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Effect of Non-Normality on Sampling Plan Using Yule's Model

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Abstract. In this paper, the effect of non-normality on sampling plan using Yule's model (second order auto regressive model {AR (2)}) represented by the Edgeworth series is studied for known σ . The effect of using the normal theory sampling plan in a non-normal situation using Yule's model is studied by obtaining the distorted errors of the first and second kind. As one will be interested in having a suitable sampling plan under Yule's model for non-normal variables the values of n and k are determined.

Keywords. Sampling plan; autoregressive process; edgeworth series; autocorrelation.

1 Introduction

In the recent years many researches have carried out the problems of sampling inspection plan due to their large applications in industries for statistical measurements. In single sampling by variable all items from the sample and all items from the remainder of rejected lot is inspected by variable. In this direction, Srivastava (1961) has studied the effect of non-normality on the sampling inspection plan by variables. Montgomery (1985) has presented a study of the effect of non-normality on variable sampling plan. Recently, Balamurali et al. (2008) and Guenther (1977) have studied the variable sampling plans with different effects.

In general the normal theory is most appraisable for spherical symmetry that provides the excellent theoretical result under normality. However, experience with real life data reveals that parent populations occurring in many

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substantive fields generally do not behave in a normal fashion. Thus to face out this problem we consider the effect of non-normality on sampling plan. Haridy and El-Shabrawy (1996) have discussed the problem of economic design of cumulative sum chart to maintain current control using non-normal mean process. Akkaya and Tiku (2001) have estimated parameters using autoregressive model in non-normal situation. Balamurali and Jun (2007) have studied the multiple dependent state sampling plans for lot acceptance based on measurement data.

Along with this, many studies show the effect of non-normality and autocorrelation on the behaviour of sampling plan (MacGregor and Harris, 1993, Amin et al., 1997, Huitema and Mckean, 2007, Shu et al., 2002, Faltin et al., 1997, Gilbert et al., 1997, Tseng and Adams, 1994, Zhang, 1966, Singh and Singh, 1982). More recently, Castagliola and Tsung (2005) have discussed the autocorrelated SPC for non-normal situations. Zou et al. (2008) have studied the problem of research monitoring autocorrelated processes using variable sampling schemes. Aminzadeh (2009) has presented the sequential and non-sequential acceptance sampling plans for autocorrelated processes using ARMA (p, q) models.

The above studies motivate us to investigate further problems in this field. Thus in the present paper, we have investigated the effect of non-normality on sampling plan using Yule's model for known σ. The present paper is distributed in four sections.

2 Model Specification and the Plan Under Yule's Model for Normal Variables

Let us define the process whose control will be investigated by

$$x_t = \mu + \xi_t, \tag{1}$$

where μ is constant, ξ_t is a stationary time series with zero mean and standard deviation σ. But now assume that the ξ_t follows a second order autoregressive scheme. In other words we express

$$\xi_t = \alpha_1 \xi_{t-1} + \alpha_2 \xi_{t-2} + \varepsilon_t, \qquad t = 1, 2, \ldots, n \tag{2}$$

where

$$(i) \qquad \varepsilon_t \sim \mathrm{N}(0, \sigma_\varepsilon^2)$$

$$(ii) \qquad \mathrm{cov}(\varepsilon_t, \varepsilon_\tau) = \begin{cases} \sigma_\varepsilon^2 & t = \tau \\ 0 & t \neq \tau. \end{cases} \tag{3}$$

The variance of Yule's model is given by

$$\sigma^2 = \left(\frac{1-\alpha_2}{1+\alpha_2}\right) \frac{\sigma_\varepsilon^2}{(1-\alpha_2)^2 - \alpha_1^2}. \tag{4}$$

Following Box and Jenkins (1976) it can be shown that for stationarity, the roots of the characteristic equation of the process in (2)

$$\phi(B) = 0, \tag{5}$$

where

$$\phi(B) = 1 - \alpha_1 B - \alpha_2 B^2$$

must lies outside the unit circle, which implies that the parameters α_1 and α_2 must satisfy the following conditions:

$$\begin{aligned} \alpha_2 + \alpha_1 &< 1 \\ \alpha_2 - \alpha_1 &< 1 \\ -1 < \alpha_2 &< 1. \end{aligned} \tag{6}$$

