MHD fully developed mixed convection in a vertical channel in the presence of radiation have been studied. It is seen that the fluid velocity is strongly affected by the radiative heat transfer as well as bouyancy force. The induced magnetic field decreases near the cool wall and it increases near the hot wall of the channel with an increase in radiation parameter. Radiation decreases the fluid temperature. It is interesting to note that the shear stress due to the flow does not vanish at the channel walls when $Gr = 0$ and hence there is no flow reversal in the absence of the buoyancy force. This model finds applications in the design of cooling systems, chemical processing equipments and the field of solar energy collections.

REFERENCES

Anjali Devi, S.P. and R. Uma Devi (2012). Effects of thermal radiation on hydromagnetic flow due to a porous rotating disk with Hall effect. *J. Applied Fluid Mechanics* 5(2), 1-7.

Aung, W. and G. Worku (1986). Developing flow and flow reversal in a vertical channel with asymmetric wall temperature. *J.Heat Transfer* 108, 299-304.

Aung, W. and G.Worku (1986). Theory of fully develoed, combined convection including flow reversal. *J.Heat Transfer* 108, 485-488.

Aung, W., L.S. Fletcher and V. Sernas (1972). Developing laminer free convection between vertical flat plates asymmetric heating. *Int.J.Heat Mass Transfer* 15, 2293-2308.

Aung, W. (1972). Fully developed laminer free convection between vertical plates heated asymmetrically. *Int.J.Heat Mass Transfer* 15, 1577-1580.

Baoku, I. G., C. Israel-Cookey and B. I. Olajuwon (2012). Influence of thermal radiation on a transient MHD Couette flow through a porous medium. *J. Applied Fluid Mechanics* 5(1), 81-87.

Barletta, A. (2002). Fully developed mixed convection and flow reversal in a vertical rectangular duct with unifrom wall heat flux. *Int.J. Heat Mass Transfer* 45, 641-654.

Boulama, K. and N. Galanis (2002). Fully developed mixed convection and flow reversal in a vertical rectangular duct with uniform wall heat flux. *Int.J. Heat Mass Transfer* 45, 641-654.

Cogley, A.C., W.C.Vincenti and S.E. Gilles (1968). Differential approximation for radiative transfer in a non-grey gas near equilibrium. *AIAAJ* ,16 ,551-553.

Datta, N. and R.N. Jana (1976). Effects of wall conductances on hydromagnetic convection of a radiating gas in a vertical channel. *Int. J. Heat Mass Transfer*, 9, 1015-1019.

Ghosh, S.K, I. Pop and D.K. Nandi (2002). MHD fully developed mixed convection flow with asymmetric heating of the walls. *Int. J. Appl. Mech. and Engineering* 7, 1211.

Ghosh, S.K. and D.K. Nandi (2000). Magnetohydrodynamic fully developed combined convection flow between vertical plates heated asymmetrically. *Journal Technical Physics* ,41, 173.

Greif, R., I.S. Habib and J.C. Lin (1971). Laminar convection of a radiating gas in a vertical channel. *J. Fluid Mech.*, 46(3), 513-520.

Grosan, T. and I. Pop (2007). Thermal radiation effect on fully develoved mixed convection flow in a vertical channel *Technische Mechanik* 27(1), 37-47.

Gupta, P.S. and A. S. Gupta (1974). Radiation effect on hydromagnetic convection in a vertical channel. *Int.J. Heat Mass Transfer* 17, 1437-1442.

Guria, M., Das, B.K., Jana, R.N. and Ghosh, S.K. (2007). Effects of wall conductance on MHD fully developed flow with asymmetric heating of the walls. *Int.J.Fluid Mech.Research* 34(6), 521-534.

Modest, M. F. (2003). *Radiative Heat Transfer* (2nd edition). Academic Press, New York.

Ogulu, A. and S. Motsa (2005). Radiative heat transfer to magnetohydrodynamic Couette flow with variable wall temperature *Physica Scripta* 71, 336-339.

