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# Modelling Supply and Demand Functions in the Water Market Under Uncertainty Conditions

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#### Abstract

Water markets have been introduced recently as an appropriate alternative to bureaucratic control and allocation of water resources. Water markets increase water use efficiency through the transfer of water to higher value uses. Several studies have been carried out to simulate hypothetical water markets under conditions of both certainty and uncertainty to show the potential gains that can be achieved by market participants. However, the effect of water supply and price risk has seldom been analyzed by the water market models presented. This study endeavours to introduce output and water input price risks into the water market models. For this purpose, an econometric mean-variance model, under output price risk and water market price risk is theoretically developed to derive demand and supply functions. This approach facilitates empirical estimation of demand and supply functions in actually formed water markets.

*Keywords*: water markets, risk, supply and demand functions, uncertainty.

مدلسازی توابع عرضه وتقاضا در بازار آب در شرایط عدم حتمیت

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#### چکیدہ

اخیراً بازار آب به عنوان یک گزینه مناسب برای نظام تخصیص و کنترل اداری منابع آب معرفی شده است. بازار آب می تواند با انتقال آب به سمت مصارف با ارزش افزوده بیشتر موجب افزایش کارائی مصرف آن شود. در مطالعات متعددی بازارهای آب فرضی جهت نشان دادن منافع بالقوه مشارکت کنندگان در بازار در شرایط حتمیت و عدم حتمیت شبیه سازی شده اند. اما ریسک قیمت وعرضه آب به ندرت در مدل های مرتبط با بازار آب وارد شده اند. این مطالعه ریسک قیمت محصول و نهاده آب را دراین ملل ها معرفی می کند. بدین منظور به طور نظری یک مدل اقتصاد سنجی " میانگین –واریانس" تحت شرایط ریسک قیمت محصول و ریسک قیمت بازاری آب جهت استخراج توابع عرضه و تقاضا ارائه می شود. نتایج این مطالعه به بر آورد تجربی توابع عرضه و تقاضا در بازارهای واقعی آب کمک خواهد نمود.

*كليدواژهها*: بازارهاي آب، ريسك، توابع عرضه وتقاضا، عدم حتميت.

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### Introduction

Water markets have been introduced recently as a viable alternative to the administrative allocation of water resources (Zekri and Easter, 2005). It has been shown that a water market increases water use efficiency through the transfer of water to uses with more value-added potential. During the last two decades several studies have been carried out to show the potential gains from water markets (Dinar and Letty, 1991; Weinberg et al., 1995; Garrido, 2000; Zekri and Easter, 2005; Gomez- Limon and Martinez, 2006). However, most studies have used a linear programming approach to simulate water markets. Linear programming models impose a fixed relationship between output and bundle of inputs. In practice, according to the law of "diminishing marginal productivity", when more of a particular input is used, holding all other inputs fixed, the associated marginal enhancement in output declines. On the other hand the linear programming approach, in contrast to econometric approach, cannot reveal differences between the VMPs of water when it is being used. This differential in fact leads to water transactions among market participants. Furthermore, these studies have investigated hypothetical water markets and have paid less attention to real markets. Thirdly, most studies have assumed a certainty condition in the water market whereas participants generally face a considerable level of risk over output price, water market price and other stochastic factors involved in agricultural production activity. These stochastic elements, in addition to farmers' attitude to risk, influence their decision making and water use.

In this paper we have tried to develop a theoretical model to derive demand and supply functions in an actually formed water market. For this purpose, at first, farmers' behaviour and the mechanism of water exchange will be explained in the agricultural water market. Then demand and supply functions will be derived under the certainty conditions. Finally, this model will be extended to include output and water market price risks.

#### **Materials and Methods**

### Water Exchange Mechanism in the Market

Consider two representative farmers who seek to maximize their profits (Figure.1). O1W0 and O2W0 represent farmers' water rights and D1 and D2 represent their water demand curves. If water exchange is not possible, farmers would allocate their water allowances to agricultural crops. Total value of production will be then  $O_1M_1BW_0$  and  $O_2M_2HW_0$ , respectively. Now we assume that farmers are allowed to exchange water through an existing market. The introduction of a water market would be expected to move water from lower (farm 2) to higher (farm 1) marginal productivity uses. The first farmer spends  $W_0 CEW_1$  Rials in the water market and increases the total value of his cropping by as much as  $W_0 BEW_1$ Rial by using purchased water. The second farmer receives  $W_0 CEW_1$  Rials in exchange for  $W_0 W_1$  unit of water but he loses as much as  $W_0 HEW_1$  Rials. Consequently, farmers gain BCE and HCE Rials from this exchange. Farmers will trade their water use rights to the point that water VMP's will be equal (point E). As can be observed in Figure 1, in an agricultural setting, if the value of the marginal product of water and initial water entitlement differ among farmers, the market mechanism induces trade of water rights and renders its use efficient