Now if G_1 and G_2 are the roots of the characteristic equation of the process given by equation (5) then

$$G_1 = \frac{\alpha_1 + \sqrt{\alpha_1^2 + 4\alpha_2}}{2}, \tag{7}$$

$$G_2 = \frac{\alpha_1 + \sqrt{\alpha_1^2 + 4\alpha_2}}{2} \tag{8}$$

when the correlation is present in the data, we have for the distribution of the sample mean $\bar{x}$, its mean and variance given by

$$\begin{aligned} E(\bar{x}) &= \mu, \\ \operatorname{var}(\bar{x}) &= \frac{\sigma^2}{n}\lambda_{ap}\left(\alpha_1, \alpha_2, n\right) \\ &= \frac{\sigma^2}{n}T^2 \qquad \text{(say)}, \end{aligned} \tag{9}$$

where $\lambda_{ap}\left(\alpha_1, \alpha_2, n\right)$ depends on the nature of the roots G_1 and G_2 and for different situations is given as follows:

(i) If G_1 and G_2 are real and distinct then we have,

$$\begin{aligned} \lambda_{ap}\left(\alpha_1, \alpha_2, n\right) &= \frac{G_1(1-G_2^2)}{(G_1-G_2)(1+G_1G_2)}\lambda(G_1, n) \\ &\quad - \frac{G_2(1-G_1^2)}{(G_1-G_2)(1+G_1G_2)}\lambda(G_2, n) \\ &= \lambda_{rd}(\alpha_1, \alpha_2, n), \end{aligned} \tag{10}$$

where

$$\lambda(G, n) = \frac{1+G}{1-G} - \frac{2G}{n}\frac{(1-G^n)}{(1-G)^2}.$$

(ii) If G_1 and G_2 are real and equal then we have,

$$\begin{aligned} \lambda_{ap}\left(\alpha_1, \alpha_2, n\right) &= \left(\frac{1+G}{1-G}\right) - \frac{2G(1-G^n)}{n(1-G)^2} \\ &\quad \times \left\{1 + \frac{(1+G)^2(1-G^n) - n(1-G^2)(1+G^n)}{(1+G^2)(1-G^n)}\right\} \\ &= \lambda_{re}\left(\alpha_1, \alpha_2, n\right). \end{aligned} \tag{11}$$

(iii) If G_1 and G_2 are complex conjugate then we have,

$$\begin{aligned} \lambda_{ap}\left(\alpha_1, \alpha_2, n\right) &= Y(d, u) + \frac{2d}{n}\{W(d, u, n) + Z(d, u, n)\} \\ &= \lambda_{cc}\left(\alpha_1, \alpha_2, n\right) \end{aligned} \tag{12}$$

where

$$Y(d, u) = \frac{1-d^4+2d(1-d^2)\cos u}{(1+d^2)(1+d^2-2d\cos u)},$$

$$W(d,u,n) = \frac{2d(1+d^2)\sin u - (1+d^4)\sin 2u - d^{n+4}\sin(n-2)u}{(1+d^2)(1+d^2-2d\cos u)^2\sin u},$$

$$Z(d,u,n) = \frac{2d^{n+3}\sin(n-1)u - 2d^{n+1}\sin(n+1)u + d^n\sin(n+2)u}{(1+d^2)(1+d^2-2d\cos u)^2\sin u},$$

$$d^2 = -\alpha_2,$$

and

$$u = \cos^{-1}\left(\frac{\alpha_1}{2d}\right).$$

We now examine the effect of the autocorrelation on the usual test criterion of single sampling plan described below:

$$\begin{cases} \text{Accept the lot} & \text{if} \quad \bar{x} + k\sigma \leqslant U \\ \text{Reject the lot,} & \quad\quad \text{otherwise,} \end{cases}$$

for a given set of values of the producer's risk α, consumer's risk β, Acceptable Quality Level (AQL) p_1 and Lot Tolerance Proportion Defective (LTPD) p_2, the values of n and k are determined by the formulae

$$n = \left(\frac{K_\alpha + K_\beta}{K_{p_1} - K_{p_2}}\right)^2, \tag{13}$$

$$k = \frac{K_\alpha K_{p_2} + K_\beta K_{p_1}}{K_\alpha + K_\beta}, \tag{14}$$

where K_{p_1}, K_{p_2}, K_α and K_β are determined by the equation

$$\int_{K_\theta}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt = \theta \tag{15}$$

for different choices of fraction defective θ. If θ is the proportion defective in the lot, we know that

$$\frac{U-\mu}{\sigma} = K_\theta. \tag{16}$$

Since the statistics $\bar{x}$ under model (1) and (2) is asymptotically normally distributed with mean μ and variance $(\sigma^2/n)\lambda_{ap}(\alpha_1, \alpha_2, n)$, the statistics $\bar{x} + k\sigma$ would also be asymptotically normally distributed with mean $\mu + k\sigma$ and variance $(\sigma^2/n)\lambda_{ap}(\alpha_1, \alpha_2, n)$.