Ozisik, M.N. (1973). *Radiative Transfer and Interactions with Conduction and Convection*. Wiley, New York.

Ozisik, M.N. (1987). *Interaction of radiation with convection. In: Handbook of Single-Phase Convection Heat Transfer* (Kakac, S., Shah, R.K. and Aung W., eds.) . Wiley, New York.

Pantokratoras, A. (2006). Fully developed laminar free covection with variable thermophysical properties between two open-ended vertical parallel plates heated asymmetrically with large temperature differences. *ASME J. Heat Tranfer* (128), 405-408.

Pathak, G., C. H. Maheshwari and S. P. Gupta (2006). Effect of radiation on unsteady free convection flow bounded by an oscillating plate with variable wall temperature *Int. J. Appl. Mech.and Engineering* 2, 371-382.

Sharma, B.K., M. Agarwal and R.C. Chaudhary (2007). Radiation effect on temperature distribution in three-dimensional Couette flow with suction or injection. *Appl. Math. Mech. (English Edition)* 28(3), 309-316.

Siegel, R. and J.R. Howell (1992). *Thermal Radiation Heat Transfer (3rd edition)*. Hemisphere, New York.

Sparrow, E.M. and R. D. Cess (1970). *Radiation Heat Transfer*. Brooks/Cole, Belmont, California.

Suneetha, S., N. Bhaskar Reddy and V. Ramachandra Prasad (2011). Radiation and mass transfer effects on MHD free convective dissipative fluid in the presence of heat source/sink. *J. Applied Fluid Mechanics* 4(1), 107-113.

Takhar, H.S., S. R. Gorla and V.M.Soundalgekar (1996). Radiation effect on MHD free convection flow of a gas past a semi-infinite vertical plate. *Int. J. Num. Methods for Heat and Fluid Flow* 2(6), 77.

APPENDIX

$$A=\begin{cases}\dfrac{M^2(1+r_T)(\cosh M-1)}{M\sinh M+2(1-\cosh M)}\left[\dfrac{1-\cosh M}{M\sinh M}-\dfrac{1-\cosh\sqrt{Ra}}{\sqrt{Ra}\sinh\sqrt{Ra}}\right], & Ra\neq M^2\\[2ex] \dfrac{(1+r_T)(1-\cosh M)^2(M-\sinh M)}{M\sinh^2 M[M\sinh M+2(1-\cosh M)]}, & Ra=M^2,\end{cases}$$

$$B=\begin{cases}\dfrac{M(1-r_T\cosh M)}{\sinh M}-\dfrac{\sqrt{Ra}(1-r_T\cosh\sqrt{Ra})}{\sinh\sqrt{Ra}}, & Ra\neq M^2\\[2ex] \dfrac{M(\cosh M-r_T)+\sinh M(r_T\cosh M-1)}{M\sinh^2 M}, & Ra=M^2,\end{cases}$$

$$A_1=\begin{cases}\dfrac{M^2(1+r_T)(1-\cosh M)}{M\sinh M+2(1-\cosh M)}\left[\dfrac{1-\cosh M}{M\sinh M}-\dfrac{1-\cosh\sqrt{Ra}}{\sqrt{Ra}\sinh\sqrt{Ra}}\right], & Ra\neq M^2\\[2ex] \dfrac{(1+r_T)(1-\cosh M)^2(\sinh M-M)}{M\sinh^2 M[M\sinh M+2(1-\cosh M)]}, & Ra=M^2,\end{cases}$$

$$B_1=\begin{cases}\dfrac{M(\cosh M-r_T)}{\sinh M}-\dfrac{\sqrt{Ra}(\cosh\sqrt{Ra}-r_T)}{\sinh\sqrt{Ra}}, & Ra\neq M^2\\[2ex] \dfrac{M(1-r_T\cosh M)+\sinh M(r_T-\cosh M)}{M\sinh^2 M}, & Ra=M^2,\end{cases}$$