## Demand Function in the Water Market

Assume that a representative farmer maximizes utility of profit under the certainty condition. Therefore, the individual farmer's optimization problem can be represented as:

Max U ( $\pi(w,x)$ ) (1)

Where  $\pi(.)$  denotes the profit function derived from production with a negative second derivative (Chambers, 1988), w is water input and x represents a vector of other inputs such as land, labour, fertilizers and machinery that are used in production. First order conditions for this problem are:

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\partial U(\pi) / \partial w = \partial U(\pi) / \partial \pi \partial \pi / \partial w = 0 \quad (2)\partial U(\pi) / \partial x = \partial U(\pi) / \partial \pi \partial \pi / \partial x = 0 \quad (3)
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Since  $\partial u(\pi) / \partial \pi$  is positive, in order to establish above relations, it is necessary that  $\partial \pi / \partial w$  and  $\partial \pi / \partial x$  be equal to zero. Therefore, under the certainty condition, we derive the water demand function from a farmer's profit maximization process. Consider a buyer farmer who uses his initial water right in addition to purchased water to produce a single crop. The profit function can be stated as follows:

$$\begin{split} &\pi = pf(w,x) + r_w \left( w_0\text{-}w \right) + r_x(x_0\text{-}x) = \\ &pf(w,x) + r_w \, w_0 + r_x \, x_0 - r_w \, w\text{-}rx \quad (4) \end{split}$$

where f(w,x) denotes production function, p is output price,  $r_w$  is water market price,  $r_x$  represents the vector of other input' price,  $w_0$  is the farmer's water right and  $x_0$  represents the vector of other inputs' ownership. First order conditions for profit maximization are:

$\partial \pi / \partial w = pf_w(w,x) - r_w = 0$	(5)
$\partial \pi / \partial x = pf_x(w,x) - r_x = 0$	(6)

where  $f_w(w,x)$  and  $f_x(w,x)$  are the marginal product of water and of other inputs, respectively. Thus, under the certainty condition we can derive the demand function in the water market from equations (5) and (6) as follows:

 $W_d = d(p, r_w, r_x)$ (7)

As mentioned before, participants in the water market are exposed to several sources of risk. According to the expected utility theorem, under the risk condition farmers maximize the expected utility of profit:

 $E[U(\pi)] = \int U(\pi_i) f_i d\pi \qquad (8)$ 

where  $\pi_i$  is the probable profit and  $f_i$  is its probability distribution function. If  $\pi_i$  has a known probability distribution, models such as mean-variance can be used to maximize expected utility of profit. The mean-variance model, that was introduced by Tobin (1959) and Markowitz (1959) is one of the most widely used approaches for describing farmers' production choices under risk (Mcquinn, 2000).

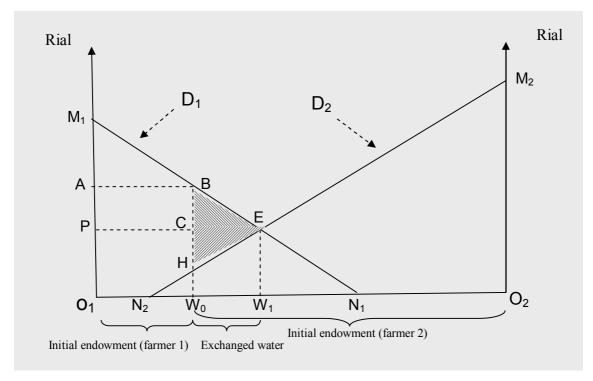


Figure 1- Mechanism of exchanging water in the market.