The OC function L_p corresponding to a fraction defective p is found as follows. Under the assumption of normality, a lot having p percent defective items will be accepted, if

$$\bar{x} + k\sigma \leqslant U = \mu + K_p\sigma,$$

where K_p is given by equation (16) for $\theta = p$. The expression for probability of acceptance

$$L_p = \Pr(\bar{x} + k\sigma \leqslant \mu + K_p\sigma),$$

is derived by recalling the normality of the statistic $(\bar{x} + k\sigma)$. The above probability after some simplification works out to be

$$L_p = \Phi\left\{\frac{\sqrt{n}}{\sqrt{\lambda_{ap}(\alpha_1, \alpha_2, n)}}(K_p - k)\right\} \tag{17}$$

where

$$\phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$$

3 Known σ Plan Under Yule's Model for Non-Normal Situations

Let the quality characteristic x follows the first four terms of an Edgeworth series, under Yule's model

$$f(x)dx = \left\{\phi(x) - \frac{\lambda_3}{6}\phi^{(3)}(x) + \frac{\lambda_4}{24}\phi^{(4)}(x) + \frac{\lambda_3^2}{72}\phi^{(6)}(x)\right\} dx. \tag{18}$$

The distribution of sample mean is given by,

$$g(\bar{x})d\bar{x} = \left\{\phi(\bar{x}) - \frac{\lambda_3 T}{6\sqrt{n}}\phi^{(3)}(\bar{x}) + \frac{\lambda_4 T^2}{24n}\phi^{(4)}(\bar{x}) + \frac{\lambda_3^2 T^2}{72n}\phi^{(6)}(\bar{x})\right\} d\bar{x} \tag{19}$$

where

$$x = \frac{X - \mu}{\sigma}, \qquad \bar{x} = \frac{\bar{X} - \mu}{\sigma T/\sqrt{n}}, \qquad \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$\phi^{(r)}(x) = \left(\frac{d}{dx}\right)^r \phi(x)$, and λ_3 and λ_4 are skewness and kurtosis respectively.

The OC function of the plan is given by

$$L'(p) = \Pr\left(\bar{x} + k\sigma \leqslant U \,\middle|\, \mu = \mu'\right)$$

where

$$\mu' = U - K'_p\sigma,$$

K'_p being the upper 100 p percent point of the standardized Edgeworthian population i.e., K'_p is given by

$$\int_{-\infty}^{K'_p} \left\{\phi(x) - \frac{\lambda_3}{6}\phi^{(3)}(x) + \frac{\lambda_4}{24}\phi^{(4)}(x) + \frac{\lambda_3^2}{72}\phi^{(6)}(x)\right\} dx = 1 - p \tag{20}$$

For a given p, the value of K'_p may be obtained by the method

$$K'_p = K_p + \frac{\lambda_3}{6}\left(K_p^2 - 1\right) + \frac{\lambda_4}{24}\left(K_p^3 - 3K_p\right) - \frac{\lambda_3^2}{72}\left(2K_p^3 - 5K_p\right). \tag{21}$$

Using the distribution of $\bar{x}$ from equation (19), the OC function of the known σ plan is obtained as

$$L'_p = \Phi(z) - \frac{\lambda_3 T}{6\sqrt{n}}\phi^{(2)}(z) + \frac{\lambda_4 T^2}{24n}\phi^{(3)}(z) + \frac{\lambda_3^2 T^2}{72n}\phi^{(5)}(z), \tag{22}$$

where

$$z = \frac{\sqrt{n}}{T}\left(K'_p - k\right).$$

The equations for determining the value of the plan parameters n and k are

$$L'(p_1) = 1 - \alpha, \tag{23}$$

and

$$L'(p_2) = \beta. \tag{24}$$

Explicit expression for n and k can not be obtained in the non-normal and Yule's model case. The equations (23) and (24) can however, be solved numerically. If n_0 and k_0 are the initial solutions then the improved solution can be obtained as $n_0+\delta n_0$ and $k_0+\delta k_0$ where δn_0 and δk_0 are the solutions of linear equations