علوم محیطی سال پنجم، شماره دوم، زمستان ۱۳۸٦ ENVIRONMENTAL SCIENCES Vol.5, No.2, Winter 2008 According to this model, if the utility function has an exponential or quadratic form or the variable is normally distributed, the expected utility can be expressed as a function of the mean and variance of the variable (Varian, 1988). In this paper we apply a mean-variance model to derive the demand function under risk. For this purpose we assume that  $\pi_i$  has normal distribution and the farmer's utility function has a negative exponential form. This functional form exhibits constant absolute risk aversion<sup>1</sup> (CARA) and has been used extensively in decision analysis (Hardaker, 1997). The expected utility of profit would be then:

$$E[U(\pi)] = -\int_{i=1}^{I} e^{-\alpha \pi_{i}} f_{i} d\pi$$
  
=  $-e^{-\alpha [\overline{\Pi} - \frac{\alpha}{2} \sigma_{\Pi}^{2}]}$  (9)

where  $\overline{\Pi}$  and  $\sigma_{\Pi}^2$  are mean and variance of profit, respectively, and  $\alpha$  is risk aversion coefficient. Equation (9) increases relative to  $\overline{\pi} - \frac{\alpha}{2} \sigma_{\Pi}^2$ . On the other hand, instead of equation (9) we can maximize the following model:

U(
$$\overline{\Pi}, \sigma_{\Pi}^{2}$$
) =  $\overline{\Pi} - \frac{\alpha}{2} \sigma_{\Pi}^{2}$  (10)

where  $U(\overline{\Pi, \sigma_{\Pi}^2})$  is the utility certainty equivalent. Model (10) is used frequently in expected utility maximization studies with constant absolute risk aversion (Coyle, 1992).

Assume that output price p and water market price  $r_w$  are stochastic variables and the price of other inputs (land, labour, fertilizers, machinery etc.), that have less fluctuation over time, are determined variables. The expected value and variance of profit are:

$$\overline{\Pi} = E(\Pi) = \overline{p} f(w_x) + \overline{r}_w w_0 + r_x x_0 - \overline{r}_w w - r_x x$$
(11)

$$\sigma_{\pi}^{2} = var(\Pi) = [f(w, x)]^{2} \sigma_{p}^{2} + (w_{0} - w)^{2} \sigma_{r_{w}}^{2} + 2(w_{0} - w)f(w, x)cov(p, r_{w})$$
(12)

where  $\mathbf{p}$  and  $\mathbf{r}_w$  are expected prices of output and water,  $\sigma_p^2$  and  $\sigma_{r_w}^2$  are variances of output price and water price and cov(p,r<sub>w</sub>) denotes covariance of p and r<sub>w</sub>. Substituting (11) and (12) into (10) gives the farmer's choice problem:

$$U^{*}(\bar{p}, \bar{r}_{w}, r_{x}, w_{0}, \Omega) = \bar{p}f(w, x) + \bar{r}_{w}w_{0}$$
  
+  $r_{x}x_{0} - \bar{r}_{w}w - r_{x}x - \alpha/2[(f(w, x))^{2}\sigma_{p}^{2} + (w_{0} - w)^{2}\sigma_{r_{W}}^{2} + 2(w_{0} - w)f(w, x)cov(p, r_{w})]$   
(13)

Where  $\Omega$  is a vector of  $\sigma_p^2, \sigma_{r_W}^2$  and  $cov(p, r_w)$ .  $U^*(\bar{p}, \bar{r}_W, r_X, w_0, \Omega)$  denotes the farmers' dual indirect utility function; that is the relation between maximum feasible utility  $U^*$  and exogenous variable  $\bar{p}$ ,  $\bar{r}_w$ ,  $r_x$ ,  $w_0$ ,  $\sigma_p^2$ ,  $\sigma_{r_W}^2$  and  $cov(p, r_w)$ .  $U^*$  is increasing in  $\bar{p}$ , decreasing in  $\bar{r}_w$ ,  $r_x$ ,  $\sigma_p^2, \sigma_{r_W}^2$  and  $cov(p, r_w)$ . Also  $U^*$  is linear, homogenous and convex in exogenous variables (Coyle, 1992). Assuming  $U^*(.)$ is differentiable, the following equation is obtained by applying the envelope theorem (Chambers, 1988):

$$\partial U^{*}(\bar{p},\bar{r}_{W},r_{X},w_{0},\Omega) / \partial \bar{r}_{W} = (w_{0} - w^{*})$$
(14)

$$\partial U^{*}(\bar{p}, \bar{r}_{W}, r_{X}, w_{0}, \Omega) / \partial \sigma_{r_{W}}^{2} = -\frac{\alpha}{2} (w_{0}^{-} w^{*})^{2}$$
(15)

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$$\partial U^{*}(\overline{p}, \overline{r}_{w}, r_{x}, w_{0}, \Omega) / \partial \operatorname{cov}(p, r_{w})$$
$$= -\alpha(w_{0} - w^{*})f^{*}(w, x)$$
(16)

Where  $\mathbf{w}^*(\mathbf{p}, \mathbf{r}_w, \mathbf{r}_x, \mathbf{w}_0, \Omega)$  is the demand

 $\partial^{2} U^{*}(\bar{p}, \bar{r}_{W}, r_{X}, w_{0}, \Omega) / \partial \bar{r}^{2} W \leq 0$ (17)  $\partial w^{*}(\bar{p}, \bar{r}_{W}, r_{X}, w_{0}, \Omega) / \partial \bar{r}_{W} \leq 0$ (18)

function of water. As  $U^*$  is convex in  $\Gamma_w$ :

On the other hand, water demand is decreasing in own-expected price and hence standard reciprocity condition is satisfied. Developing relation (15) and (16) by using (14) gives:

$$\partial U^{*}(.)/\partial \overline{a}_{\overline{r}_{W}}^{2} = -\frac{\alpha}{2} [\partial u^{*}(.)/\partial \overline{r}_{W}]^{2}$$
(19)
$$\partial U^{*}(.)/\partial cov(p, r_{W}) = -\alpha [\partial U^{*}(.)/\partial \overline{r}_{W} \partial U^{*}(.)/\partial \overline{p}]$$
(20)

Derivatives of U<sup>\*</sup>(.) with respect to variance of water market price  $\sigma_{r_W}^2$  and prices covariance cov(p,r<sub>w</sub>) are simple nonlinear functions of derivatives with respect to expected prices  $\vec{p}$  and  $\vec{r}_w$ . Thus, as Coyle (1992) mentioned, due to restrictions (19) and (20) it is practically easier to specify functional form for the water demand function compared to the standard price certain models, where it is simpler to specify a functional form for the dual and then derive factor demand equation using the envelope theorem.

After estimating the water demand function  $w^{*}(\bar{p}, \bar{r}_{w}, r_{x}, w_{0}, \Omega)$ , the hypothesis of risk neutrality of farmers can be tested. If farmers are risk neutral, all coefficients of prices variance and covariance in water demand function will be insignificant and naturally, the dual utility function (13) reduces to the standard profit function (4). If the hypothesis of risk neutrality is rejected, coefficient of risk aversion can be calculated from estimated demand function.

It is necessary to mention that all coefficients of dual utility function (13) except  $\alpha$  appear directly in the water demand function. To calculate  $\alpha$ , we differentiate (15) with respect to  $\mathbf{r}_{w}$  and use (14):

$$\partial^{2} U^{*}(.) / \partial \sigma_{r_{W}}^{2} \partial \bar{r}_{W} = -\alpha (w - w_{0}) \frac{\partial w^{*}(.)}{\partial \bar{r}_{W}}$$
$$= -\alpha \frac{\partial U^{*}(.)}{\partial \bar{r}_{W}} \frac{\partial^{2} U^{*}(.)}{\partial \bar{r}_{w}^{2}}$$
(21)

By rearranging above relation,  $\alpha$  can be calculated from the estimated water demand function as follows:

$$\alpha = -\frac{\partial^2 U^*(.)/\partial \bar{r}_W \partial \sigma_{r_W}^2}{\partial U^*(.)/\partial \bar{r}_W \partial^2 U^*(.)/\partial \bar{r}_W^2}$$
(22)

As has already been mentioned,  $\overline{p}$ ,  $\overline{r}_w$ ,  $r_x$ ,  $\sigma_p^2$ ,  $\sigma_r^2$  and  $cov(p,r_w)$  are exogenous variables. In empirical studies the lag of output and water market prices can be used as expected prices  $\overline{p}$  and  $\overline{r}_w$ . The variance and covariance of farmers' subjective probability distribution of prices are:

$$\sigma_{pt}^{2} = \sum_{i=1}^{I} \beta_{t-i} [p_{t-i} - E_{t-(i+1)} p_{t-i}]^{2}$$
(23)

$$cov(p, r_{W}) = \sum_{i=1}^{I} \beta_{t-i} [p_{t-i} - E_{t-(i+1)} p_{t-i}) (rw_{t-i} - E_{t-(i+1)} rw_{t-i}]$$
(24)

علوم محيطى سال پنجم، شماره دوم، زمستان ١٣٨٦ ENVIRONMENTAL SCIENCES Vol.5, No.2, Winter 2008 The current variance or covariance equals the sum of squares of prediction errors of the pervious i years, weighting with  $\beta_{t-i}$ . Many authors (Coyle, 1992, 1999; Chavas and Holt, 1990) have assumed that i is equal to three and  $\beta_{t-i}$  is equal to 0.5, 0.33 and 0.17 with a decreasing rate.