$$A(p_1)\delta n_0 - B(p_1)\delta k_0 = 1-\alpha - C(p_1) \tag{25}$$

and

$$A(p_2)\delta n_0 - B(p_2)\delta k_0 = \beta - C(p_2) \tag{26}$$

where

$$\begin{aligned} A(p) = \frac{T^2}{2n_0}\Bigg[& z_0\Phi(z_0) - \frac{\lambda_3 T}{6\sqrt{n_0}}\left\{z_0\phi^{(3)}(z_0) - \phi^{(2)}(z_0)\right\} \\ & + \frac{\lambda_4 T^2}{24n_0}\left\{z_0\phi^{(4)}(z_0) - 2\phi^{(3)}(z_0)\right\} \\ & + \frac{\lambda_3^2 T^2}{72n_0}\left\{z_0\phi^{(6)}(z_0) - 2\phi^{(5)}(z_0)\right\}\Bigg], \end{aligned} \tag{27}$$

$$B(p) = \frac{\sqrt{n_0}}{T}\left\{\phi(z_0) - \frac{\lambda_3 T}{6\sqrt{n_0}}\phi^{(3)}(z_0) + \frac{\lambda_4 T^2}{24n_0}\phi^{(4)}(z_0) + \frac{\lambda_3^2 T^2}{72n_0}\phi^{(6)}(z_0)\right\}, \tag{28}$$

$$C(p) = \left\{\phi(z_0) - \frac{\lambda_3 T}{6\sqrt{n_0}}\phi^{(2)}(z_0) + \frac{\lambda_4 T^2}{24n_0}\phi^{(3)}(z_0) + \frac{\lambda_3^2 T^2}{72n_0}\phi^{(5)}(z_0)\right\}. \tag{29}$$

and

$$z_0 = \frac{\sqrt{n_0}}{T}\left(K'_p - k_0\right).$$

The required value of n and k can be obtained by taking the normal theory values as the initial solution and repeating the process of iteration for equations (25) and (26) till the desired accuracy is obtained.

4 Discussion of Numerical Results and Conclusions

For the purpose of illustrating the effect of non-normality and Yule's model on the error of the first and second kind and the plan parameters n and k, we have determined the values of these quantities for $p_1 = 0.05$, $p_2 = 0.30$, $\alpha = 0.05$, $\beta = 0.10$, with different values of λ_3, λ_4 and for different roots using Yule's model.

Table 1 shows the actual errors of the first and second kind when normal theory known σ plan is used under Yule's model and non-normal condition. It is evident from the Table 1 that, for leptokurtic, platykurtic population under the root of (real and distinct, real and equal) the error of first and second kind increases while in the case of complex conjugate errors are coincides approximately the same result as in error free case. This shows that non-normality and Yule's model effect seriously on the errors.

Table 2 shows the value of n and k for Yule's model under non-normal situation. The values of n are rounded up and the values of k are given up to four decimal places which is correct up to the third places of decimal. It can be seen from the table that for leptokurtic, platykurtic population the value of n and k increase for different roots while for negative skewness the value of n and k decrease.

Therefore, it may be inferred that the use of normal and independent sampling plan in non-normal and Yule's model is not valid. Even when there is slight departure from independencies and normality, it is advisable to take into account the dependence and non-normality of the parent population while choosing the sampling plan parameters n and k.

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Table 1. Values of α' and β' (underlines) for known σ plan under Yule's model $\alpha = 0.05$, $\beta = 0.10$, $p_1 = 0.05$, $p_2 = 0.30$

$\lambda_3 \rightarrow$	0				-.5				.5			
	Real and Distinct	Read and Equal	Complex Conjugate	Error Free	Real and Distinct	Real and Equal	Complex Conjugate	Error Free	Real and Distinct	Real and Equal	Complex Conjugate	Error Free
$\lambda_4 \downarrow$	(.3, .6, 7)	(.8, −.16, 7)	(.8, −.6, 7)		(.3, .6, 7)	(.8, −.16, 7)	(.8, −.6, 7)		(.3, .6, 7)	(.8, −.16, 7)	(.8, −.6, 7)	
0	.2358 .2877	.3483 .3821	.0606 .1131	.0478 .0973	.4198 .4500	.4440 .4330	.1146 .1478	.0972 .1388	.1631 .1576	.3012 .3091	.0346 .0960	.0256 .0740
-.5	.2415 .2936	.3709 .3724	.0585 .1278	.0452 .1126	.4220 .4607	.4623 .4273	.1117 .1665	.0932 .1573	.1715 .1518	.3198 .2934	.0323 .1061	.0239 .0877
.5	.2300 .2860	.3298 .3939	.0638 .1013	.0480 .0959	.4177 .4397	.4300 .4409	.1176 .1327	.0976 .1370	.1547 .1648	.2755 .3227	.0370 .0851	.0258 .0727
1.5	.2219 .2808	.2904 .4233	.0691 .0799	.0555 .0615	.4140 .4146	.3998 .4632	.1258 .1033	.1098 .0932	.1374 .1803	.2317 .3558	.0402 .0678	.0311 .0430
2.5	.2110 .2769	.2529 .4615	.0743 .0599	.0607 .0450	.4108 .2429	.3717 .4954	.1342 .0797	.1186 .0708	.1200 .1969	.1863 .3972	.0442 .0514	.0349 .0298