## Supply Function in the Water Market

To derive supply function in the water market, we consider a water right owner with the initial allotment  $W_0$ . He may be a farmer, a retired farmer or an heir. The water right holder is assumed to gain utility from water reserves and income from its sale. In Figure 2, the line  $M_0W_0$  indicates the accessible combination of income and asset reservation. Point E, where the utility indifference curve is tangent to  $M_0W_0$ , indicates the optimal choice. Hence, a rational water right holder supplies  $OW_0$ - $OR_1$  to the market.

Now we derive mathematically the supply function in the water market under the certainty condition. In the water market, a supplier is faced with the following optimization problem:

 $\begin{array}{ll} \mbox{Max U(M,R)} & (25) \\ \mbox{Subject to } M=r_w(W_0-R)=r_wS & (26) \end{array}$ 

where U(.) is the utility function, *S* is supplied water in the market, *M* is income, resulting from water sale, *R* is water reservation and  $r_w$  is water market price. By using (26), U(M,R) can be rewritten as: U(M,R)=U( $r_wS$ ,  $W_0$ -S) (27)

The first order condition for maximizing (27) is:  $\partial U(.)/\partial S = 0$  (28)

From condition (28), the water supply function  $s(r_w, W_0)$  would be derived under the certainty condition.

Now we assume that water market price is a stochastic variable and the participant must make a

decision under risk. As already mentioned, under risk conditions instead of the utility function the expected utility function would be maximized. By using (26), (25) can be stated as:

 $U(M,R)=U[(W_0-R) r_w, R]$  (29)

According to the mean-variance model, the indirect utility function of income can be stated as follows:

$$U^{*}(W_{0}, \overline{r}_{W}, \sigma_{M}^{2}) = \overline{M} - \beta / 2\sigma_{M}^{2}$$
(30)

where  $U^*(W_0, \bar{r}_w, \sigma_M^2)$  is the utility certainty equivalent,  $\overline{M}$  and  $\sigma_M^2$  are the mean and variance of income and  $\beta$  is the suppliers' risk aversion coefficient. Probable income can be denoted as:

$$\mathbf{M} = (\bar{\mathbf{r}}_{\mathbf{W}} + \varepsilon)(\mathbf{W}_{\mathbf{0}} - \mathbf{R}) \tag{31}$$

 $\bar{r}_{W}$  is expected price of water. Then  $\overline{M}$  and  $\sigma_{M}^{2}$  can be calculated as:  $\overline{M} = \bar{r}_{W}(W_{0} - R)$  (32)

$$\operatorname{var}(M) = \sigma_{M}^{2} = (W_{0} - R)^{2} \sigma_{r_{W}}^{2}$$
 (33)

Substituting (32) and (33) into (30) yields:

$$U^{*}(W_{0}, \overline{r}_{w}, \sigma_{r_{W}}^{2}) = \overline{r}_{w}(W_{0}^{-}R) - \beta/2(W_{0}^{-}R)^{2}\sigma_{r_{W}}^{2}$$
(34)

Applying the envelope theorem, the water supply function under uncertainty conditions in the water market would be obtained as follows:

$$U^{*}(W_{0}, \overline{r}_{W}, \sigma_{\overline{r}_{W}}^{2})/\partial \overline{r}_{W} = s(W_{0}, r_{W}, \sigma_{\overline{r}_{W}}^{2})$$
(35)

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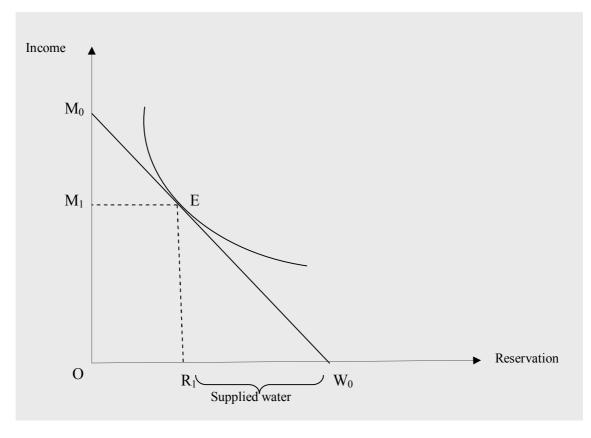


Figure 2-Mechanism of supply water in the market.