	(.5, 7, 0)	(.8, 7, 0)	Error Free	(.5, 7, 0)	(.8, 7, 0)	Error Free	(.5, 7, 0)	(.8, 7, 0)	Error Free
0	.1423 .2033	.2148 .2709	.0478 .0973	.2407 .2840	.3732 .4094	.0972 .1388	.1370 .1629	.1629 .1637	.0256 .0740
-.5	.1433 .2137	.2192 .2741	.0452 .1126	.2351 .3102	.3745 .4263	.0932 .1573	.1672 .1631	.1693 .1592	.0239 .0877
.5	.1435 .1933	.2133 .2651	.0480 .0959	.2415 .2616	.3777 .3988	.0976 .1370	.1363 .1627	.1564 .1689	.0258 .0727
1.5	.1436 .1742	.2046 .2578	.0555 .0615	.2528 .2243	.3756 .3691	.1098 .0932	.1343 .1618	.1432 .1808	.0311 .0430
2.5	.1457 .1560	.1990 .2484	.0607 .0450	.2596 .1892	.3800 .3423	.1186 .0908	.1311 .1595	.1295 .1928	.0349 .0298

Table 2. Values of n (integers) and k for known σ plan under Yule's model $\alpha = 0.05$, $\beta = 0.10$, $p_1 = 0.05$, $p_2 = 0.30$

$\lambda_3 \rightarrow$	0				-.5				.5			
	Real and Distinct	**Read and Equal**	**Complex Conjugate**	**Error Free**	**Real and Distinct**	**Real and Equal**	**Complex Conjugate**	**Error Free**	**Real and Distinct**	**Real and Equal**	**Complex Conjugate**	**Error Free**
$\lambda_4 \downarrow$	$(.3, .6, 7)$	$(.8, -.16, 7)$	$(.8, -.6, 7)$		$(.3, .6, 7)$	$(.8, -.16, 7)$	$(.8, -.6, 7)$		$(.3, .6, 7)$	$(.8, -.16, 7)$	$(.8, -.6, 7)$	
0	9 1.2433	9 1.1835	8 1.0403	7 1.0154	9 1.1655	10 1.1455	9 1.0514	11 1.0032	9 1.3180	9 1.2212	7 1.0014	6 1.0342
-.5	10 1.4879	9 1.2030	8 1.0655	8 1.0374	9 1.2046	9 1.0905	9 1.0687	11 1.0243	9 1.3338	9 1.2367	7 1.0384	6 1.0573
.5	9 1.2169	8 1.0751	7 1.0078	7 1.0132	9 1.1491	10 1.1264	8 1.0355	11 1.0011	8 1.1943	8 1.0251	6 .9641	6 1.0319
1.5	9 1.1642	9 1.1244	6 .9428	7 .9487	9 1.0857	9 1.0782	7 .9995	10 .9395	8 1.2546	8 1.1654	5 .9062	5 .9635
2.5	9 1.1123	8 1.0198	5 .8997	6 .9033	9 .9845	9 1.0350	7 .9580	9 .8697	8 1.1931	8 1.1253	5 .7517	5 .9150

	$(.5, 7, 0)$	$(.8, 7, 0)$	Error Free	$(.5, 7, 0)$	$(.8, 7, 0)$	Error Free	$(.5, 7, 0)$	$(.8, 7, 0)$	Error Free
0	9 1.0565	9 1.3087	7 1.0154	8 1.0550	8 1.0964	11 1.0032	10 1.3901	16 2.5098	6 1.0342
-.5	9 1.2248	8 1.2436	8 1.0374	11 1.3114	8 1.1162	11 1.0243	10 1.4022	15 2.5007	6 1.0573
.5	9 1.1364	9 1.2008	7 1.0132	10 1.2144	8 1.0768	11 1.0011	8 1.0725	23 3.5787	6 1.0319
1.5	8 1.100	8 1.1370	7 .9487	11 1.2903	8 1.0625	10 .9395	8 1.2165	13 2.8162	5 .9635
2.5	8 1.0626	8 1.1069	6 .9033	11 .7976	10 .6851	9 .8967	8 1.1333	12 1.6724	5 .9150

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