## Conclusion

Recently, the water market has been introduced as an allocation mechanism for water resource management. Although trading pattern forms is based on differences in the value of the marginal product of water among farmers, it is affected by risk and uncertainty, too. Under such a condition, the value of the marginal product of water at equilibrium exceeds its expected shadow price (see appendix). Furthermore, water is used less intensively than under certainty conditions. Then, ignoring risk or uncertainty in the water market analysis may depict the participants' behaviour incorrectly and lead to incorrect decisions by policymakers. This paper explains that how water demanders (farmers) and water suppliers (including farmers, retired farmers or heirs) behave in the agriculture water market. By using the mean-variance model, an econometric model under output price risk

and water market price risk is theoretically developed to derive demand and supply functions.

#### Appendix

In this section, according to Calatrava (2002), it is be indicated that under a water price risk condition in the water market, the value of the marginal product of water (VMP) at the optimum exceeds its expected shadow price (in a similar fashion, output price risk can be considered). The expected utility of profit for a farmer in a water market can be stated as:  $E[U(\pi)=E[U(py-r(w-w_0)]$  (1)

where *p* is price of output, *w* is the volume of water used in cropping,  $w_0$  is initial allotment, *r* is water market price and *y* is amount of output. First order condition for maximizing  $E(U(\pi))$  is:

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\partial E(U(\pi))/\partial w = E[\partial U(\pi)/\partial \pi \partial \pi/\partial w]
=E[U'(\pi)(VMP-r)]=0
                                                         (2)
or
E[U'(\pi)VMP]=E[U'(\pi)r]
                                                               (3)
```

Subtracting  $E[U'(\pi)\overline{r}]$  from both sides of (3) ( $\overline{r}$  is expected of water price) imply:

 $E[U'(\pi)VMP] - E[U'(\pi)r]$ =E[U'( $\pi$ )r]-E[U'( $\pi$ )r] (4)or

 $E[U'(\pi)(VMP-r)] = E[U'(\pi)(r-r)]$ (5)

Now, for a moment let us set aside above relation. Consider the expectation of profit:

 $E(\pi) = py - r (w - w_0)$ (6)

Adding  $(\bar{r} - r)(w - w_0)$  to both sides of (6) yields:  $E(\pi)+(\overline{r}-r)(w-w_0)$ =py- $r(w-w_0)+(r-r)(w-w_0)$ (7)

Rearranging (7) would imply that:

```
E(\pi)+(\vec{r}-r)(w-w_0)=py-r(w-w_0)
                                          (8)
or
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\pi = E(\pi) + (r - r)(w - w_0)
                                                       (9)
```

Since the farmer is a buyer in the water market then w>w<sub>0</sub>. If  $\overline{r}$  >r then, according to (9),  $\pi > E(\pi)$  and from convexity property of the utility function we can conclude that:

U'( $\pi$ )<U'[E( $\pi$ )] (10)

and therefore:

 $U'(\pi)$  (r - r)  $< U'[E(\pi)]$  (r - r) (11)

Note that if  $r < r_{(11)}$  can be established again because  $\pi < E(\pi)$  and then  $U'(\pi) > U'[E(\pi)]$  and, consequently, U'( $\pi$ ) (r -r)<U'[E( $\pi$ )] (r -r).

Taking expectation from both sides (11) would imply:

 $E[U'(\pi)(\overline{r} - r)] \leq E[U'(E(\pi))(\overline{r} - r)]$ (12)or  $E[U'(\pi)(\overline{r} - r)] \leq U'[E(\pi)]E(\overline{r} - r)$ (13)As  $E(\mathbf{r} - \mathbf{r}) = 0$  then (13) states that:  $E[U'(\pi)(\bar{r}-r)] < 0$ (14)Now we compare (14) with (5), consequently:  $E[U'(\pi)(VMP-r)] > 0$ (15)Which would imply:  $VMP > \overline{r}$ (16)

## Notes

1- After estimating the demand function, forms of risk aversion can be tested (see Pope (1991).

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